Alpha Blending and Introduction to Geometry

- Alpha Blending

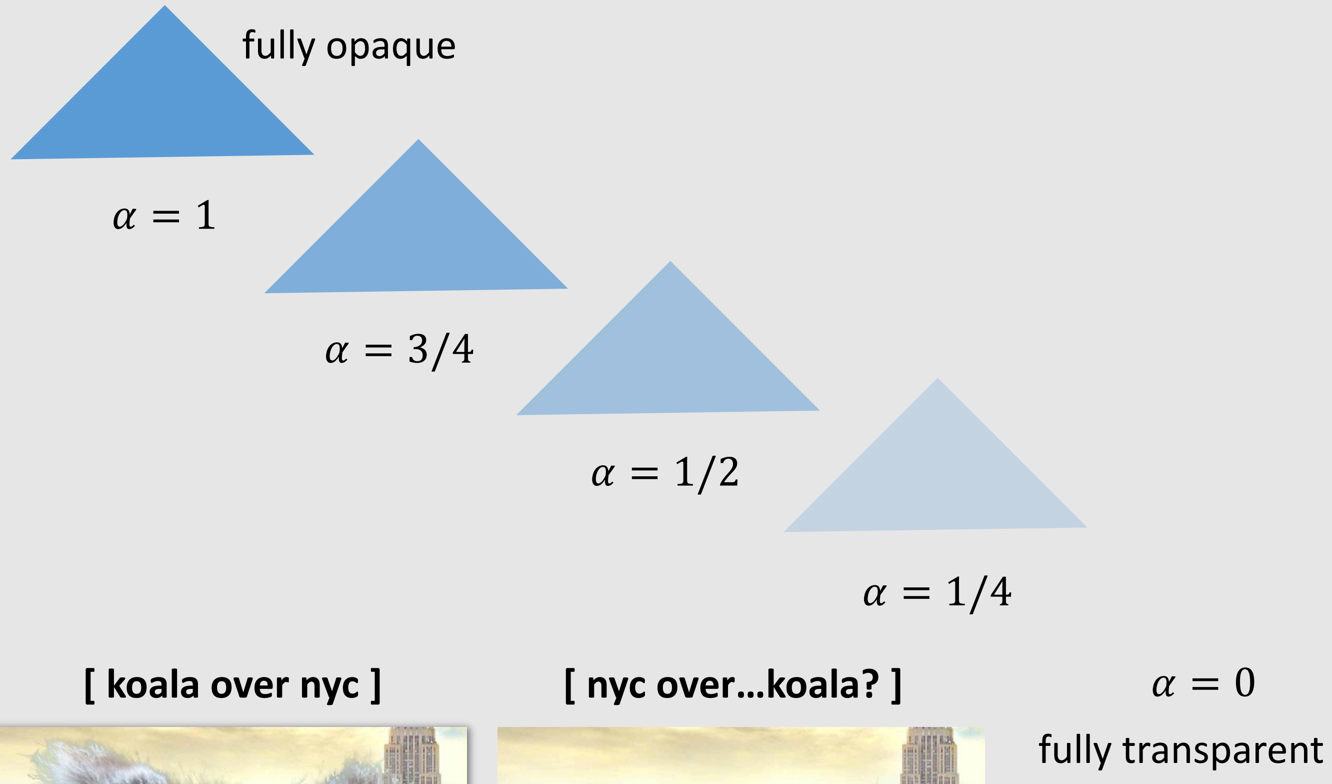
- The Graphics Pipeline Revisited

- Introduction to Geometry

Alpha Values

- Another common image format: RGBA
  - Alpha channel specifies 'opacity' of object
  - Basically how transparent it is
  - Most common encoding is 8-bits per channel (0-255)

- Compositing A over B != B over A
  - Consider the extreme case of two opaque objects…



[koala over nyc]

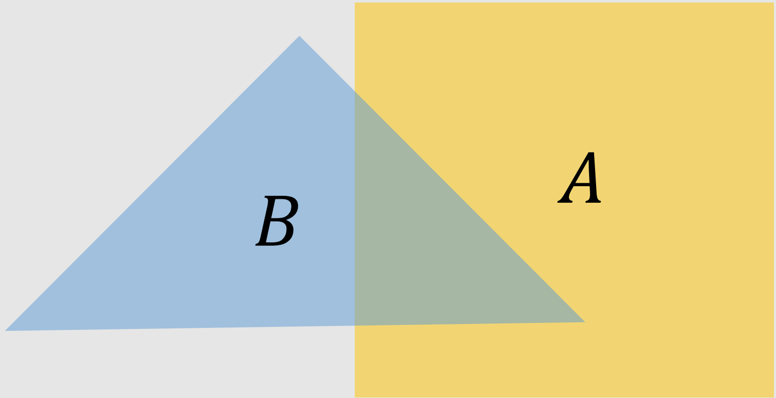
[nyc over…koala?]

where is the koala…

Non-Premultiplied Alpha

- **Goal:** Composite image B with alpha α_B over image A with alpha α_A

$$A = (A_r, A_g, A_b)$$
$$B = (B_r, B_g, B_b)$$



B over A

- Composite RGB:

what B lets through

$$C = \alpha_B B + (1 - \alpha_B)\alpha_A A$$

appearance of semi-transparent B

appearance of semi-transparent A

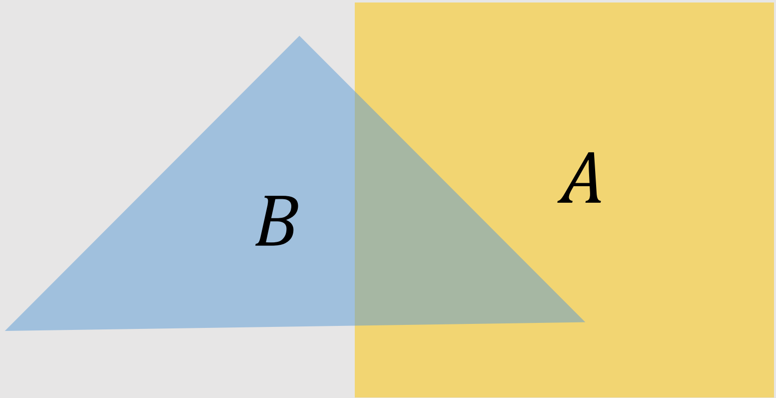
- Composite Alpha:

$$\alpha_C = \alpha_B + (1 - \alpha_B)\alpha_A$$

Two different equations is inefficient!!

Premultiplied Alpha

- **Goal:** Composite image B with alpha α_B over image A with alpha α_A



B over A

$$A' = (\alpha_A A_r, \alpha_A A_g, \alpha_A A_b, \alpha_A)$$
$$B' = (\alpha_B B_r, \alpha_B B_g, \alpha_B B_b, \alpha_B)$$

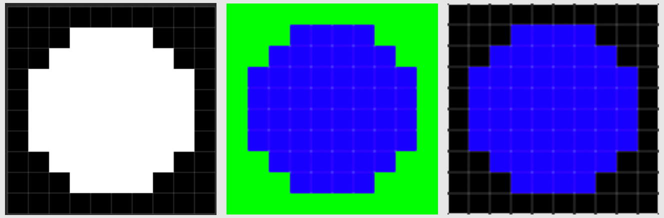
- <u>Composite RGBA:</u>

$$C' = B' + (1 - \alpha_B)A'$$

- <u>Un-Premultiply for Final Color:</u>

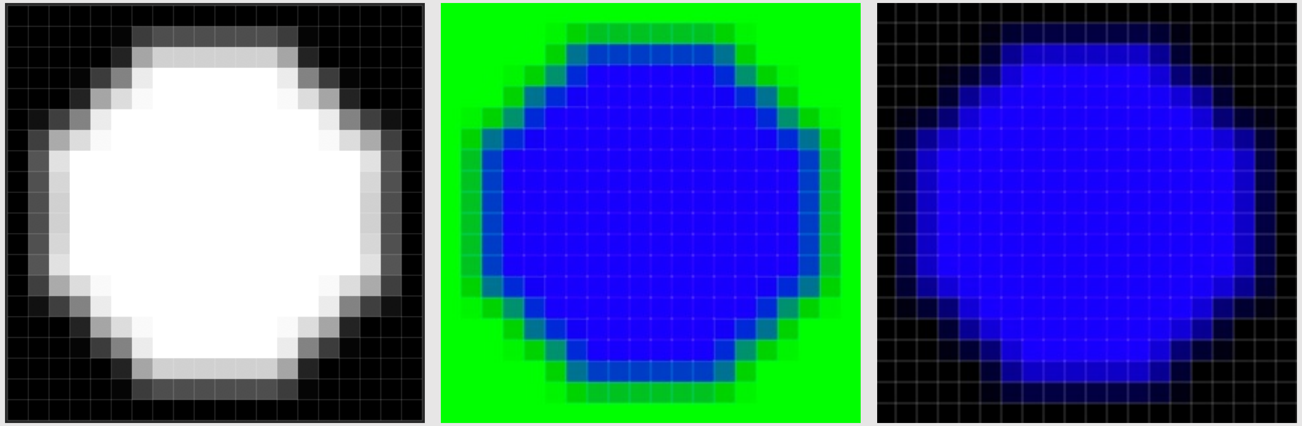
$$(C_r, C_g, C_b, \alpha_C) \Longrightarrow (C_r/\alpha_C, C_g/\alpha_C, C_b/\alpha_C)$$

Why Premultiplied Matters [Upsample]



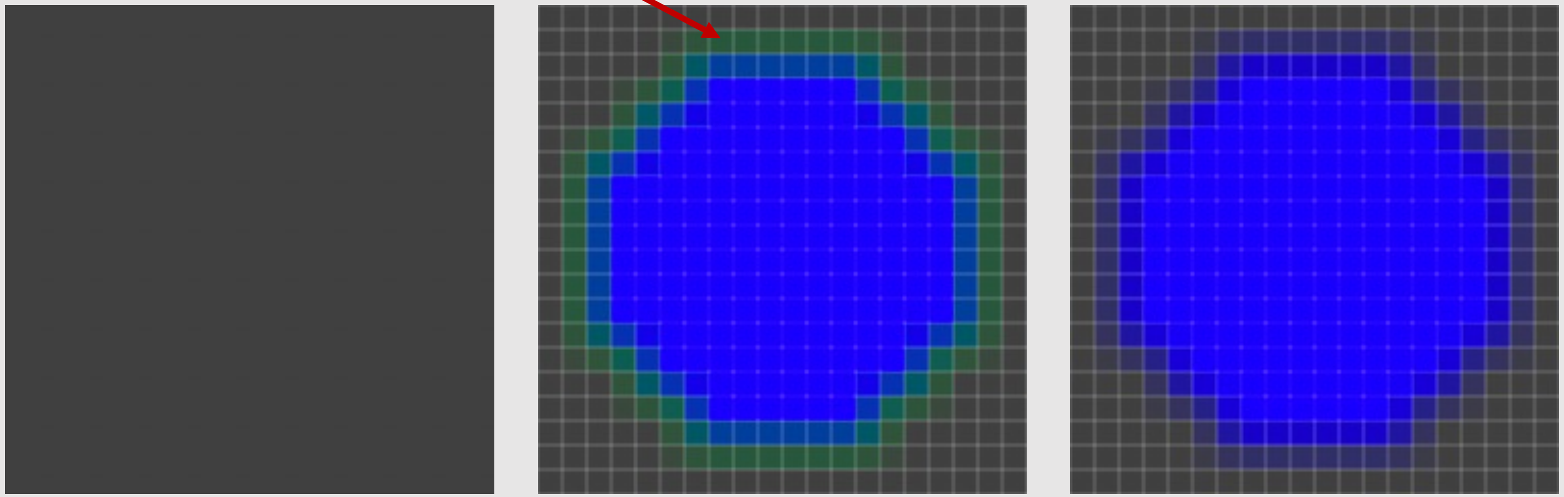
alpha color premultiplied

upsample →



upsampled alpha | upsampled color | upsampled premultiplied

Known as fringing



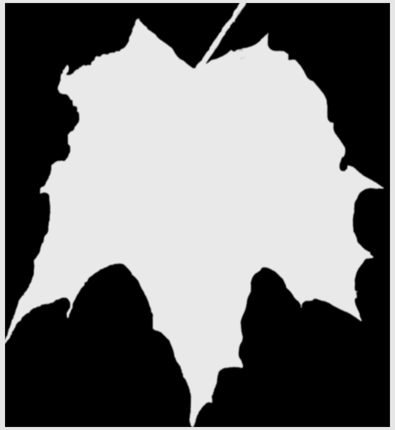
new background A ($\alpha_A = 1$) | B over A | B over A (premultiplied)

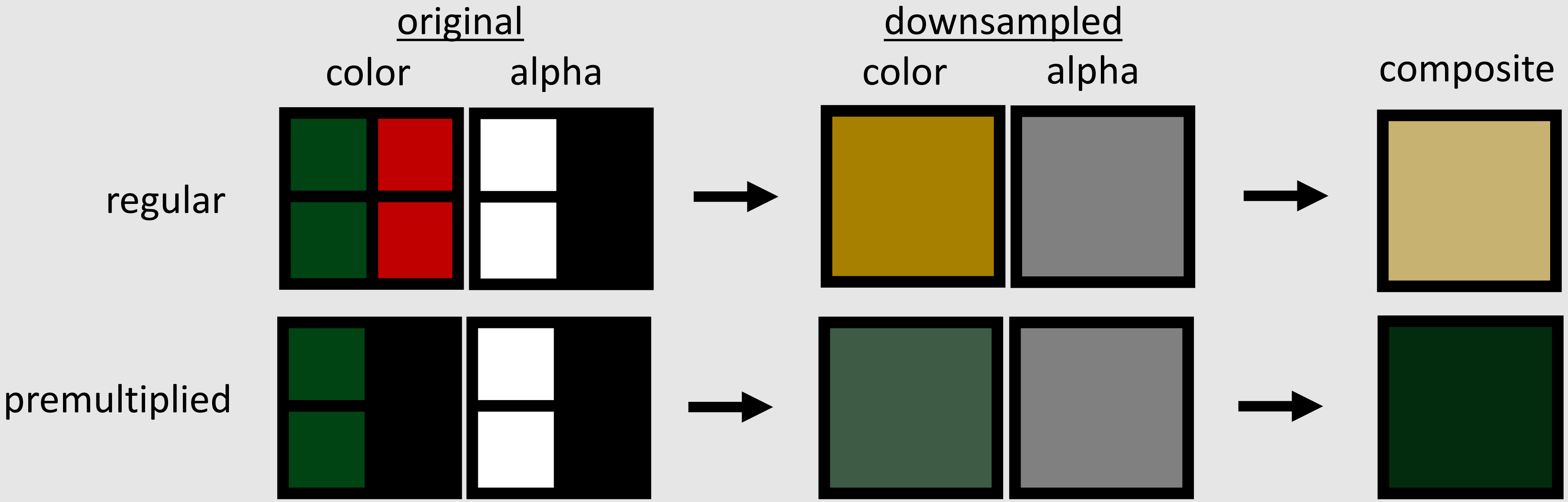
Something isn't right…

Why Premultiplied Matters [Downsample]

