Midterm Review

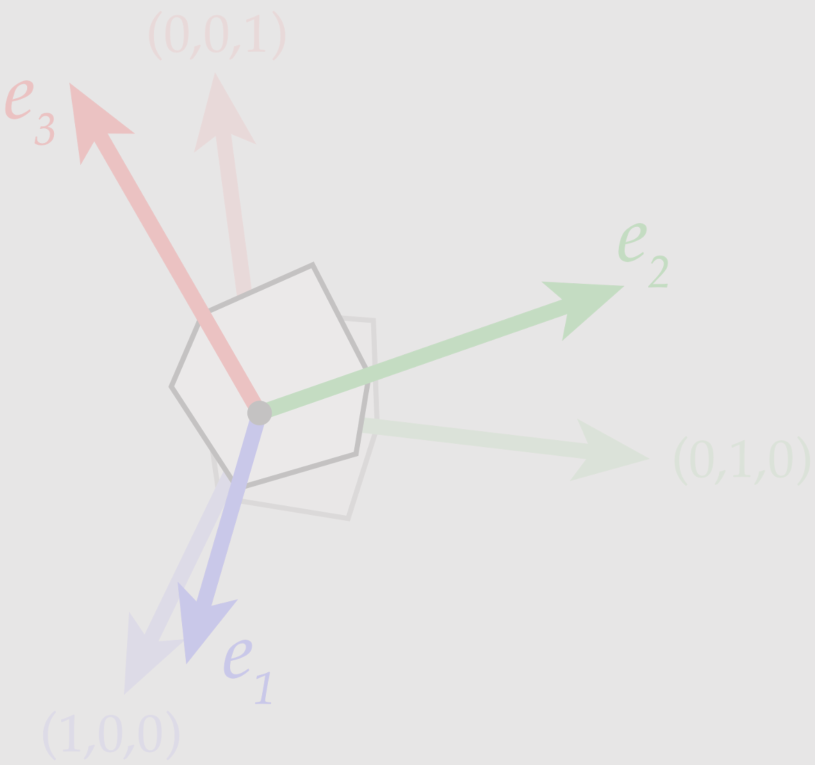
Midterm Overview

- 80 minutes, during class this Wednesday
    - Basic mathematical questions, no intense calculation
    - Know your definitions and be able to apply them!
    - No pseudocode
    - Review slides are a good hint as to what might be on the exam :)

- Cheat sheet: one 3x3 inch note (about the size of a post it note) front and back

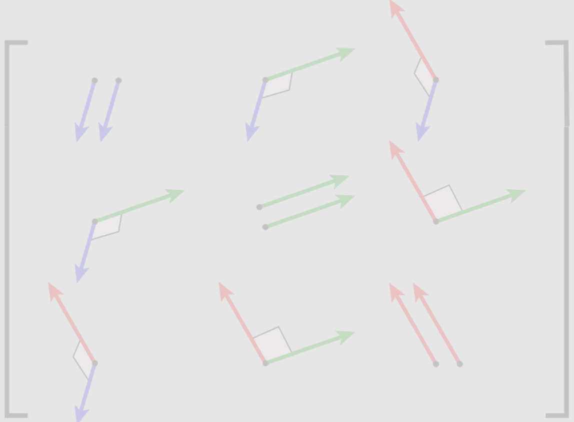
- **Please bring a blue/black pen to write your solutions**

3D Inverse Rotations



$$\begin{array}{cc} R^{\top} & R \\ \begin{bmatrix} — e_1^T — \\ — e_2^T — \\ — e_3^T — \end{bmatrix} & \begin{bmatrix} | & | & | \\ e_1 & e_2 & e_3 \\ | & | & | \end{bmatrix} \end{array}$$

$=$



$$= \begin{bmatrix} e_1^T e_1 & e_1^T e_2 & e_1^T e_3 \\ e_2^T e_1 & e_2^T e_2 & e_2^T e_3 \\ e_3^T e_1 & e_3^T e_2 & e_3^T e_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^T R = I \;\Rightarrow\; R^T = R^{-1}$$

If you need to review any slides more in depth, look here for which lecture it came from

- Transformations Review
- Rasterization Review
- Geometry Review
- Spatial Data Structures Review

Transformations

- Homogeneous coordinates
- 3D Translation
- 3D Scale
- 3D Rotation
    - Axis-Aligned rotation
    - Axis-Angle rotation
    - Rotations from orthonormal bases

3D Transforms in Homogeneous Coordinate

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

[point in 3D]

Matrix representations of 3D linear transformations just get
an additional identity row/column:

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & s & 0 \\ 0 & 1 & t & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & u \\ 0 & 1 & 0 & v \\ 0 & 0 & 1 & w \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[rotate around y by θ]　[shear by z in (s,t) direction]　[scale by a,b,c]　[translate by (u,v,w)]

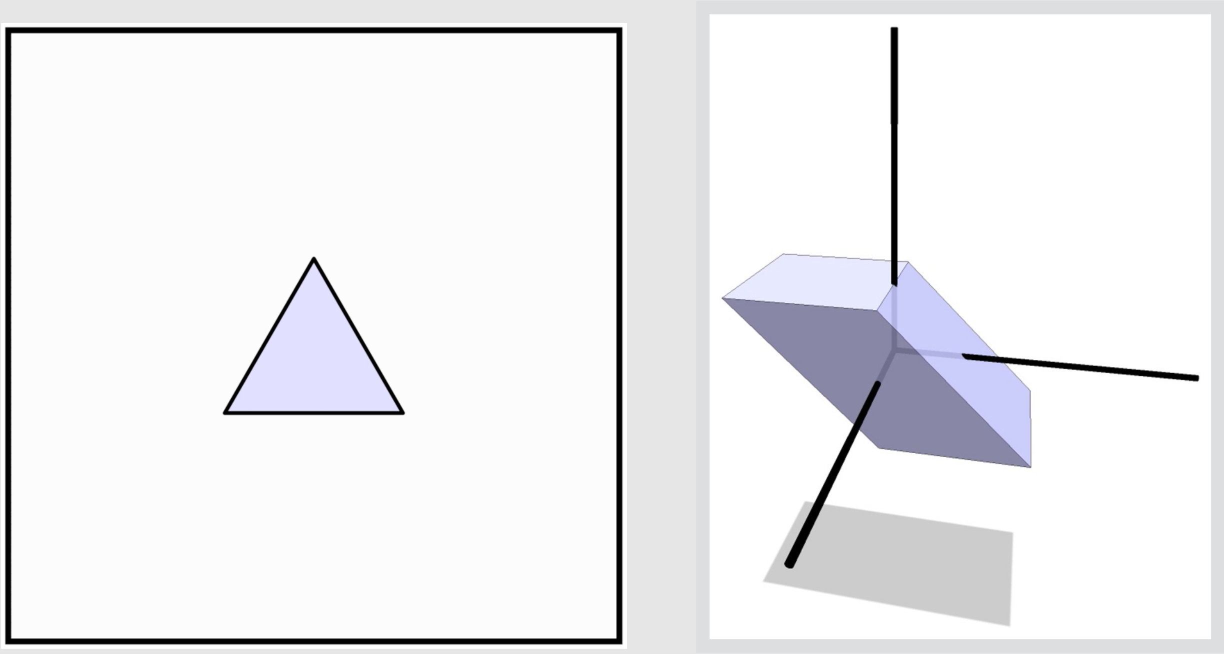
Translation in Homogeneous Coordinates

- A 2D translation is similar to a 3D shear
  - Moving a slice up/down the shear moves the shape
- Recall shear is written as:

$$f_{\mathbf{u},\mathbf{v}}(\mathbf{x}) = \mathbf{x} + \langle \mathbf{v}, \mathbf{x} \rangle \mathbf{u}$$

$$f_{\mathbf{u},\mathbf{v}}(\mathbf{x}) = \left(I + \mathbf{u}\mathbf{v}^\top\right)\mathbf{x}$$

- In our case, $v = (0,0,1)$, so**



$$\begin{bmatrix} 1 & 0 & u_1 \\ 0 & 1 & u_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cp_1 \\ cp_2 \\ c \end{bmatrix} = \begin{bmatrix} c(p_1 + u_1) \\ c(p_2 + u_2) \\ c \end{bmatrix} \overset{1/c}{\Longrightarrow} \begin{bmatrix} p_1 + u_1 \\ p_2 + u_2 \end{bmatrix}$$

**most often in this class we will also use c = 1

Non-Uniform Scaling

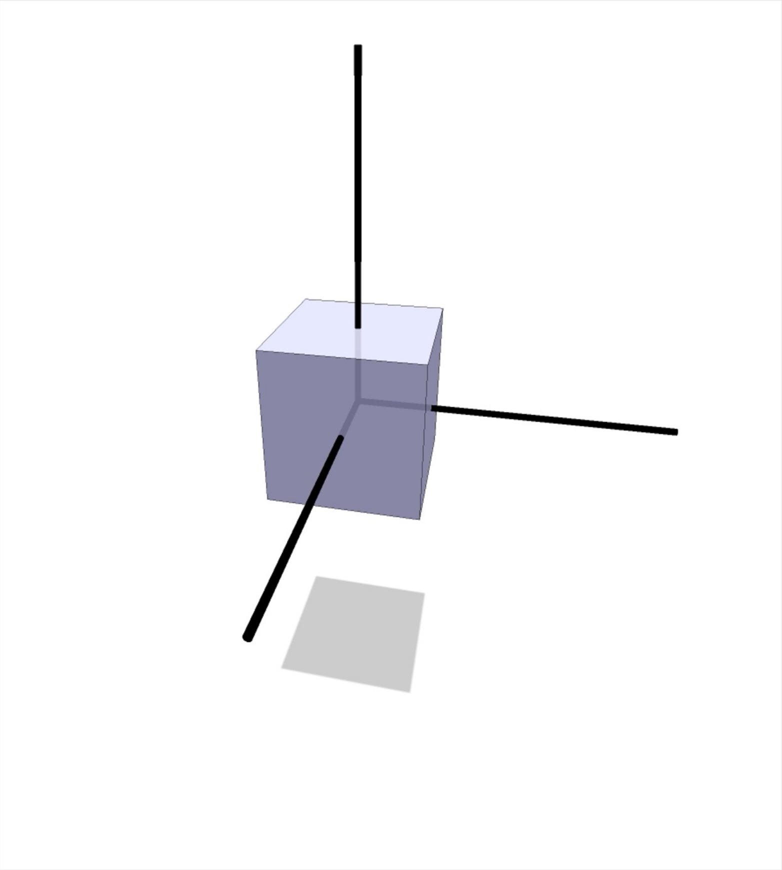
- To scale a vector u by a non-uniform amount (a, b, c):

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} au_1 \\ bu_2 \\ cu_3 \end{bmatrix}$$

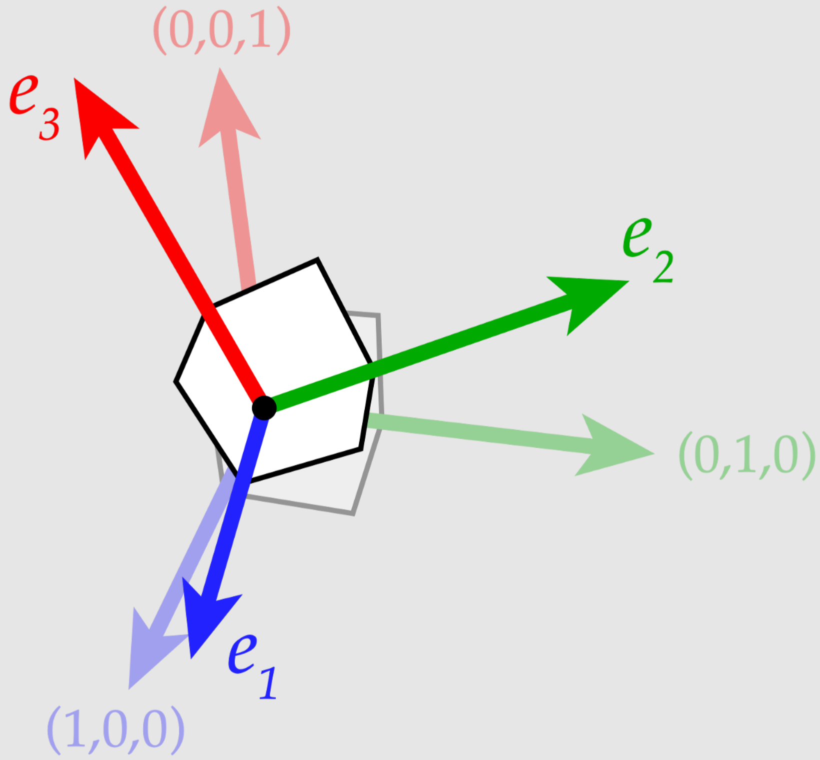
- The above works only if scaling is axis-aligned. What if it isn't?
- Idea:
  - Rotate to a new axis R
  - Perform axis-aligned scaling D
  - Rotate back to original axis R^T

$$A := R^T D R$$

  - Resulting transform A is a symmetric matrix

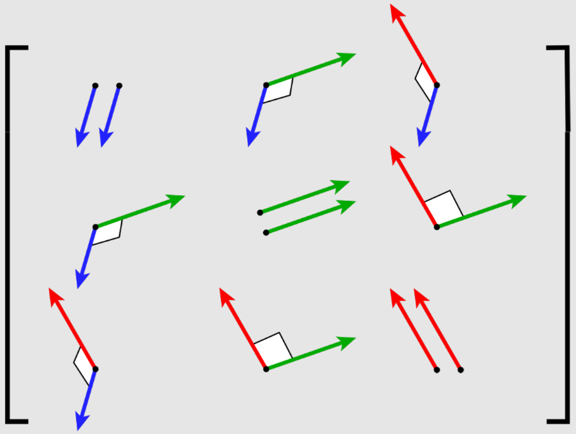


3D Inverse Rotations



$$\begin{array}{cc} R^{\mathsf{T}} & R \\ \begin{bmatrix} — & e_1^T & — \\ — & e_2^T & — \\ — & e_3^T & — \end{bmatrix} & \begin{bmatrix} | & | & | \\ e_1 & e_2 & e_3 \\ | & | & | \end{bmatrix} \end{array}$$

$=$

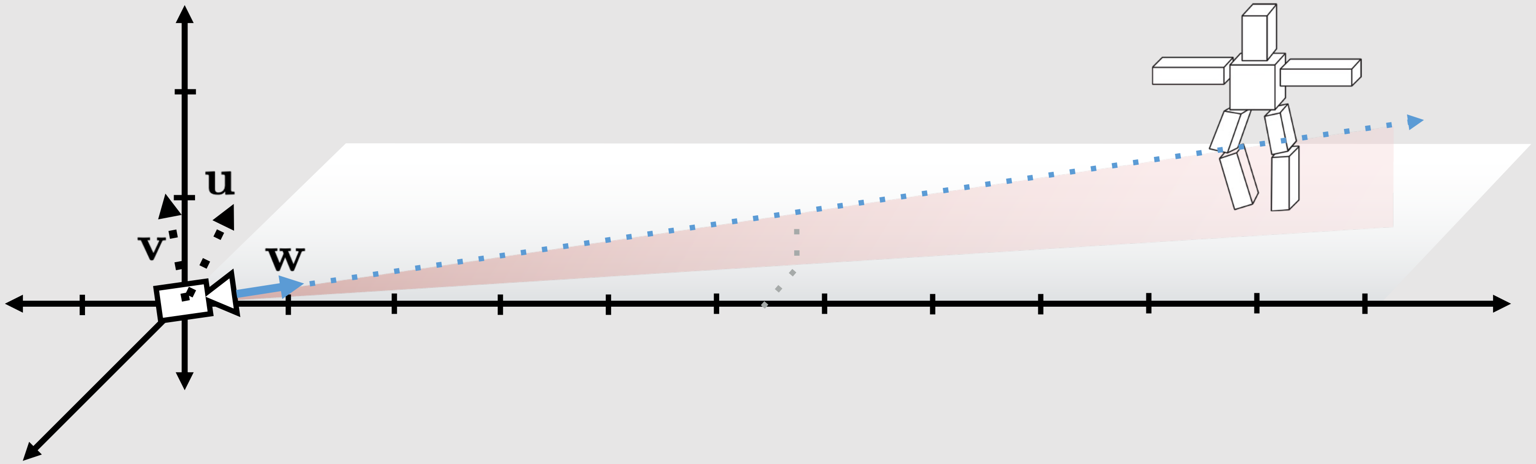


$$= \begin{bmatrix} e_1^T e_1 & e_1^T e_2 & e_1^T e_3 \\ e_2^T e_1 & e_2^T e_2 & e_2^T e_3 \\ e_3^T e_1 & e_3^T e_2 & e_3^T e_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{R^T R = I \;\Rightarrow\; R^T = R^{-1}}$$

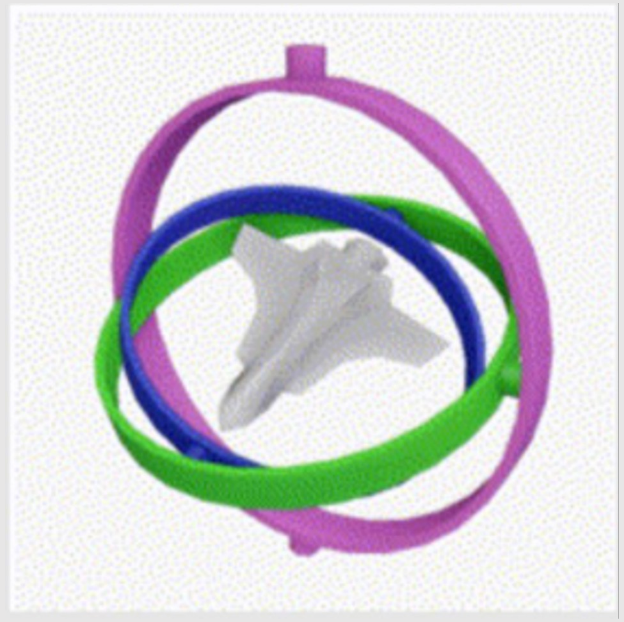
Rotations From Orthonormal Bases



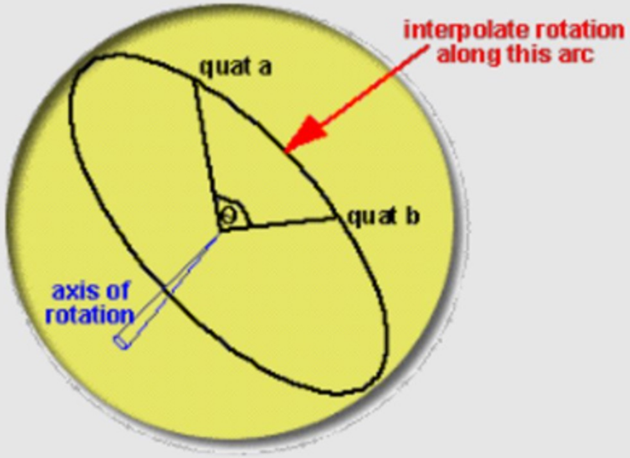
$$R = \begin{bmatrix} -u_x & v_x & -w_x \\ -u_y & v_y & -w_y \\ -u_z & v_z & -w_z \end{bmatrix} \qquad R^{-1} = \begin{bmatrix} -u_x & -u_y & -u_z \\ v_x & v_y & v_z \\ -w_x & -w_y & -w_z \end{bmatrix}$$

Other Ways to Rotate - Euler Angles and Quaternions

- **Euler Angles:** Rotate by combining rotation matrices for each major axis
  - Rx = rotation about x axis, Ry = rotation about y axis, Rz =rotation about z axis
    - RxRyRz = final rotation matrix
  - Fairly intuitive, but prone to gimbal lock!
  - Interpolating between Euler angles can create odd looking path, non-uniform rotation speed, etc.



