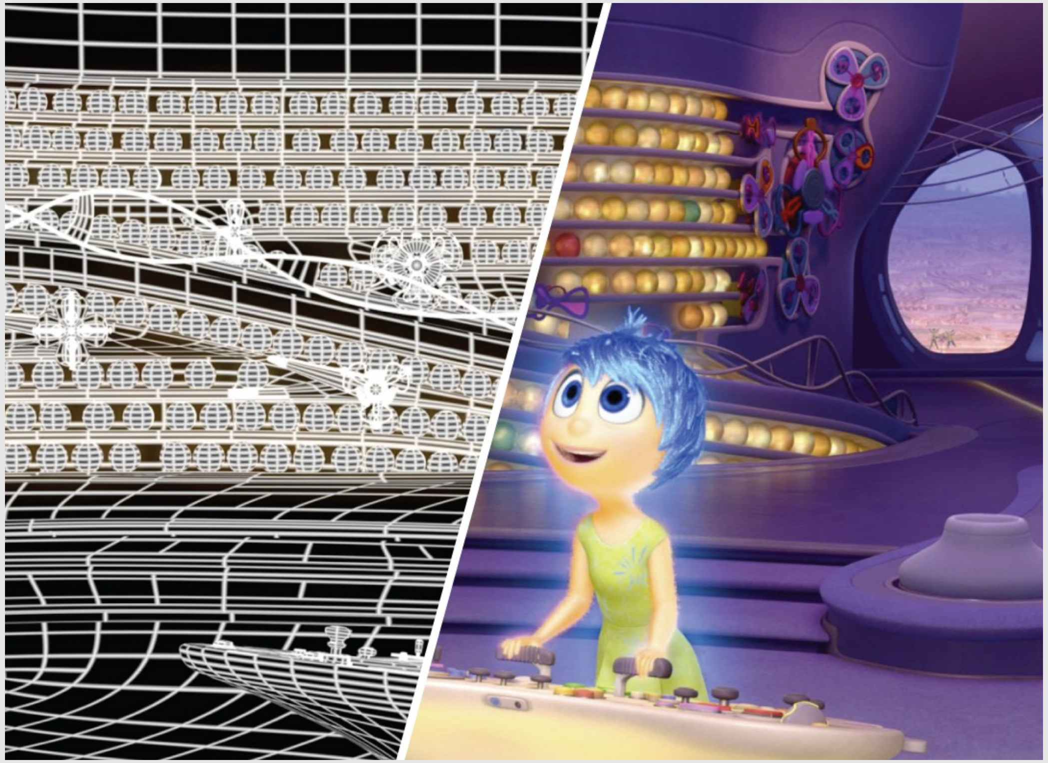
Radiometry

- Introduction to Rendering
- Radiometry
- Solid Angle

What is Rendering

- **Rendering** is the process of converting 2D or 3D data into a buffer of colors
  - The resulting buffer is saved as an image/video, commonly referred to as a **render**
  - Metamers are useful for matching color outputs from different displays
- No one correct way to make a render
  - Using different render algorithms will produce different results, even if the same input scene is used



Inside Out (2015) Pixar

Review: Rasterization

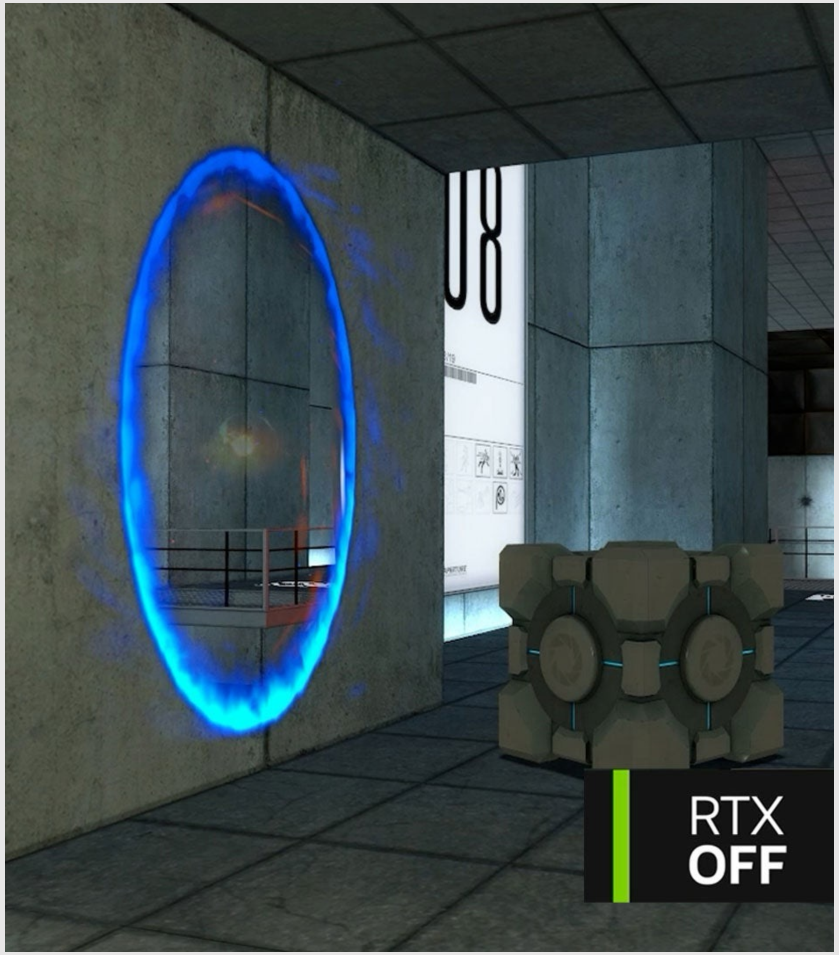
- Rasterization is a form of rendering:
  - Converting 3D (or even 2D) scenes into pixel buffers that we save out as images

- Input:
  - 2D/3D shapes

- Algorithm:
  - Check if pixel intersects shape
  - Shade pixel if passes intersection/depth test
  - Repeat

- Output:
  - An image

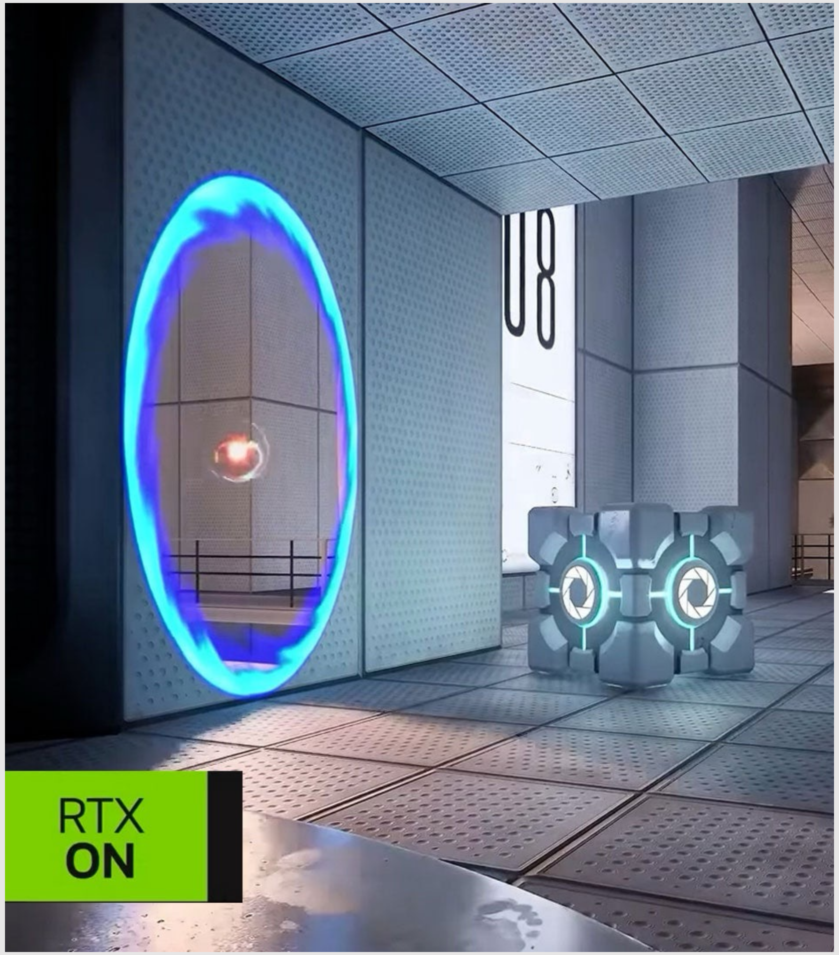
- Fits the definition of a renderer



Portal RTX (2022) Valve & Nvidia

New: Path Tracing

- Path Tracing is a form of rendering:
    - Converting 3D scenes into pixel buffers that we save out as images
- Input:
    - 3D shapes
- Algorithm:
    - Trace light rays into scene
    - Rays pick up color info from scene
    - Rays report color back to camera pixels
- Output:
    - An image
- We will explore this algorithm in depth today

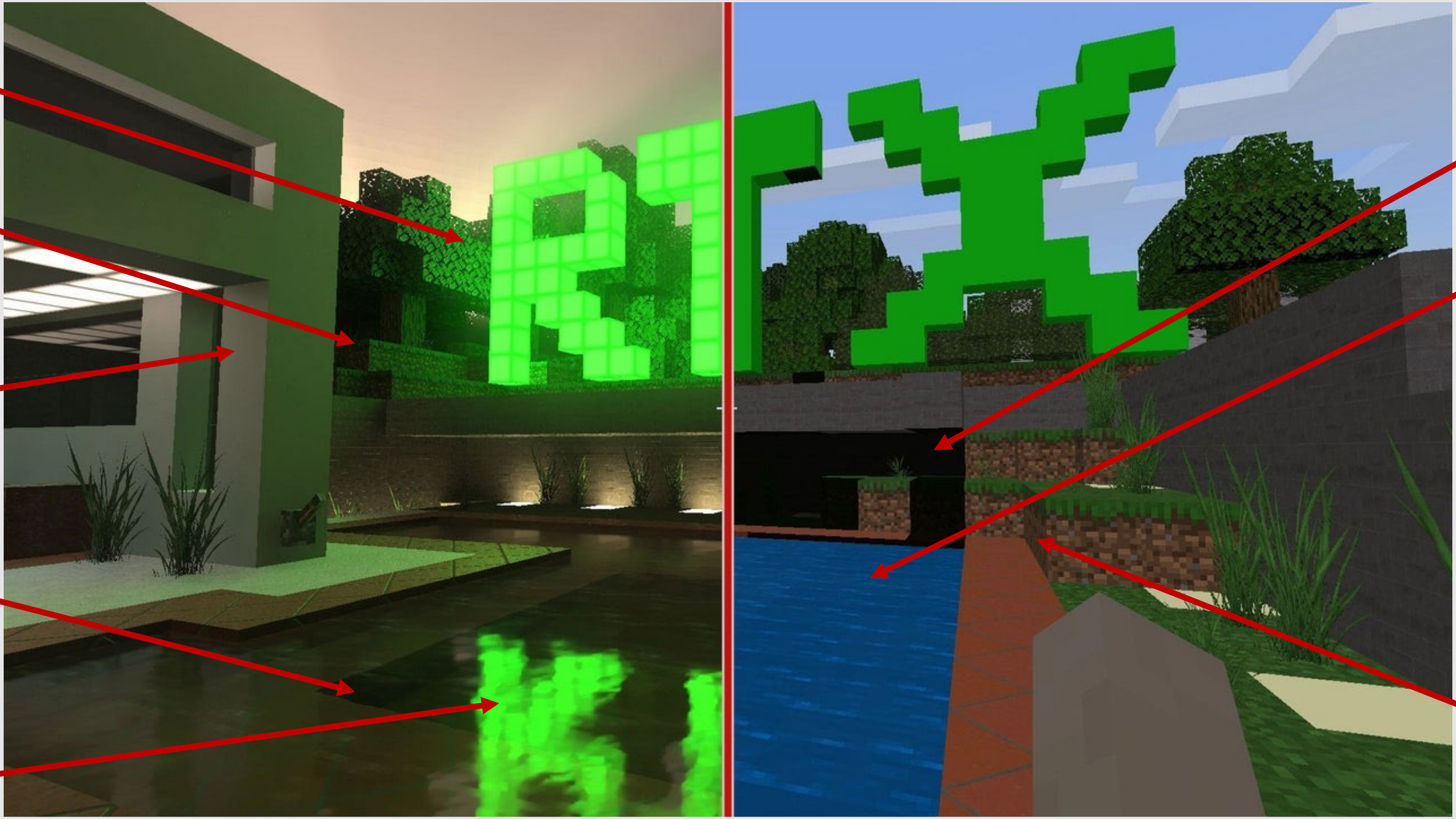


Portal RTX (2022) Valve & Nvidia

Path Tracing vs. Rasterization

Minecraft RTX (2020) Microsoft & Nvidia

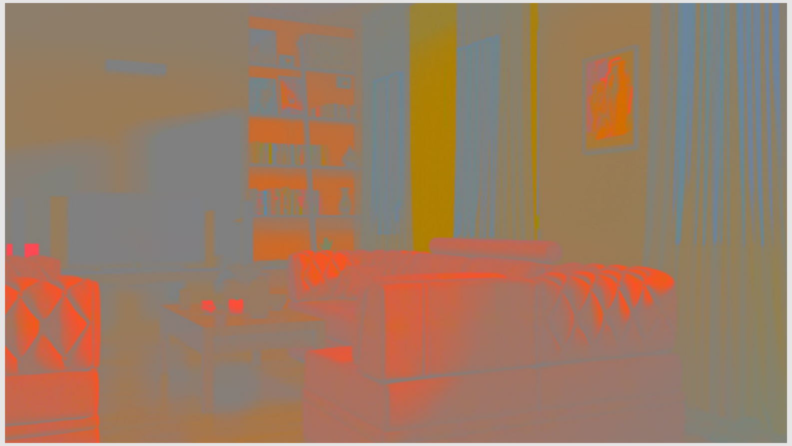
Path Tracing vs. Rasterization



Minecraft RTX (2020) Microsoft & Nvidia

Components of a Render

[last lecture]