[RGB]

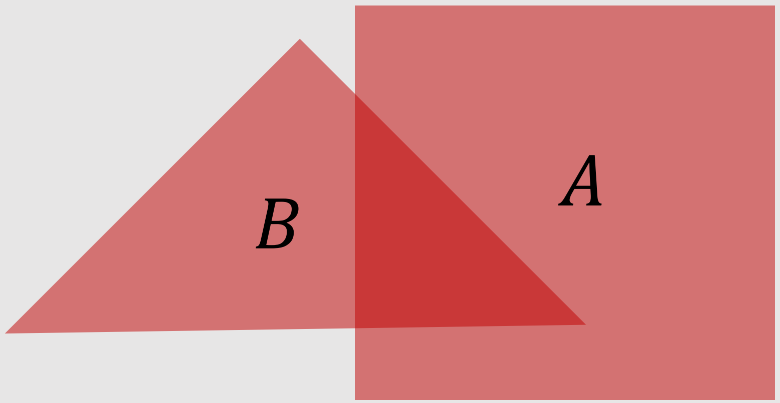
[A]





Closed Under Composition

- **Goal:** Composite bright red image B with alpha 0.5 over bright red image A with alpha 0.5



B over A

$$A = (1, 0, 0, 0.5)$$
$$B = (1, 0, 0, 0.5)$$

- Non-Premultiplied:

color

$$0.5 * (1,0,0) + (1 - 0.5) * 0.5 * (1,0,0)$$
$$\downarrow$$
$$(0.75, 0, 0)$$

alpha

$$0.5 + (1 - 0.5) * 0.5 = 0.75$$

- Premultiplied:

$$0.5 * (0.5,0,0,0.5) + (1 - 0.5) * (0.5,0,0,0.5)$$
$$\downarrow$$
$$(0.75, 0, 0, 0.75)$$
$$\downarrow \text{ divide out alpha}$$
$$(1, 0, 0)$$

Blend Methods

When writing to color buffer, can use any blend method

$D_{RGBA} = S_{RGBA} + D_{RGBA}$	Blend Add
$D_{RGBA} = S_{RGBA} - D_{RGBA}$	Blend Subtract
$D_{RGBA} = -S_{RGBA} + D_{RGBA}$	Blend Reverse Subtract
$D_{RGBA} = \min(S_{RGBA}, D_{RGBA})$	Blend Min
$D_{RGBA} = \max(S_{RGBA}, D_{RGBA})$	Blend Max
$D_{RGBA} = S_{RGBA} + D_{RGBA} * (1 - S_A)$	Blend Over

S_{RGBA} and D_{RGBA} are pre-multiplied

Updated Depth Buffer (Z-buffer) Sample Code

Should we still be doing depth writes for alpha primitives?

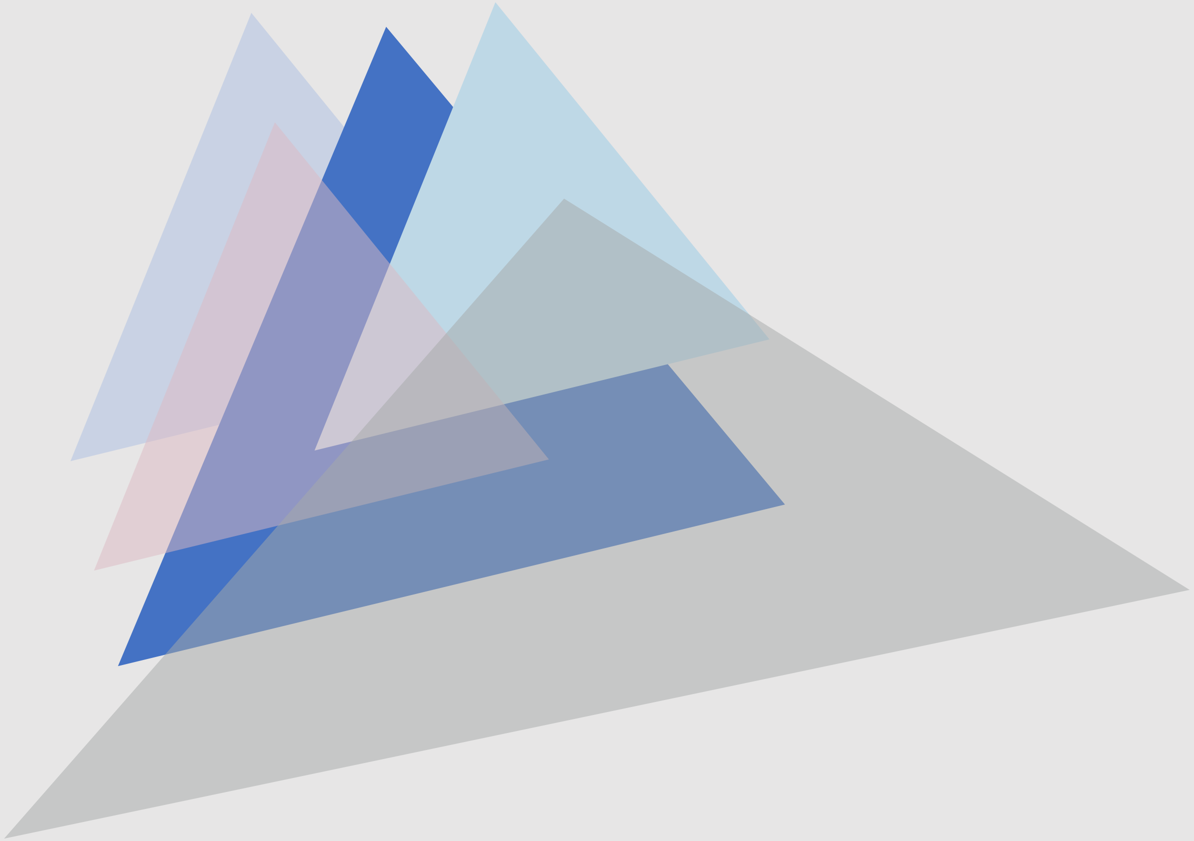
```
draw_sample(x, y, d, c) //new depth d & color c at (x,y)
{
  if (d < zbuffer[x][y])
  {
    // triangle is closest object seen so far at this
    // sample point. Update depth and color buffers.
    zbuffer[x][y] = d;
    color[x][y] = c.rgba + (1-c.a) * color[x][y];
  }
  // otherwise, we've seen something closer already;
  // don't update color or depth
}
```

Assumes color[x][y] and c are both premultiplied.

Triangles must be rendered back to front!
A over B != B over A

Blend Render Order

- For mixtures of opaque and transparent triangles:
  - **Step 1:** render opaque primitives (in any order) using depth-buffered occlusion
    - If pass depth test, triangle overwrites value in color buffer at sample
    - Depth **READ** and **WRITE**
  - **Step 2:** disable depth buffer update, render semi-transparent surfaces in back-to-front order.
    - If pass depth test, triangle is composited OVER contents of color buffer at sample
    - Depth **READ** only



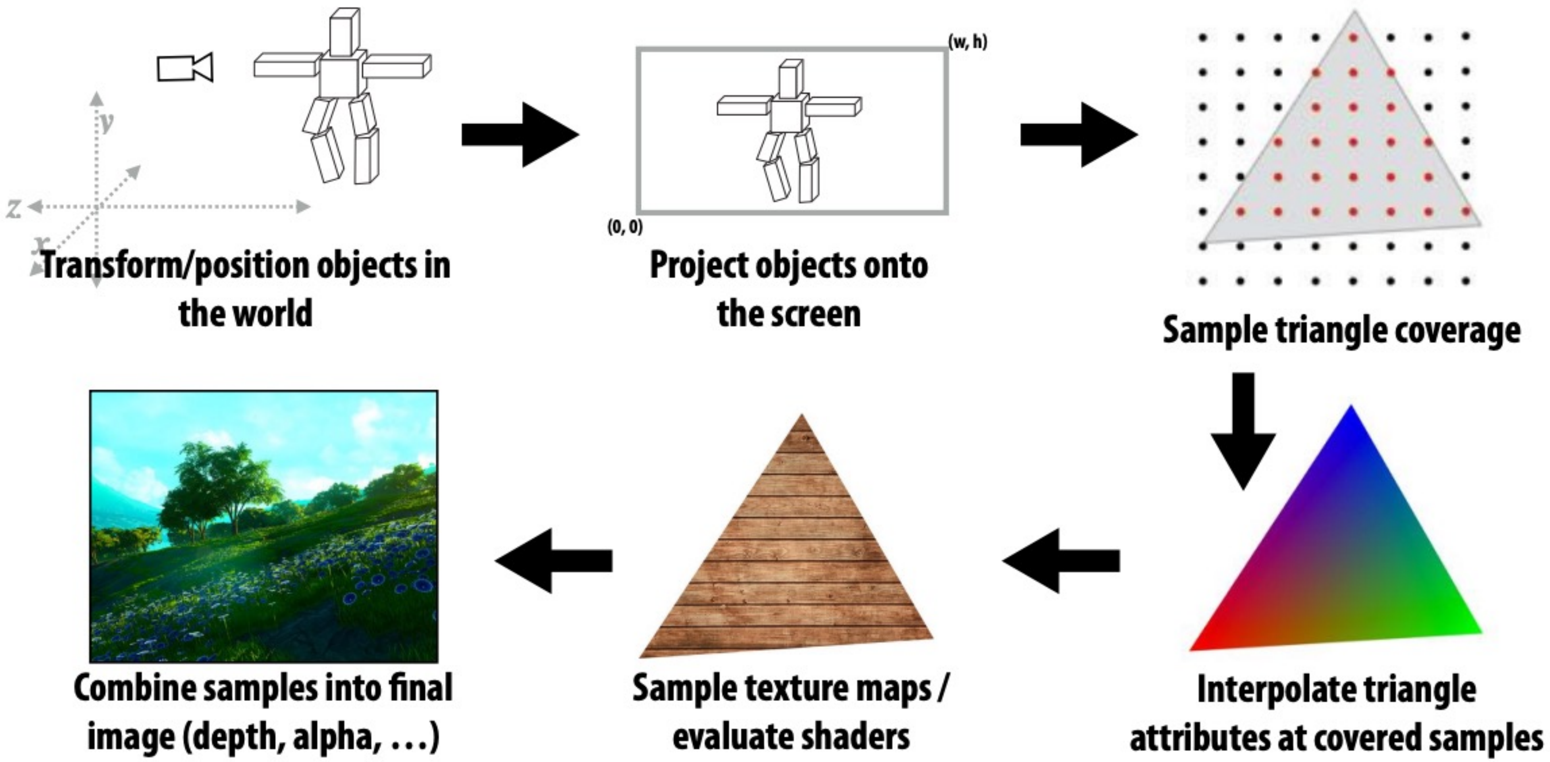
- ~~Alpha Blending~~

- The Graphics Pipeline Revisited

- Introduction to Geometry

The "Simpler" Graphics Pipeline

Now Let's Put It All Together!