- **Quaternions**: 4 coordinates, 1 real and 3 complex
  - Defined by $i^2 = j^2 = k^2 = ijk = -1$
  - Not prone to gimbal lock! But not very intuitive to think about :(
  - Can interpolate between them correctly with SLERP



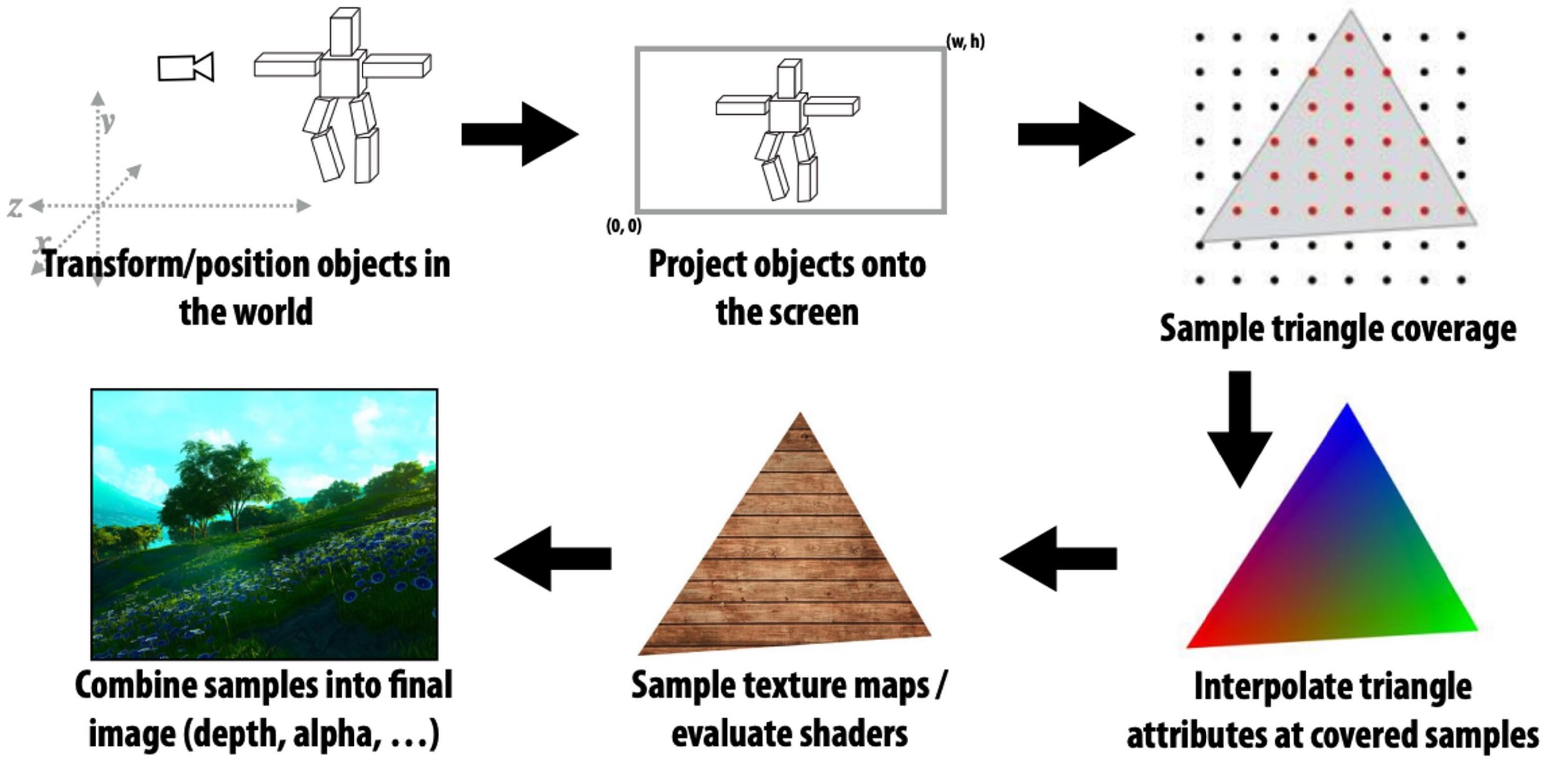
Animating Rotation with Quaternion Curves (1985) Shoemake

- Transformations Review
- Rasterization Review
- Geometry Review
- Spatial Data Structures Review

Rasterization

- The “simpler” graphics pipeline
- Scene graph
- Clipping
- Rasterization
    - Sampling
    - Point-in-triangle tests
    - Barycentric coordinates
- Textures
- Depth and Alpha blending

The “Simpler” Graphics Pipeline



Transform/position objects in the world

Project objects onto the screen

Sample triangle coverage

Interpolate triangle attributes at covered samples

Sample texture maps / evaluate shaders

Combine samples into final image (depth, alpha, …)

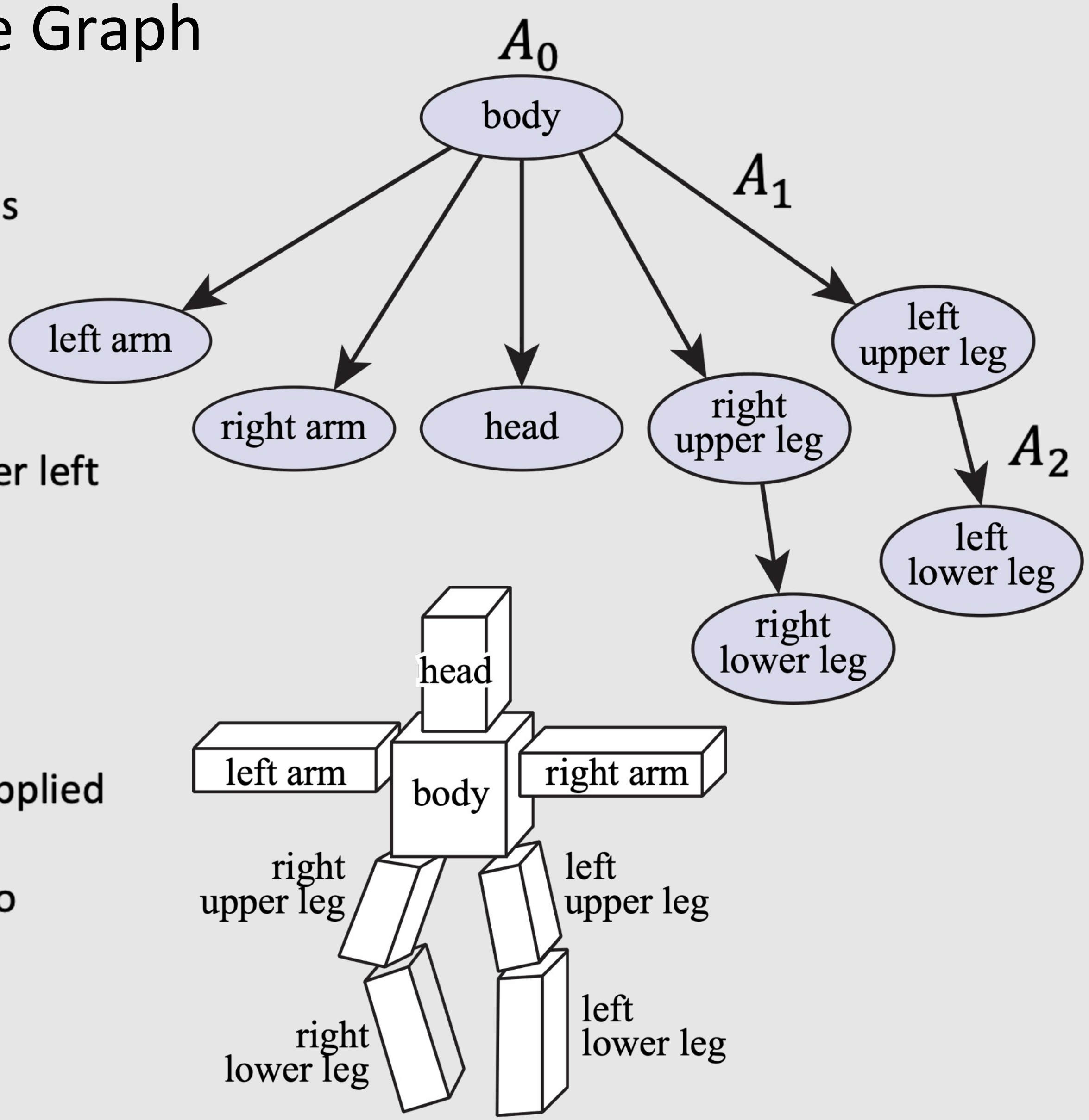
Scene Graph

- Suppose we want to build a skeleton out of cubes

- **Idea:** transform cubes in world space
  - Store transform of each cube

- **Problem:** If we rotate the left upper leg, the lower left leg won't track with it
  - **Better Idea:** store a hierarchy of transforms
    - Known as a scene graph
    - Each edge (+root) stores a linear transformation
    - Composition of transformations gets applied to nodes
      - Keep transformations on a stack to reduce redundant multiplication

- **Lower left leg transform:** $A_2 A_1 A_0$

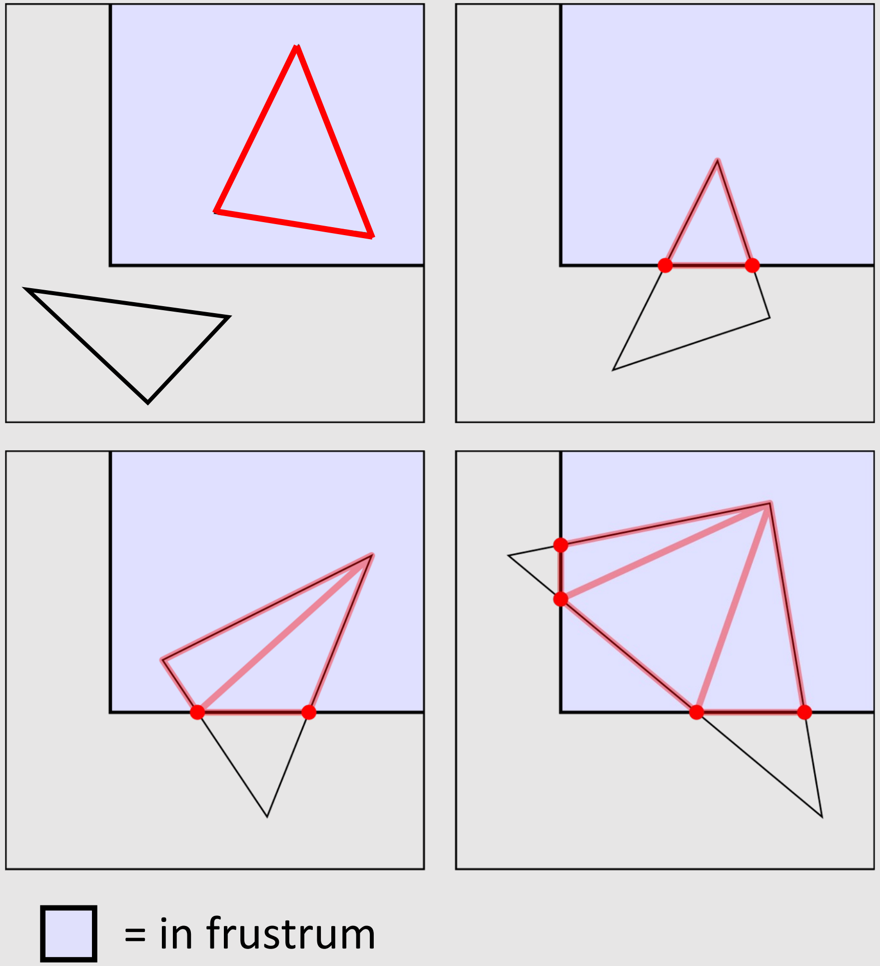


Clipping

- Clipping eliminates triangles not visible to the camera (not in view frustum)
  - Don’t waste time rasterizing primitives you can’t see!
  - Discarding individual fragments is expensive
    - “Fine granularity”
  - Makes more sense to toss out whole primitives
    - “Coarse granularity”

- What if a primitive is **partially clipped?**
  - Partially enclosed primitives are triangulated into non-overlapping smaller triangles that fit in the frustrum

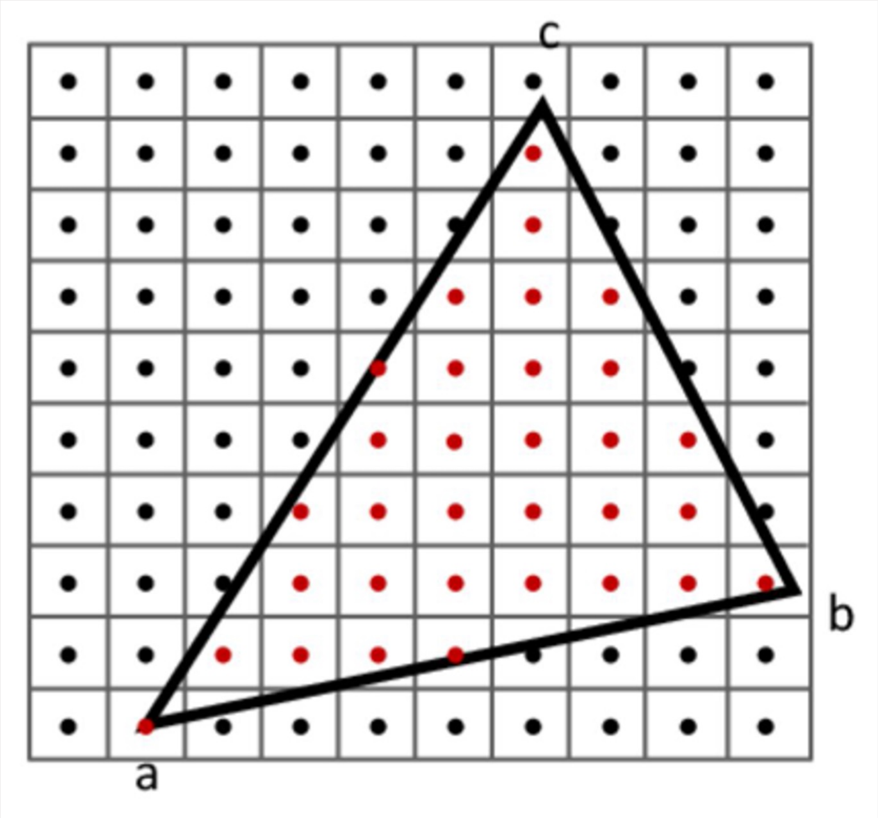
- If part of a triangle is outside the frustrum, it means at least one of its vertices are outside the frustrum
  - **Idea:** check which side of halfspaces the vertices are at


☐ = in frustrum

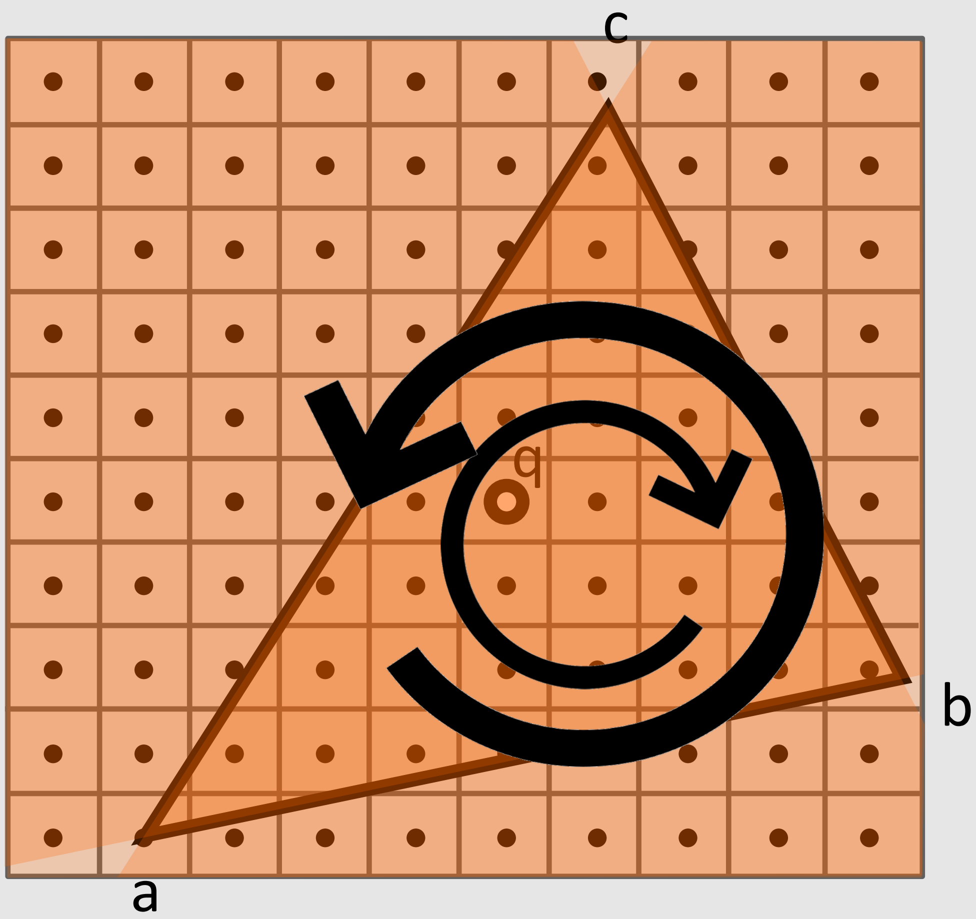
Rasterization

- Triangle
  - Bounding box
  - Incremental triangle traversal
  - Hierarchical coverage

- For each **Primitive** (Triangle):
  - For each **Pixel**:
    - If **Pixel** in **Primitive**:
      - Pixel color = Interpolated triangle color



Point-In-Triangle Test



- **Measurements must all either be positive or negative** for point to be in triangle

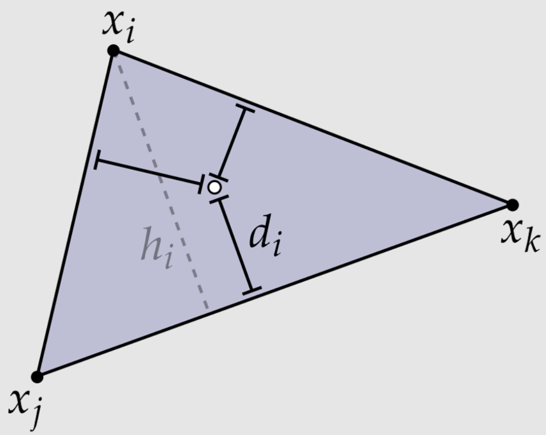
$$
\begin{gathered}
(\overrightarrow{ac} \times \overrightarrow{ab}) \cdot (\overrightarrow{ac} \times \overrightarrow{aq}) > 0 \,\&\& \\
(\overrightarrow{cb} \times \overrightarrow{ca}) \cdot (\overrightarrow{cb} \times \overrightarrow{cq}) > 0 \,\&\& \\
(\overrightarrow{ba} \times \overrightarrow{bc}) \cdot (\overrightarrow{ba} \times \overrightarrow{bq}) > 0
\end{gathered}
$$

OR

$$
\begin{gathered}
(\overrightarrow{ab} \times \overrightarrow{ac}) \cdot (\overrightarrow{ac} \times \overrightarrow{aq}) < 0 \,\&\& \\
(\overrightarrow{ca} \times \overrightarrow{cb}) \cdot (\overrightarrow{cb} \times \overrightarrow{cq}) < 0 \,\&\& \\
(\overrightarrow{bc} \times \overrightarrow{ba}) \cdot (\overrightarrow{ba} \times \overrightarrow{bq}) < 0
\end{gathered}
$$

- Orientation no longer matters
  - Just be consistent!

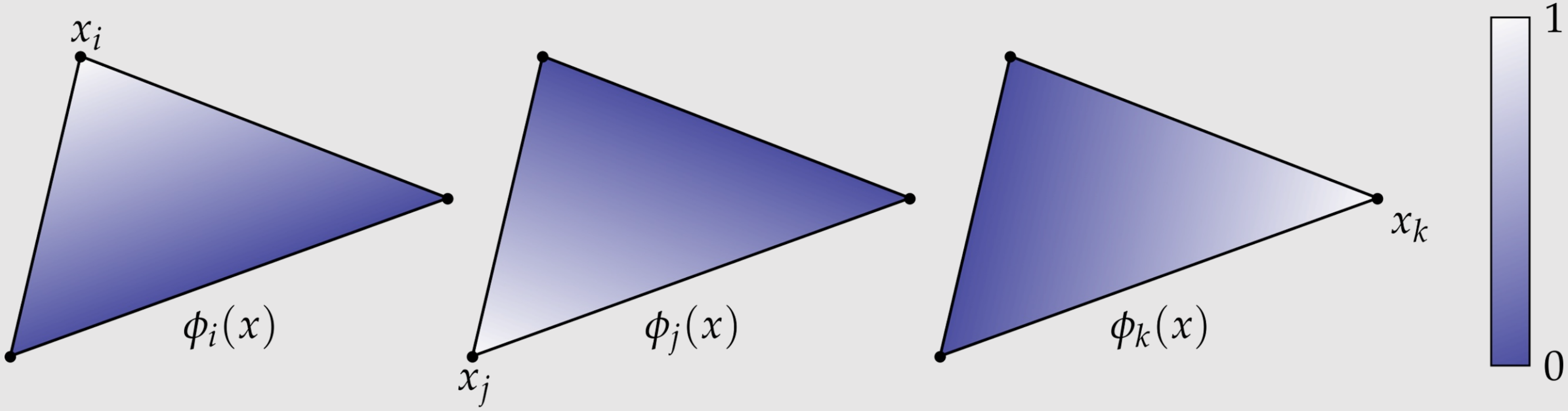
Barycentric Coordinates



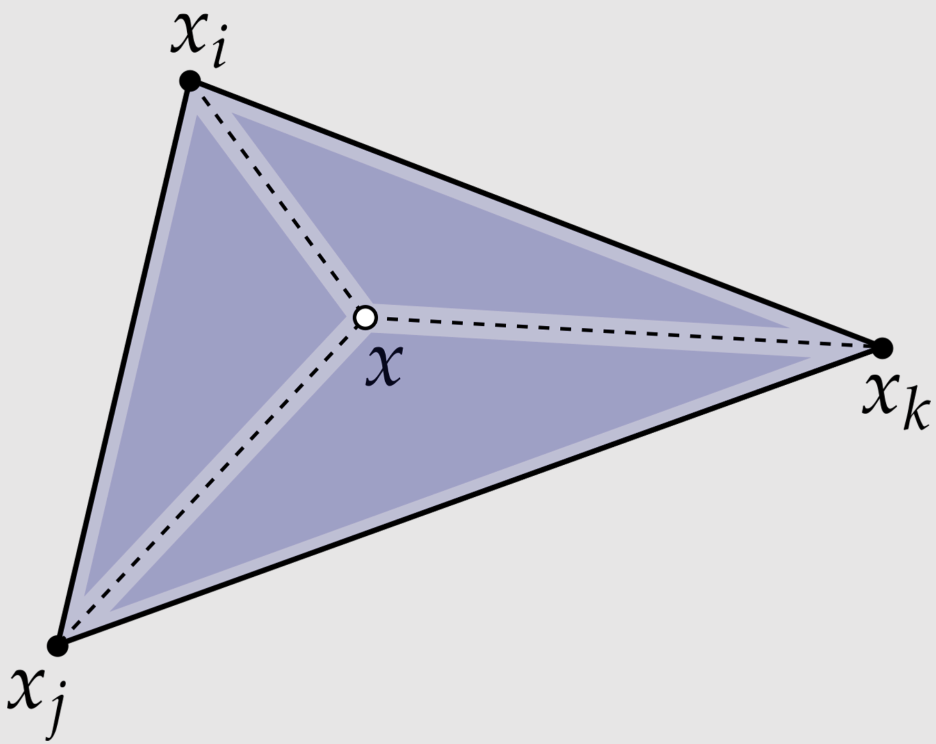
- Inversely proportional to the signed distance between the target point and a point within the triangle
- Can be computed as:

$$\phi_i(x) = d_i(x)/h_i$$

- How would you compute h_i? $d_i(x)$?