[color]

\+

[this lecture]

[light]

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[next lecture]

[render]

Recall: light helps us carry color information in a scene

- ~~Introduction to Rendering~~
- Radiometry
- Solid Angle

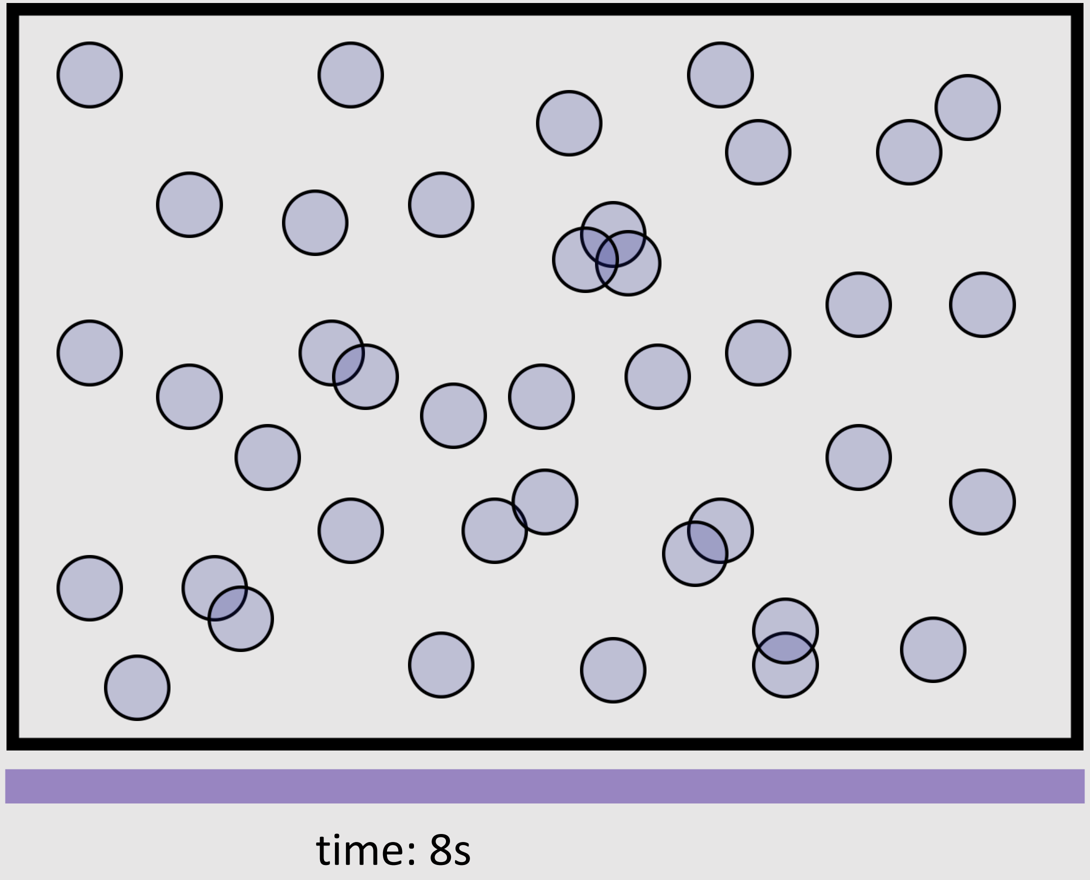
The Light Source

Kirby & The Forgotten Land (2022) Nintendo

- Light sources emit electromagnetic radiation
  - In this class, we will treat light as a particle
  - Nice property: light paths are **ray-like**
    - We know how to work with rays
- Adding light into our scenes allow us to illuminate color
  - **A scene without lights will be just black**
  - Light bounces off objects (emittance), until it hits a sensor (eyes, camera, etc.)
- **Radiometry** is the measure of light

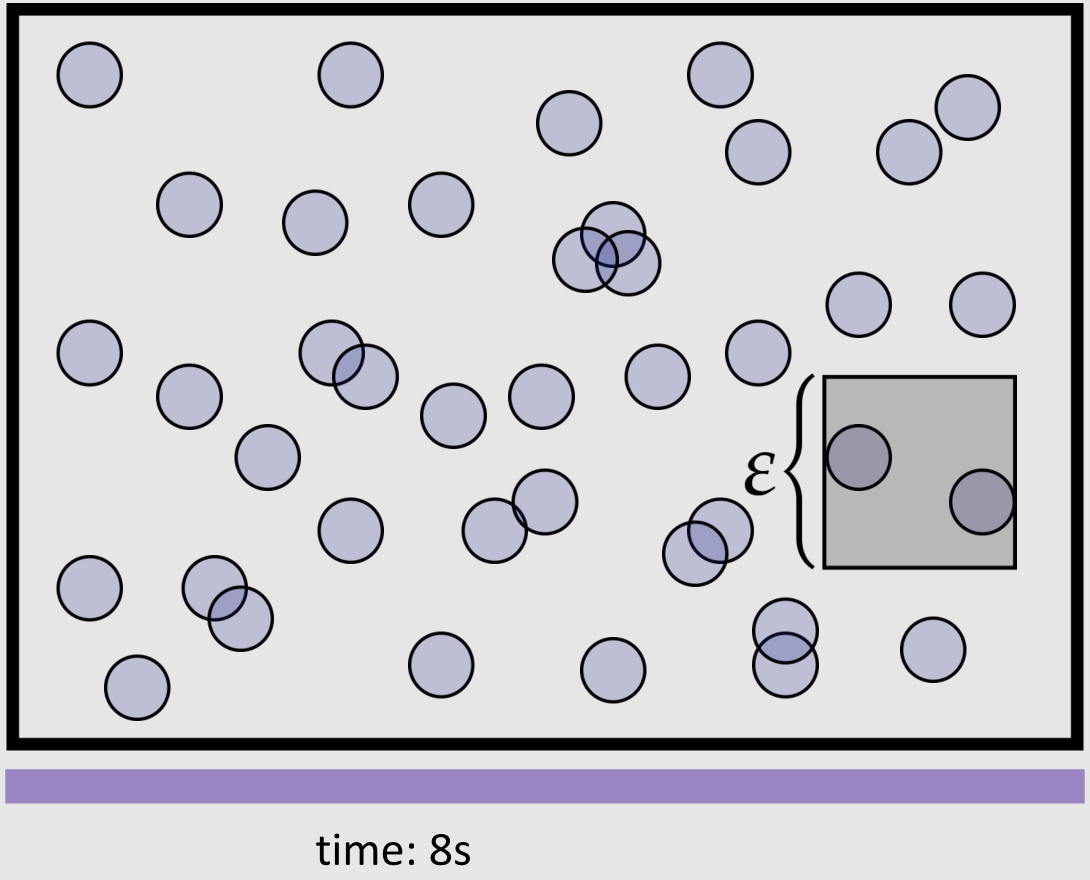
If Radiometry is the study of measuring light,
then how do we measure light?

Radiant Energy



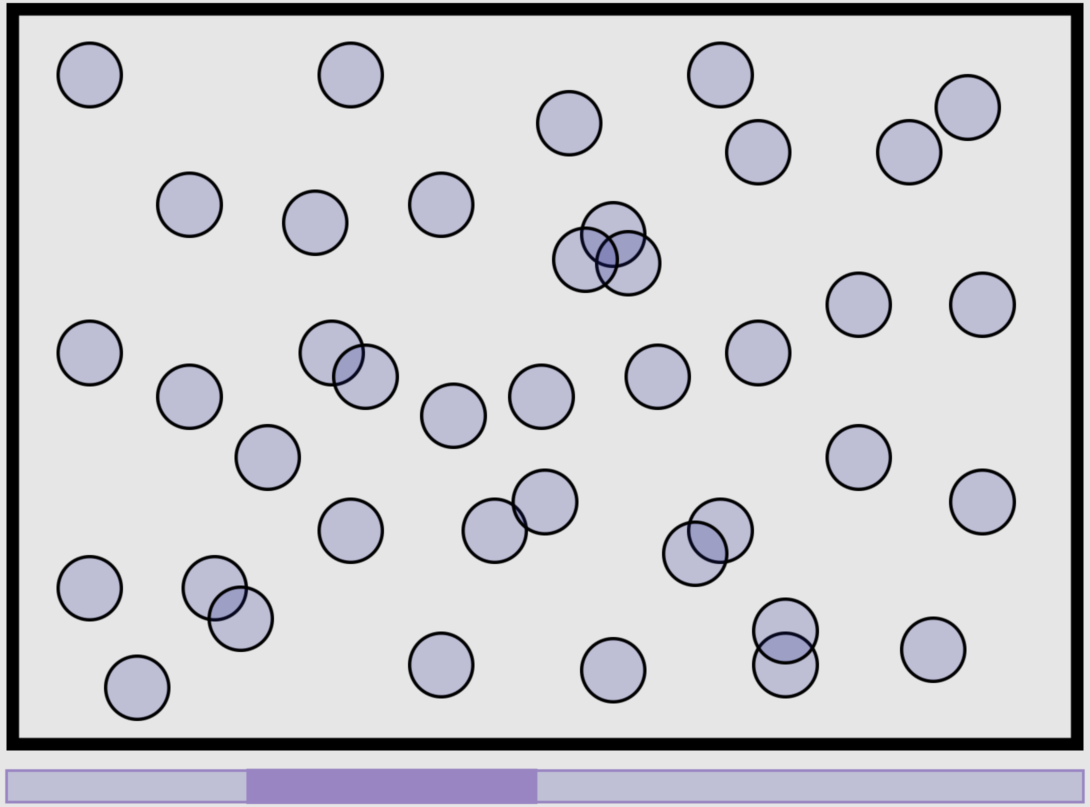
- **Radiant Energy** is total number of hits over the complete duration of the scene
    - This quantity captures the total energy of all the photons hitting the scene
- **Joules** is an energy measurement for photons
- **Example:** Radiant Energy: 40 Joules
    - 40 *(J)*

Radiant Density



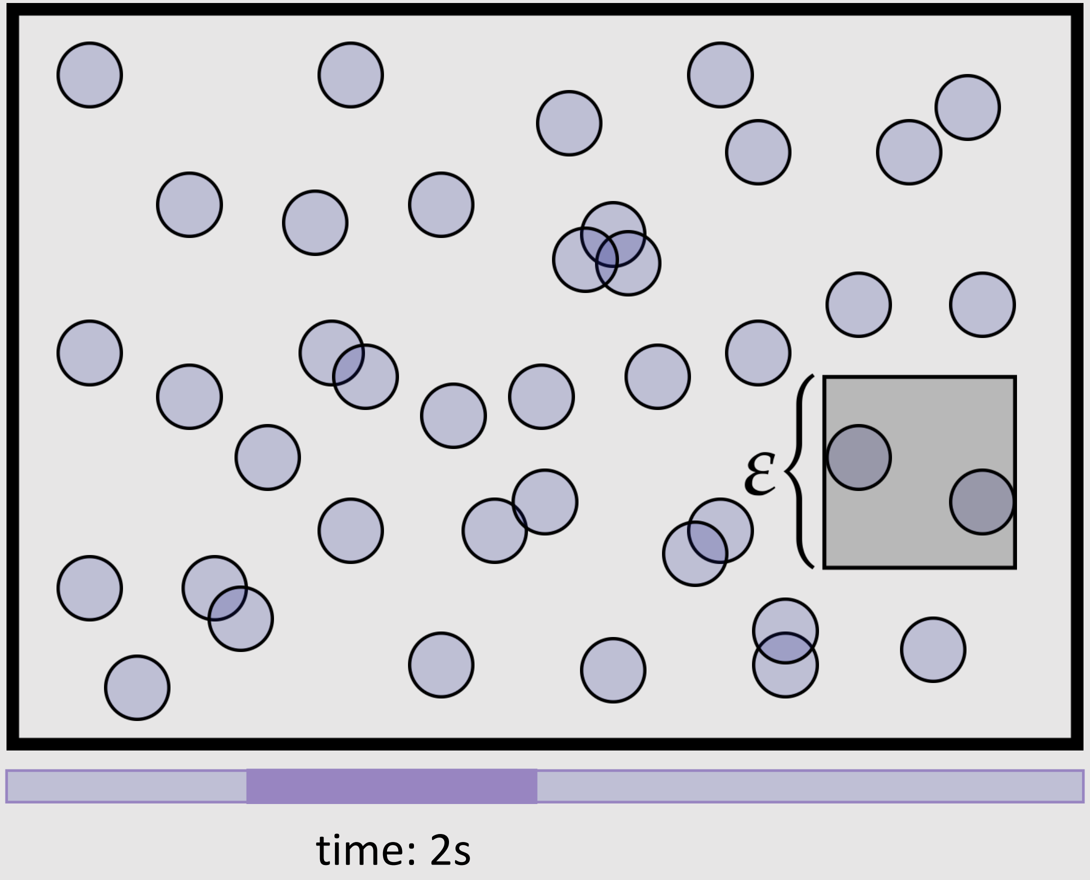
- **Problem:** Larger sensor window allows for more light to enter (not a fair comparison!)
- **Radiant Density** is total number of hits per unit area
  - Compute hits per second in some "really small" area, divided by area
- **Example:** Radiant Density: 40 J / 10 m^2
  - 4 (J/m^2)