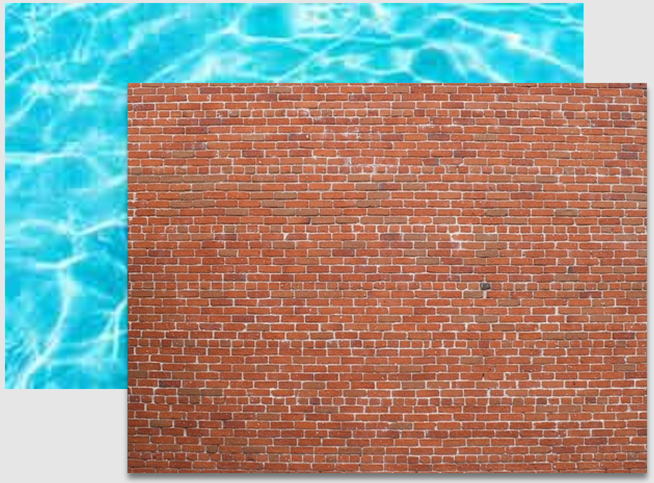


The Inputs

```
positions = {
    v0x, v0y, v0z,
    v1x, v1y, v1x,
    v2x, v2y, v2z,
    v3x, v3y, v3x,
    v4x, v4y, v4z,
    v5x, v5y, v5x
};
```

```
texcoords ={
    v0u, v0v,
    v1u, v1v,
    v2u, v2v,
    v3u, v3v,
    v4u, v4v,
    v5u, v5v
};
```

[vertices]



[textures]

Object-to-camera-space transform $T \in \mathbb{R}^{4\times4}$

Perspective projection transform $P \in \mathbb{R}^{4\times4}$

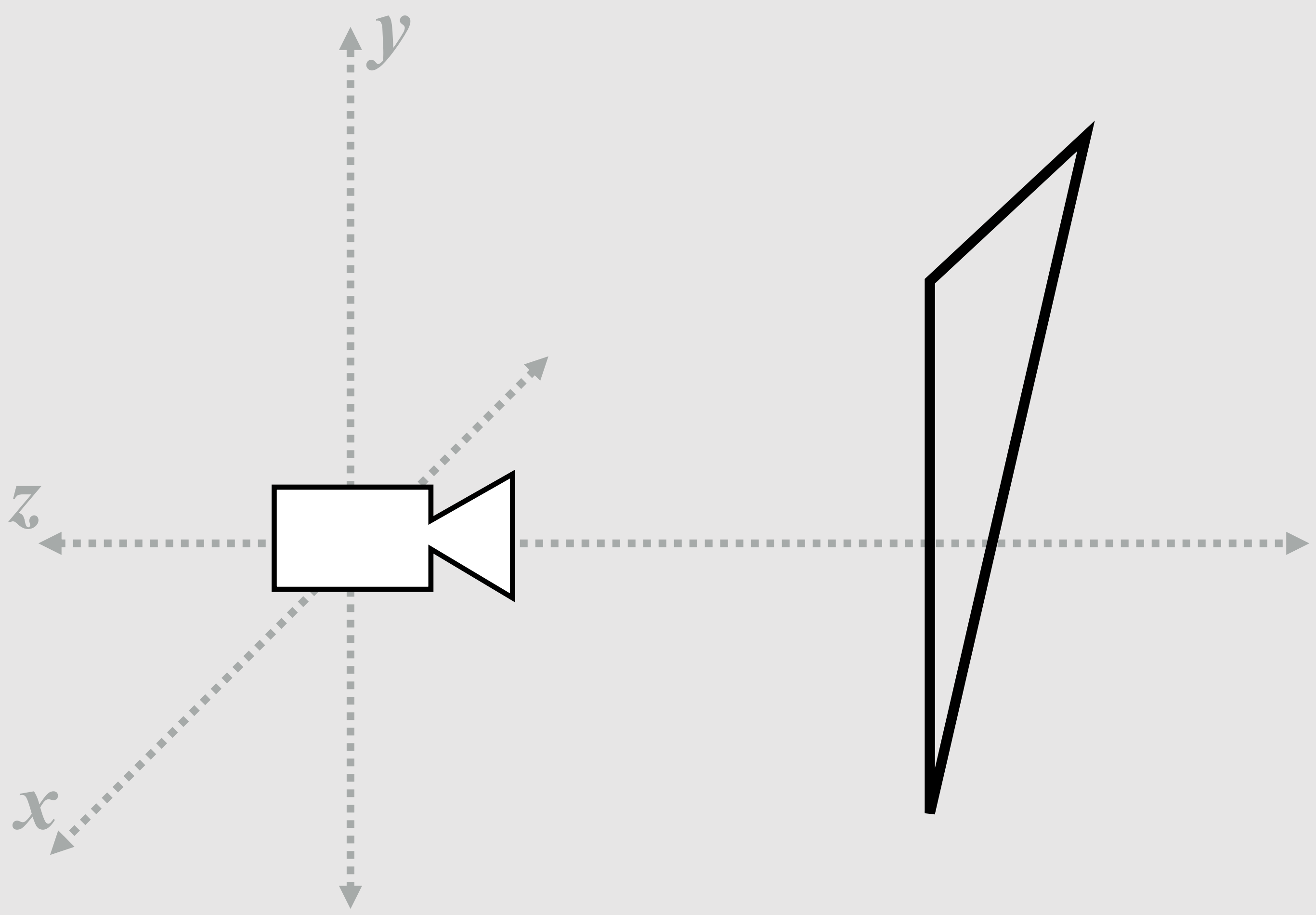
Output image (W, H)

[camera properties]

[machine]

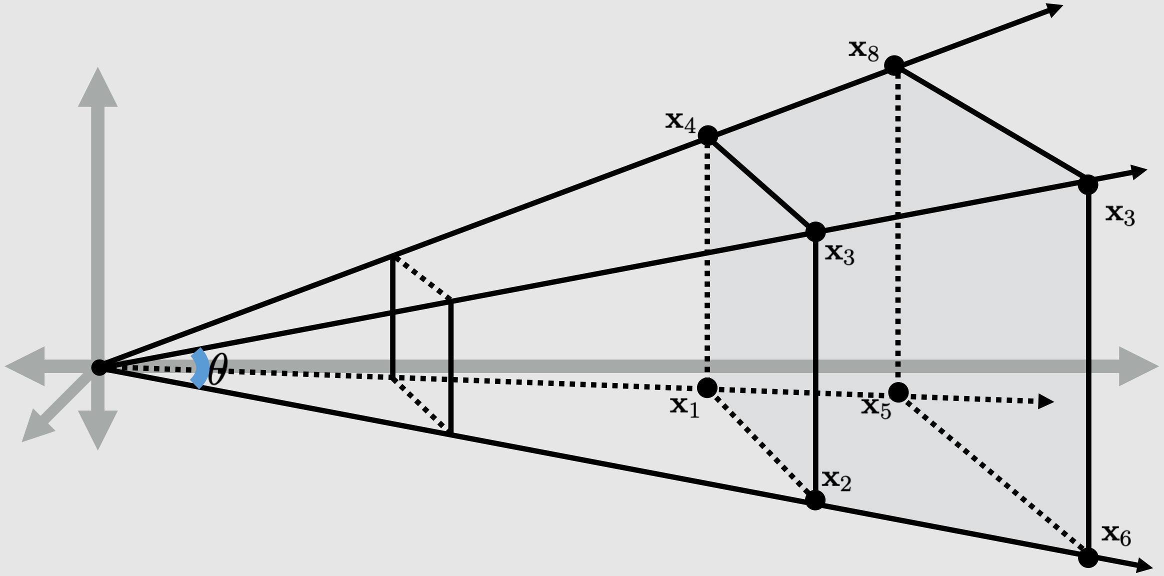
Step 1: Transform

Transform triangle vertices into camera space

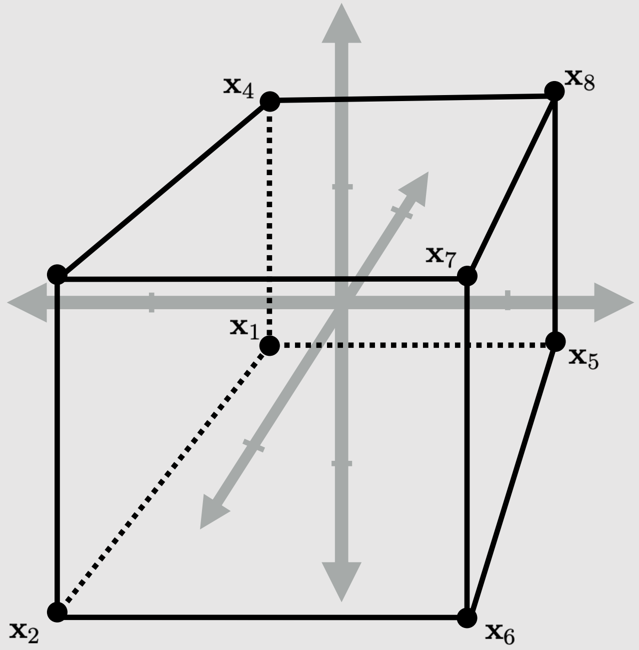


Step 2: Perspective Projection

Apply perspective projection transform to transform triangle vertices into normalized coordinate space



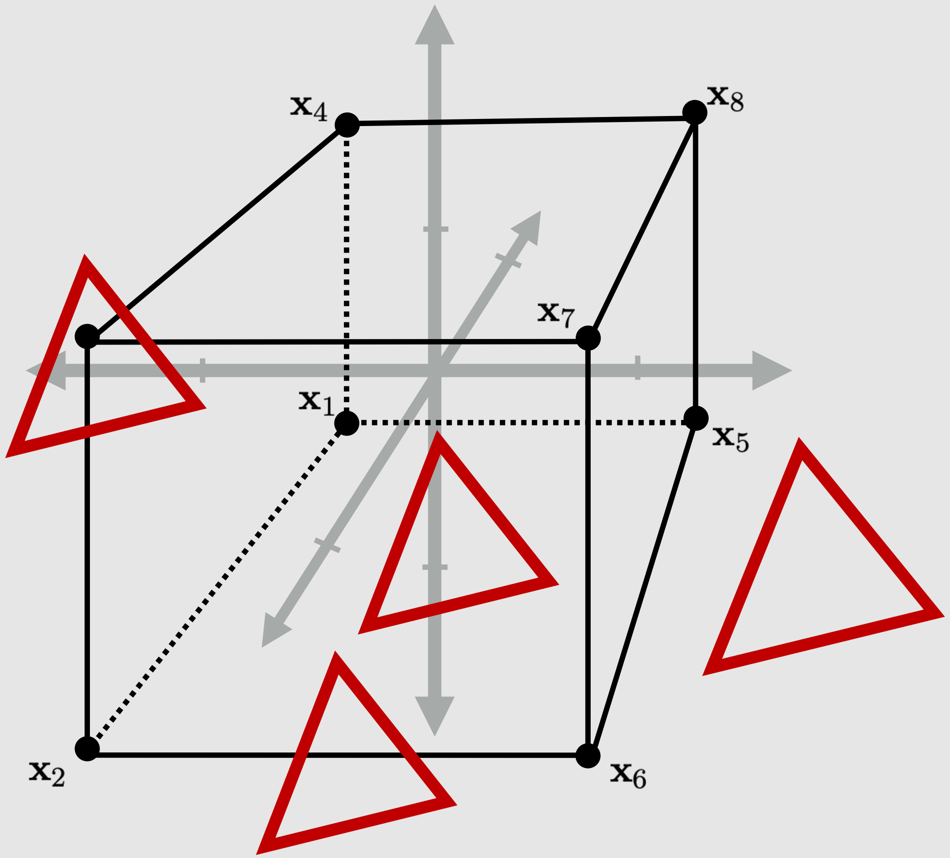
[3D camera space position]



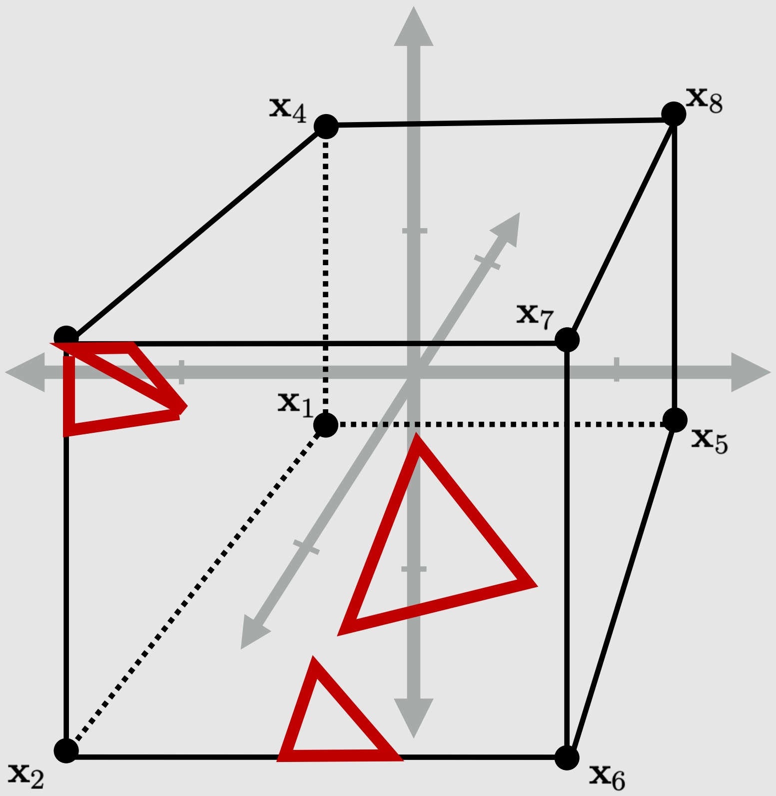
[normalized space position]

Step 3: Clipping

Discard triangles completely outside cube.
Clip triangles partially in cube.



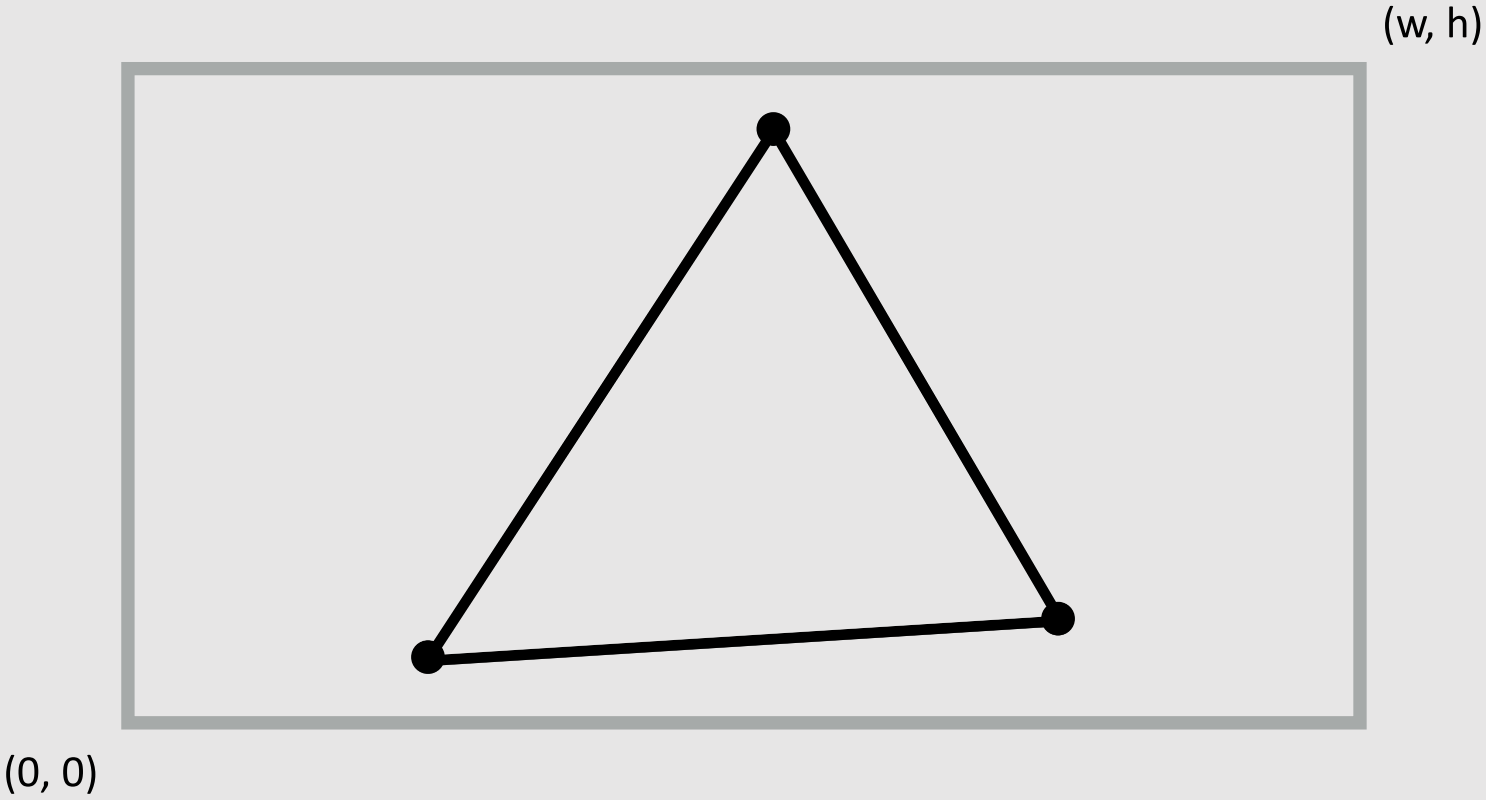
[pre-clipping]