Barycentric Coordinates [Another Way]



- Directly proportional to the signed area created by the triangle composed of the other two target points and a point within the triangle

- Can be computed as:

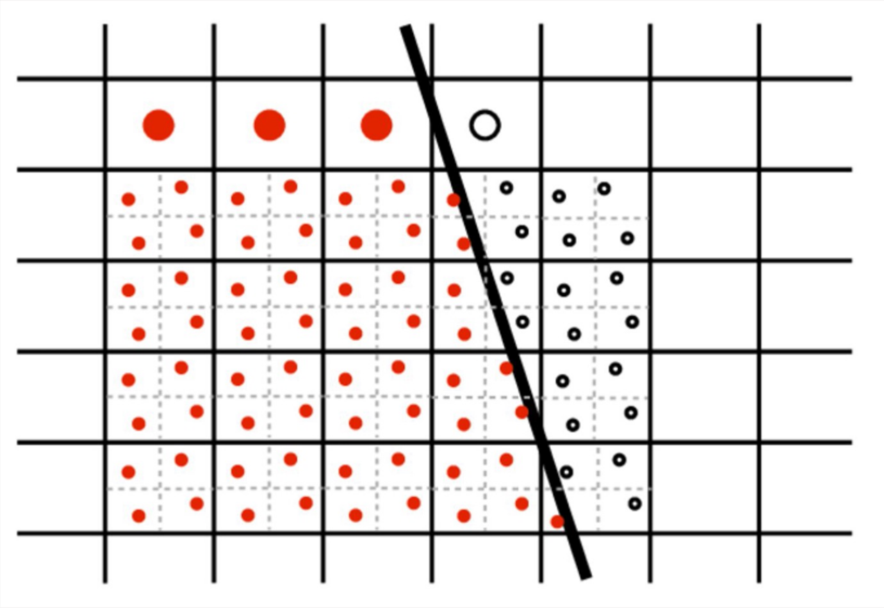
$$\phi_i(x) = \frac{\text{area}(x, x_j, x_k)}{\text{area}(x_i, x_j, x_k)}$$

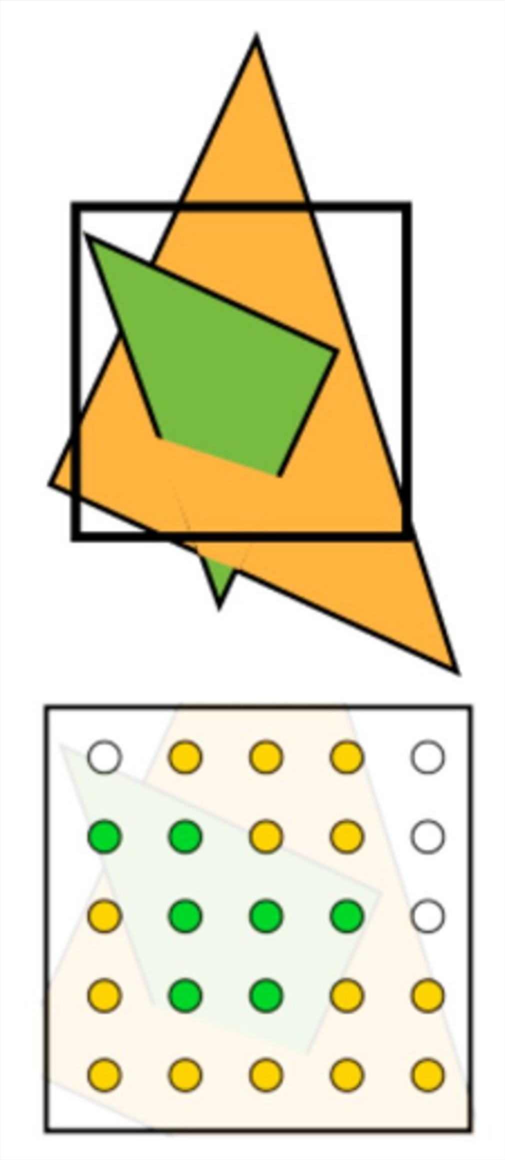
- Note that signed distance / area implies barycentric coordinates can be negative, but they will still sum to 1! (if on the same plane, otherwise we project point to the plane containing our triangle)

** Interesting read of barycentric coordinates for n-gons: https://www.inf.usi.ch/hormann/barycentric/

Coverage via Samples

- Sample : Discrete measurement of a signal
  - Multisampling vs Supersampling
- Approximate the coverage of the area of a pixel by taking *n* samples
  - Per sample coverage & depth test + texture lookup + alpha blending



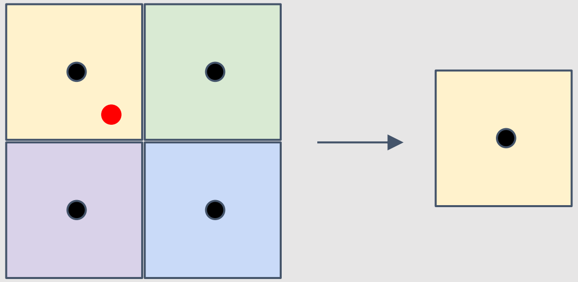


Nearest Neighbor Sampling

- **Idea:** Grab texel nearest to requested location in texture
- **Requires:**
    - 1 memory lookup
    - 0 linear interpolations

$$x' \leftarrow round(x - 0.5), \qquad y' \leftarrow round(y) - 0.5$$

$$t \leftarrow tex.lookup(x', y')$$



Bilinear Interpolation Sampling

- **Idea:** Grab nearest 4 texels and blend them together based on their inverse distance from the requested location
    - Blend two sets of pixels along one axis, then blend the remaining pixels

- **Requires:**
    - 4 memory lookup
    - 3 linear interpolations

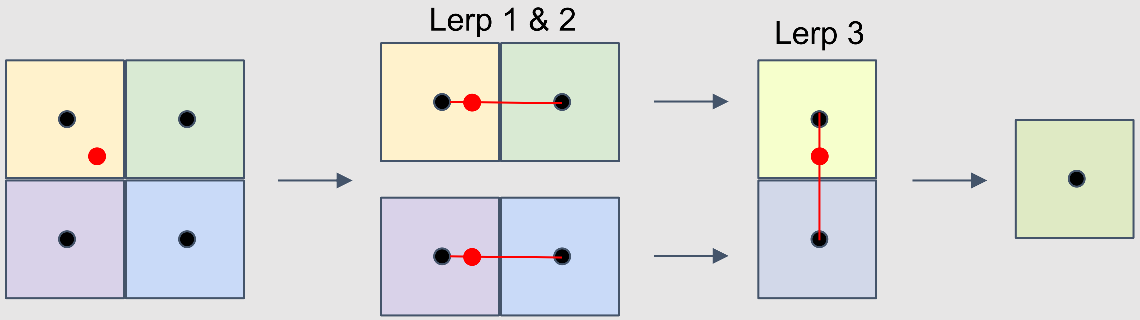
$$x' \leftarrow floor(x - 0.5), \qquad y' \leftarrow floor(y - 0.5)$$

$$\Delta x \leftarrow (x - 0.5) - x'$$
$$\Delta y \leftarrow (y - 0.5) - y'$$

$$t_{(x,y)} \leftarrow tex.lookup(x', y')$$
$$t_{(x+1,y)} \leftarrow tex.lookup(x' + 1, y')$$
$$t_{(x,y+1)} \leftarrow tex.lookup(x', y' + 1)$$
$$t_{(x+1,y+1)} \leftarrow tex.lookup(x', +1\ y' + 1)$$

$$t_x \leftarrow (1 - \Delta x) * t_{(x,y)} + \Delta x * t_{(x+1,y)}$$
$$t_y \leftarrow (1 - \Delta x) * t_{(x,y+1)} + \Delta x * t_{(x+1,y+1)}$$

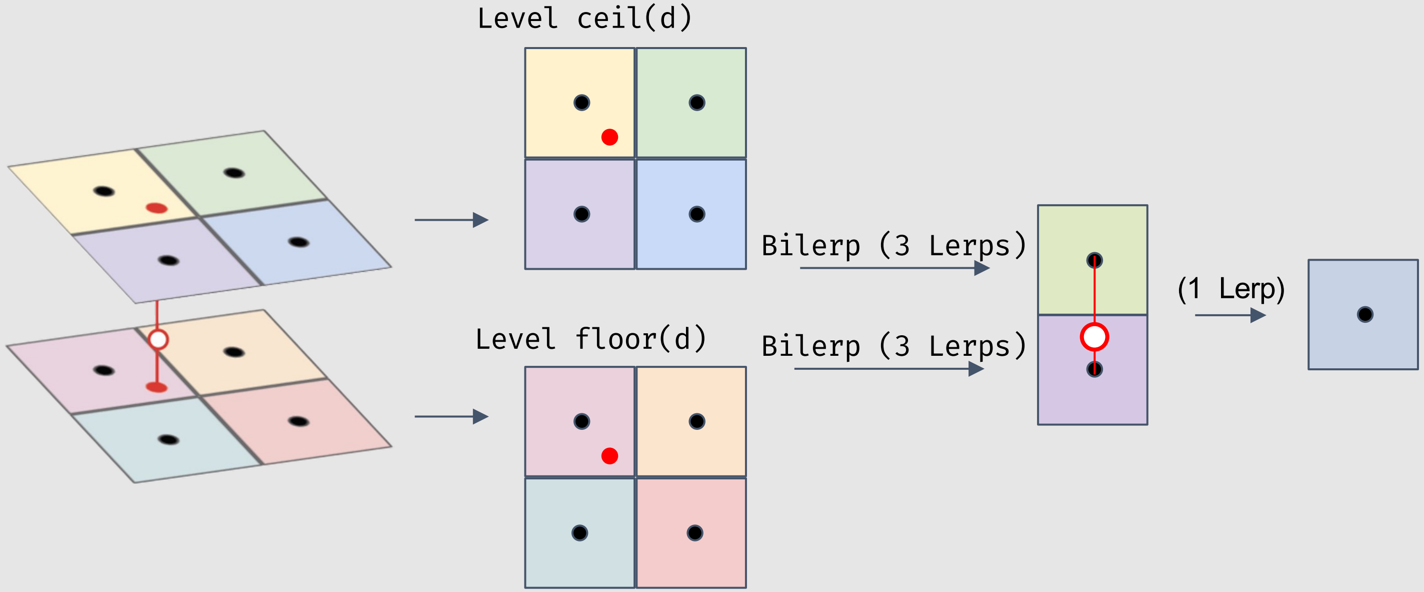
$$t \leftarrow (1 - \Delta y) * t_x + \Delta y * t_y$$



Trilinear Interpolation Sampling

- **Idea:** Perform bilinear interpolation on two layers of the mip-map that represents proper minification/magnification, blending the results together

- **Requires:**
    - 8 memory lookup
    - 7 linear interpolations



$$
\begin{aligned}
{L_x}^2 &\leftarrow \frac{du^2}{dx} + \frac{dv^2}{dx} \\
{L_y}^2 &\leftarrow \frac{du^2}{dy} + \frac{dv^2}{dy}
\end{aligned}
$$

$$
\begin{aligned}
L &\leftarrow \sqrt{\max({L_x}^2, {L_y}^2)} \\
d &\leftarrow log_2 L
\end{aligned}
$$

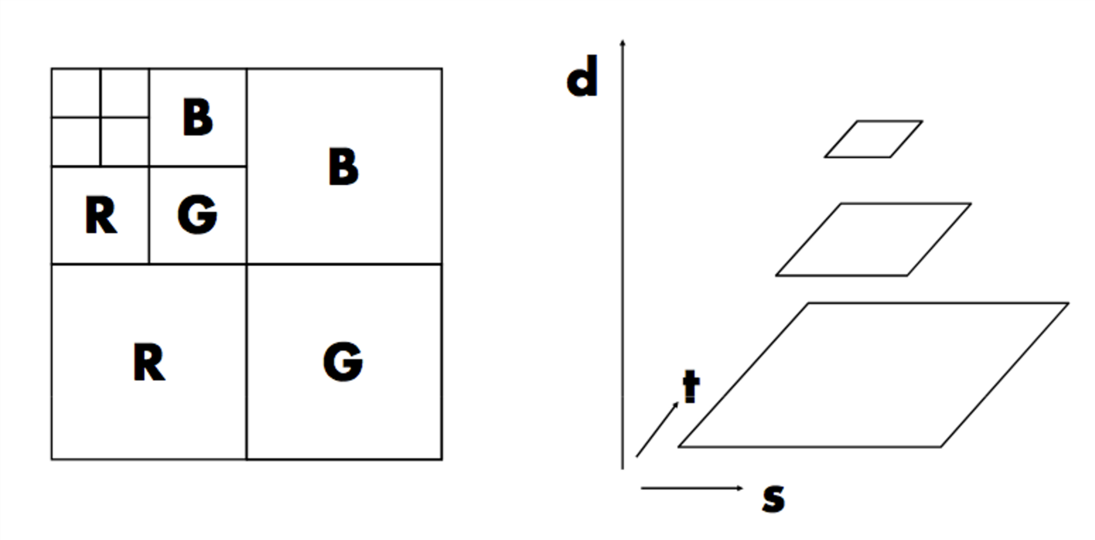
$$
\begin{aligned}
d' &\leftarrow floor(d) \\
\Delta d &\leftarrow d - d'
\end{aligned}
$$

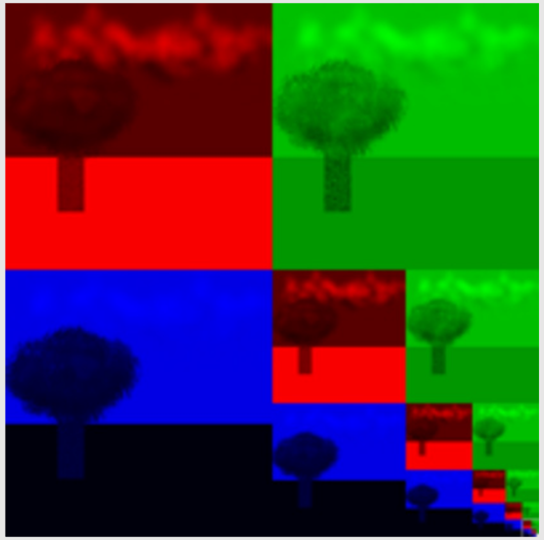
$$
\begin{aligned}
t_d &\leftarrow tex[d'].bilinear(x, y) \\
t_{d+1} &\leftarrow tex[d' + 1].bilinear(x, y) \\
t &\leftarrow (1 - \Delta d) * t_d + \Delta d * t_{d+1}
\end{aligned}
$$

Mip-Map [L. Williams '83]

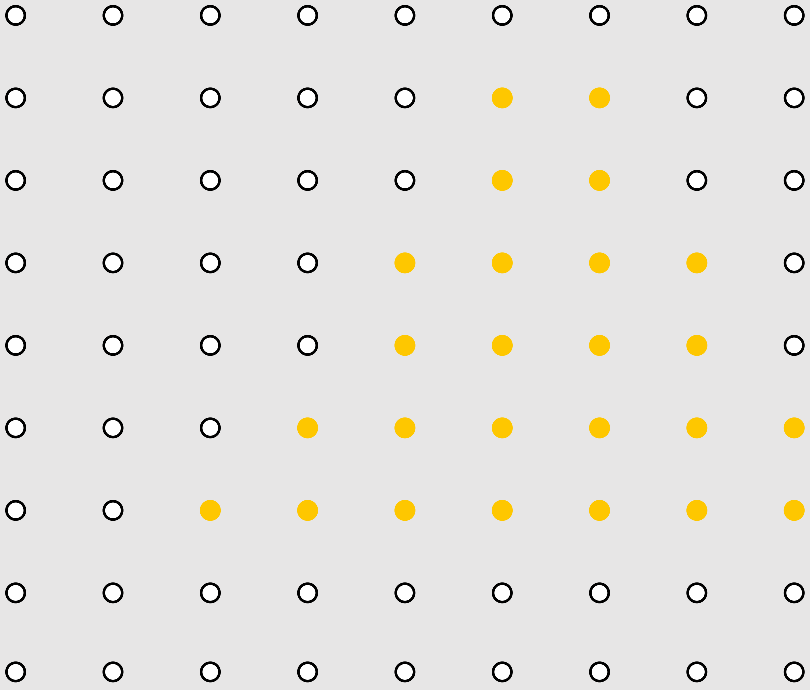
- Storing an RGB Mip-Map can be fit into an image twice the width and twice the height of the original image
  - See diagram for proof :)
  - Does not work as nicely for RGBA!

- **Issue:** bad spatial locality
  - Requesting a texel requires lookup in 3 very different regions of an image



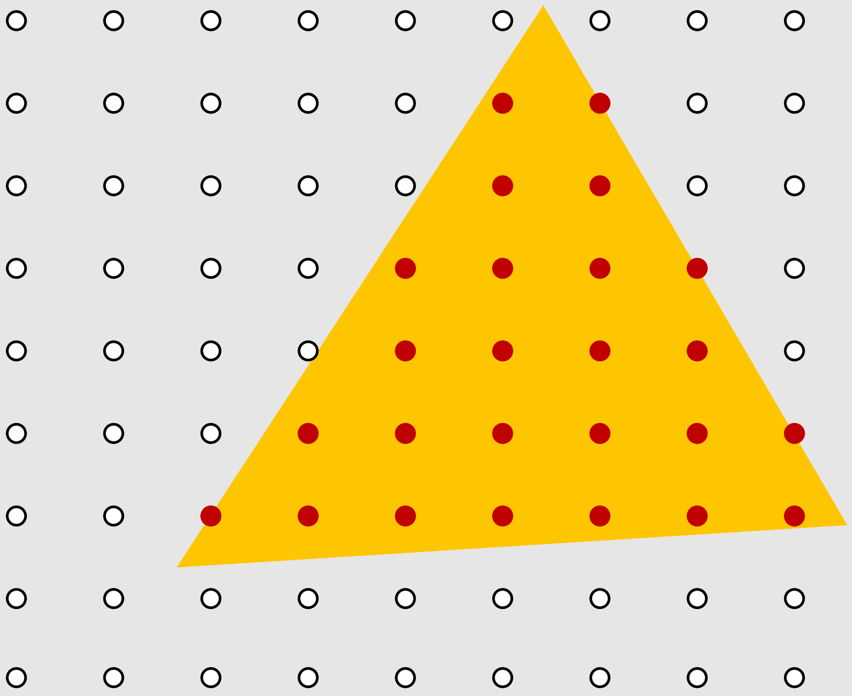


Depth Buffer (Z-buffer)



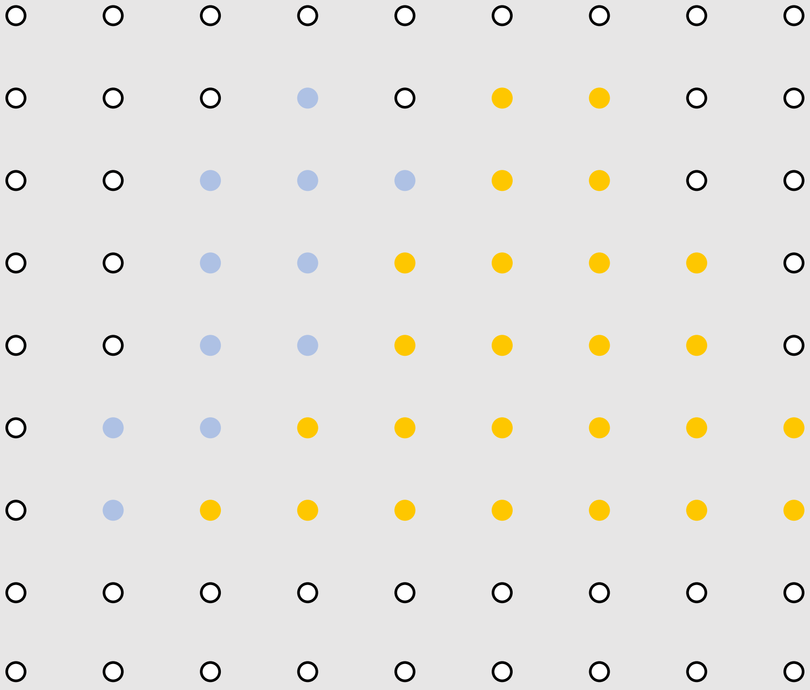
[color buffer]