Radiant Flux



- **Problem:** Longer exposure allows for more light to enter (not a fair comparison!)
- **Radiant Flux** is total number of hits per second
  - Rather than record total energy over some (arbitrary) duration, may make more sense to record total hits per second
- **Watts** *(W)* measures Joules / second
- **Example:** Radiant Flux: 40 J / 2 s
  - 20 *(J/s)* = 20 *(W)*

Irradiance



- **Problem:** Larger sensor window + Longer Exposure
- **Irradiance** is total number of hits per second per unit area
  - Solves both issues
- **Example:** Irradiance: 40 J / 2 s / 10 m^2
  - 2 $(J/s/m^2)$ = 2 (W/m^2)

Radiant Recap

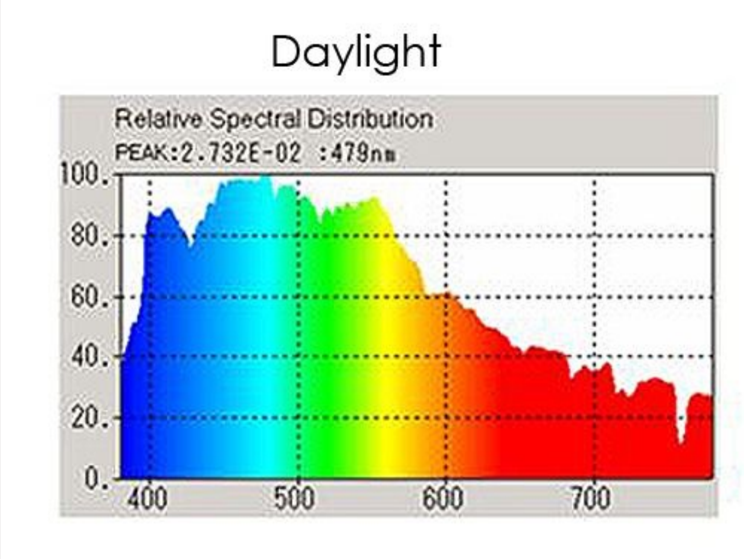
Radiant Energy (total number of hits) *Joules (J)*	**Radiant Energy Density** (hits per unit area) *Joules per sq meter* (J/m^2)
Radiant Flux (total hits per second) *Watts (W)*	**Radiant Flux Density** **a.k.a. *Irradiance*** (hits per second per unit area) *Watts per sq meter* (W/m^2)

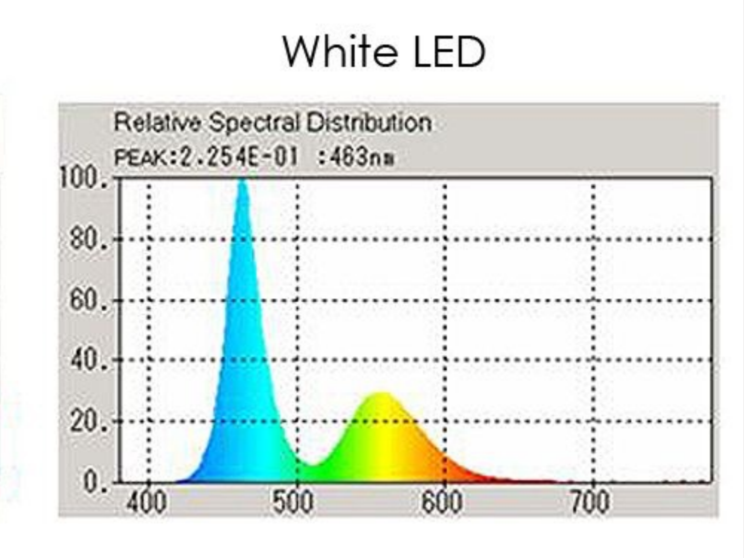
Varying Wavelengths

- We defined radiant energy as **total number of hits**
  - Yet we measure radiant energy in Joules (i.e., as **total energy)**
  - This assumes all photons have the same energy
- In actuality, photon energy (Q) is inversely proportional to wavelength (λ)
  - Planck's constant (h) and speed of light (c) are both constants
  - Higher wavelengths (red) have lower energy

$$Q = \frac{hc}{\lambda}$$

- No longer can assume radiant energy as just **total number of hits**
  - Instead need to measure **radiant energy per wavelength**
    - Dividing by time length of the measurement helps us build a **Spectral Power Distribution**



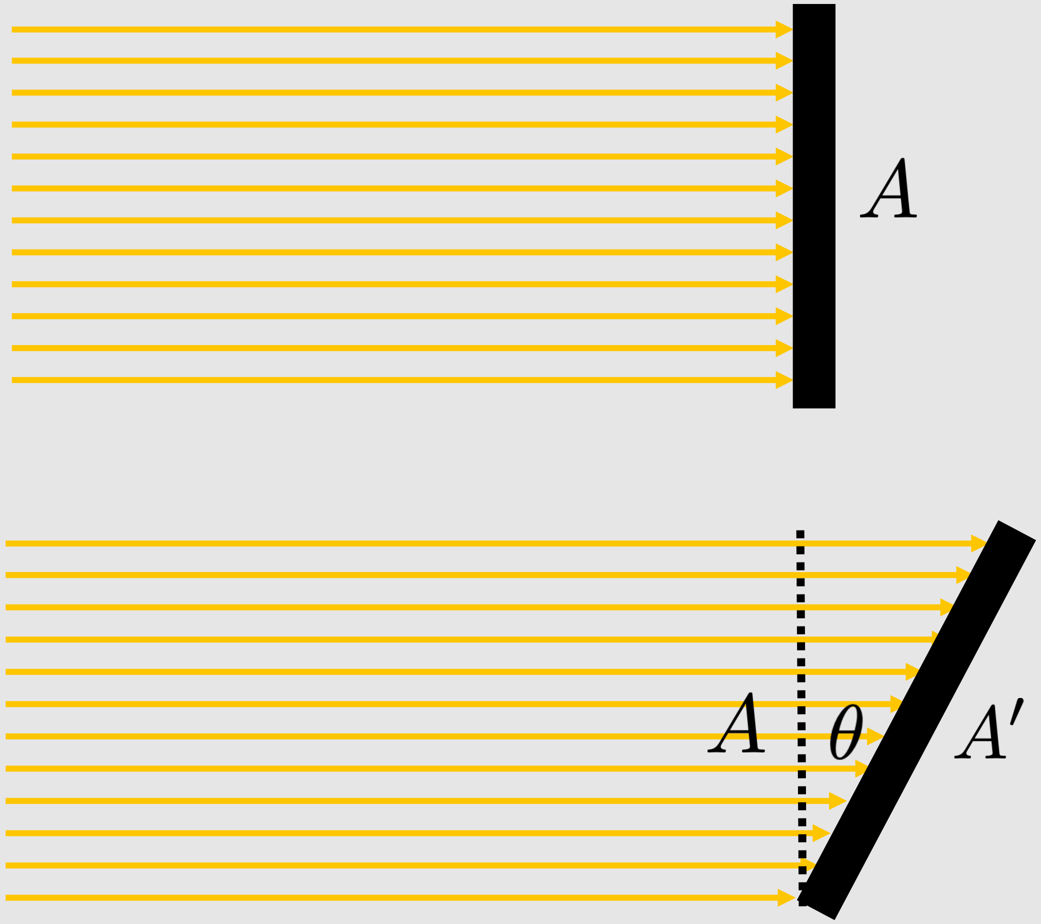


Lambert's Law

- Irradiance (E) at surface is proportional to the flux (Φ) and the cosine of angle (θ) between light direction and surface normal:

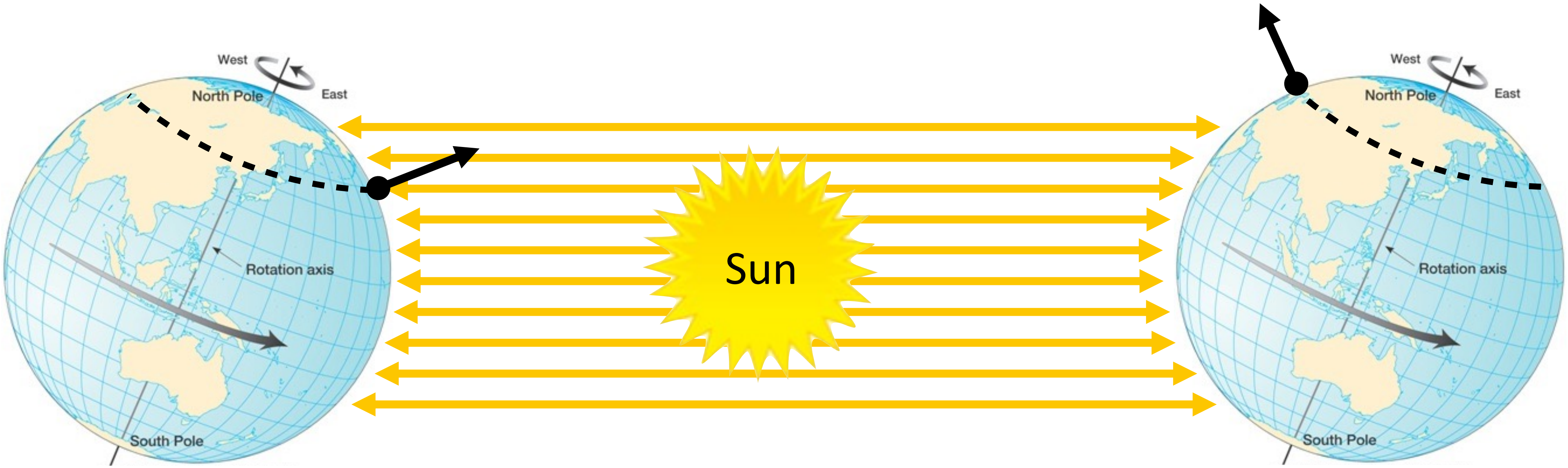
$$E = \frac{\Phi}{A'} = \frac{\Phi \cos\theta}{A}$$

- Consider rotating a plane away from light rays
  - Plane will darken until it is perpendicular to light rays, then it will be completely black



$$A = A' \cos\theta$$

Lambert's Law



[Summer]
Norther Hemisphere

[Winter]
Norther Hemisphere

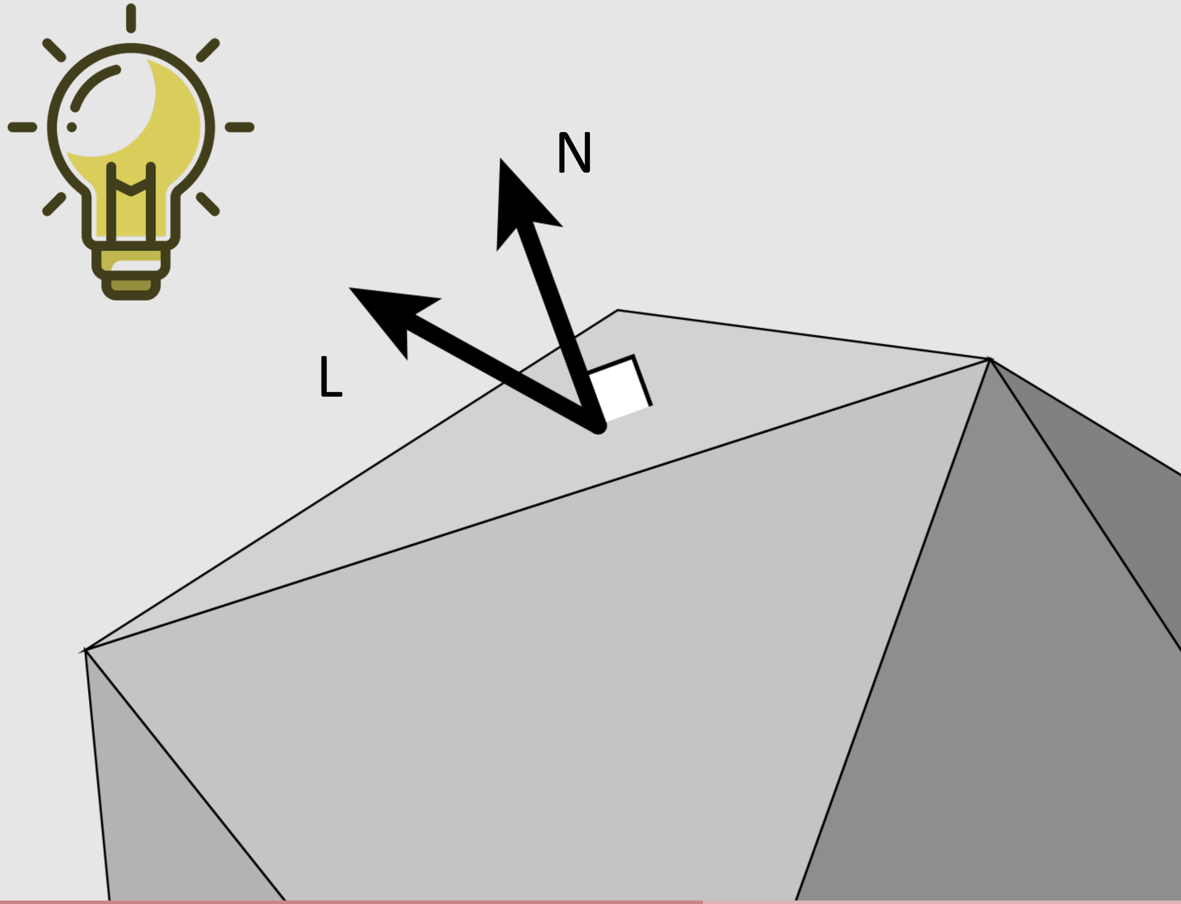
Explains why Pittsburgh is so cold all the time…