[post-clipping]

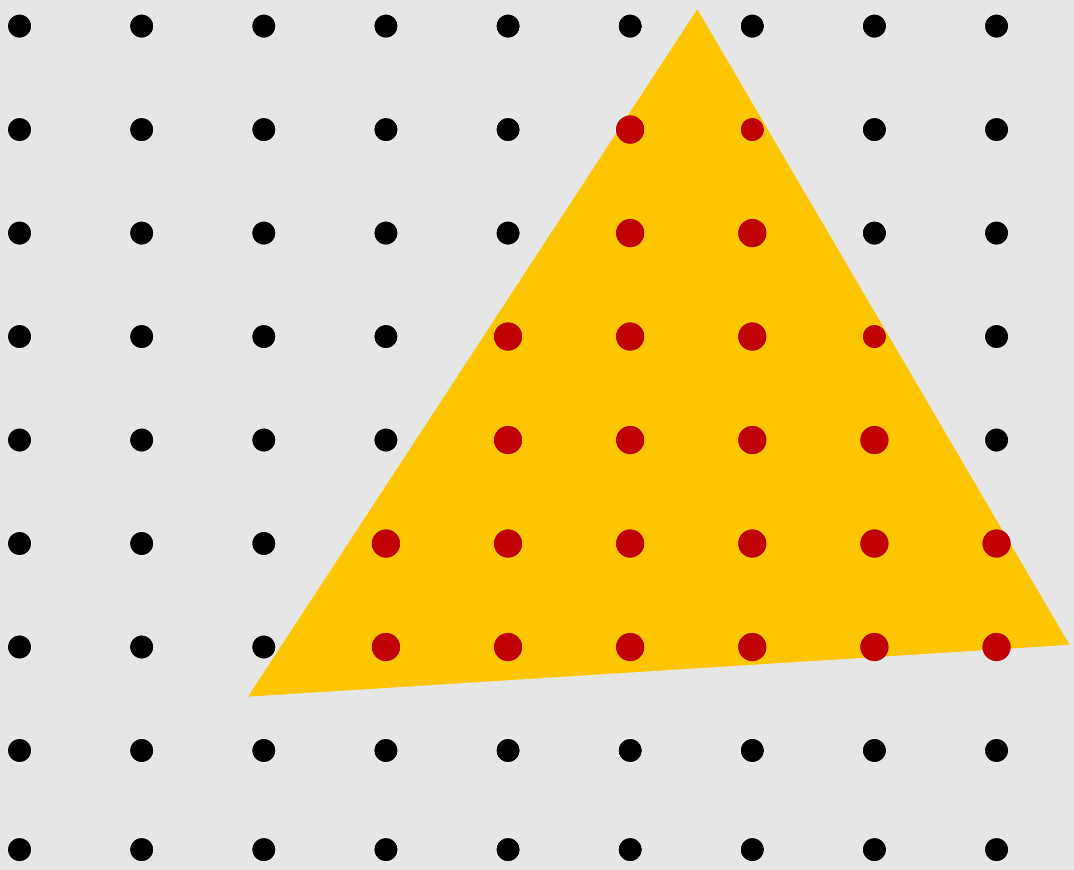
Step 4: Transform To Screen Coordinates

Perform homogeneous divide.
Transform vertex xy positions from normalized coordinates
into screen coordinates (based on screen [w, h]).



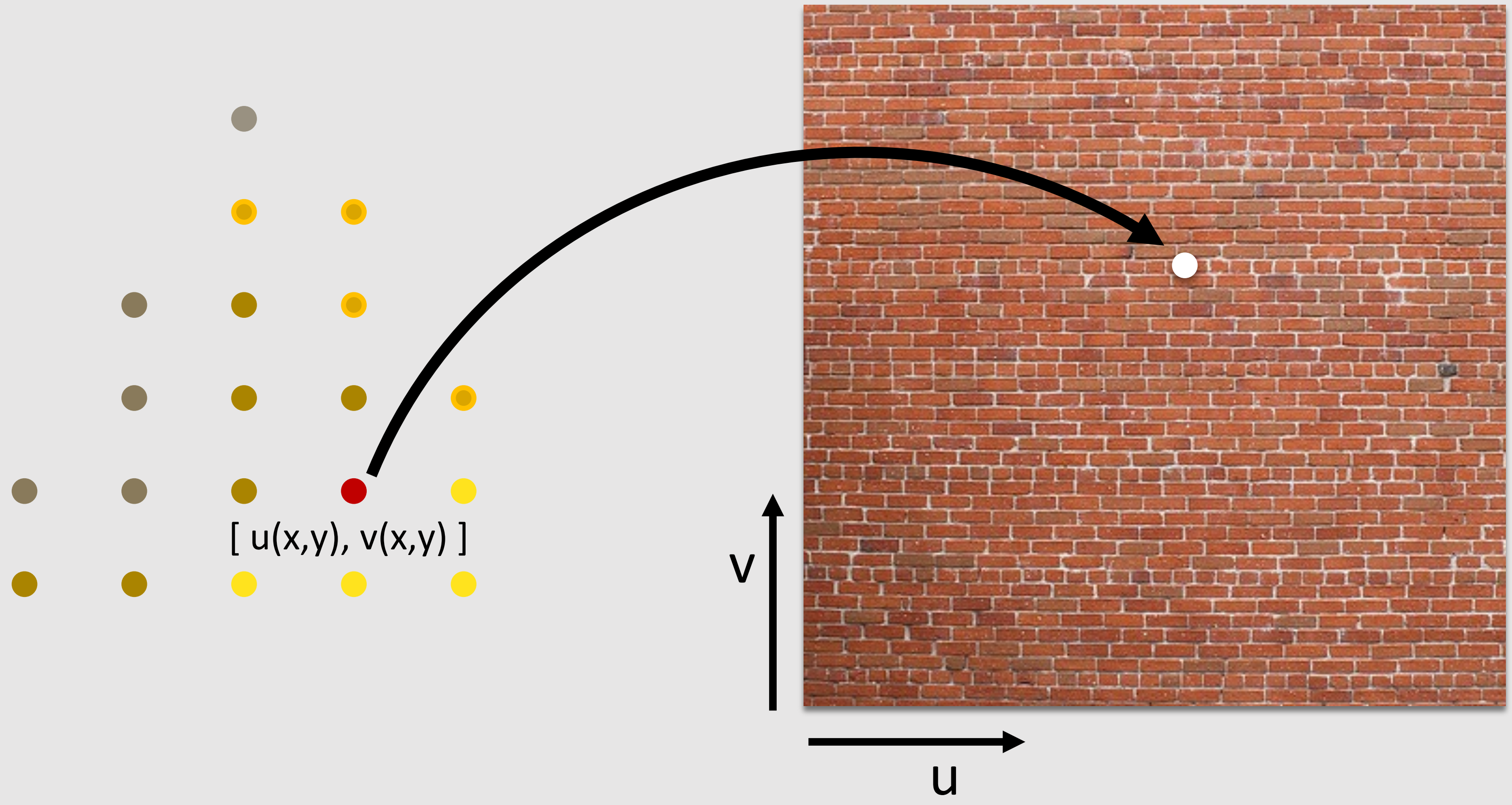
Step 5: Sample Coverage

Check if samples lie inside triangle.
Evaluate depth and barycentric coordinates at all passing samples.



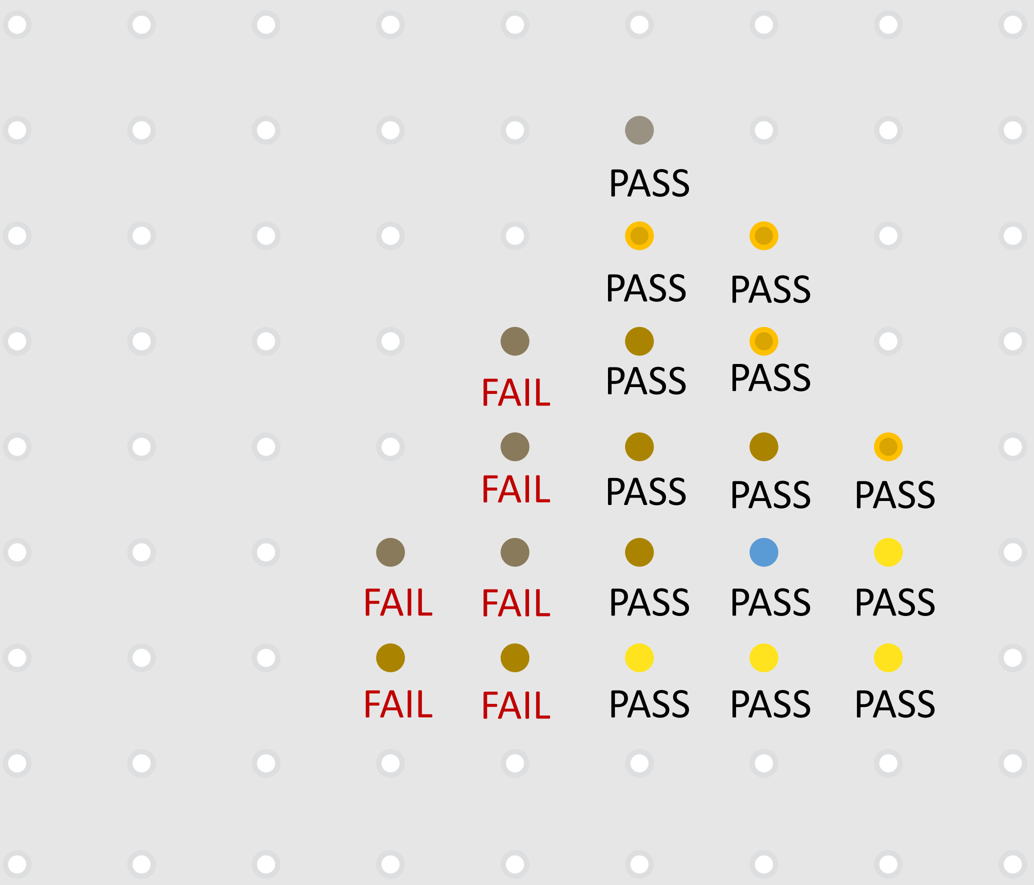
Step 6: Compute Color

Texture lookups, color interpolation, etc.



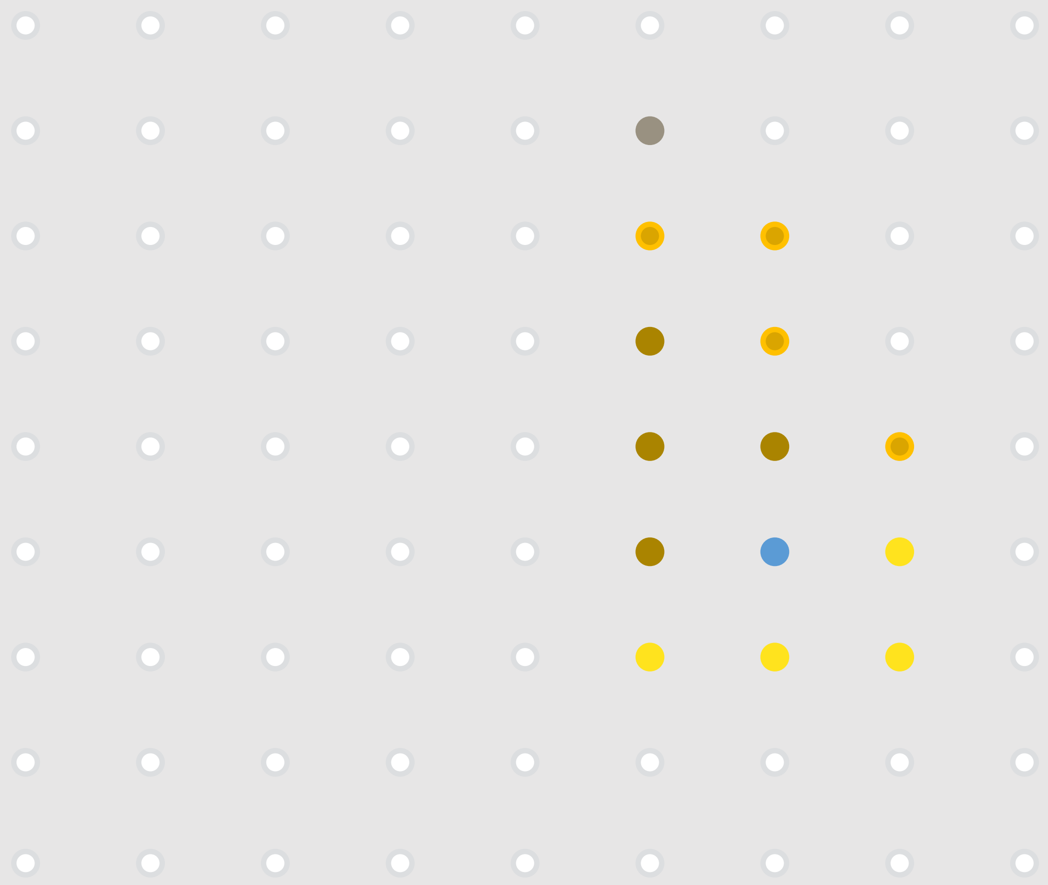
Step 7: Depth Test

Check depth and update depth if closer primitive found.
(can be disabled)