● — sample passed depth test

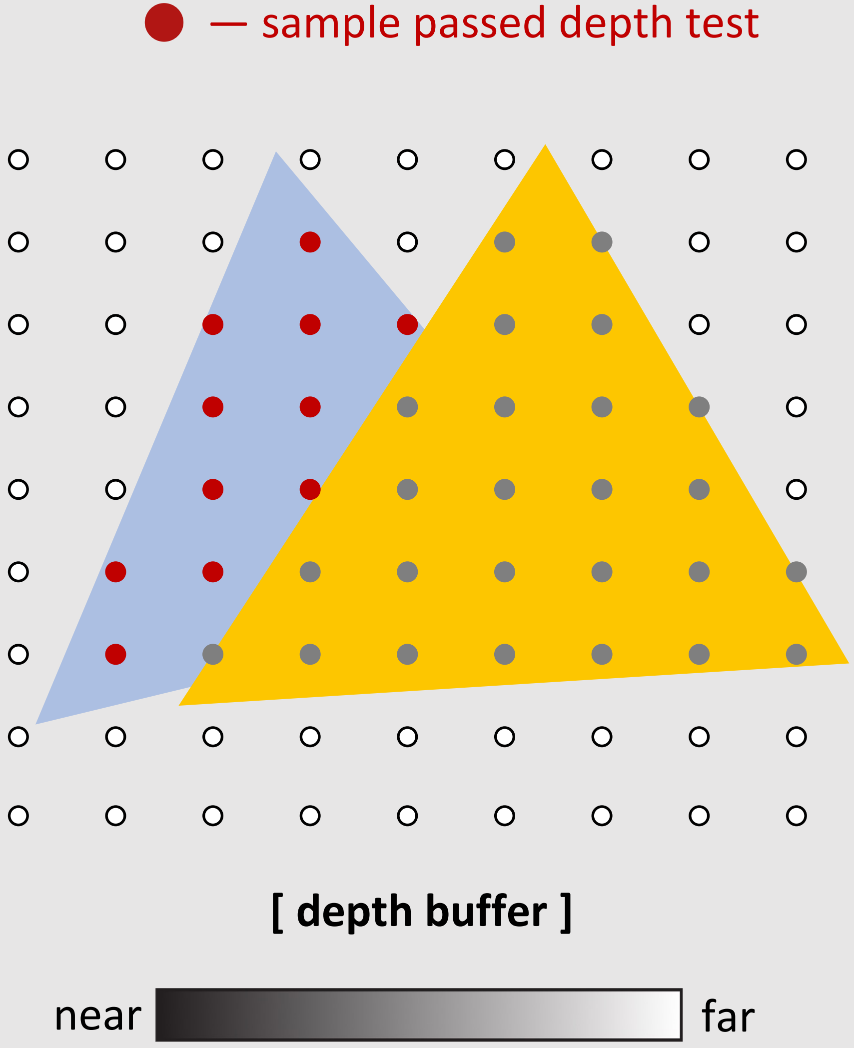


[depth buffer]

Depth Buffer (Z-buffer)

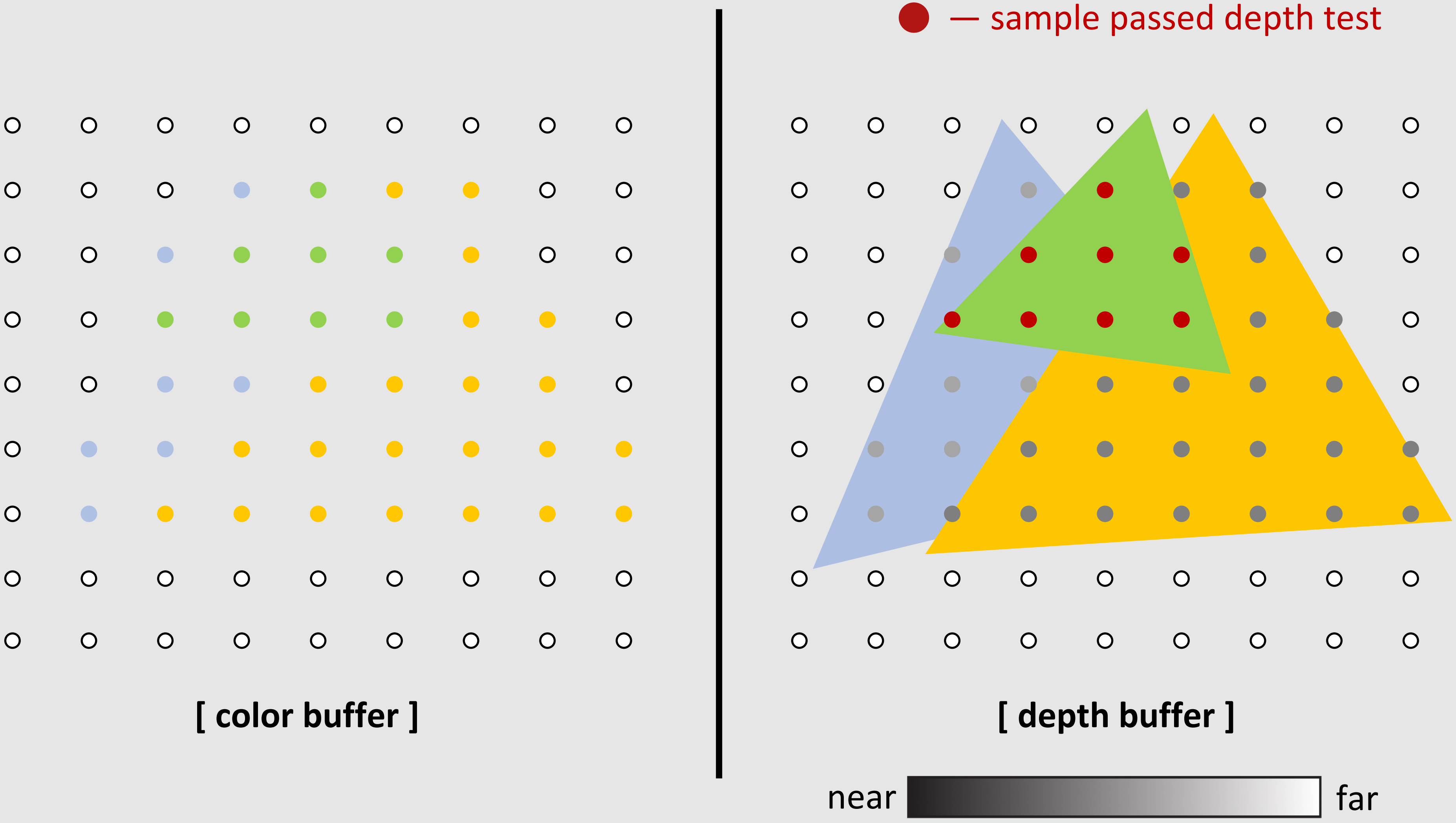


[color buffer]



[depth buffer]

Depth Buffer (Z-buffer)



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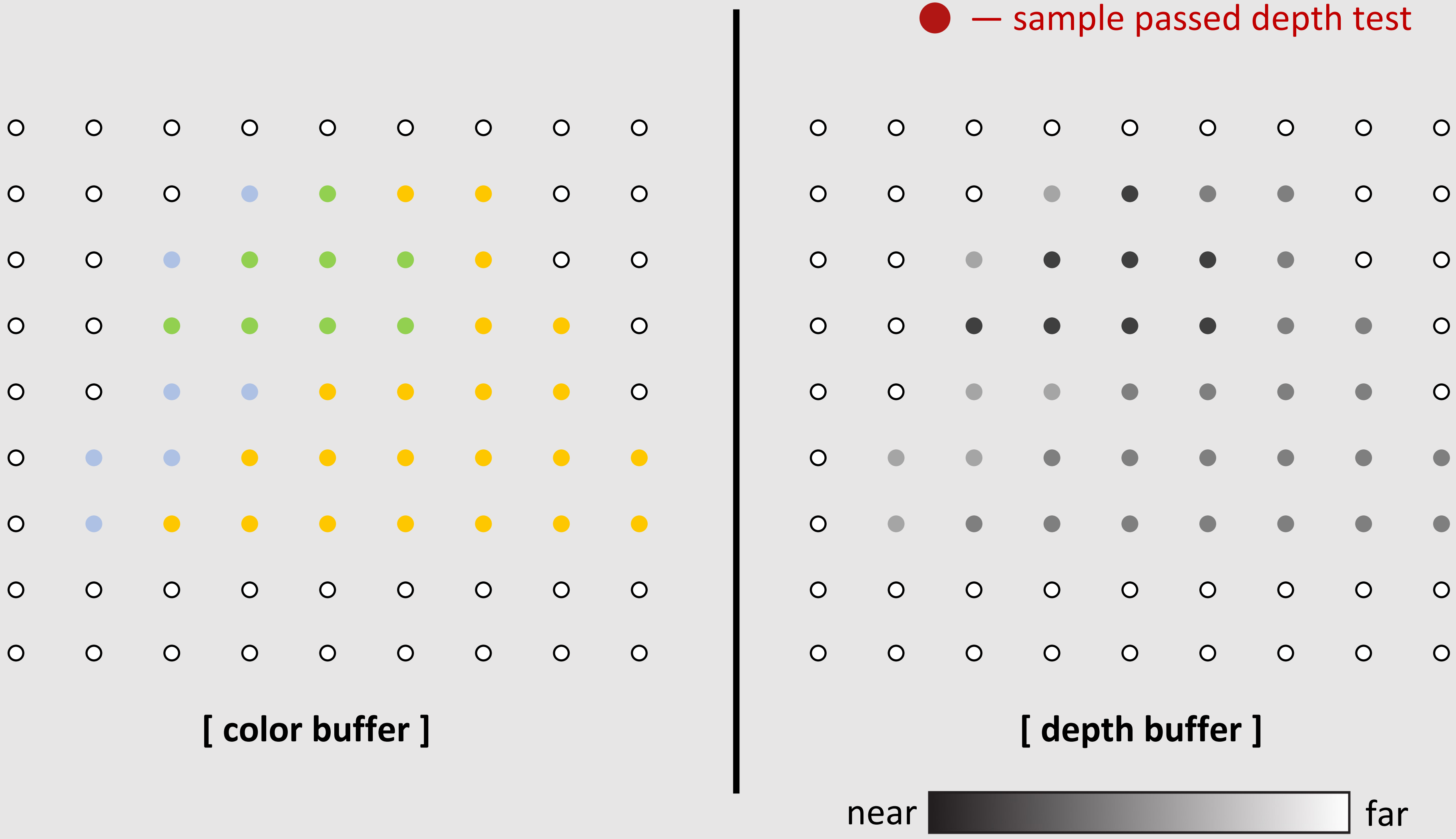


● — sample passed depth test

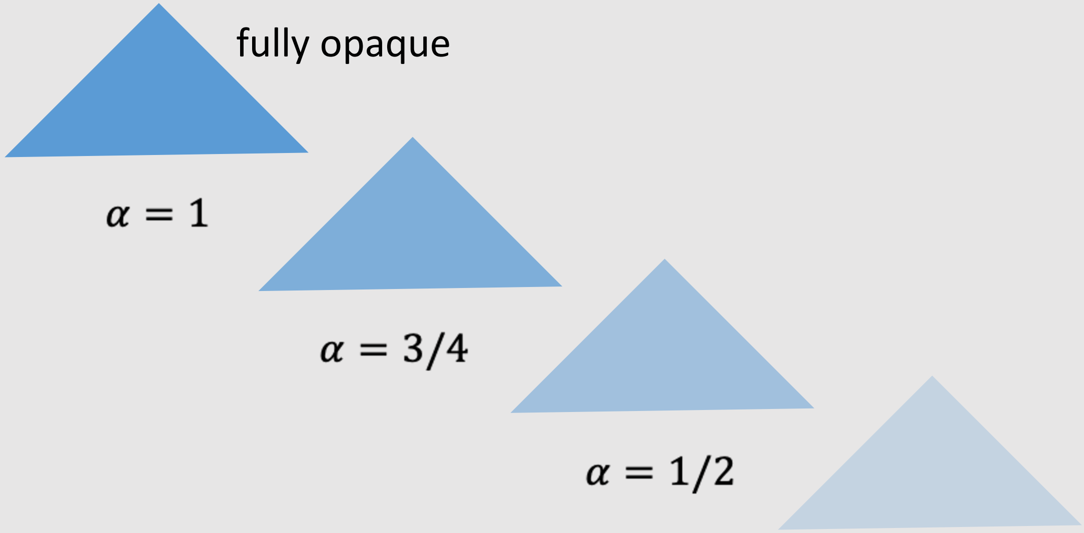
[color buffer]

[depth buffer]

near

far

Alpha Values



fully opaque

$\alpha = 1$

$\alpha = 3/4$

$\alpha = 1/2$

$\alpha = 1/4$

$\alpha = 0$

fully transparent

- Common image format: RGBA
  - Alpha channel specifies ‘opacity’/transparency of object
  - Most common encoding is 8-bits per channel
- Compositing A over B != B over A
  - Consider the extreme case of two opaque objects…
- Non-premultiplied alpha vs Premultiplied alpha

$C = \alpha_B B + (1 - \alpha_B)\alpha_A A$

$C' = B' + (1 - \alpha_B)A'$

[koala over nyc]

[nyc over…koala?]

where is the koala…

- Transformations Review
- Rasterization Review
- Geometry Review
- Spatial Data Structures Review

Geometry Types

- What is implicit geometry
  - Algebraic surfaces
  - Constructive solid geometry
  - Signed distance fields

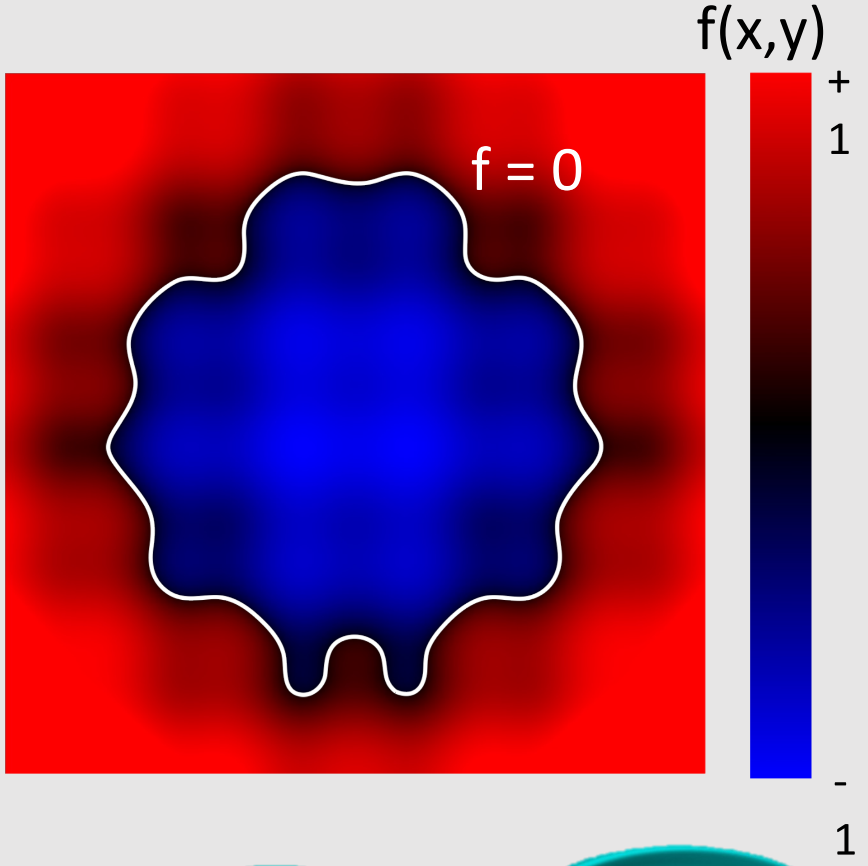
- What is explicit geometry
  - Point clouds
  - Triangle meshes

- Be able to compare the pros and cons of implicit and explicit geometry

- Manifold mesh requirements

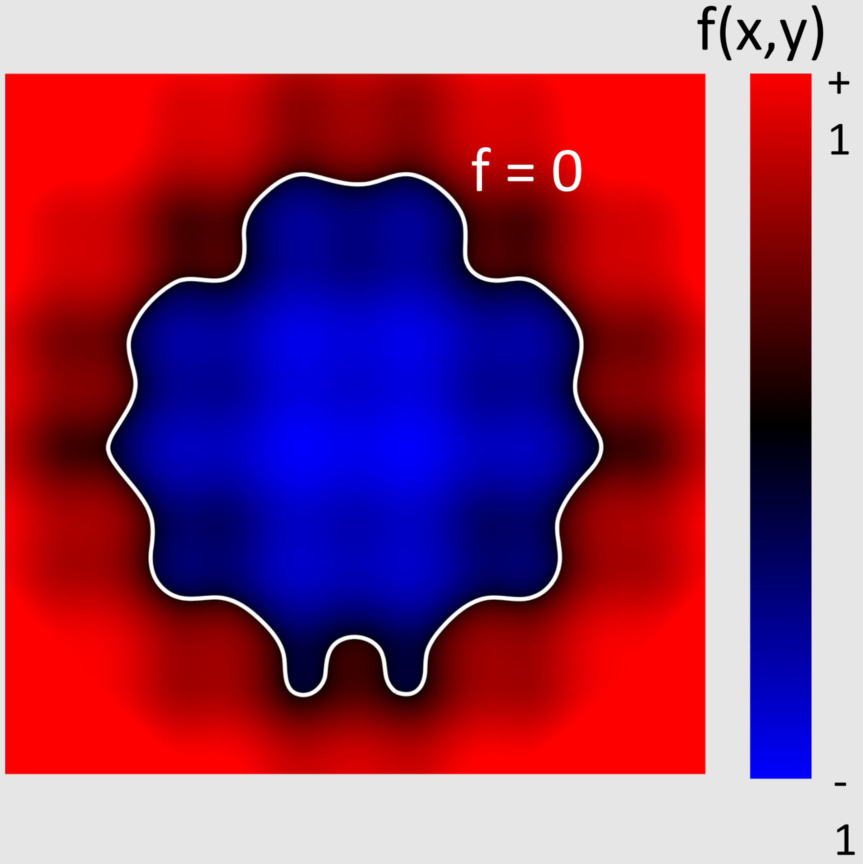
Implicit Geometry

- Points aren’t known directly, but satisfy some relationship
  - Example: unit sphere is all points such that $x^2+y^2+z^2=1$
- More generally, in the form f(x,y,z) = 0
- Finding example points is **hard**
  - Requires solving equation
- Checking if points are inside/outside is **easy**
  - Just evaluate the function with a given point



Signed Distance Fields

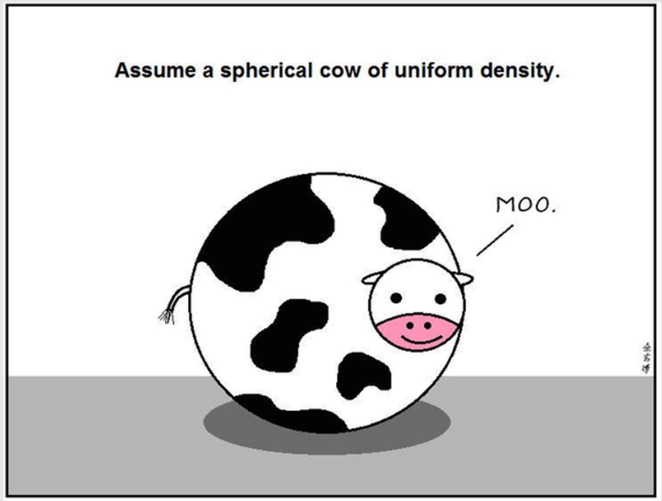
- Signed distance fields are implicit functions $f(x, y, z)$ that tell us the sign (inside/outside) and the distance away from the boundary
  - Gradient $\nabla f(x, y, z)$ makes finding the boundary easier
- SDFs make it easy to check where and how far a point is from a surface



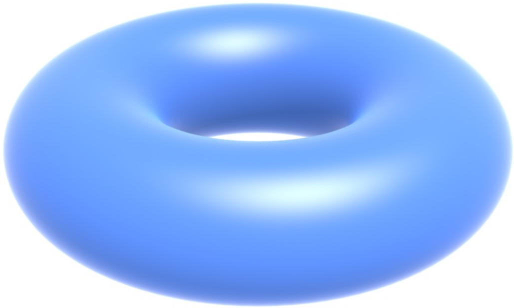
Algebraic Surfaces [Implicit]

- Simple way to think of it: a surface built with algebra [math]
  - Generally thought of as a surface where points are some radius r away from another point/line/surface

- Easy to generate smooth/symmetric surfaces
  - Difficult to generate impurities/deformations



$$x^2+y^2+z^2=1$$

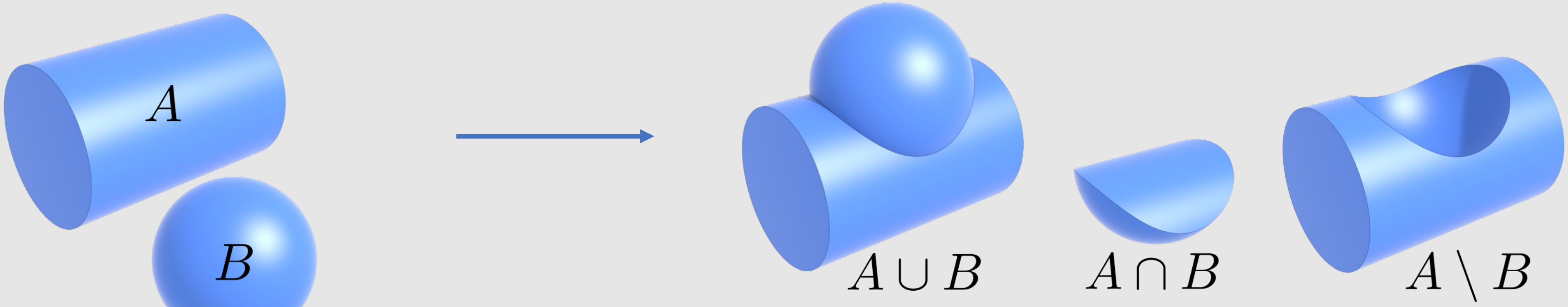


$$(R-\sqrt{x^2+y^2})^2+z^2=r^2$$

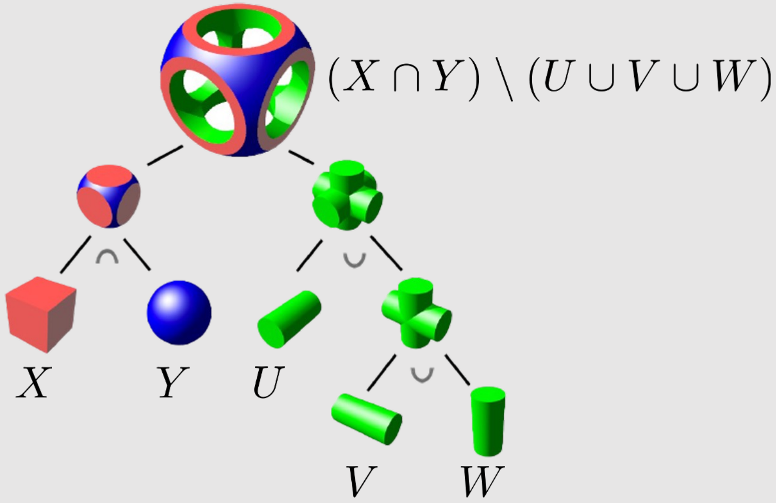
$$(x^2+\frac{9y^2}{4}+z^2-1)^3 = x^2z^3+\frac{9y^2z^3}{80}$$