N-Dot-L Lighting

- Our first (and most basic) way to shade a surface
  - Inspired by Lambert's Law

- **Algorithm:** take dot product of unit surface normal (N) and unit direction to light (L)

```
// compute contribution of light onto surface
double surfaceColor( Vec3 N, Vec3 L )
{
   float f = dot(N,L);
   return f;
}
```

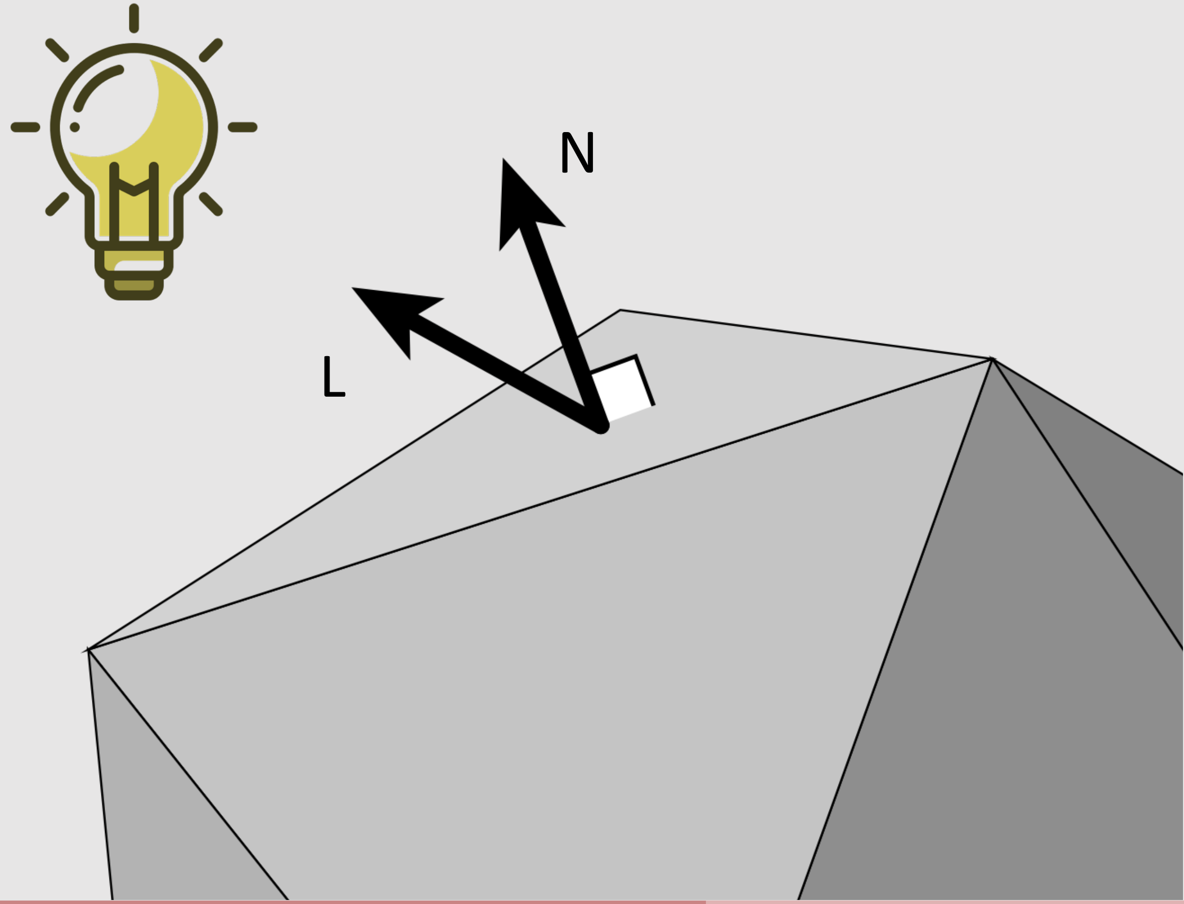


N-Dot-L Lighting

- **Problem:** what if light source is on other side of primitive?
  - Previous algorithm would light primitive, even if facing wrong direction

- **Solution:** ensure dot product sign is positive
  - Orientations match

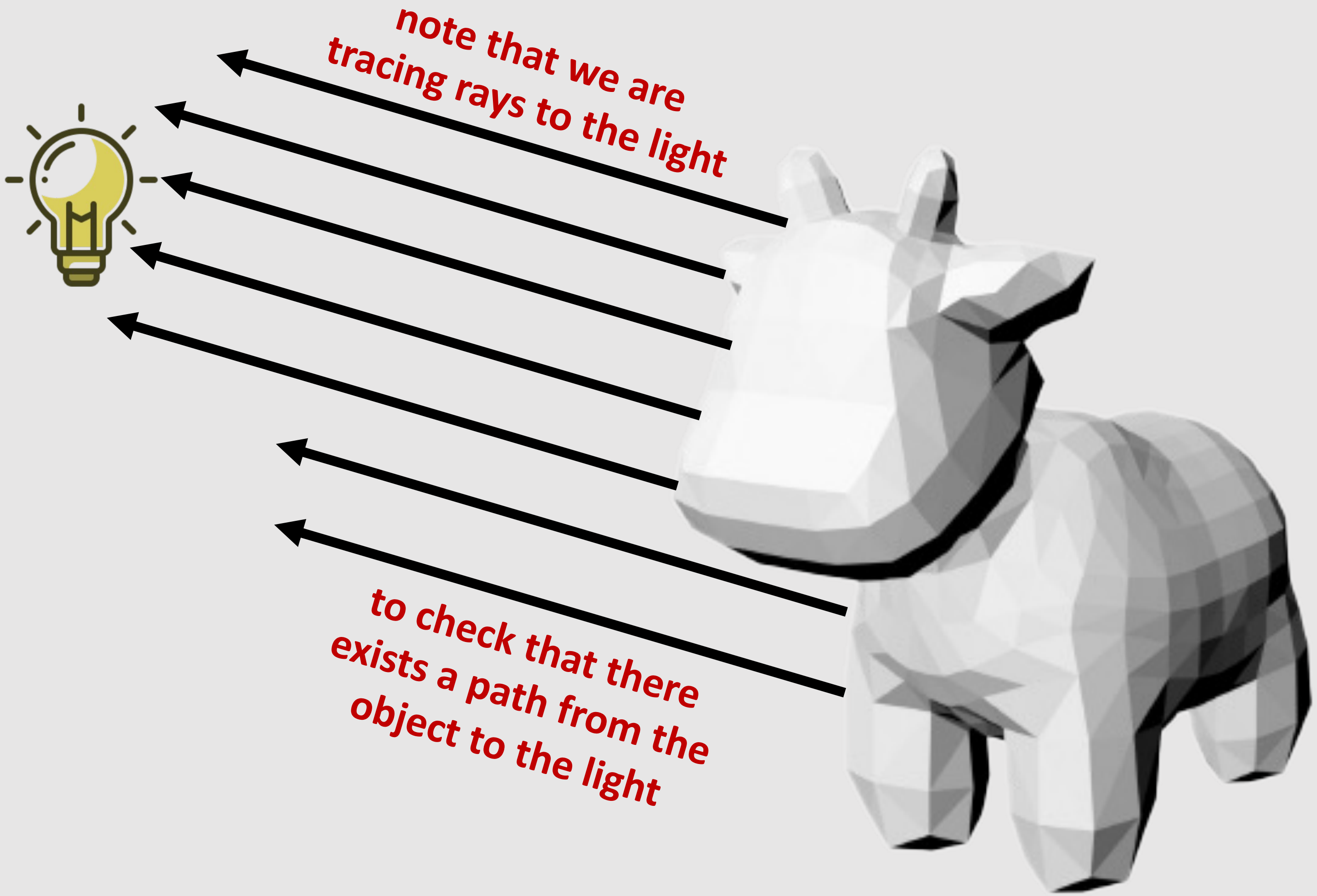
```
// compute contribution of light onto surface
double surfaceColor( Vec3 N, Vec3 L )
{
   float f = dot(N,L);
   return max(0,f);
}
```



What kind of lights do we have?

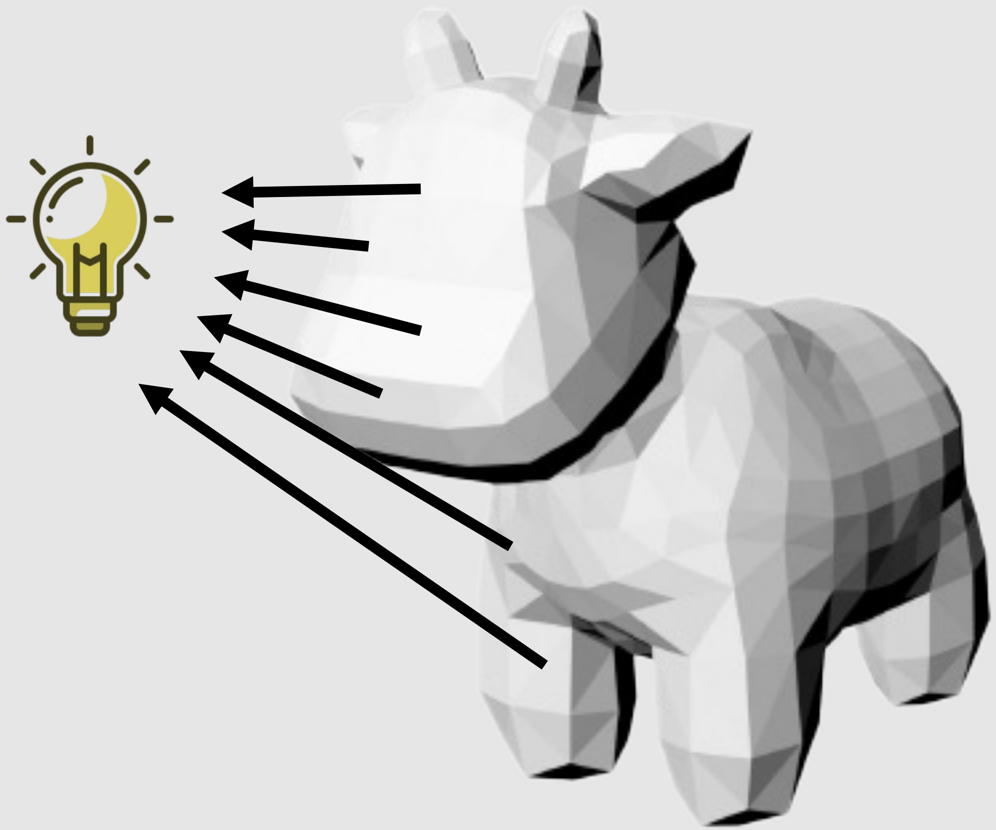
Directional Light

- **Abstraction:** infinitely bright light source "at infinity"
  - All light directions (L) are therefore identical
  - All planes with the same orientation get the same contribution
- "infinitely bright" means light does not get weaker with distance
- **Example:** the sun