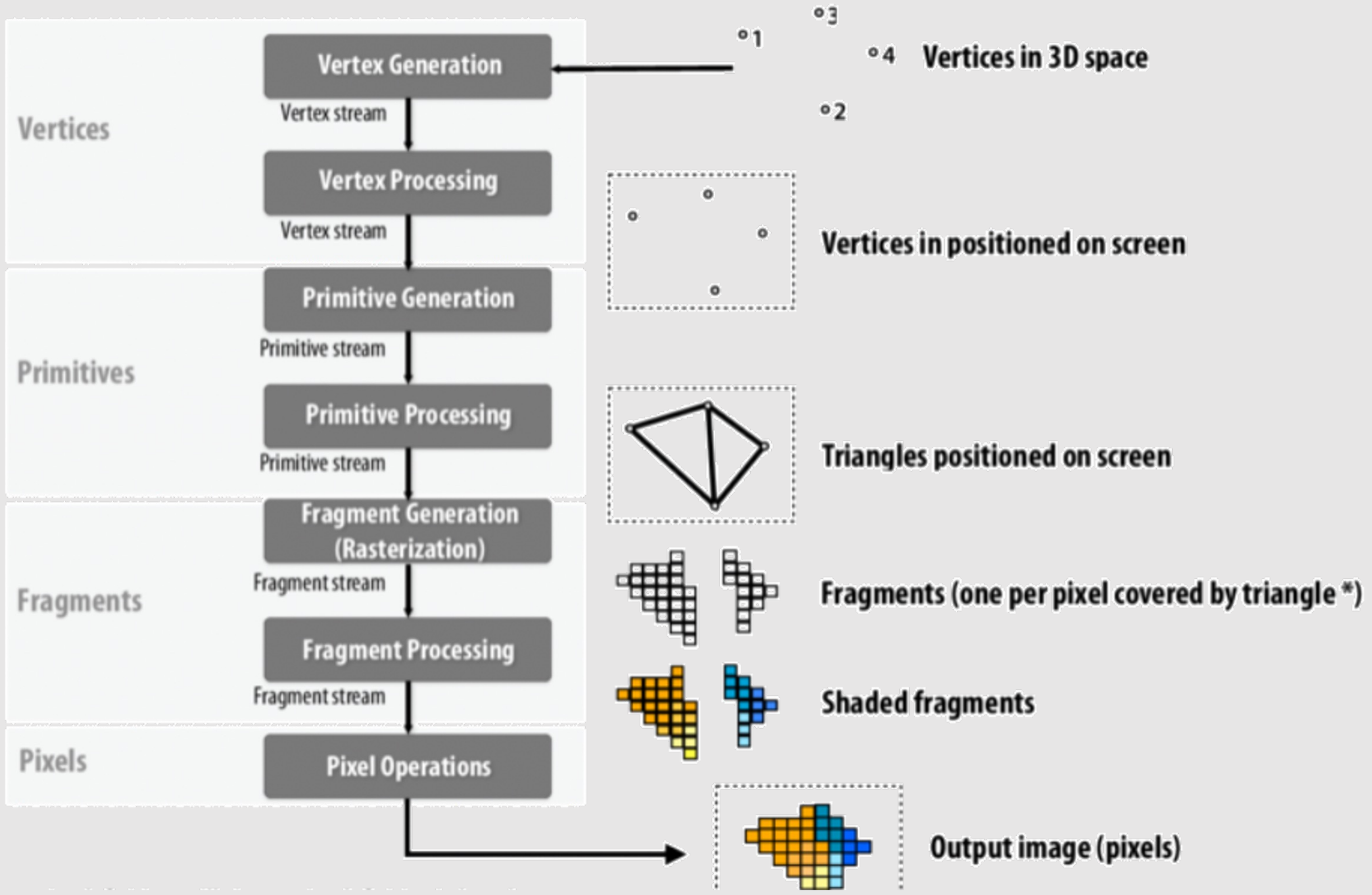


Step 8: Color Blending

Update color buffer with correct blending operation.



The “Real” Graphics Pipeline



Doesn’t look much different than what we discussed…

- ~~Alpha Blending~~
- ~~The Graphics Pipeline Revisited~~

- Introduction to Geometry

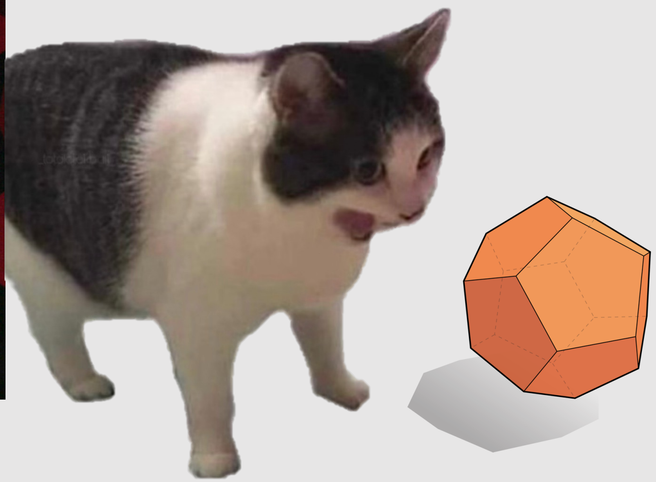
- ~~Alpha Blending~~
- ~~The Graphics Pipeline Revisited~~

- Introduction to Geometry
  - Implicit & Explicit Geometry
  - Manifold Geometry
  - Local Geometric Operations

Some Motivation

"I hate meshes.
I cannot believe how hard this is.
Geometry is hard."

"why won't you subdivide"



-- David Baraff

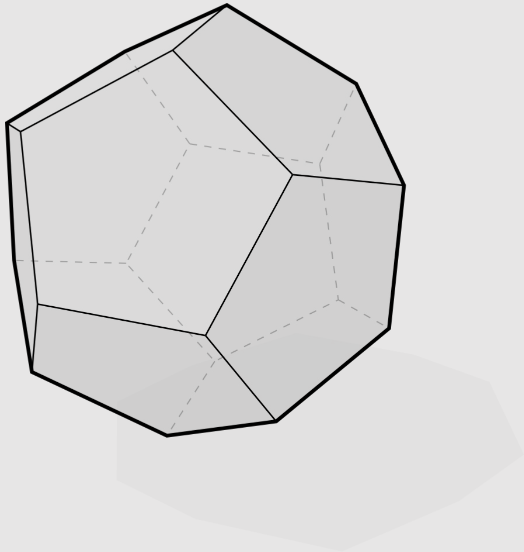
Senior Research Scientist
Pixar Animation Studios
(also a former CMU prof.)

What Is Geometry?

"Earth" "measure"

ge•om•et•ry /jēˈämətrē/ *n.*

1. The study of shapes, sizes, patterns, and positions.
2. The study of spaces where some quantity (lengths, angles, etc.) can be *measured*.



Remember that Computer Graphics is just operating on a bunch of numbers.
If we can measure it, we can represent it as numbers on our computer!

How To Represent Geometry

[IMPLICIT]

$$x^2 + y^2 = 1$$

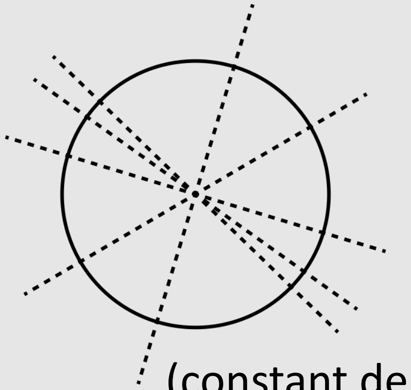
[LINGUISTIC]

"unit circle"

[EXPLICIT]

$$(\underbrace{\cos\theta}_{x}, \underbrace{\sin\theta}_{y})$$

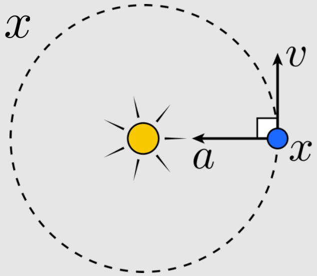
[TOMOGRAPHIC]



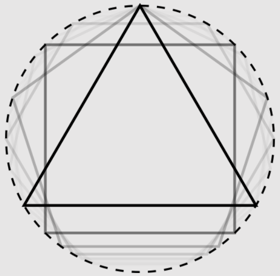
(constant density)

[DYNAMIC]

$$\frac{d^2}{dt^2}x = -x$$



[DISCRETE]

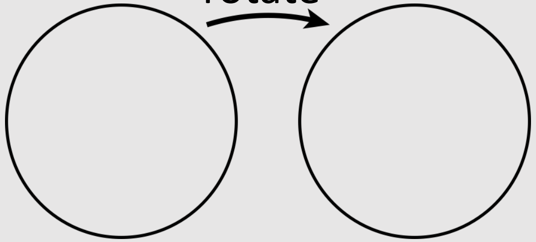


n ➞ ∞

[CURVATURE]

$$\kappa = 1$$

[SYMMETRIC]