Constructive Solid Geometry [Implicit]

- Build more complicated shapes via Boolean operations
  - Basic operations:

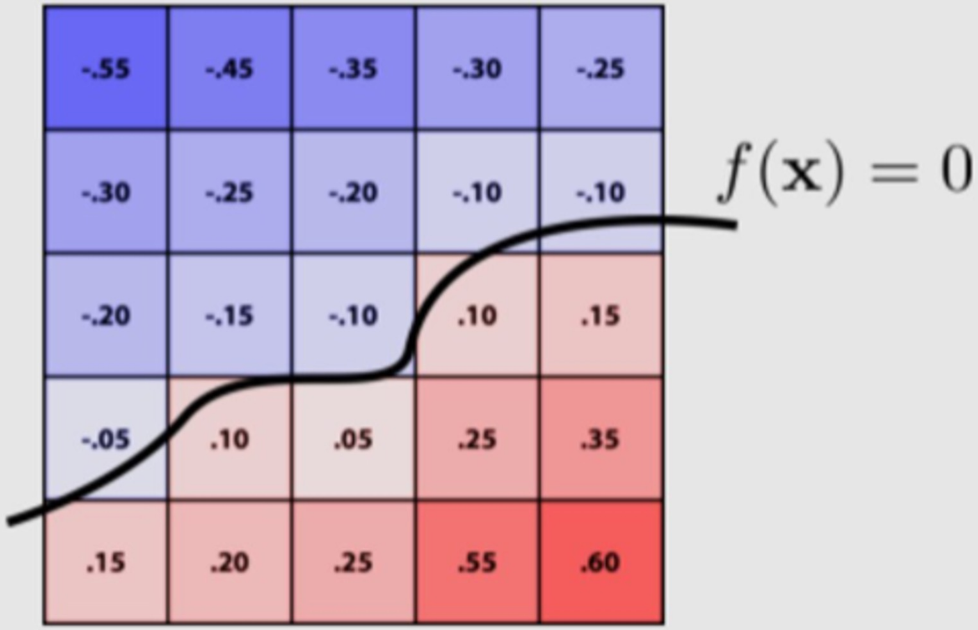


- Can be used to form complex shapes!

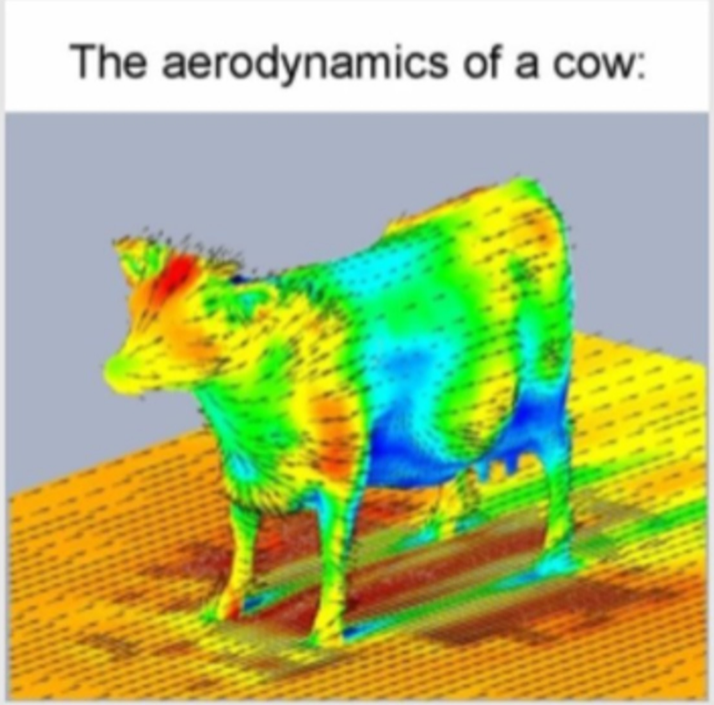


Level Sets [Implicit]

- Store a grid of values approximating function



-.55	-.45	-.35	-.30	-.25
-.30	-.25	-.20	-.10	-.10
-.20	-.15	-.10	.10	.15
-.05	.10	.05	.25	.35
.15	.20	.25	.55	.60



The aerodynamics of a cow:

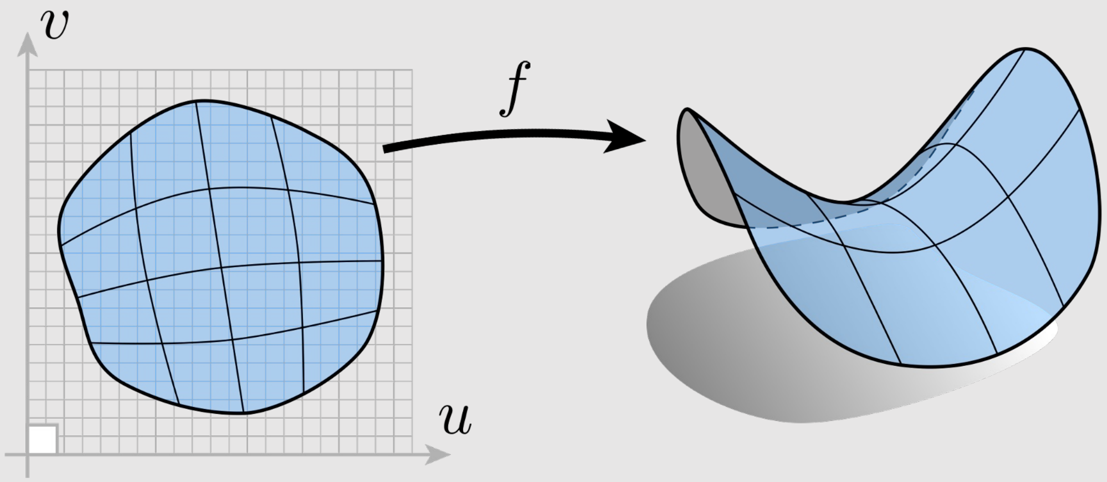
- Surface is found where interpolated values equal zero
  - How do find this? Bilinear interpolation!
- [+] Provides much more explicit control over shape
- [-] Runs into problems of aliasing!

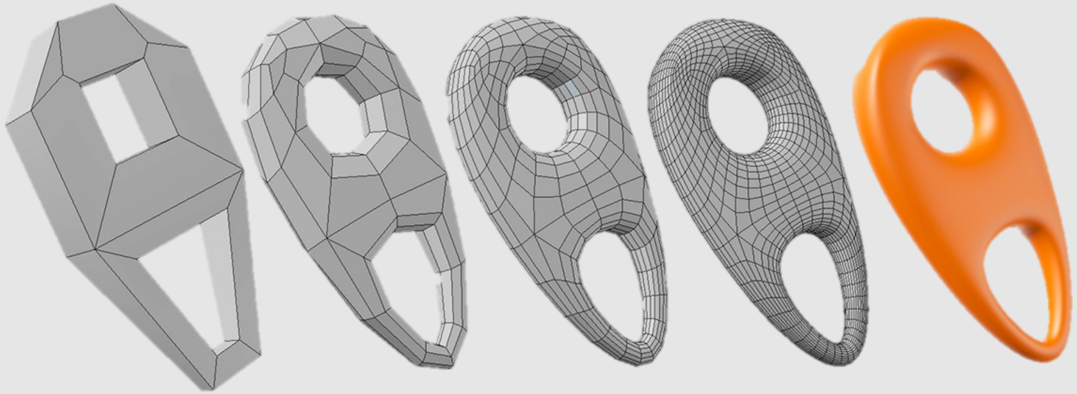
Explicit Geometry

- All points are given directly
  - The polygons we were given during rasterization is an example of explicit geometry
- More generally:

$$f : \mathbb{R}^2 \to \mathbb{R}^3 ; (u, v) \mapsto (x, y, z)$$

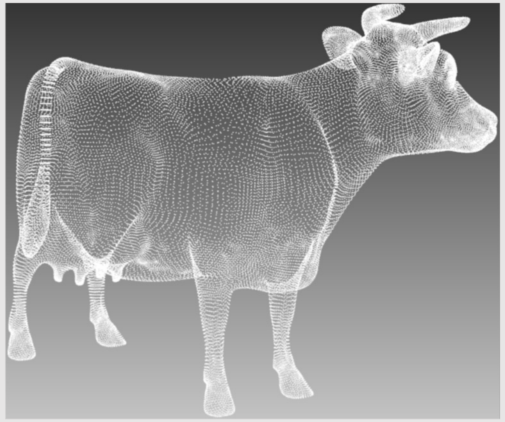
  - Given any (u, v), we can find a point on the surface
  - Can limit (u, v) to some range
    - Example: triangle with barycentric coordinates
- Finding example points is **easy**
  - We are given them for free
- Checking if points are inside/outside is **hard**
  - We are given the output values and need to find input values that satisfy the geometry





Point Cloud [Explicit]

- Easiest representation: list of points (x, y, z)
  - Often augmented with normal
- Easily represent any kind of geometry
- Easy to draw dense cloud (>>1 point/pixel)
- Easy for simulation
- Large lookup time
- Large memory overhead
  - Hard to interpolate undersampled regions
  - Hard to do processing / simulation /
  - Result is just as good as the scan

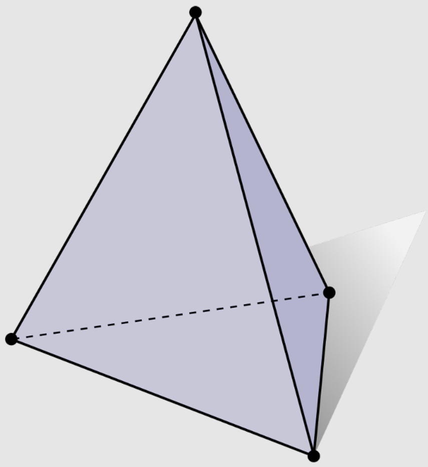


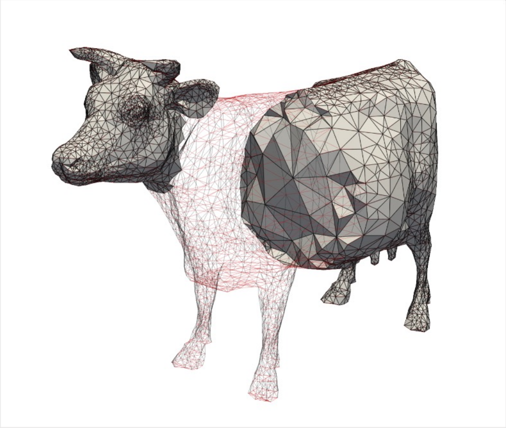
Triangle Mesh [Explicit]

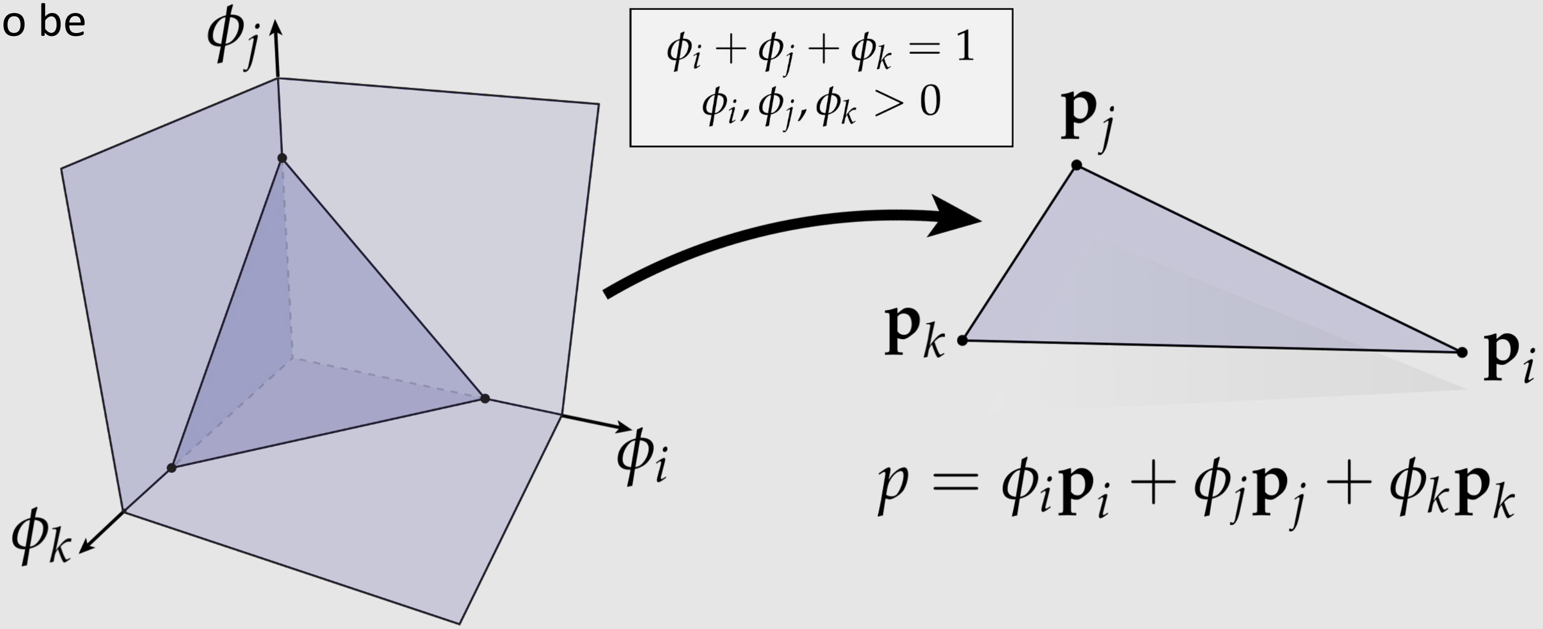
- Large memory overhead
    - Store vertices as triples of coordinates *(x,y,z)*
    - Store triangles as triples of indices *(i,j,k)*

- Easy interpolation with good approximation
    - Use barycentric interpolation to define points inside triangles

- Polygonal Mesh: shapes do not need to be triangles
    - Ex: quads







$$\boxed{\begin{gathered}\phi_i + \phi_j + \phi_k = 1 \\ \phi_i, \phi_j, \phi_k > 0\end{gathered}}$$

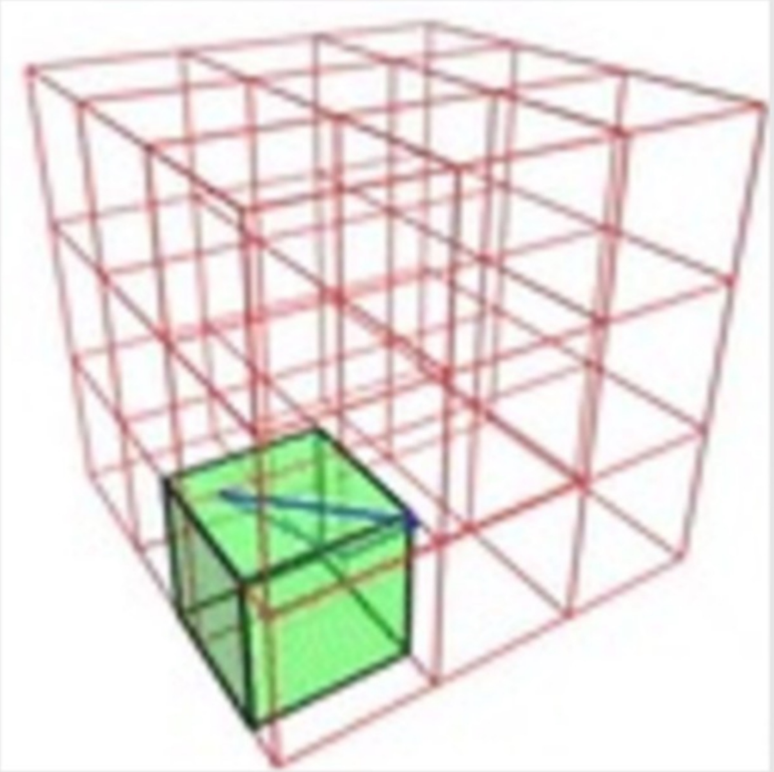
$$p = \phi_i \mathbf{p}_i + \phi_j \mathbf{p}_j + \phi_k \mathbf{p}_k$$

Marching Cubes

- Marching cubes is an algorithm for converting implicit geometry to explicit
  - Adds both positional (vertices) and connectivity (edges)

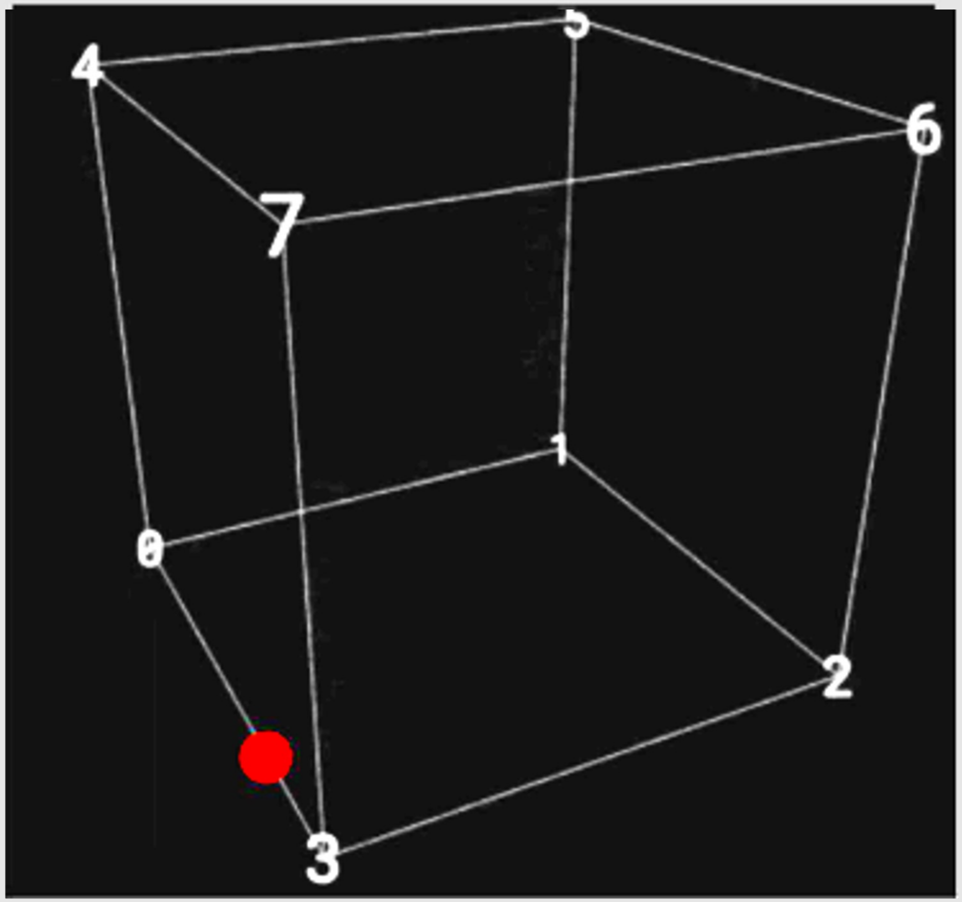
- **Idea:** march a cube though the scene, checking if each of the vertices in the cube lie inside or outside the implicit function $f(x, y, z)$
  - 8 vertices, 8 checks
  - Can encode as an 8-bit number
  - Generate geometry that makes sure inside vertices are enclosed by the geometry, and outside geometry are kept out

- **Issue:** how big of a cube to use
  - A smaller cube leads to finer details
  - A smaller cube also requires more samples