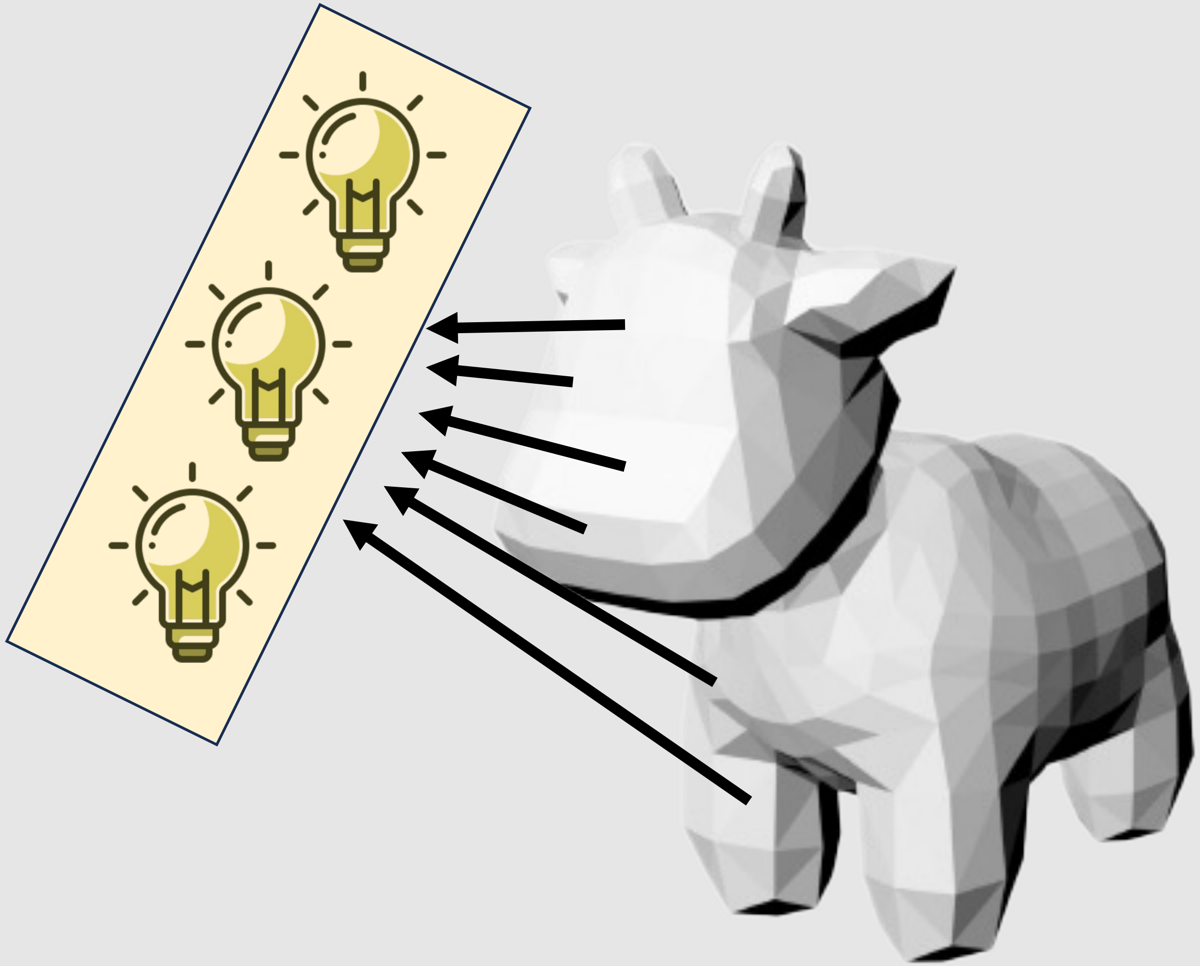
Point Light

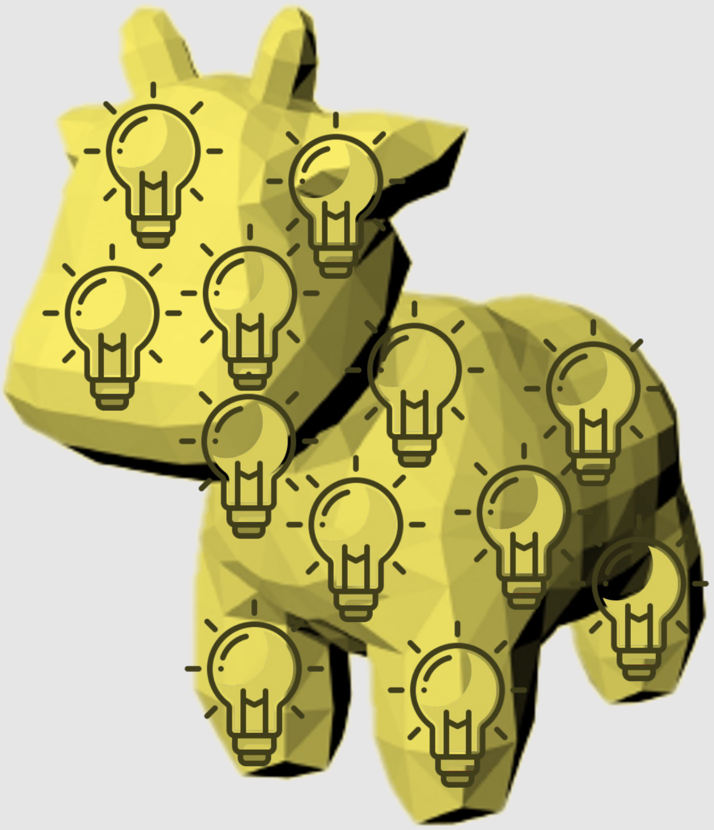
- **Abstraction:** point light with no volume placed in 3D space
  - Varying light directions (L)
- Light gets weaker with distance
- **Example:** a lightbulb



Area Light

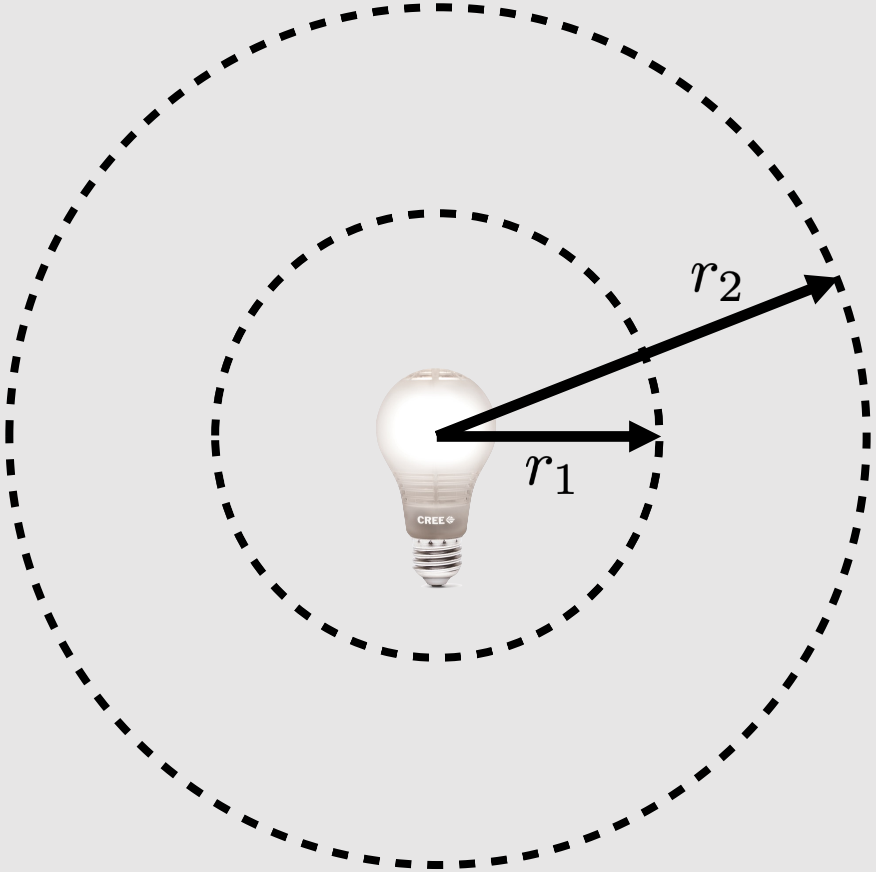
- **Abstraction:** geometry with volume that emits light
  - Varying light directions (L)
    - When constructing L, can pick any point on light geometry
- Point lights are not physically accurate
  - Everything in life has volume
  - Point lights are approximated to take up infinitely small space
- Light gets weaker with distance
- **Example:** a square ceiling light





Can make a light source out of any geometry
Even a cow…

Irradiance Falls Off With Distance



- As light moves away from a light source, it "spreads out"
  - Weakens the light the farther from the light source
- Radiant flux (Φ) spread out over spherical surface:

$$E_1 = \frac{\Phi}{4\pi r_1^2} \rightarrow \Phi = 4\pi r_1^2 E_1$$

$$E_2 = \frac{\Phi}{4\pi r_2^2} \rightarrow \Phi = 4\pi r_2^2 E_2$$

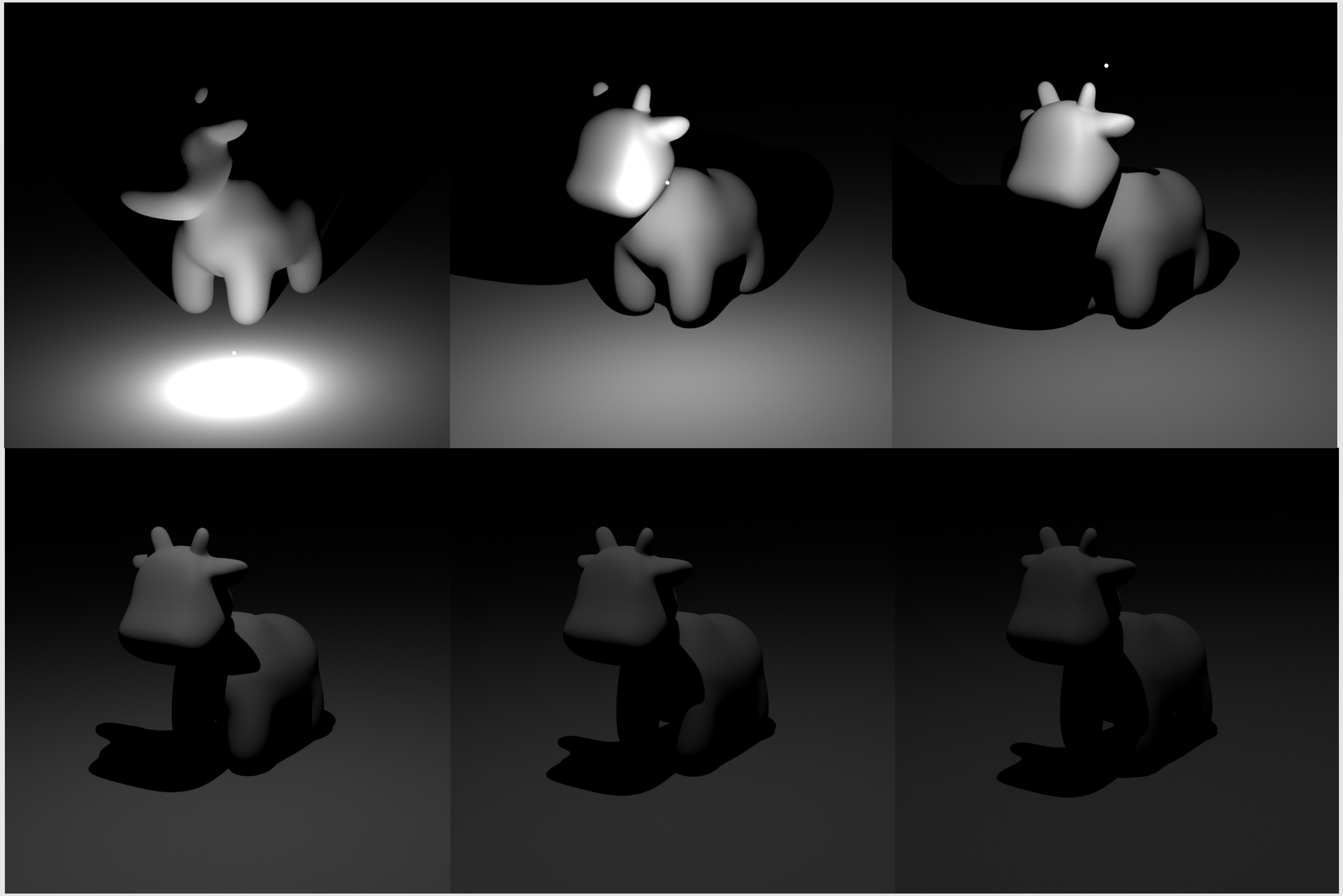
$$\frac{E_2}{E_1} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

  - Irradiance (E) gets quadratically darker with distance

Irradiance Falls Off With Distance

- **Analogy:** throwing a stone in water
  - Ripples spread out from origin, getting smaller as they spread farther
- Same energy spread out over larger area, leads to smaller ripples the farther out

Quadratic Falloff of Lights



[light moving in 1mm increments]

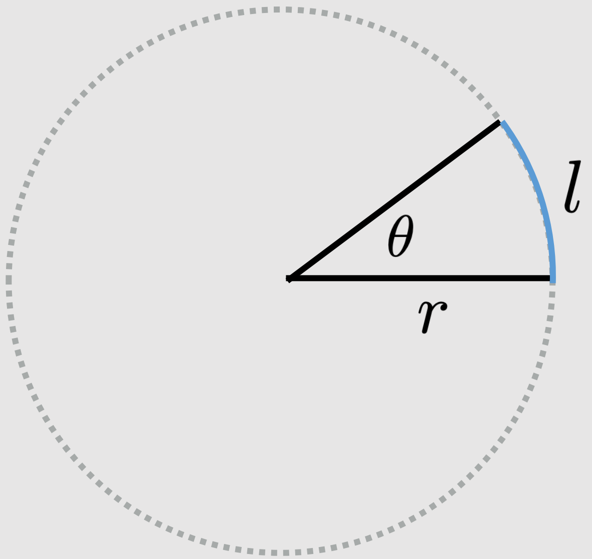
- ~~Introduction to Rendering~~
- ~~Radiometry~~
- Solid Angle

Solid Angle

- **Angle:** ratio of subtended arc length on circle to radius

$$\theta = \frac{l}{r}$$

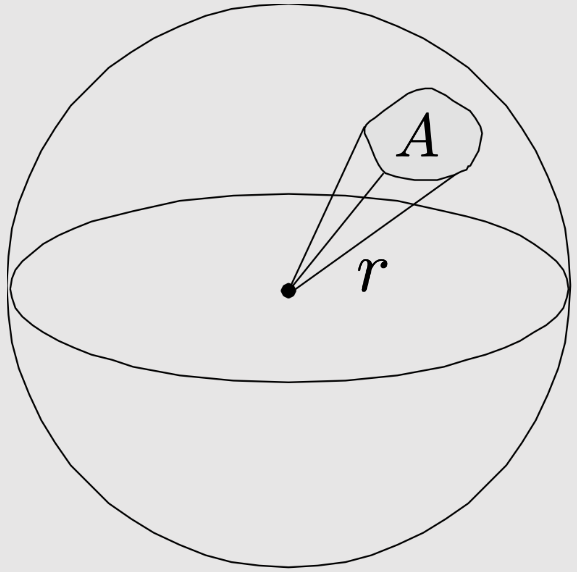
  - Circle has 2π radians (r)