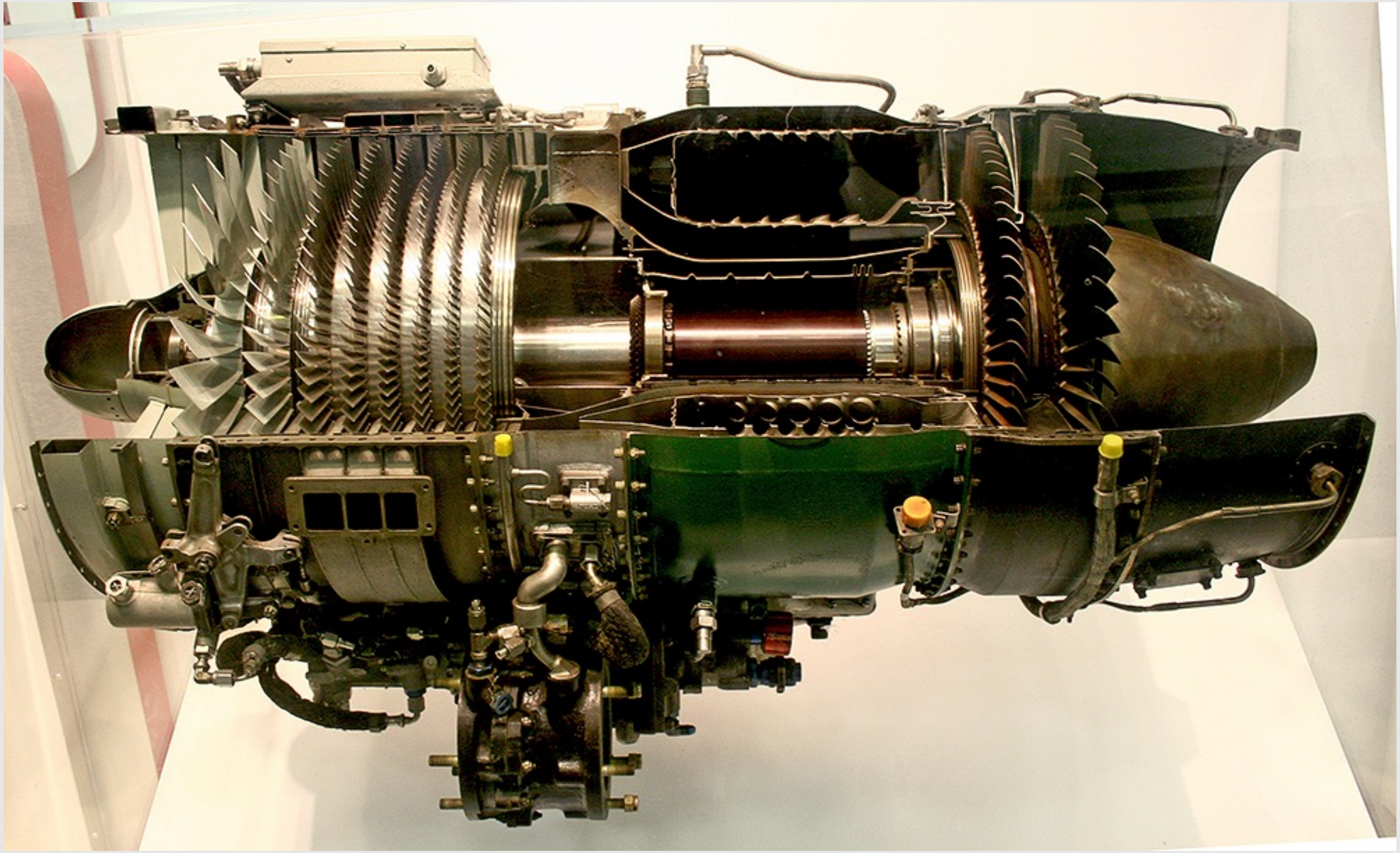


How To Represent Humans

How To Represent Water

How To Represent Cloth

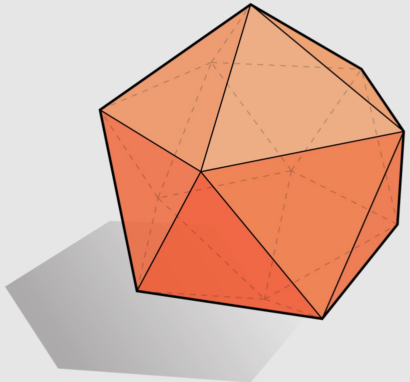
How To Represent Machines

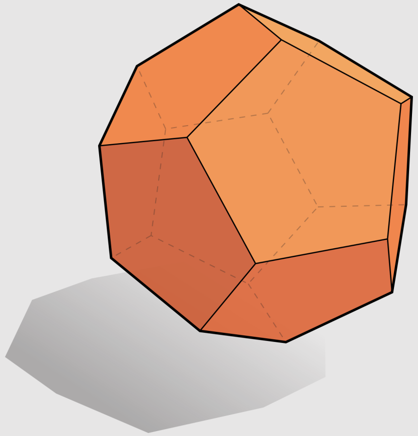


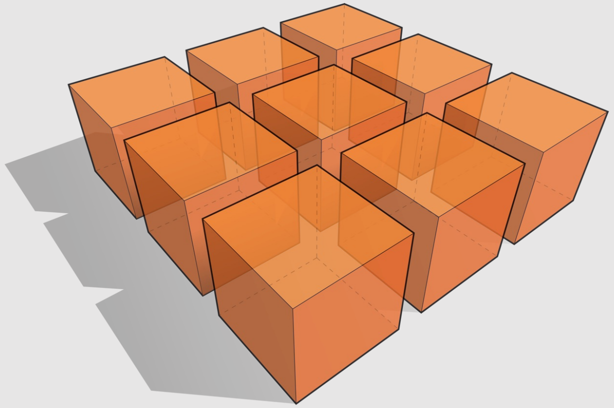
How To Represent This Thing

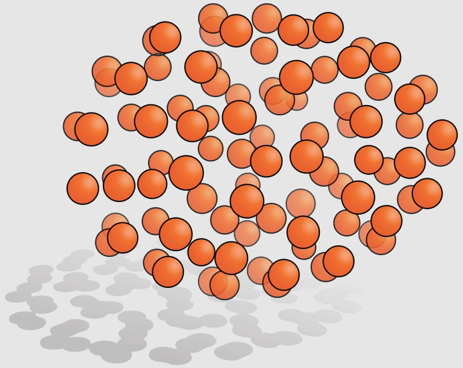
Many Ways To Encode Geometry

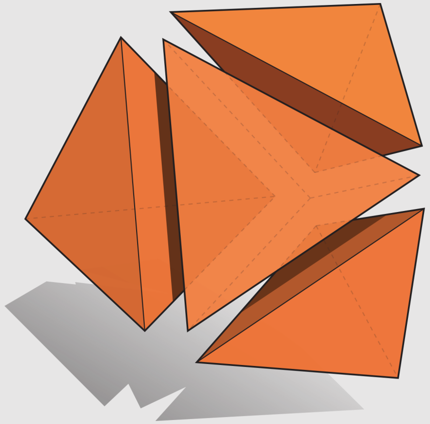
- Explicit:
  - point cloud
  - polygon meshes
  - subdivision surfaces
  - NURBS
- Implicit:
  - level set
  - constructive solid geometry
  - algebraic surface
  - L-systems
  - Fractals
- Not one best geometric representation!
  - Each is suited for a different task
  - Tradeoffs between:
    - Accuracy
    - Memory
    - Performance (searching/operating)





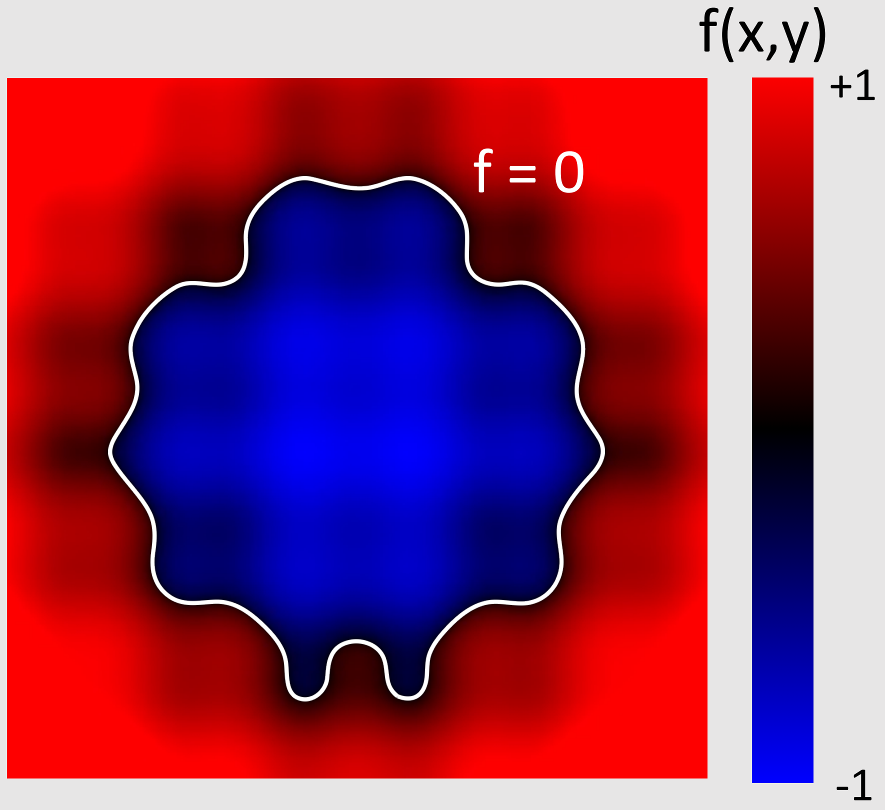






Implicit Geometry

- Points aren't known directly, but satisfy some relationship
  - Example: unit sphere is all points such that $x^2+y^2+z^2=1$
- More generally, in the form $f(x,y,z) = 0$
- Finding example points is **<u>hard</u>**
  - Requires solving equation
- Checking if points are inside/outside is **<u>easy</u>**
  - Just evaluate the function with a given point

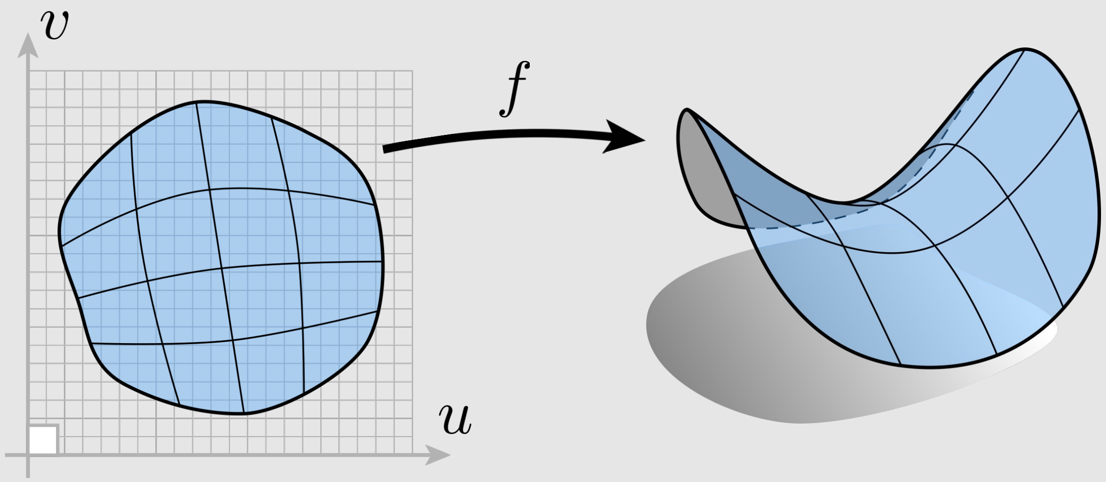


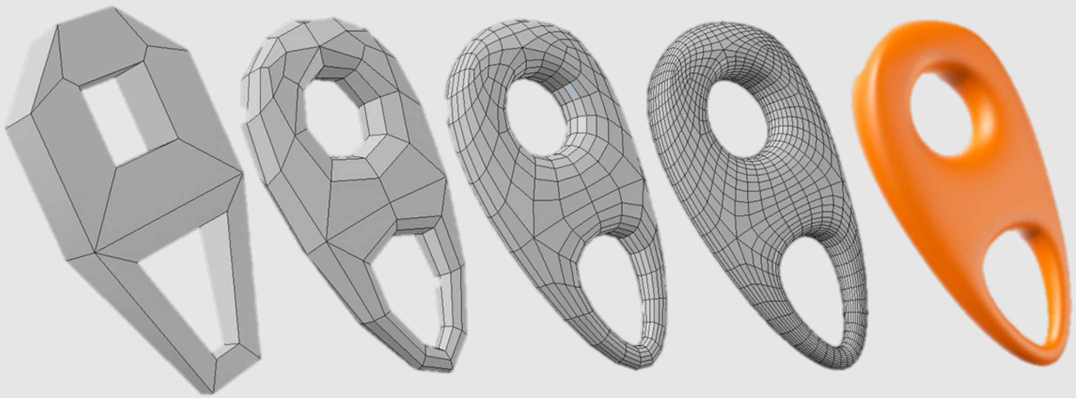
Explicit Geometry

- All points are given directly
- More generally:

$$f : \mathbb{R}^2 \to \mathbb{R}^3; (u, v) \mapsto (x, y, z)$$

  - Given any (u, v), we can find a point on the surface
  - Can limit (u, v) to some range
    - **Example:** triangle with barycentric coordinates
- Finding example points is **<u>easy</u>**
  - We are given them for free
- Checking if points are inside/outside is **<u>hard</u>**
  - We are given the output values and need to find input values that satisfy the geometry





What does **easy** and **hard** mean?

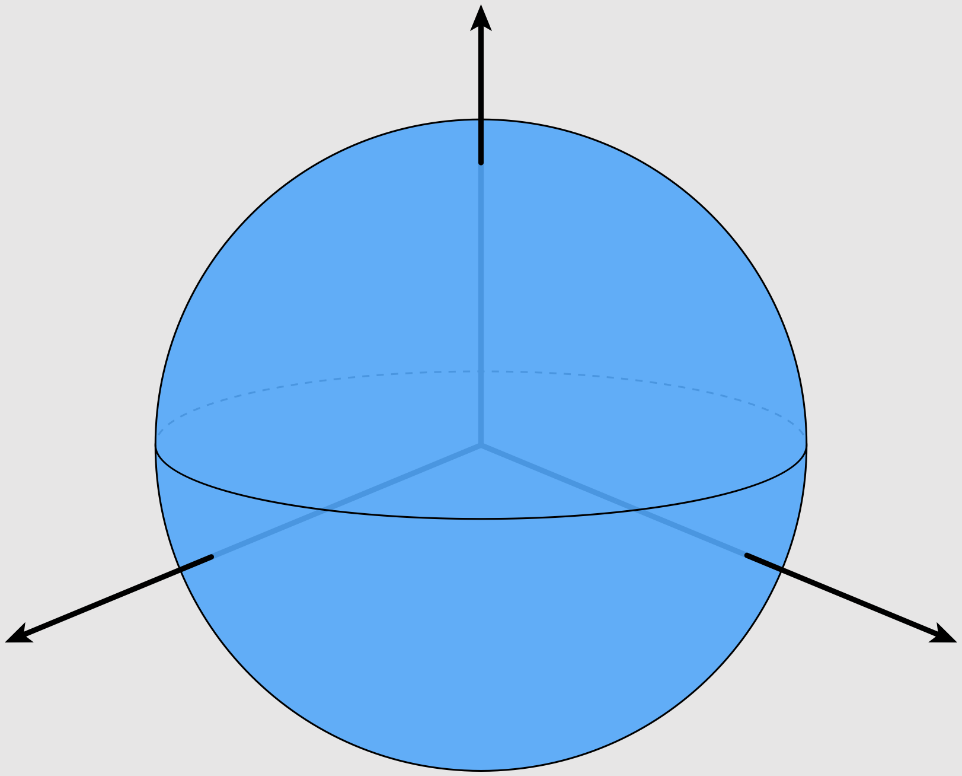
Implicit Geometry [Hard]

- Given the unit sphere:

$$f(x, y, z) = x^2 + y^2 + z^2 = 1$$

- Find a point that exists on it.

- **Answer:** (1,0,0)
    - Not so difficult, but how did you arrive at the answer?
    - We are given a constraint, and need to find parameters (x, y, z) that satisfy the constraint
        - Keep guessing and checking



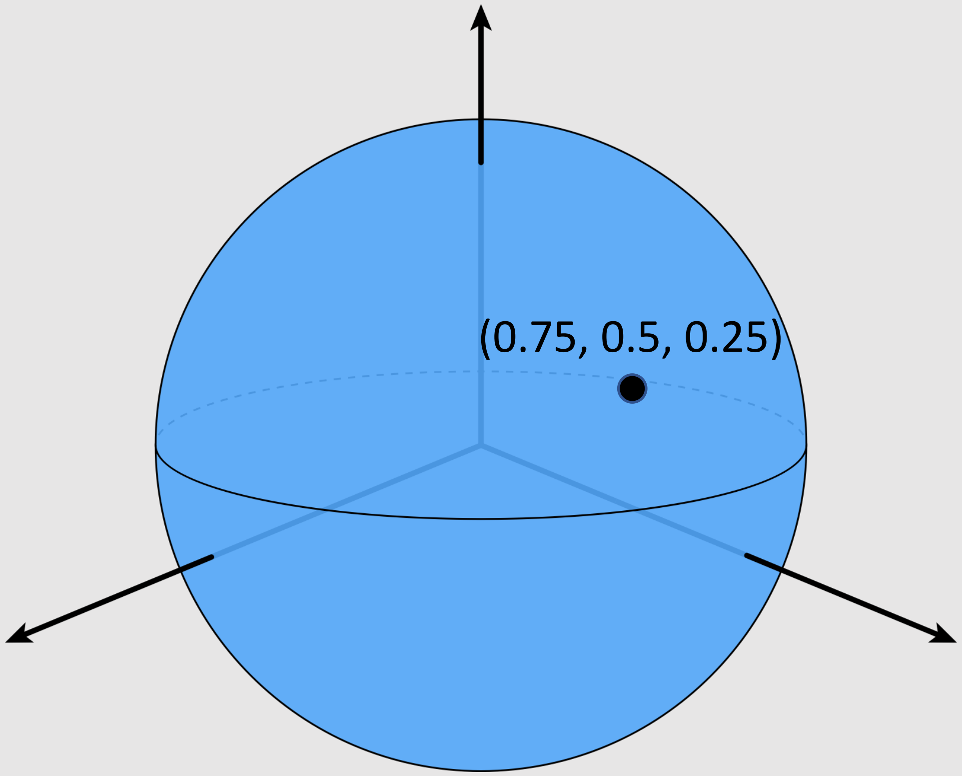
Implicit Geometry [Easy]

- Given the unit sphere:

$$f(x, y, z) = x^2 + y^2 + z^2 = 1$$

- Find if the point (0.75, 0.5, 0.25) lives inside it.