Marching Cubes Linear Interpolation

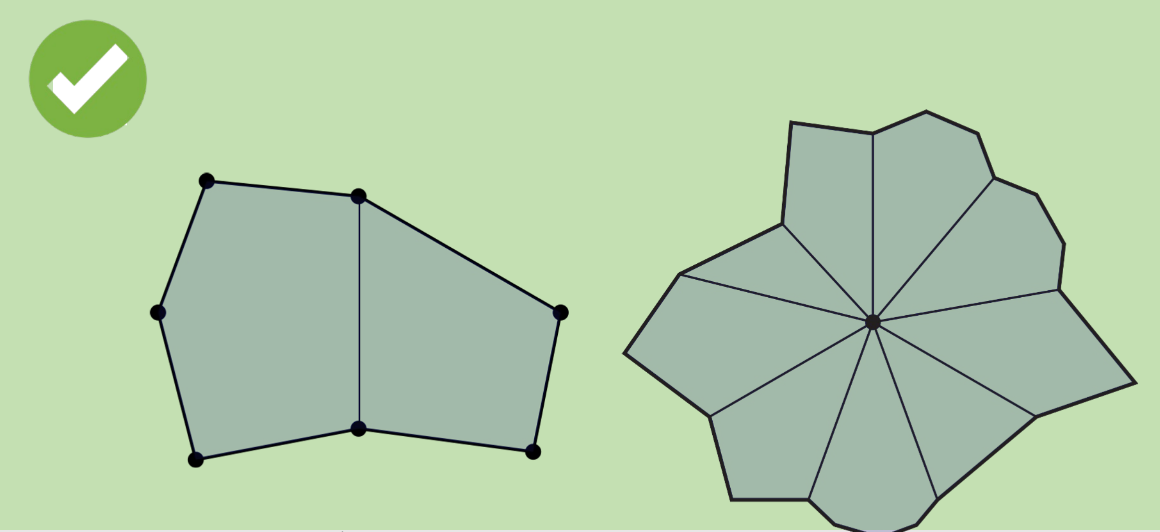
- **Issue:** lookup table only tells us on what edges to place vertices and how to connect them
  - Does not tell us the specific location of vertices
- When placing vertices, can linearly interpolate them on the edges depending on the evaluated values on the cube vertices
- Example:
  - $f(x_0, y_0, z_0) = -0.75$
  - $f(x_3, y_3, z_3) = +0.25$
    - Vertex is placed ¼ distance away from corner 3, ¾ distance from corner 0

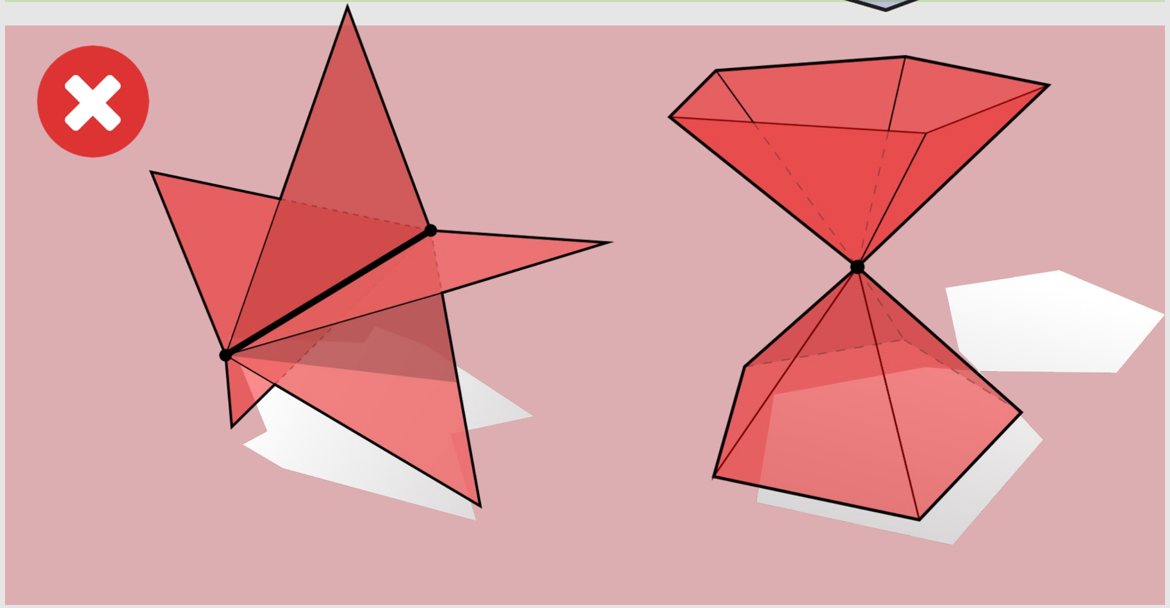


- **What if we want to interpolate across a face?**
  - **Bilinear Interpolation!**
  - **Between texture sampling, generating mipmaps, level sets, marching cubes etc… Bilinear interpolation is very useful!**

Manifold Properties

- For polygonal surfaces, we will check for **“fins”** and **”fans”**
- Every edge is contained in only two polygons (no **“fins”**)
  - The extra 3rd or 4th or 5th or so forth polygon is the fin of a fish
- The polygons containing each vertex make a single **“fan”**
  - We should be able to loop around the faces around a vertex in a clear way





Halfedge Mesh

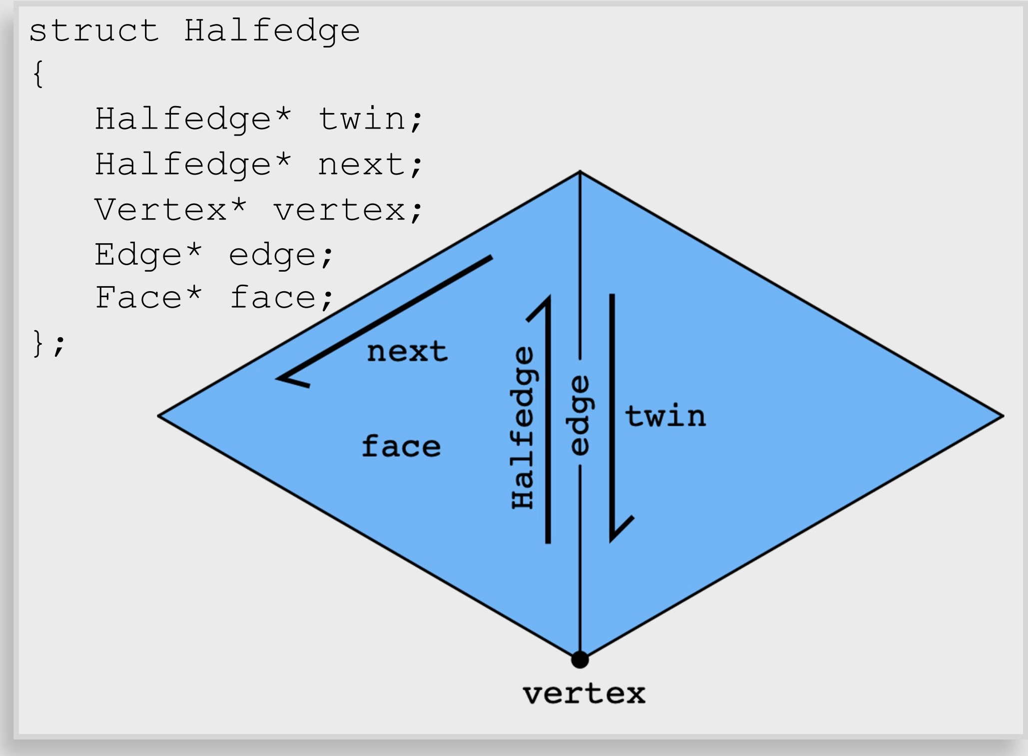
- What are the components of a halfedge mesh
- How to traverse around a vertex? A face? An edge?
- Why can we not represent a non-manifold mesh using halfedge geometry?
- What makes a good mesh?

Halfedge Data Structure

- Let’s store a little, but not a lot, about our neighbors:
  - Halfedge data structure added to our geometry
  - Each edge gets 2 halfedges
    - Each halfedge ”glues” an edge to a face

- **Pros:**
  - [+] No duplicate coordinates
  - [+] Finding neighbors is O(1)
  - [+] Easy to traverse geometry
  - [+] Easy to change mesh connectivity
  - [+] Easy to add/remove mesh elements
  - [+] Easy to keep geometry manifold

- **Cons:**
  - [-] Does not support nonmanifold geometry



```
struct Halfedge
{
   Halfedge* twin;
   Halfedge* next;
   Vertex* vertex;
   Edge* edge;
   Face* face;
};
```

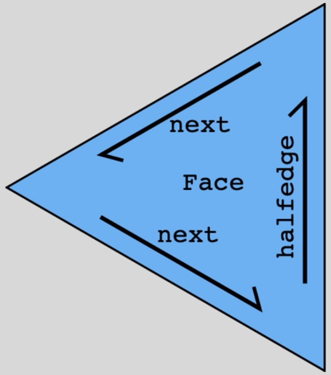
Halfedge Data Structure

- Makes mesh traversal easy
  - Use “twin” and “next” pointers to move around the mesh
  - Use “vertex”, “edge”, and “face” pointers to grab element

```
struct Halfedge
{
   Halfedge* twin;
   Halfedge* next;
   Vertex* vertex;
   Edge* edge;
   Face* face;
};
```

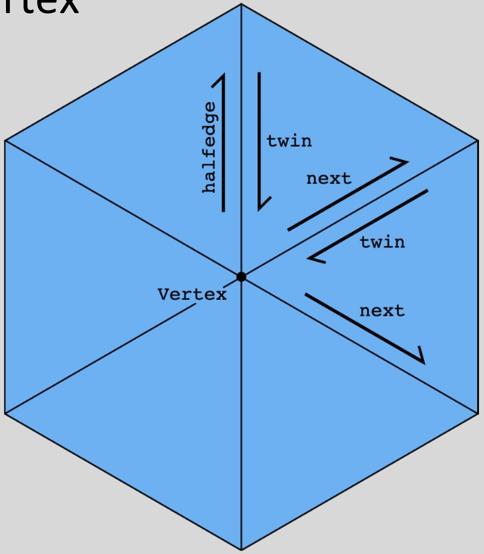
Example: visit all vertices in a face

```
Halfedge* h = f->halfedge;
do {
   h = h->next;
   // do something w/ h->vertex
}
while( h != f->halfedge );
```



Example: visit all neighbors of a vertex

```
Halfedge* h = v->halfedge;
do {
   h = h->twin->next;
}
while( h != v->halfedge );
```



Note: only makes sense if mesh is manifold!

Halfedge Data Structure

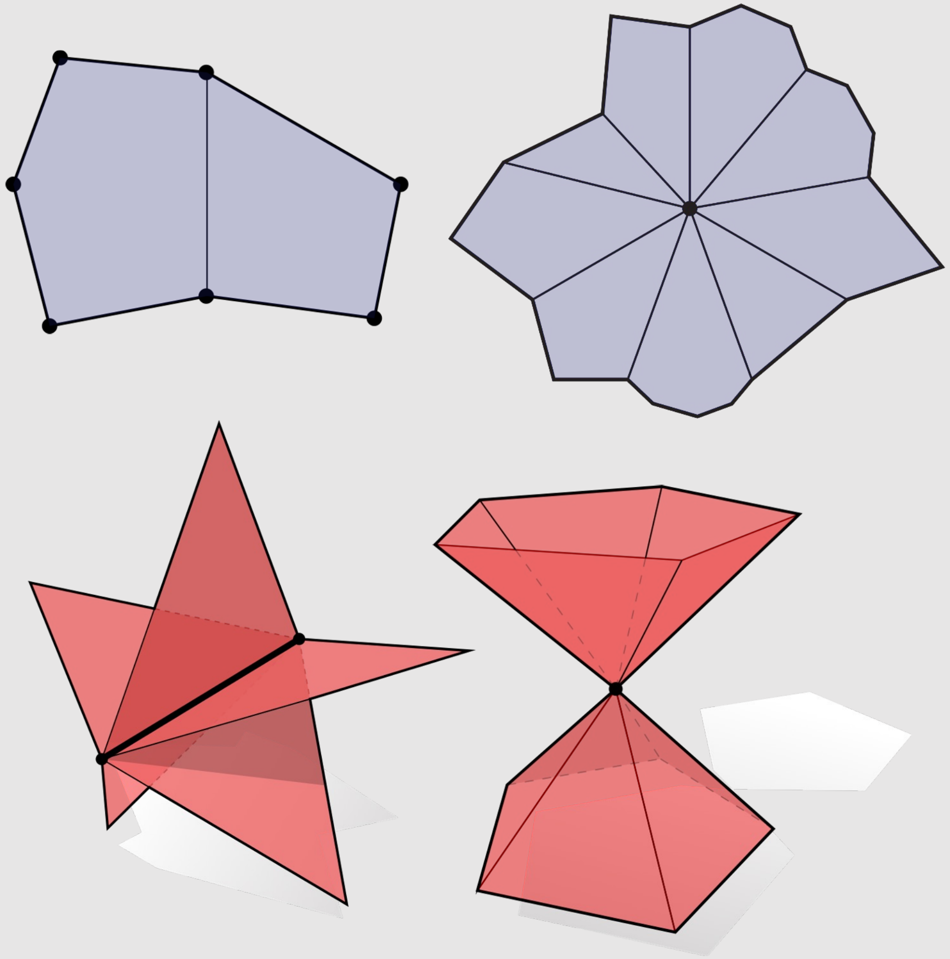
- Halfedge meshes are always manifold!

- Halfedge data structures have the following constraints:

```
h->twin->twin == h // my twin’s twin is me
h->twin != h // I am not my own twin
h2->next = h //every h’s is someone’s “next”
```

- Keep following **next** and you’ll traverse a face
- Keep following **twin** and you’ll traverse an edge
- Keep following **next->twin** and you’ll traverse a vertex

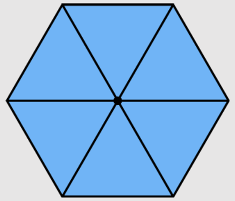
- Q: Why, therefore, is it impossible to encode the red figures?
  - First shape violates first 2 conditions
  - Second shape violates 3rd condition



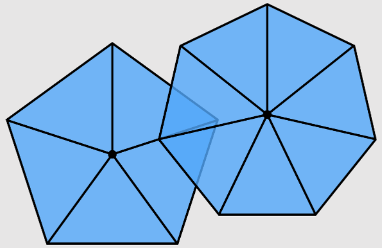
A Good Mesh Has…

- Good approximation of original shape
  - Keep only elements that contribute information about shape
  - More elements where curvature is high

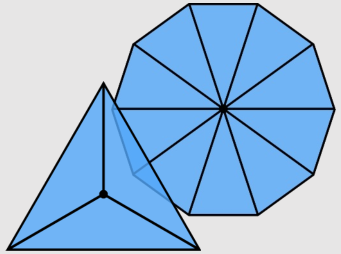
- Regular vertex degree
  - Degree 6 for triangle mesh, 4 for quad mesh
    - Better polygon shape
    - More regular computation
    - Smoother subdivision



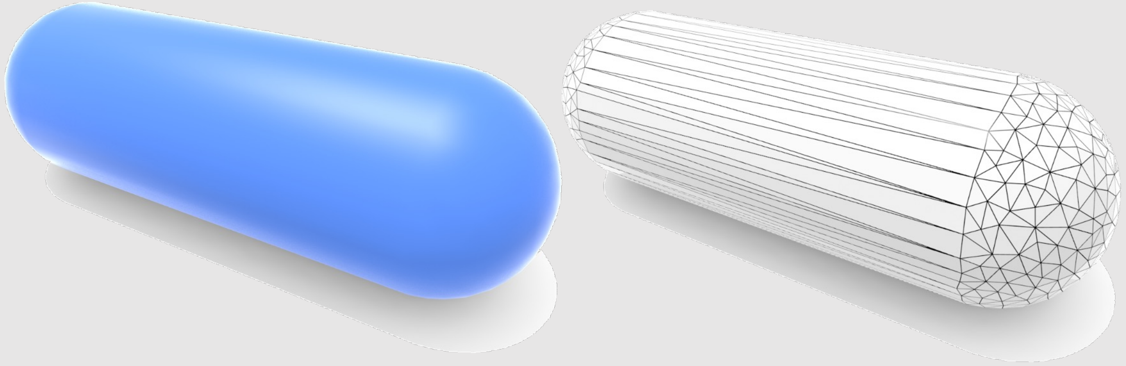
[good]

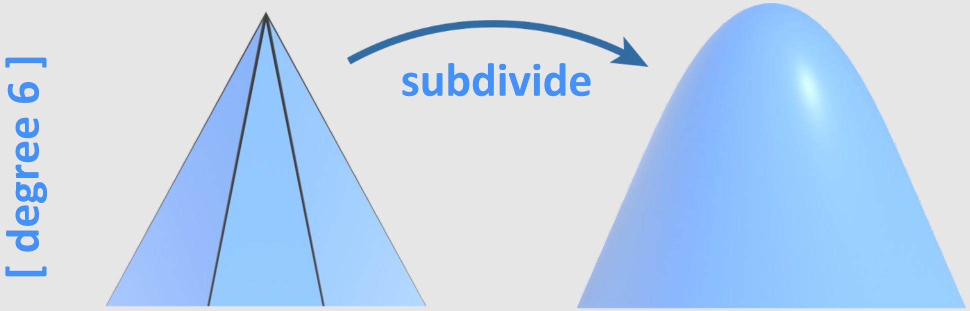


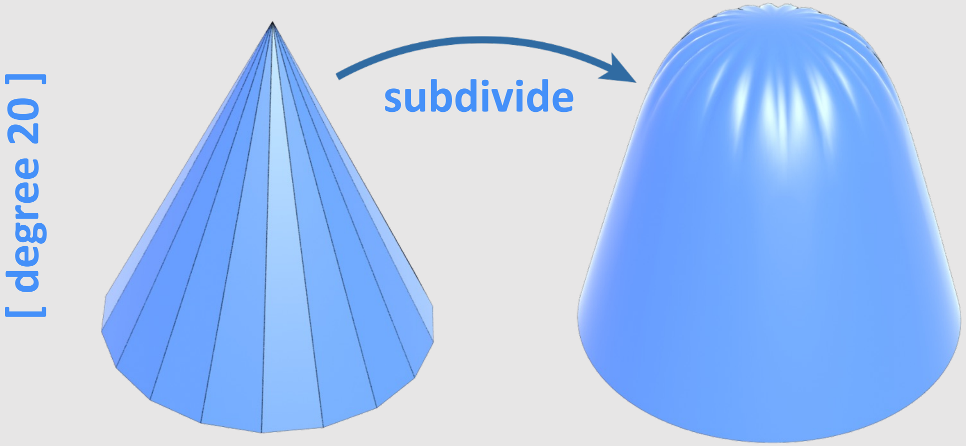
[okay]



[bad]





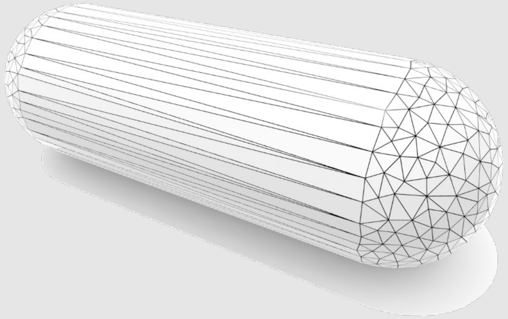


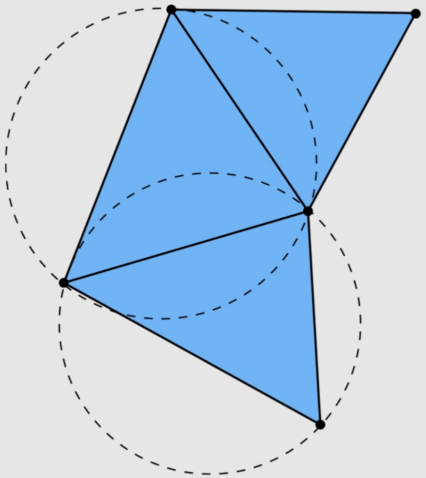
A Good Mesh Has…

- Good triangle shape
  - All angles close to 60 degrees

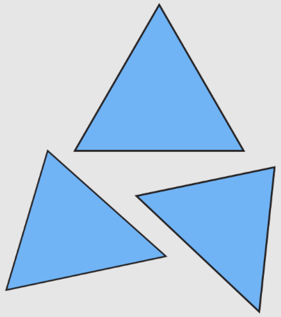
- More sophisticated condition: Delaunay
  - For every triangle, the unique circumcircle (circle passing through all vertices of the triangle) does not encase any other vertices
  - Many nice properties:
    - Maximizes minimum angle
    - Smoothest interpolation

- **Tradeoff:** sometimes a mesh can be approximated best with long & skinny triangles
  - Doesn't make the mesh Delaunay anymore
  - Example: cylinder





[delaunay]



[good]

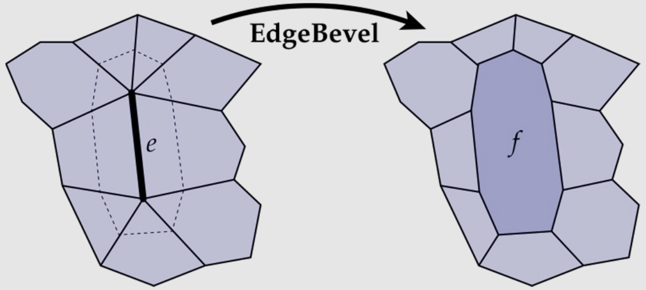
[bad]

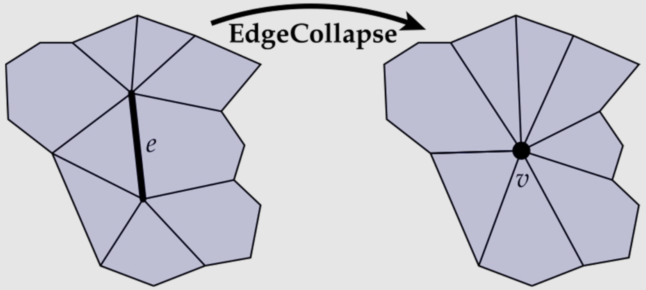
Halfedge Mesh Operations

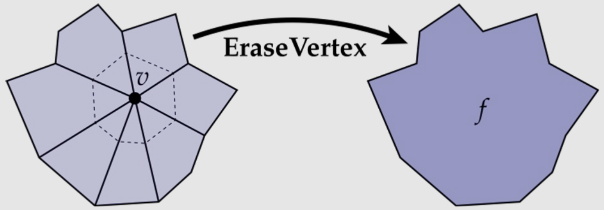
- Local Operations
    - EdgeBevel
    - EdgeCollapse
    - EraseVertex
    - FaceBevel
    - EdgeFlit
    - EdgeVertexSplit
    - VertexBevel
    - EraseEdge
    - FaceCollapse
    - …