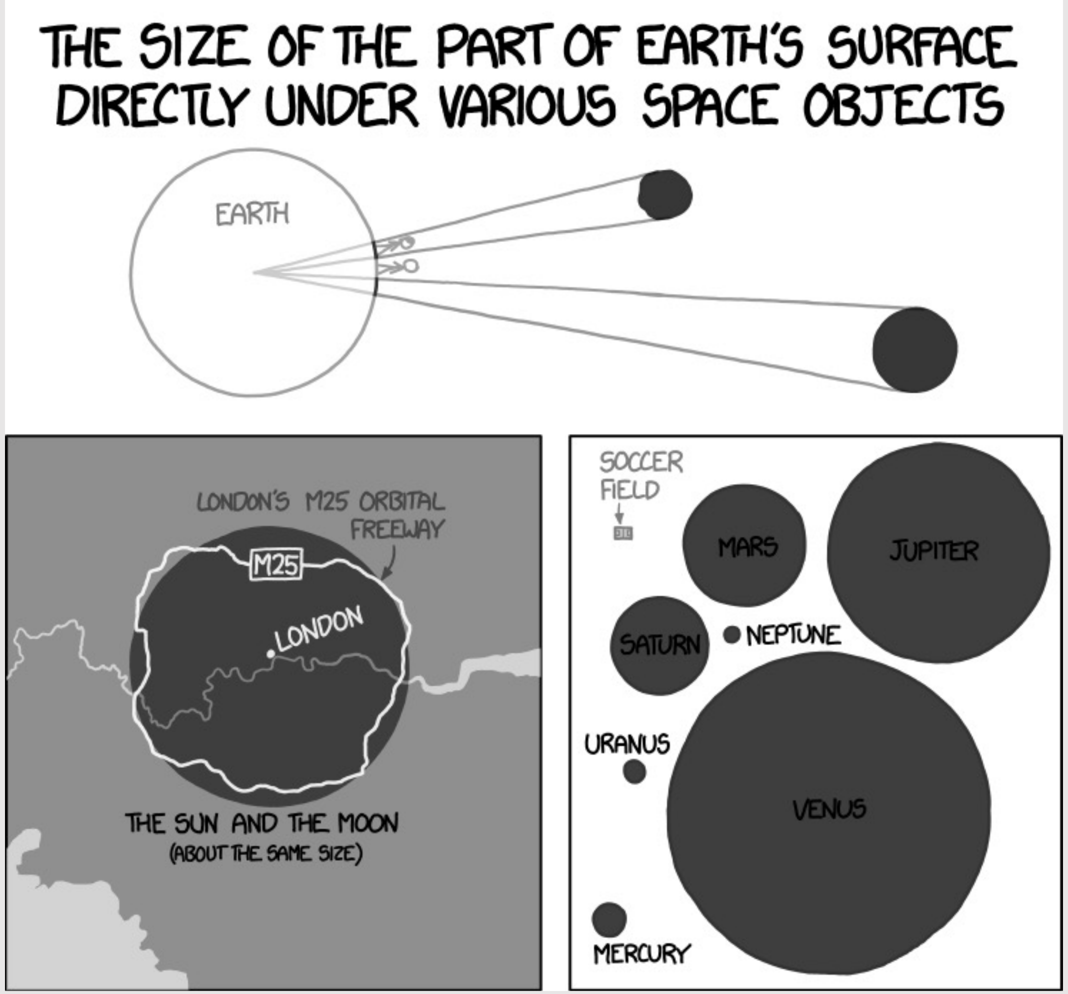
- **Solid Angle:** ratio of subtended area on sphere to radius squared

$$\Omega = \frac{A}{r^2}$$

  - Sphere has 4π steradians (sr)
    - $A = 4\pi r^2$, divide out r^2



Solid Angle in Astronomy

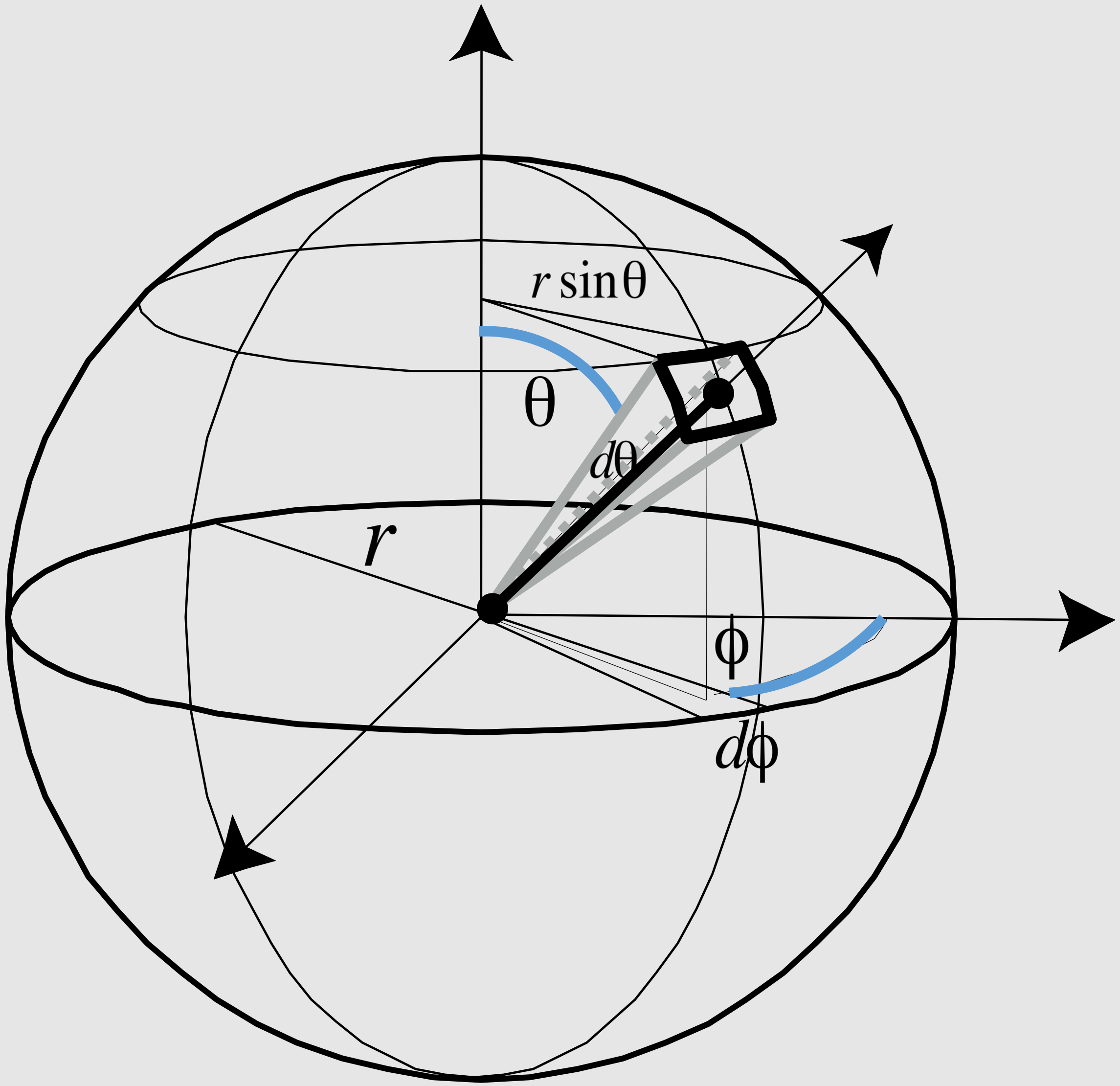


http://xkcd.com/1276/

- Sun and moon both subtend ~60µ sr as seen from Earth
  - Even though they vary greatly in size, they also vary greatly in distance
- Surface area of earth: ~510M km2
- Projected area:

$$510\text{Mkm}^2 \frac{60\mu\text{sr}}{4\pi\text{sr}} = 510\frac{15}{\pi} \approx 2400\text{km}^2$$

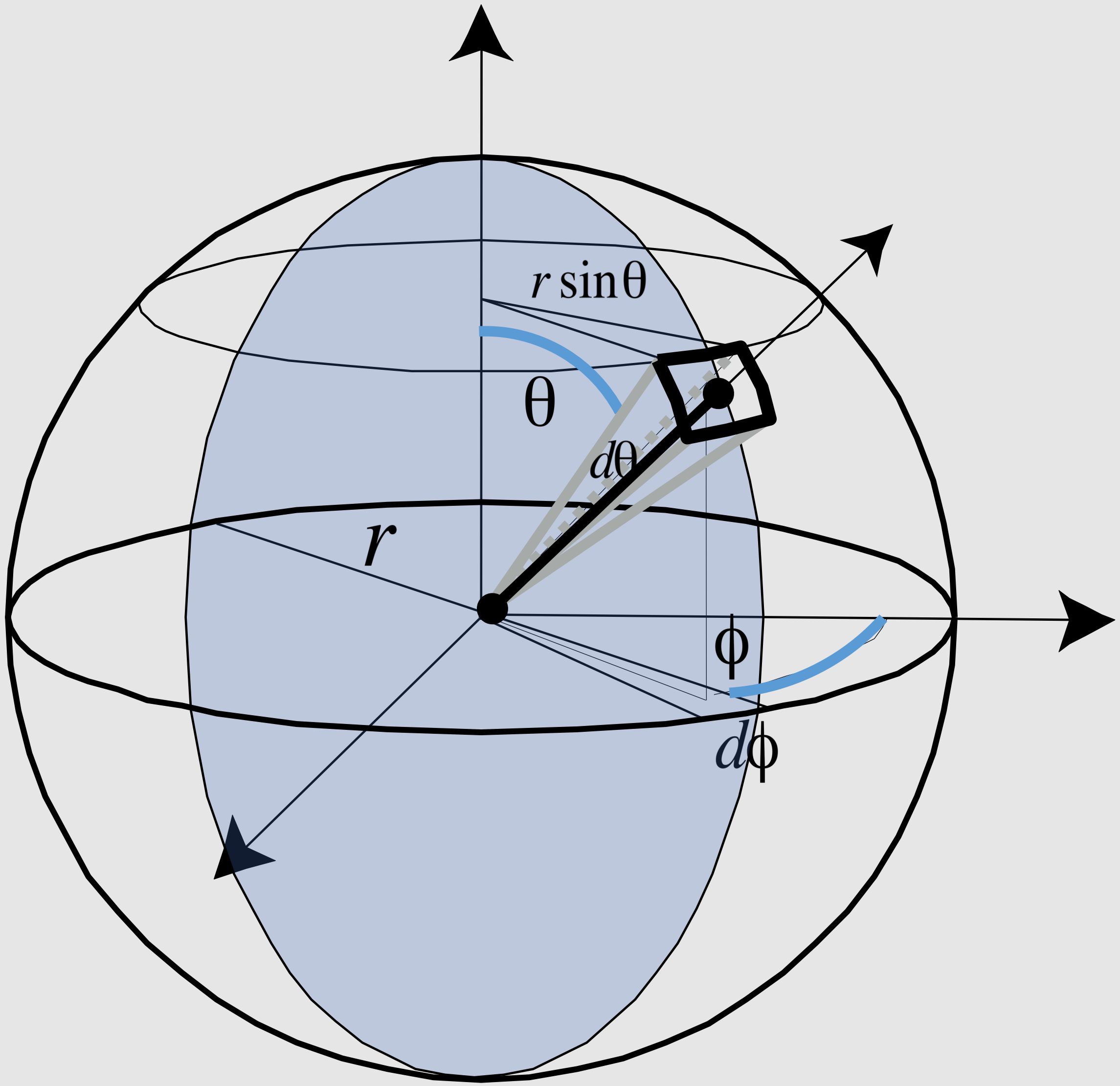
Differential Solid Angle



- **Goal:** when parameterizing a unit sphere in terms of (θ, φ), how does a small change $\mathrm{d}\theta$ or $\mathrm{d}\varphi$ affect the solid angle?

$$\begin{aligned} \mathrm{d}A &= (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi) \\ &= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \end{aligned}$$

Differential Solid Angle



- **Goal:** when parameterizing a unit sphere in terms of (θ, φ), how does a small change $\mathrm{d}\theta$ or $\mathrm{d}\varphi$ affect the solid angle?