- **Answer:** yes!
  - $f(0.75, 0.5, 0.25) = 0.75^2 + 0.5^2 + 0.25^2 = 0.875 < 1$
  - Easy to check! Just evaluate the sign of the function at the desired point



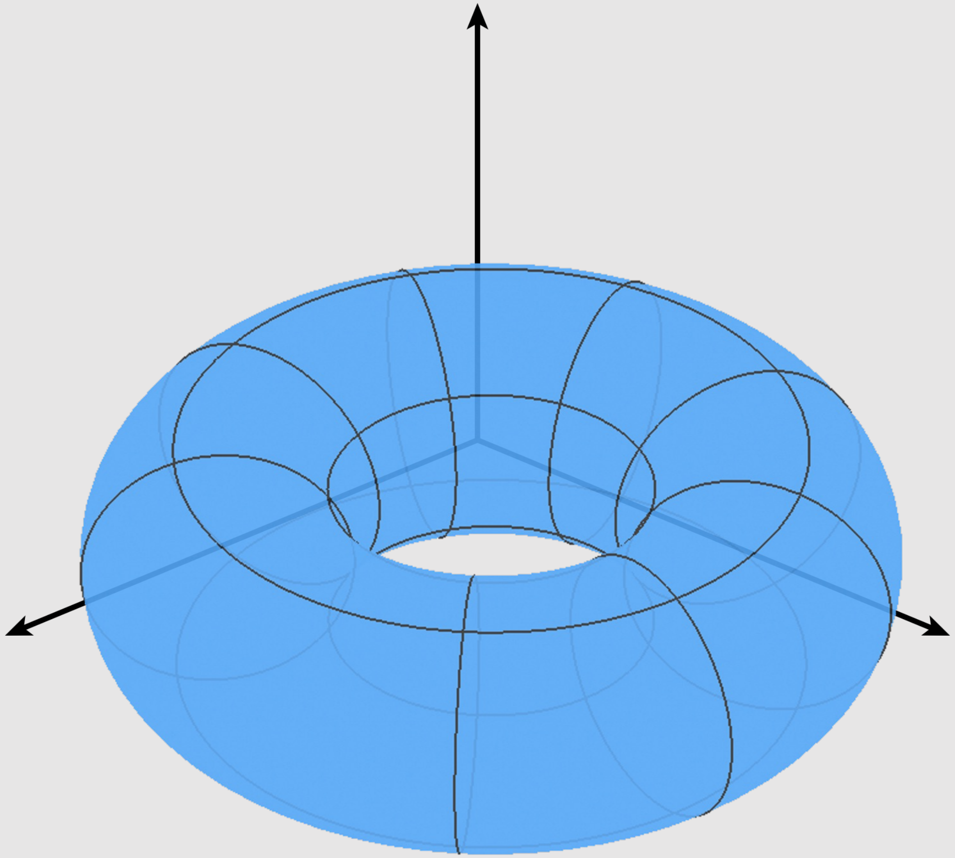
Explicit Geometry [Easy]

- Given the torus:

$$f(u,v) = ((2+\cos u)\cos v\,, (2+\cos u)\sin v\,, \sin u)$$

- Find a point that exists on it.

- **Answer:** (3,0,0)
  - Just plug in any value of (u,v)!
    - We plugged in $(u,v) = (0,0)$



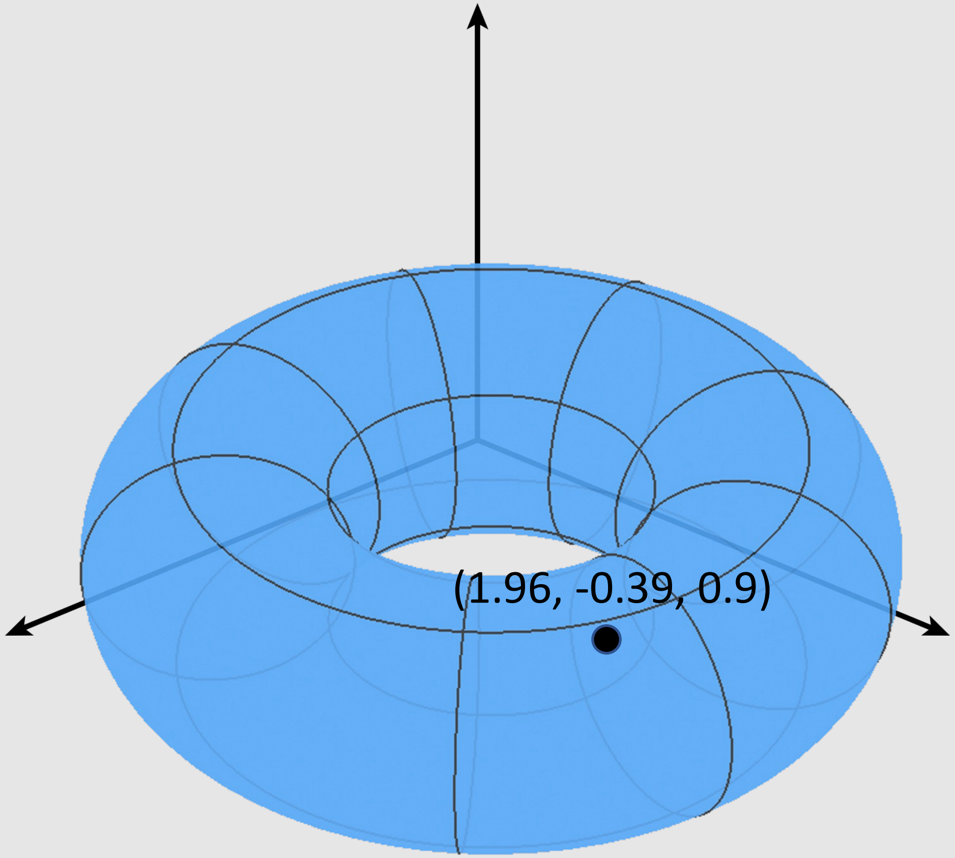
Explicit Geometry [Hard]

- Given the torus:

$$f(u, v) = ((2 + \cos u)\cos v\,, (2 + \cos u)\sin v\,, \sin u)$$

- Find if the point (1.96, -0.39, 0.9) lives inside it.

- **Answer:** no, I'm not computing that
  - We are given a constraint, and need to find parameters (u, v) that satisfy the constraint
    - Keep guessing and checking

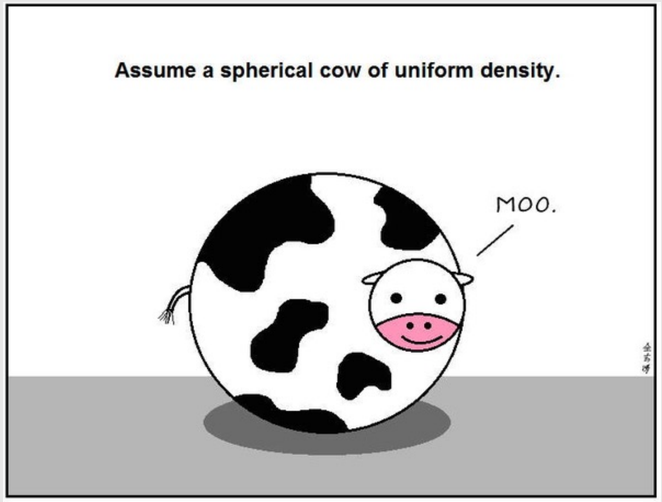


Let’s look at some implicit examples…

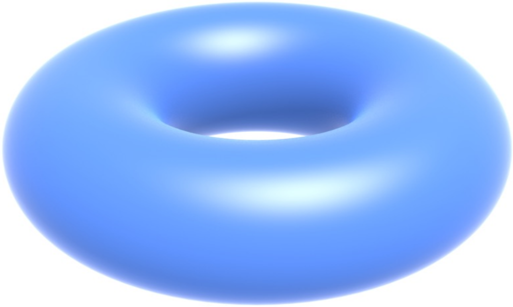
Algebraic Surfaces [Implicit]

- A surface built with algebra
  - Generally thought of as a surface where points are some radius r away from another point/line/surface

- [+] Generates smooth/symmetric surfaces
- [-] Cannot generate impurities/deformations



$$x^2+y^2+z^2=1$$

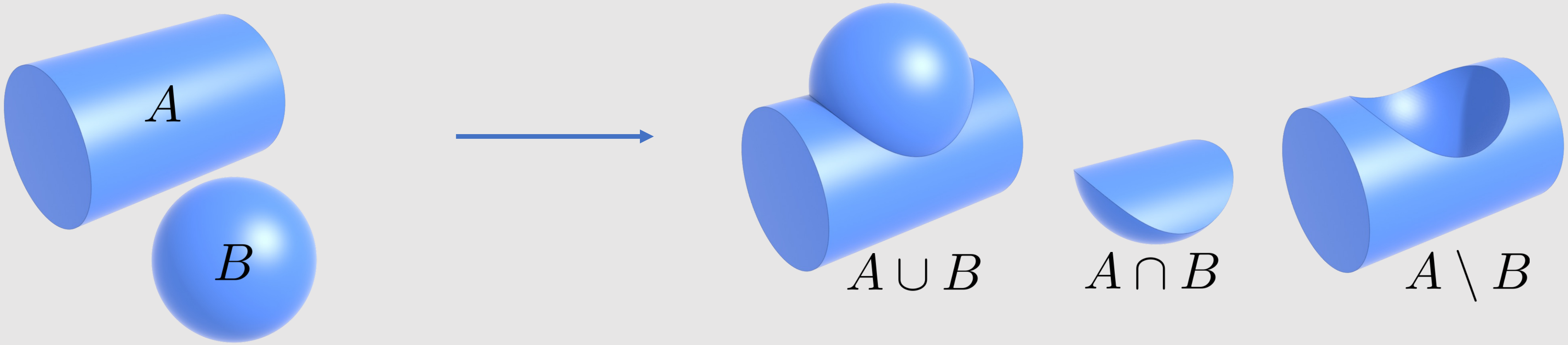


$$(R-\sqrt{x^2+y^2})^2+z^2=r^2$$

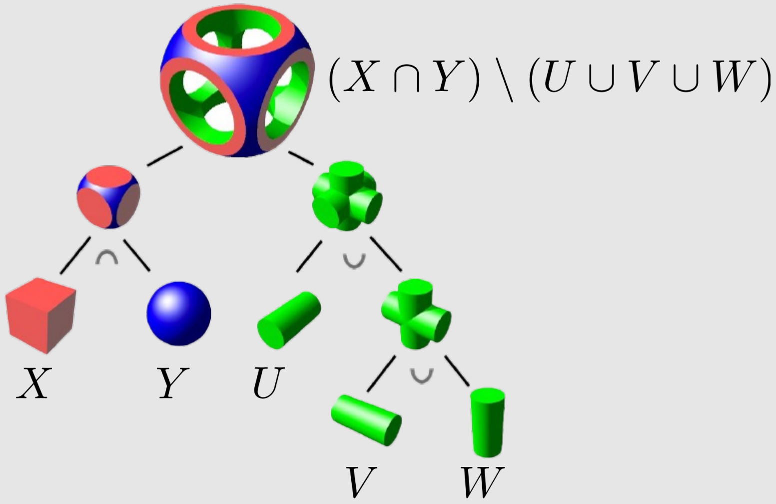
$$(x^2+\frac{9y^2}{4}+z^2-1)^3 = x^2z^3+\frac{9y^2z^3}{80}$$

Constructive Solid Geometry [Implicit]

- Build more complicated shapes via Boolean operations
  - Basic operations:

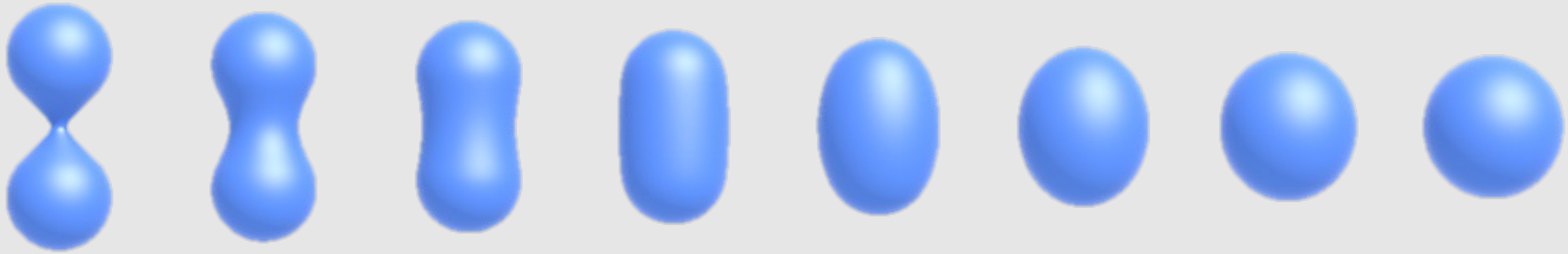


- Can be used to form complex shapes!