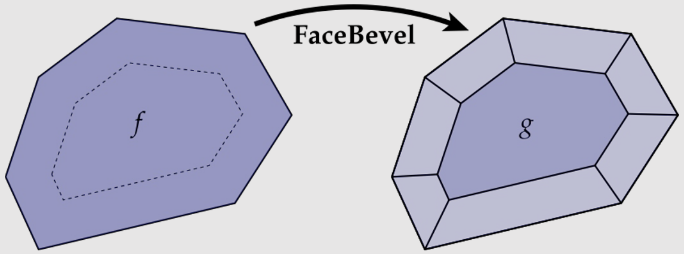
- Global Operations
    - Loop Subdivision
    - Isotropic Remeshing
    - Simplification
    - …

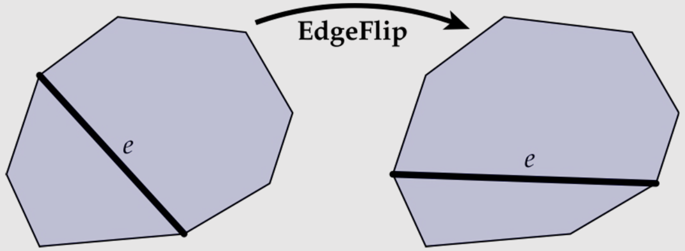
Local Operations

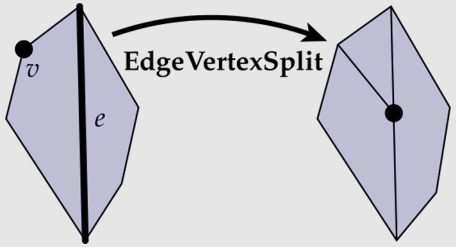


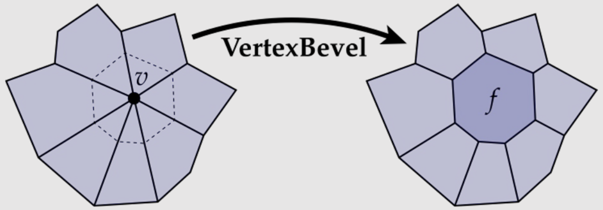


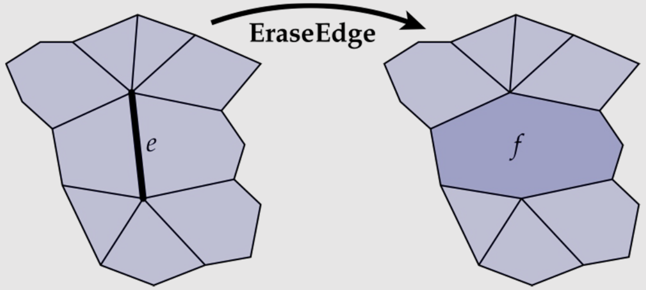


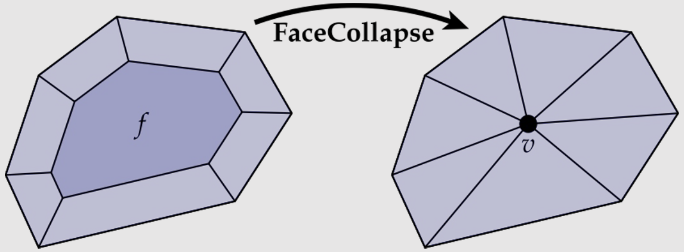






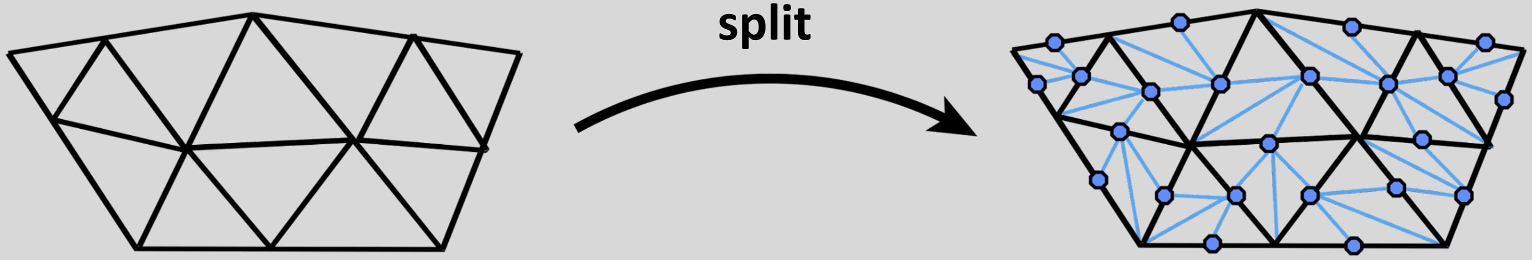




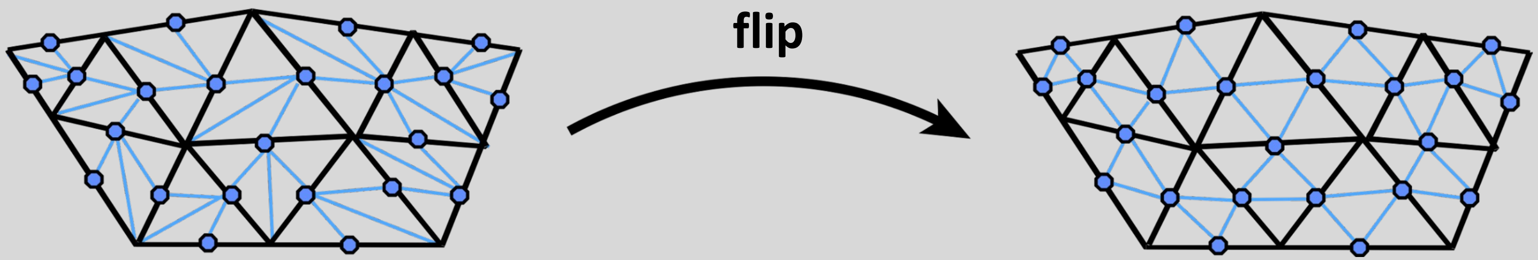


Loop Subdivision Using Local Ops

Step 1:
Split all edges in any order

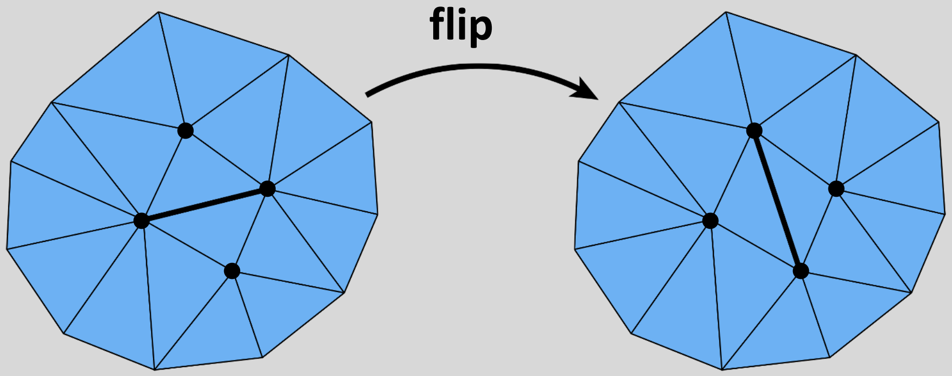


Step 2:
Flip new edges until they touch two new vertices

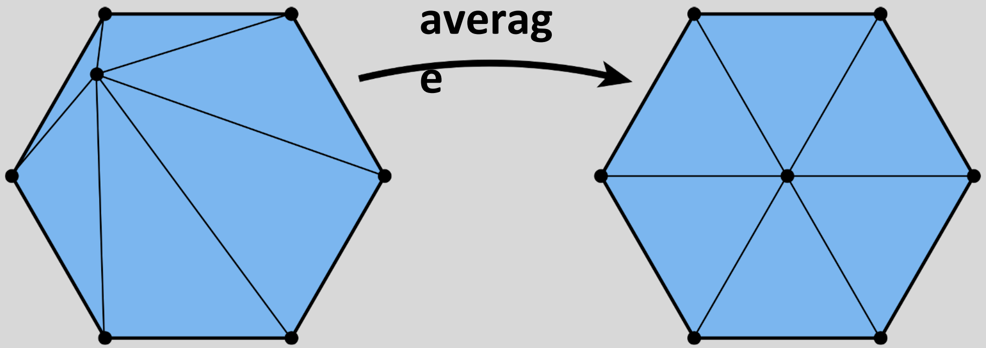


Isotropic Remeshing

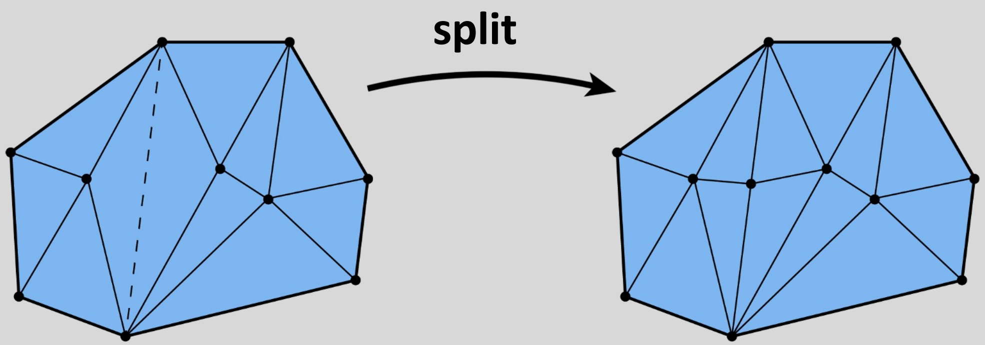
Step 1:



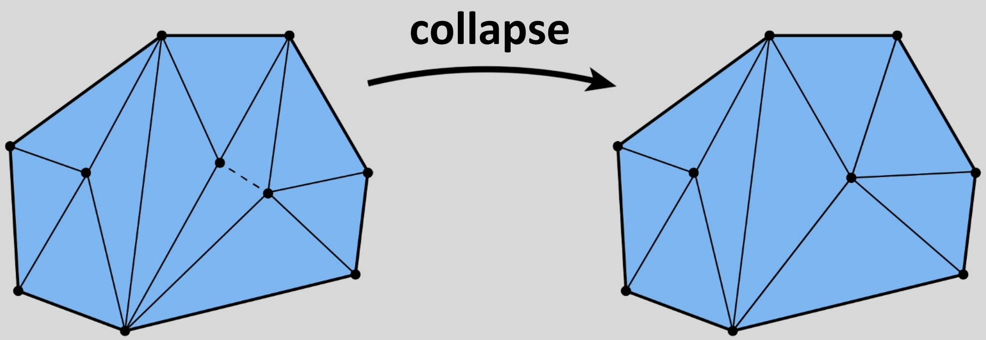
Step 2:



Step 3:



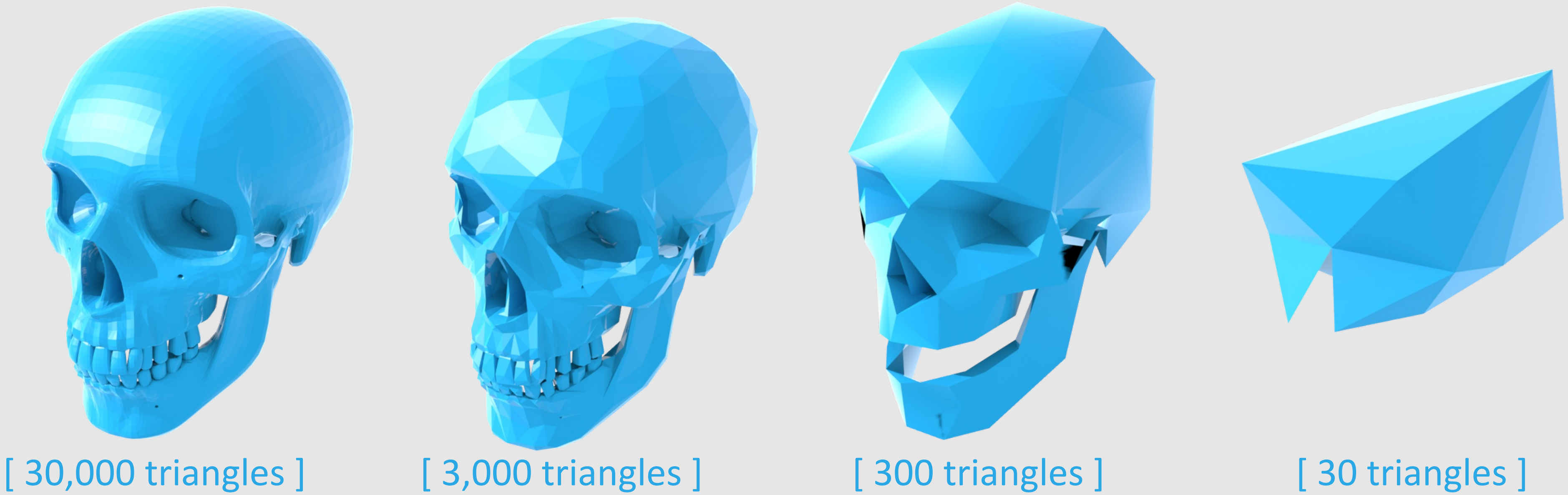
Step 4:



Simplification Algorithm Basics

- Greedy Algorithm:
  - Assign each edge a cost
  - Collapse edge with least cost
  - Repeat until target number of elements is reached
- Particularly effective cost function: quadric error metric**

No need to remember the specific metric!



**invented at CMU (Garland & Heckbert 1997)

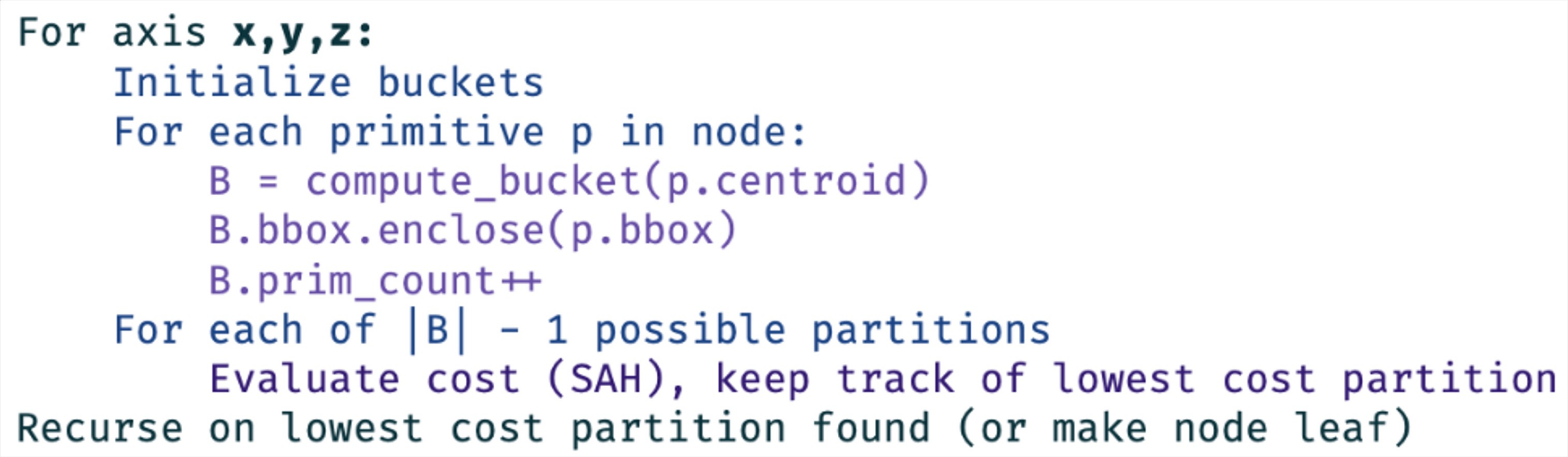
- Transformations Review
- Rasterization Review
- Geometry Review
- Spatial Data Structures Review

Spatial Data Structures

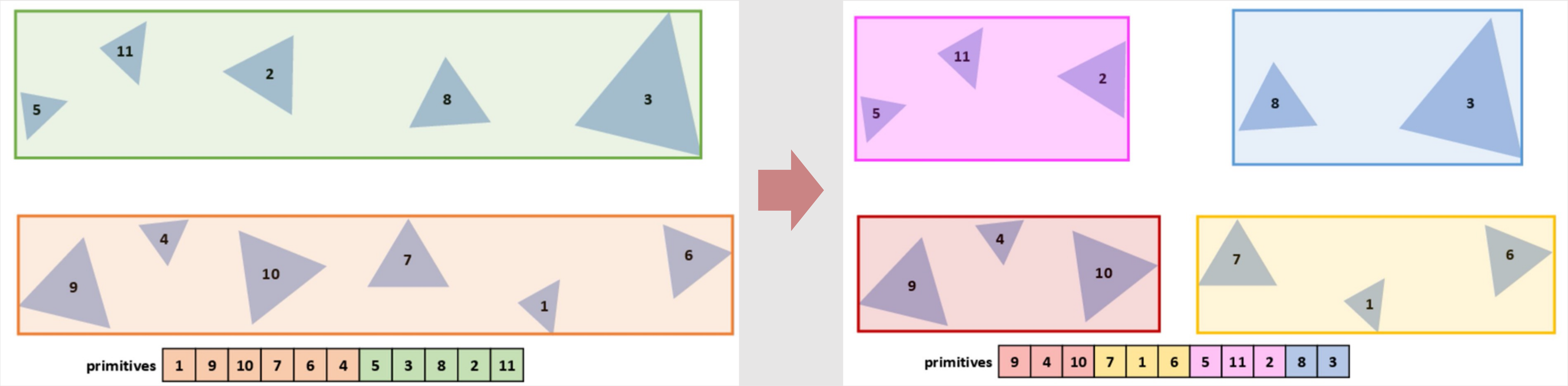
- Primitive-partitioning acceleration structure:
  - Partitions node’s primitives into disjoint sets (but sets may overlap in space)
  - Bounding Volume Hierarchy
    - How to construct a BVH
    - How to traverse a BVH
    - Axis-aligned vs non-axis aligned BVHs

- Space-partitioning acceleration structures:
  - Partitions space into disjoint regions (but primitives may be contained in multiple regions)
  - K-D Trees
  - Uniform Grids
  - Quad/OctTreees

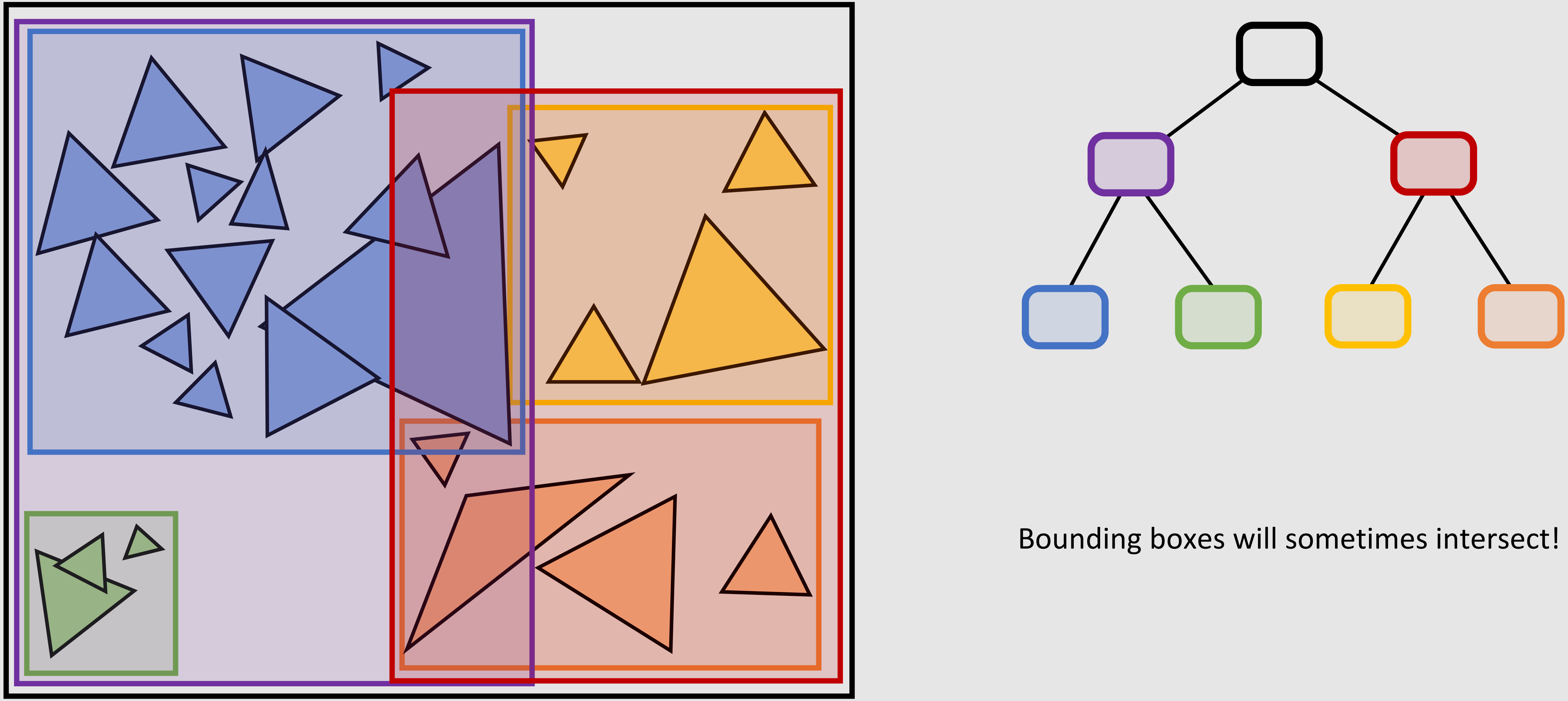
BVH Construction



```
For axis x,y,z:
    Initialize buckets
    For each primitive p in node:
        B = compute_bucket(p.centroid)
        B.bbox.enclose(p.bbox)
        B.prim_count++
    For each of |B| - 1 possible partitions
        Evaluate cost (SAH), keep track of lowest cost partition
Recurse on lowest cost partition found (or make node leaf)
```



BVH Example



Bounding boxes will sometimes intersect!