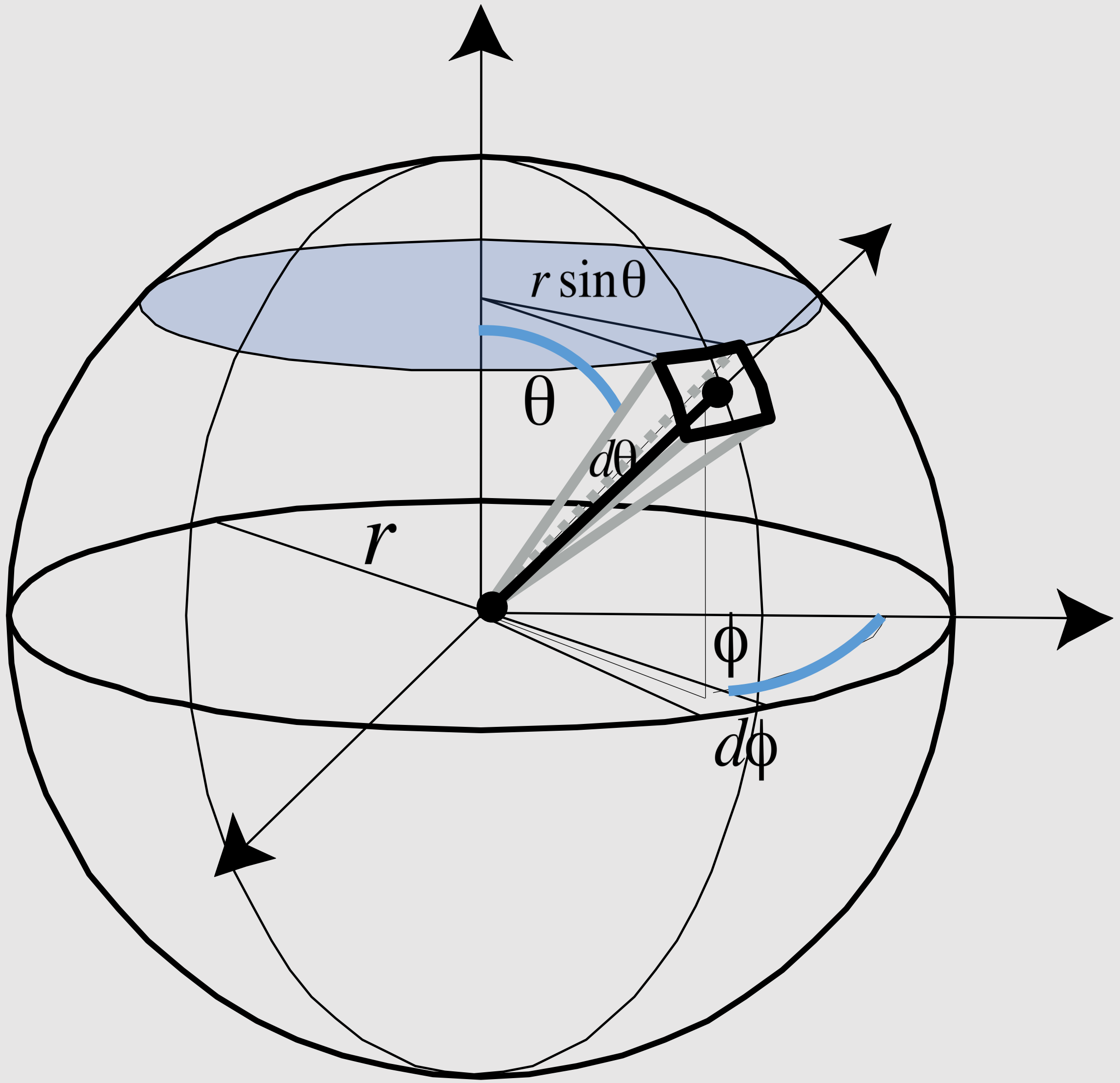
$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r \sin\theta\,\mathrm{d}\phi)$$
$$= r^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

- Recall:

$$\theta = \frac{l}{r}$$

  - Longitude of subtended area is $r\theta$

Differential Solid Angle



- **Goal:** when parameterizing a unit sphere in terms of (θ, φ), how does a small change $\mathrm{d}\theta$ or $\mathrm{d}\varphi$ affect the solid angle?

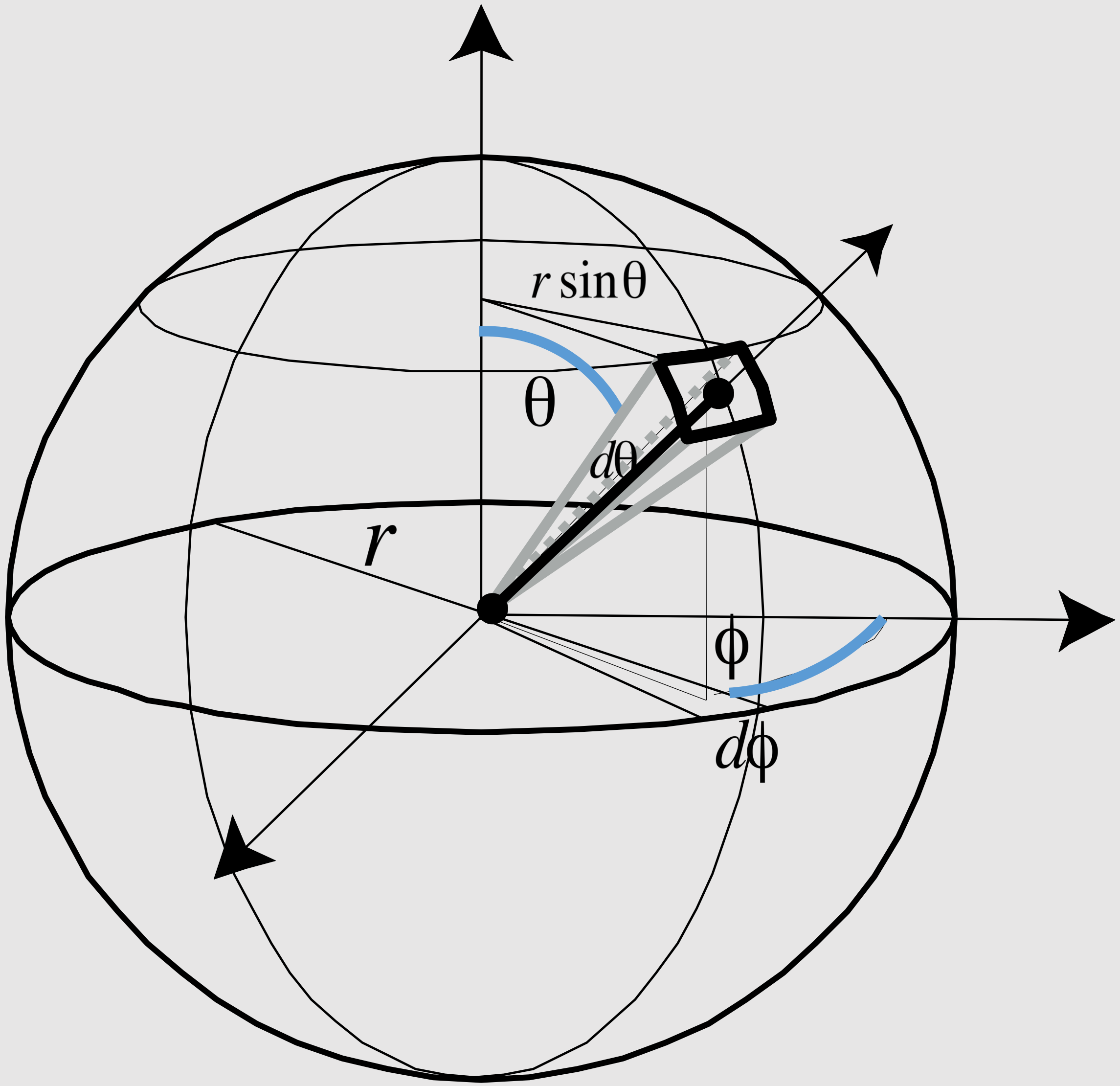
$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi)$$
$$= r^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

- Recall:

$$\theta = \frac{l}{r}$$

  - Latitude of subtended area is $r'\varphi$
  - r' is really $rsin\theta$ because we reparametrize in terms of φ

Differential Solid Angle



- **Goal:** when parameterizing a unit sphere in terms of (θ, φ), how does a small change $d\theta$ or $d\varphi$ affect the solid angle?

$$\begin{aligned} dA &= (r\, d\theta)(r \sin\theta\, d\phi) \\ &= r^2 \sin\theta\, d\theta\, d\phi \end{aligned}$$

- Differential solid angle is then:

$$d\omega = \frac{dA}{r^2} = \sin\theta\, d\theta\, d\phi$$

Radiance

- **Radiance** is the measure of radiant energy
  - *per unit time*
  - *per unit area*
  - *per unit solid angle*

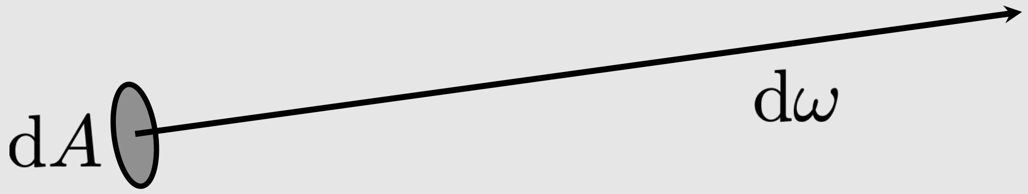
- Easier way to express: **Radiance** is the measure of irradiance
  - *per unit solid angle*

- Even easier way to express: **Radiance** is power along a ray defined by an origin point and direction
  - *per unit area* describes starting location
  - *per unit solid angle* describes location light is heading

$$L(p,\omega) = \lim_{\Delta \to 0} \frac{\Delta E_\omega(p)}{\Delta\omega} = \frac{dE_\omega(p)}{d\omega}$$

$$E(p) = \lim_{\Delta \to 0} \frac{\Delta \phi(p)}{\Delta A} = \frac{d\phi(p)}{dA}$$

$$L(p,\omega) = \frac{dE_\omega(p)}{d\omega} = \frac{d^2\phi(p)}{dA\, d\omega} \left[\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{sr}}\right]$$

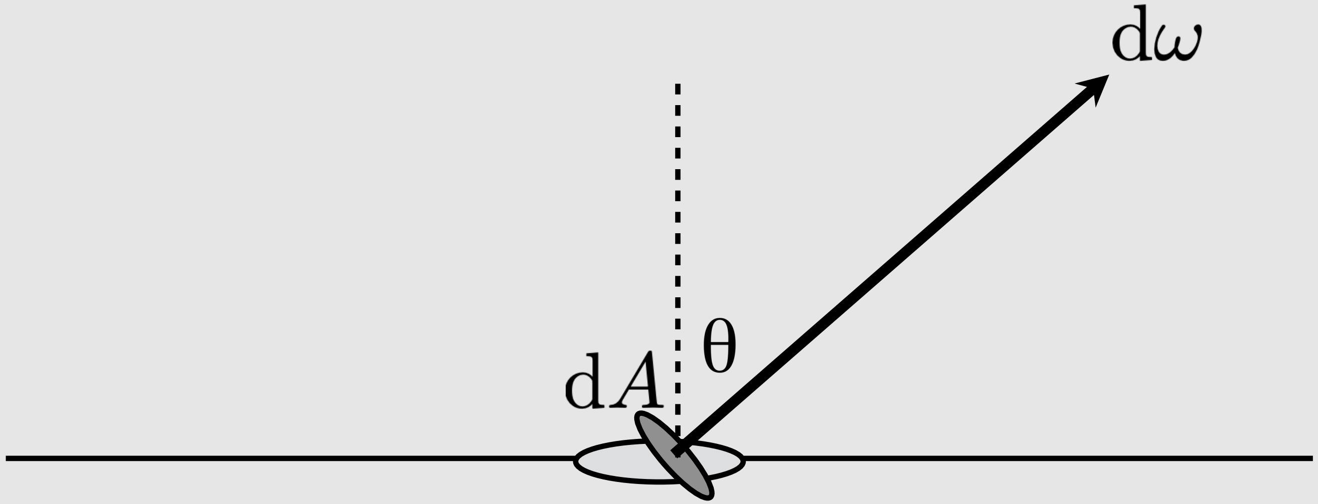


Surface Radiance

- **Issue:** what if dA is not perpendicular to surface normal?
  - **Solution:** Lambert's Law!

$$L(p,\omega) = \frac{dE_\omega(p)}{d\omega} = \frac{d^2\phi(p)}{dA\, d\omega} = \frac{d^2\phi(p)}{dA' d\omega \cos\theta}$$

$$A = A' \cos\theta$$



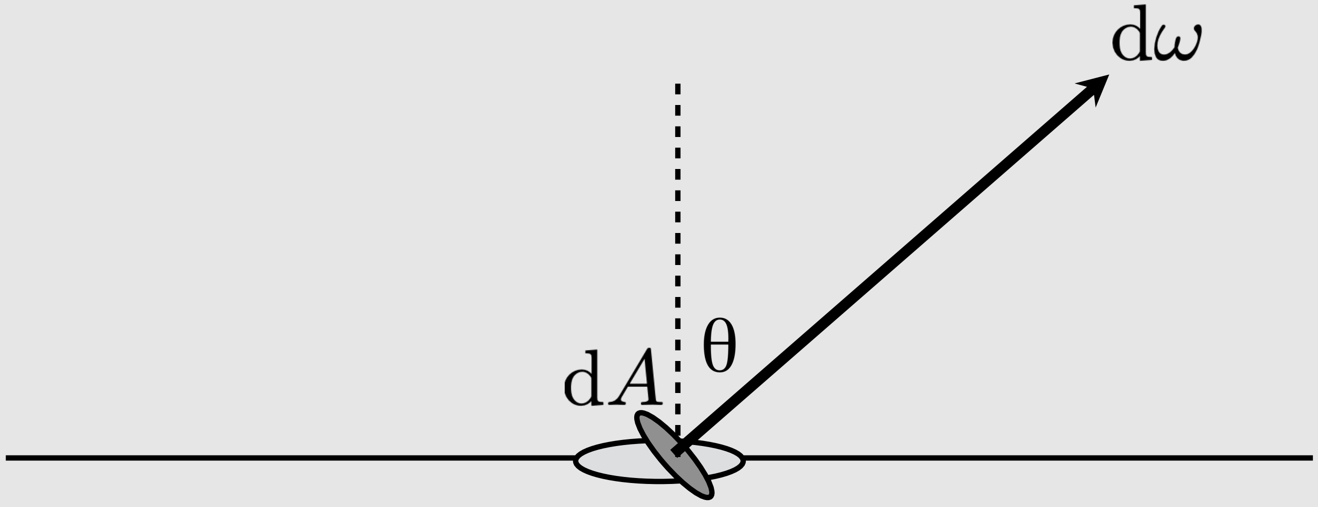
Surface Radiance (Flipped)