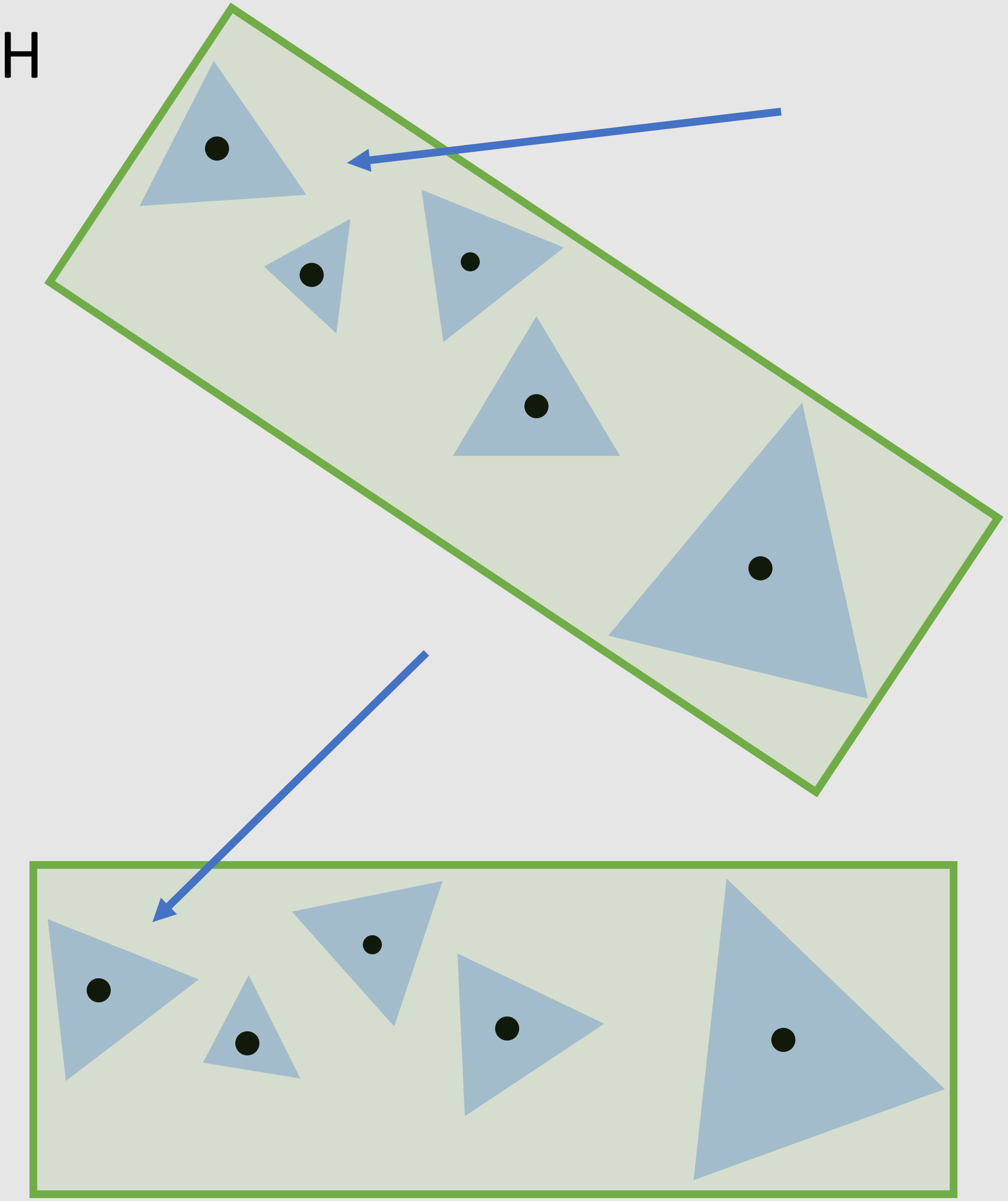
Axis-Aligned BVH

- **Are non-axis-aligned BVHs actually faster?**
  - Yes, and no.

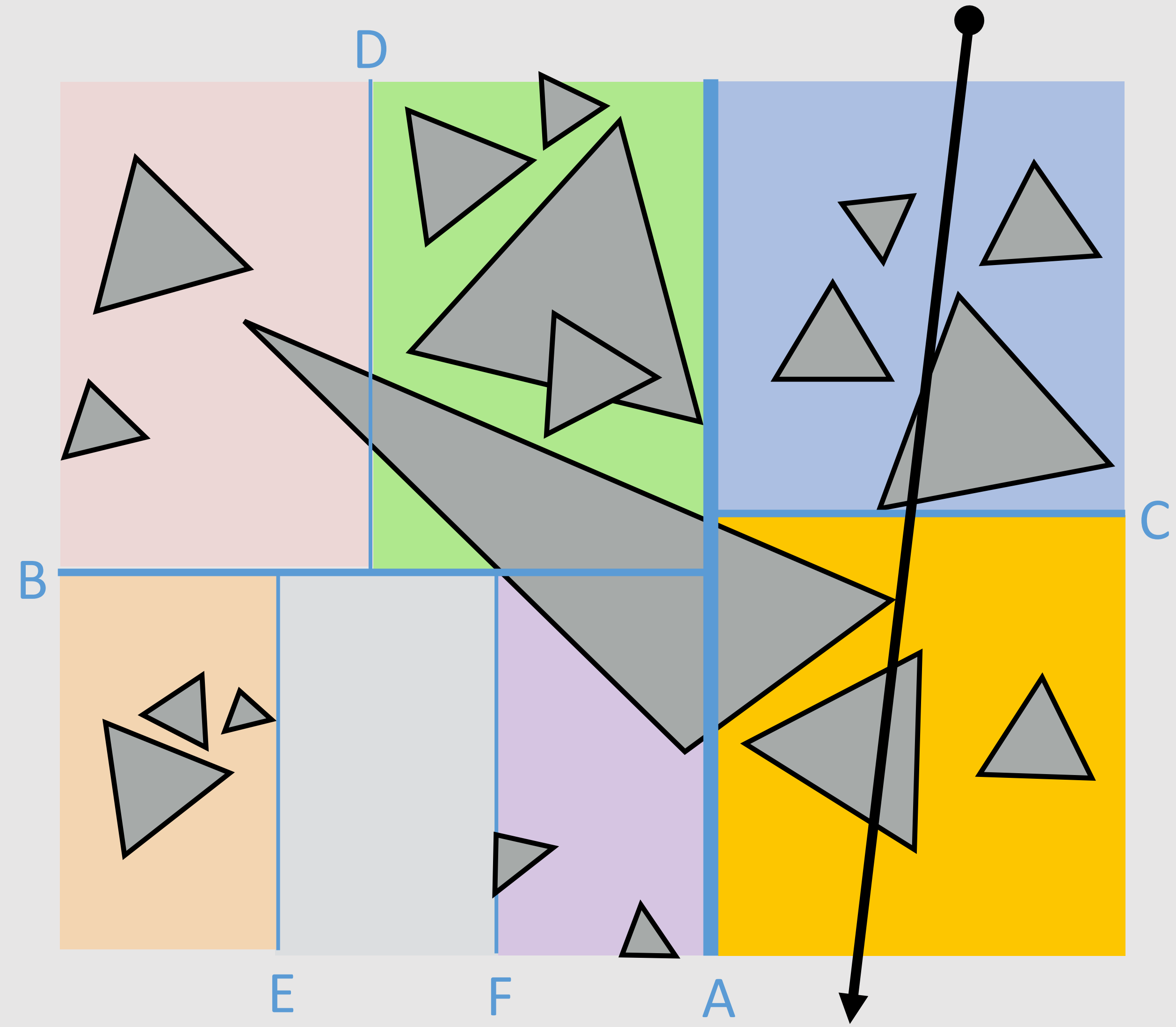
$$C = C_{trav} + \frac{S_A}{S_C} N_A C_{tri} + \frac{S_B}{S_C} N_B C_{tri}$$

  - Surface area ratio $\frac{S_A}{S_C}$ decreases with better-fitting bboxes
  - Bounding box intersection cost C_{trav} increases with more compute required to check unaligned bbox

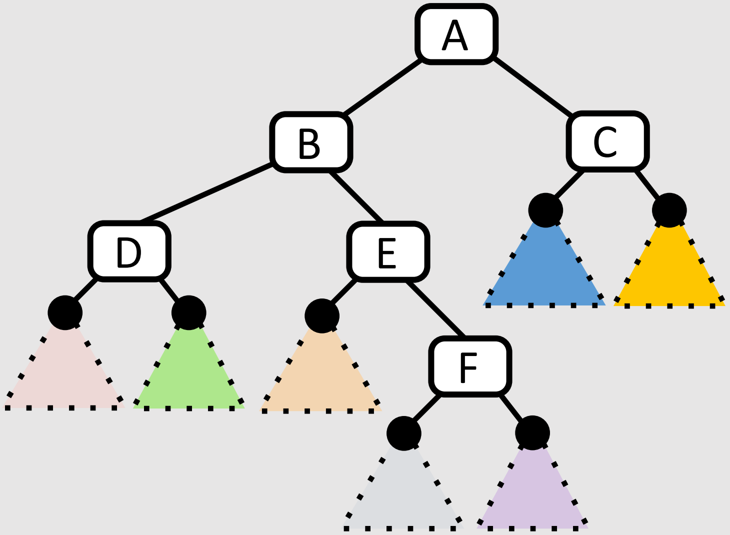
- **How to check for intersection with non-axis-aligned bbox?**
  - Bbox now has an extra transform matrix T taking it from its local space to its parent space
    - Apply the **inverse** transform to the **ray** and compute axis-aligned intersections
  - Larger memory overhead, now need to store the transform with each node



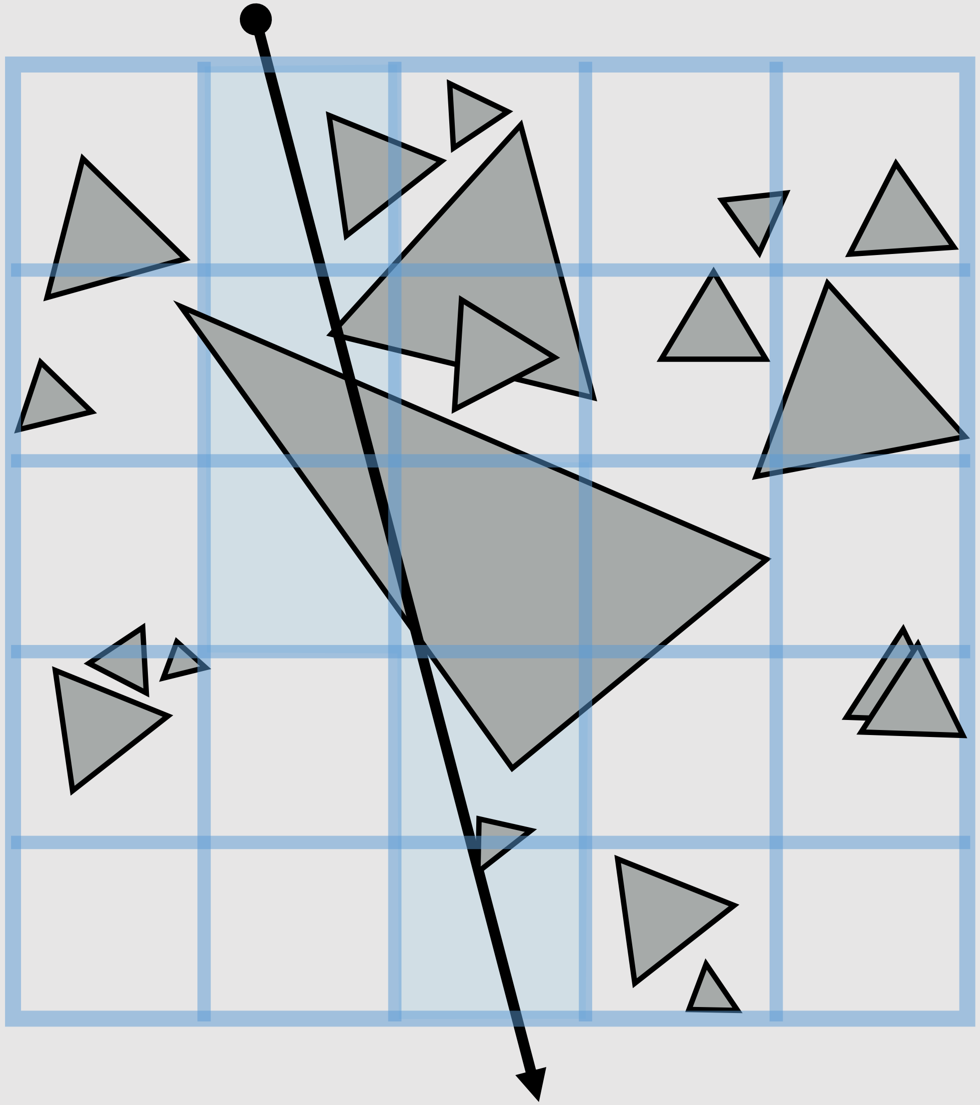
K-D Trees



- Recursively partition space via axis-aligned partitioning planes
  - Interior nodes correspond to spatial splits
  - Node traversal proceeds in front-to-back order
  - Unlike BVH, can terminate search after first hit is found
  - Still $O(\log(N))$ performance



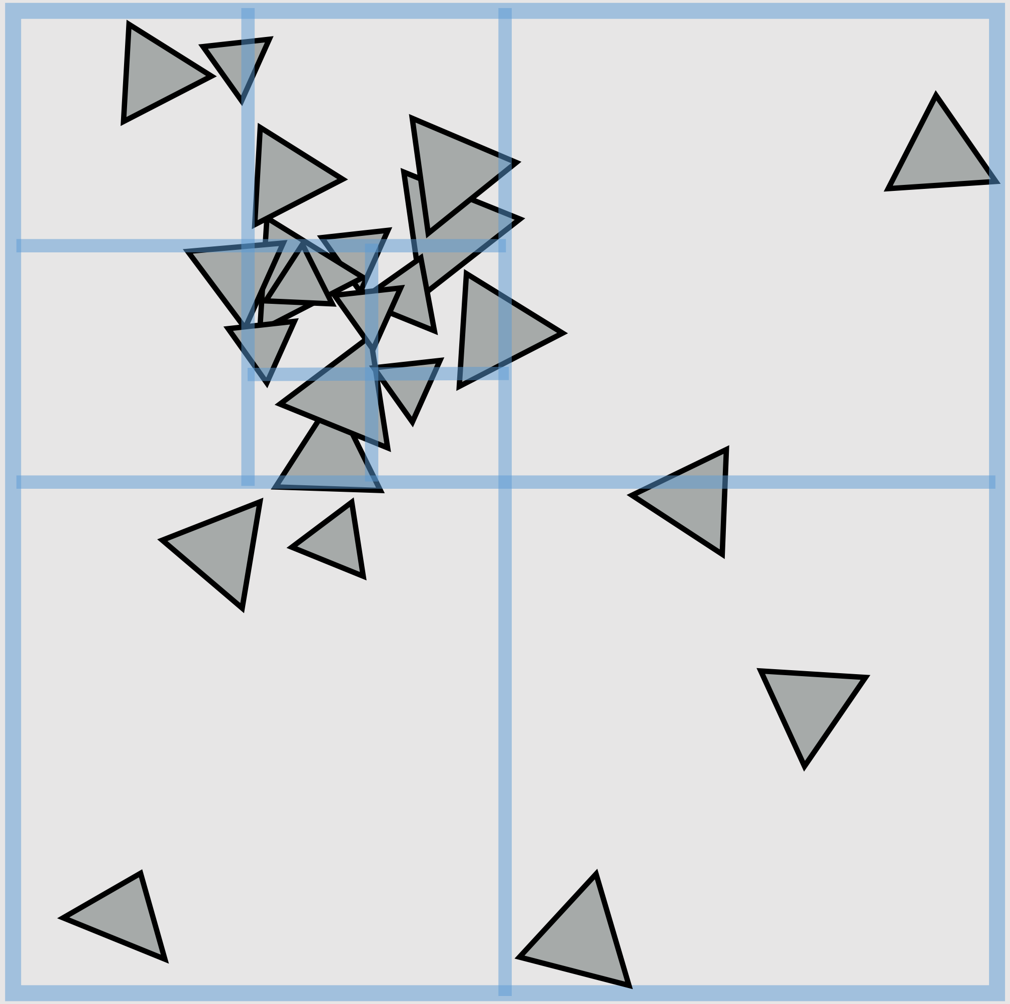
Uniform Grid



- Partition space into equal sized volumes (volume-elements or “voxels”)
- Each grid cell contains primitives that overlap voxel. (very cheap to construct acceleration structure)
- Walk ray through volume in order
  - Very efficient implementation possible (think: 3D line rasterization)
  - Only consider intersection with primitives in voxels the ray intersects

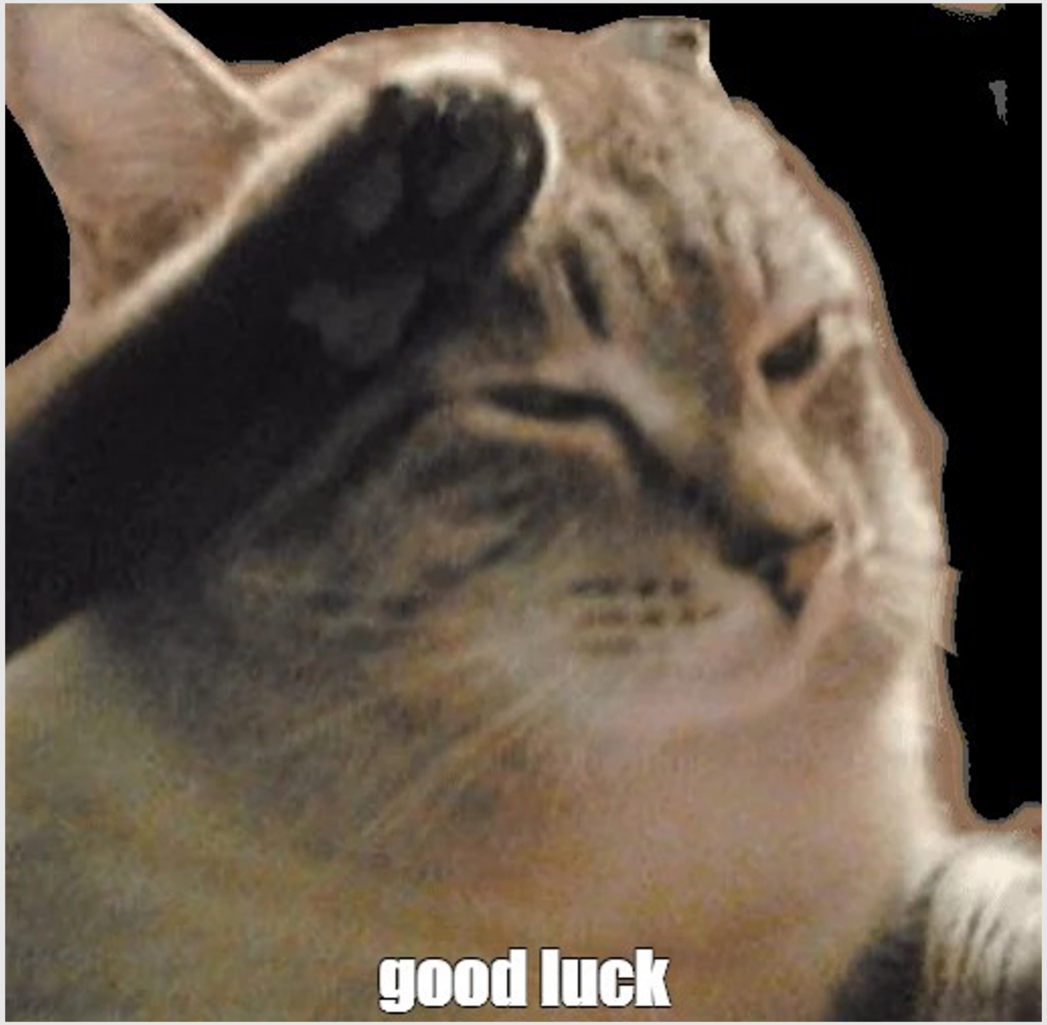
- What is a good number of voxels?
  - Should be proportional to total number of primitives N
  - Number of cells traversed is proportional to $O(\sqrt[3]{N})$
    - A line going through a cube is a cubed root
    - Still not as good as $O(\log(N))$

Quad-Tree/Octree



- Like uniform grid, easy to build
- Has greater ability to adapt to location of scene geometry than uniform grid
  - Still not as good adaptability as K-D tree

- **Quad-tree:** nodes have 4 children
  - Partitions 2D space
- **Octree:** nodes have 8 children
  - Partitions 3D space


good luck