- **Issue:** what if we have irradiance E in terms of A'?
  - **Solution:** Flip Lambert's Law!

$$L(p,\omega) = \frac{dE_\omega(p)}{d\omega} = \frac{d^2\phi(p)}{dA'\,d\omega} = \frac{d^2\phi(p)\cos\theta}{dA\,d\omega}$$

$$L(p,\omega) = L'(p,\omega)\cos\theta$$

$$A' = A\,/\cos\theta$$

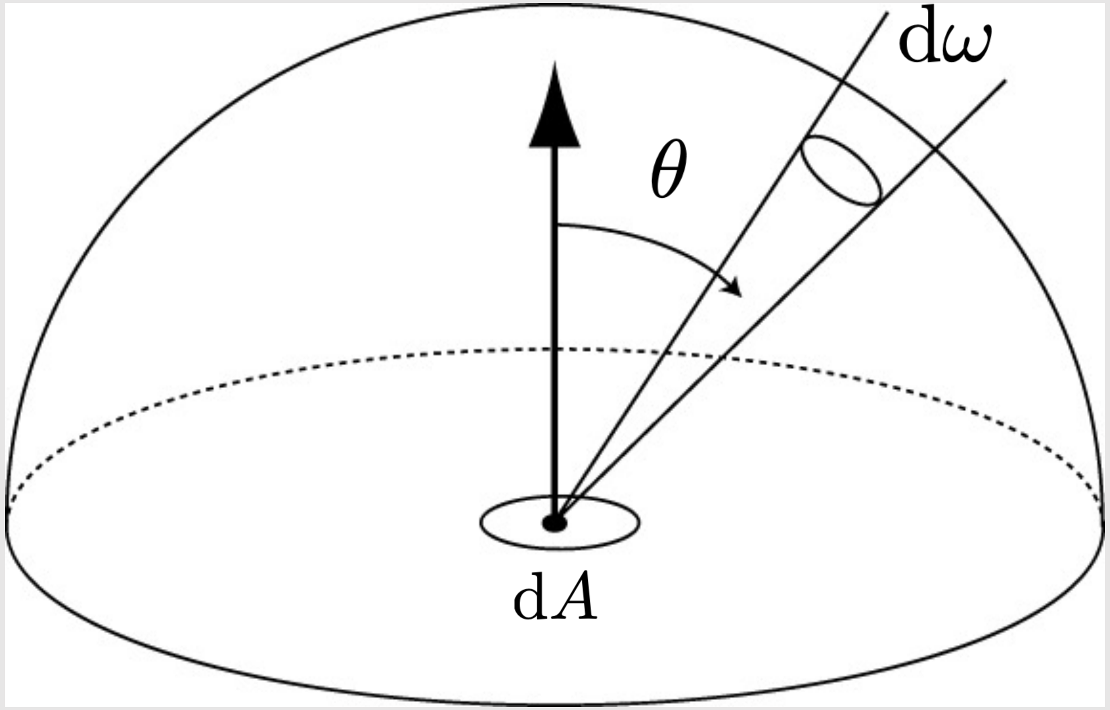


Irradiance From The Environment

- In rendering, we want to measure all the incoming light into a point
- Computing irradiance (E) on surface, due to incoming light from all directions:

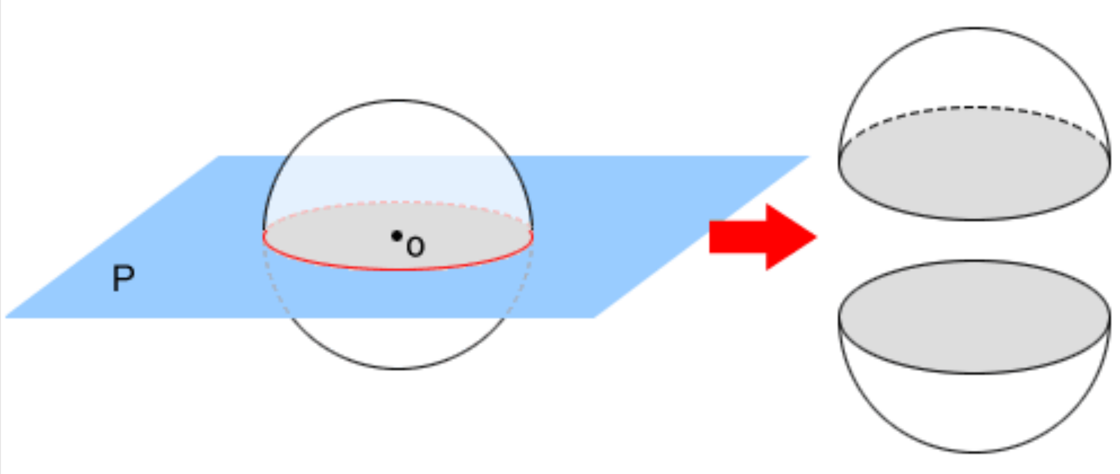
$$E(\mathsf{p}) = \int_{H^2} L_i(\mathsf{p}, \omega) \cos\theta d\omega$$

  - Incoming light is incident onto dA'
  - Lambert's Law adds $\cos\theta$ to convert it to dA



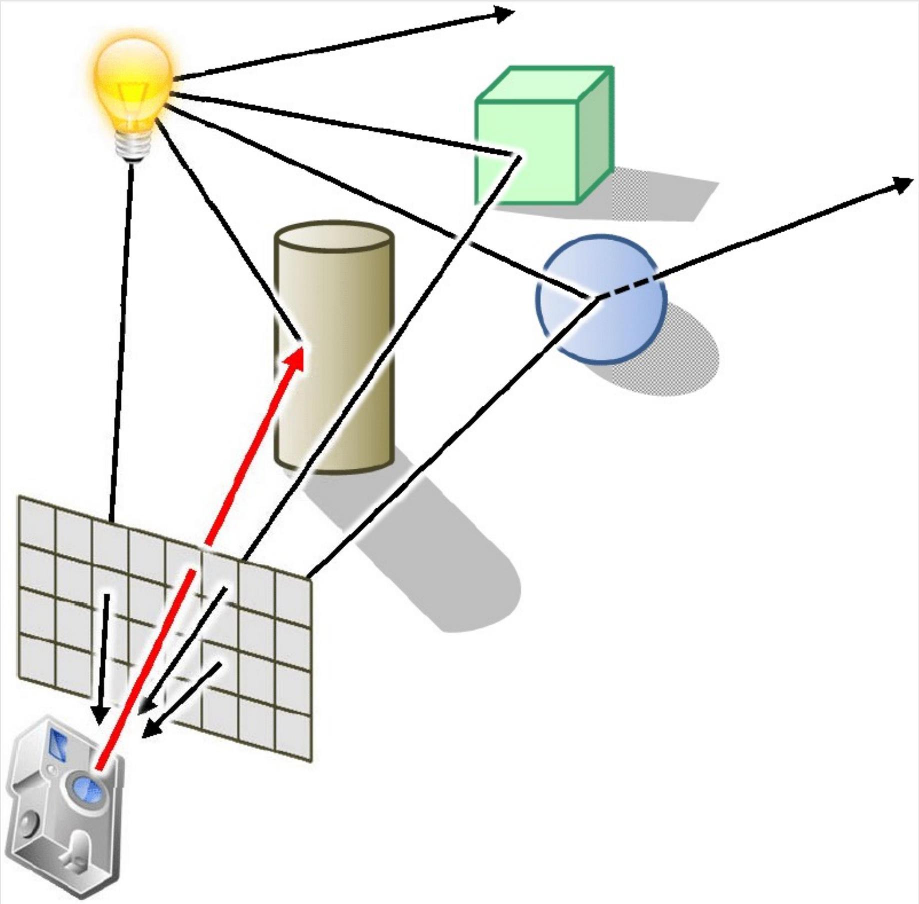
Irradiance From The Environment

- Why do we use hemispheres for radiance calculations?
  - Recall we approximate scene geometry using explicit primitives
    - Each primitive has a planar representation
- When light enters or exits the primitive, it can only do so on one side of the primitive
  - Hence, all possible ray directions are limited to a hemisphere on the primitive

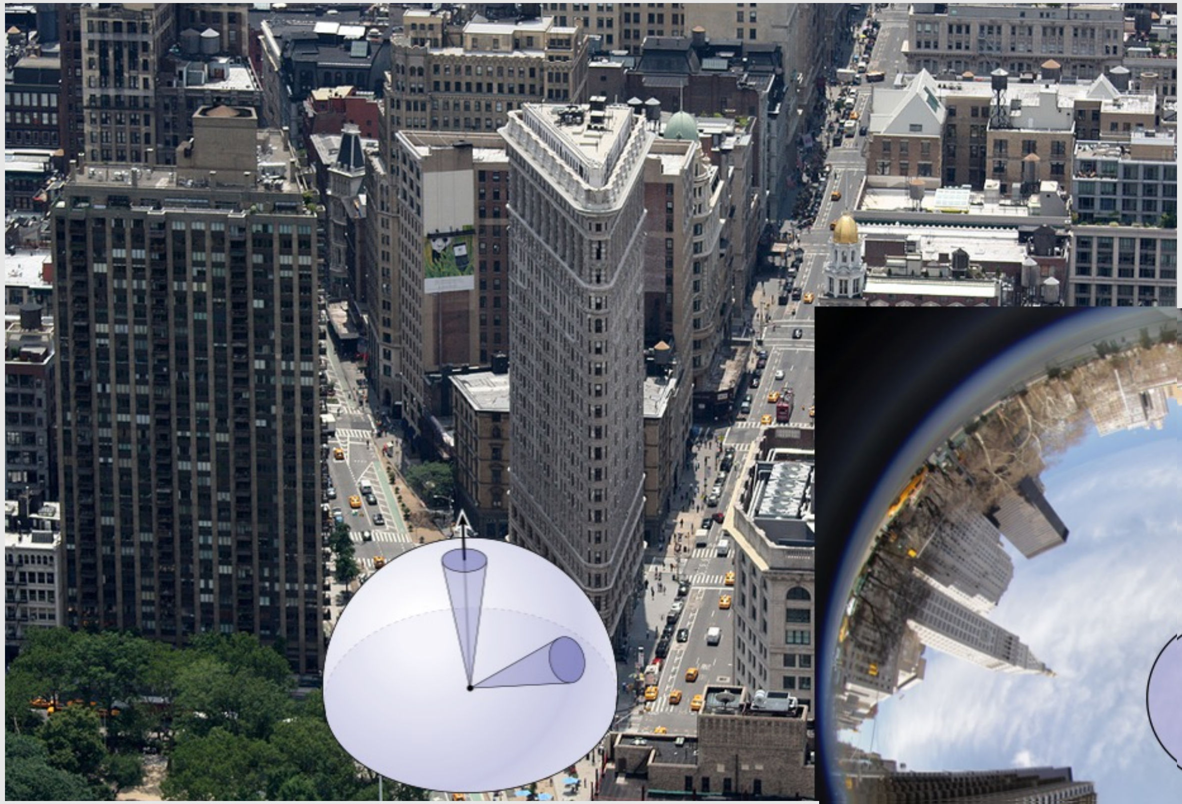


Radiance In Rendering

- Rendering is all about computing radiance
  - We will use rays to simulate light interactions in a scene
    - Integrating over these light rays gives us a way to render a scene
- Radiance is constant (and linear) in a **vacuum**
  - Normally air friction causes radiance values to scatter/lose energy due to particle-particle interactions
    - We will be rendering in vacuums of space to ignore these imperfections :)



Radiance In Rendering



irradiance

radiance in direction ω

$$E = \int_{\mathrm{H}^2} L(\omega) \cos\theta \, d\omega$$

angle between ω and n

Radiometric & Photometric Terms

Physics	Radiometry	Photometry
Energy	Radiant Energy	Luminous Energy
Flux (Power)	Radiant Power	Luminous Power
Flux Density	Irradiance (incoming) Radiosity (outgoing)	Illuminance (incoming) Luminosity (outgoing)
Angular Flux Density	Radiance	Luminance
Intensity	Radiant Intensity	Luminous Intensity

Photometric Units

Photometry	**MKS**	**CGS**	**British**
Luminous Energy	Talbot	Talbot	Talbot
Luminous Power	Lumen	Lumen	Lumen
Illuminance Luminosity	Lux	Phot	Footcandle
Luminance	Nit, Apostlib, Blondel	Stilb Lambert	Footlambert
Luminous Intensity	Candela	Candela	Candela

"Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?" —James Kajiya