Blobby Surfaces [Implicit]

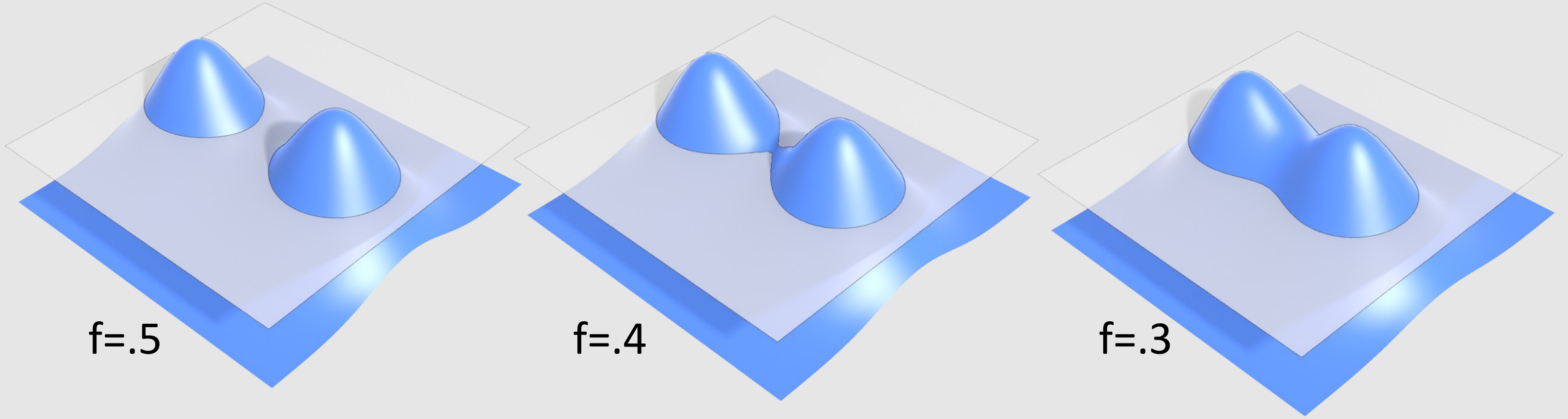
- Instead of Booleans, gradually blend surfaces together:



- Easier to understand in 2D:

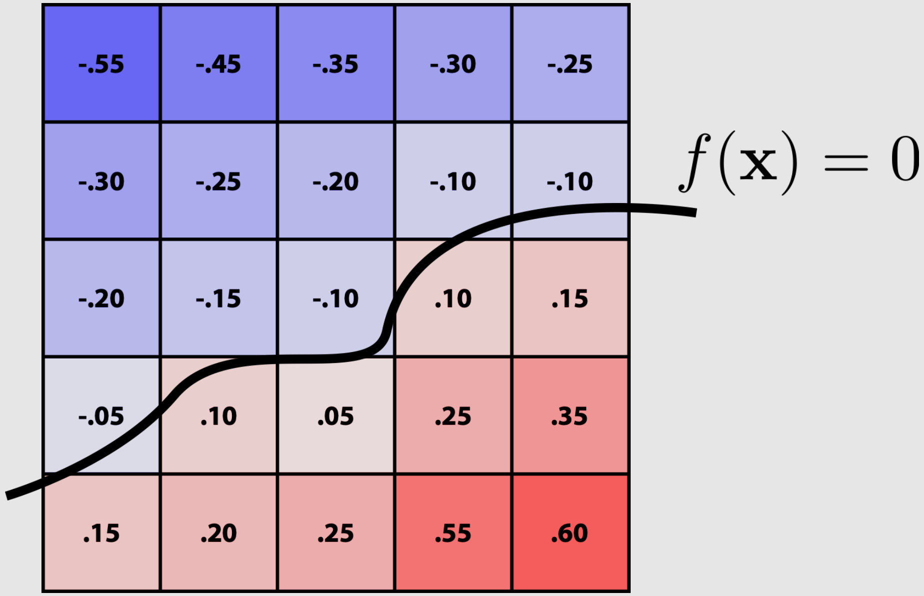
$$\phi_p(x) := e^{-|x-p|^2}$$ (Gaussian centered at p)

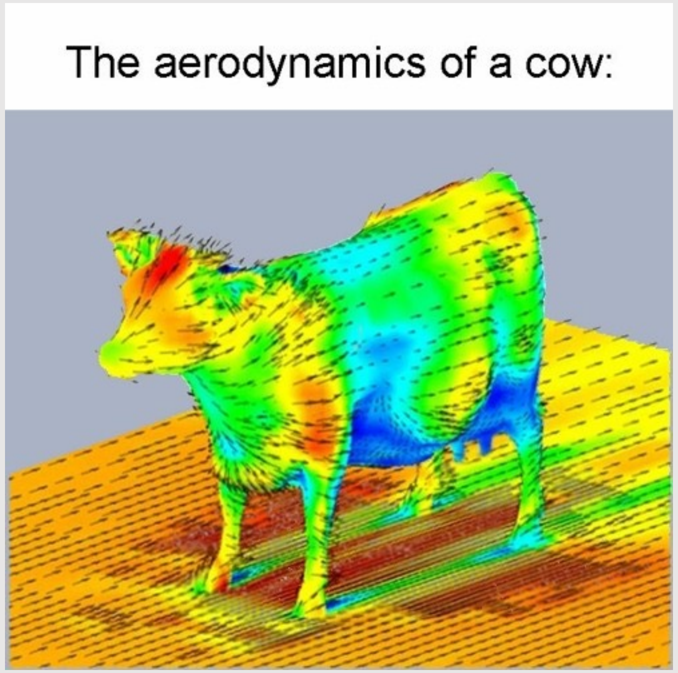
$$f := \phi_p + \phi_q$$ (Sum of Gaussians centered at different points)



Level Set Methods [Implicit]

- Store a grid of values approximating function





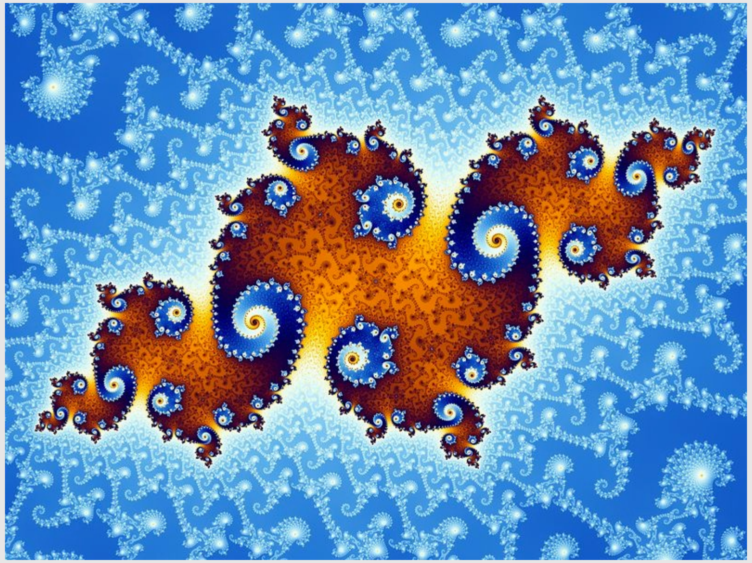
- Surface is found where interpolated values equal zero

- [+] Provides much more explicit control over shape
- [-] Runs into problems of aliasing!

Fractals [Implicit]

- No precise definition; exhibit self-similarity, detail at all scales

- [+] New "language" for describing natural phenomena
- [-] Hard to control shape!

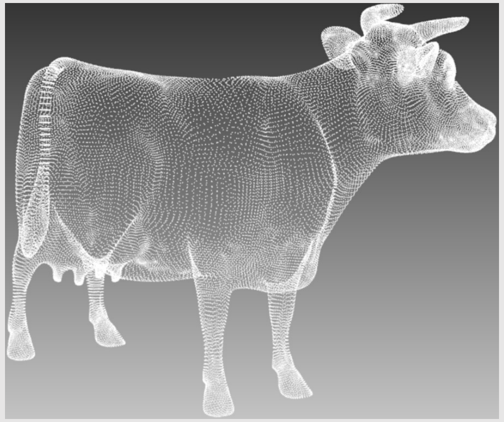


Let’s look at some explicit examples…

Point Cloud [Explicit]

- A list of points (x, y, z)
  - Often augmented with normals

- [+] Easily represent any kind of geometry
- [+] Easy to draw dense cloud (>>1 point/pixel)
- [+] Easy for simulation
- [-] Large lookup time
- [-] Large memory overhead
  - Hard to interpolate undersampled regions
  - Hard to do processing / simulation /
  - Result is just as good as the scan

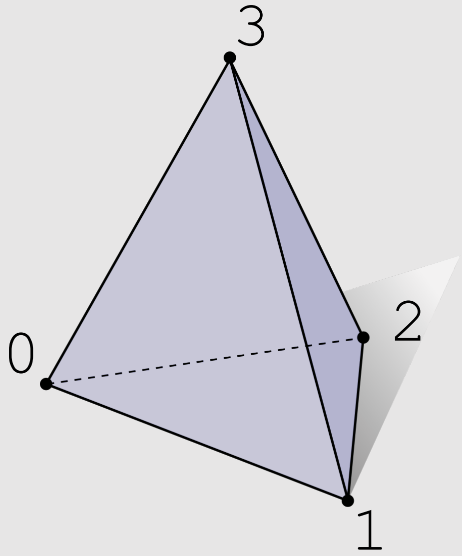


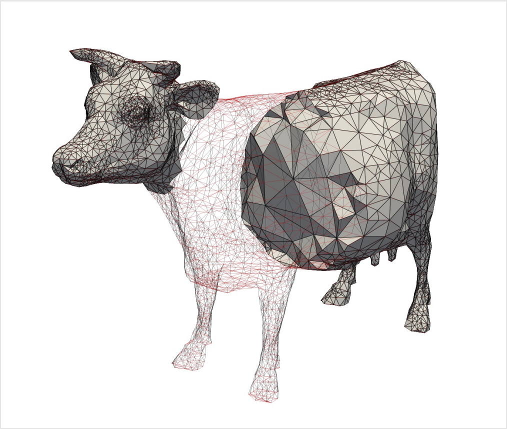
Triangle Mesh [Explicit]

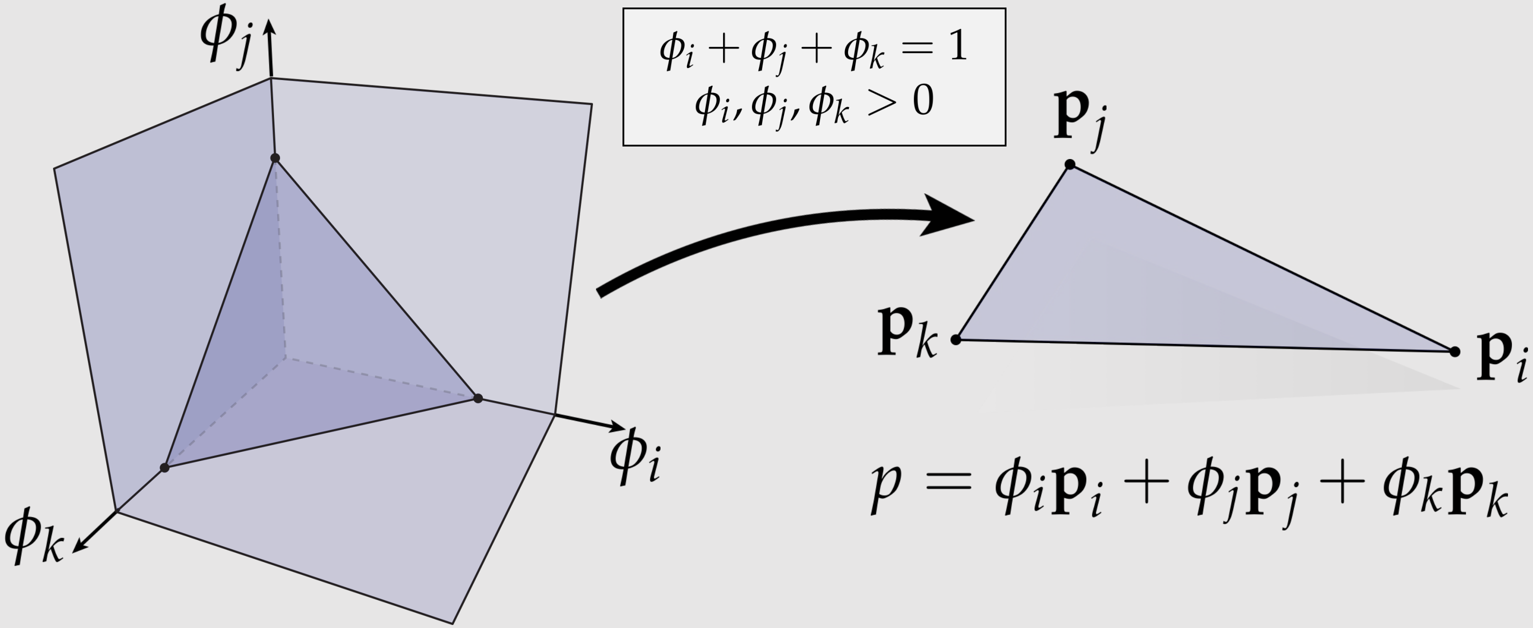
- [+] Easy interpolation with good approximation
  - Use barycentric interpolation to define points inside triangles
- [-] Large memory overhead
  - Store vertices as triples of coordinates *(x,y,z)*
  - Store triangles as triples of indices *(i,j,k)*

- Polygonal Mesh: shapes do not need to be triangles
  - Ex: quads

	[VERTICES]			[TRIANGLES]		
	x	**y**	**z**	**i**	**j**	**k**
0:	-1	-1	-1	0	2	1
1:	1	-1	1	0	3	2
2:	1	1	-1	3	0	1
3:	-1	1	1	3	1	2







$$\phi_i + \phi_j + \phi_k = 1$$
$$\phi_i, \phi_j, \phi_k > 0$$

$$p = \phi_i \mathbf{p}_i + \phi_j \mathbf{p}_j + \phi_k \mathbf{p}_k$$

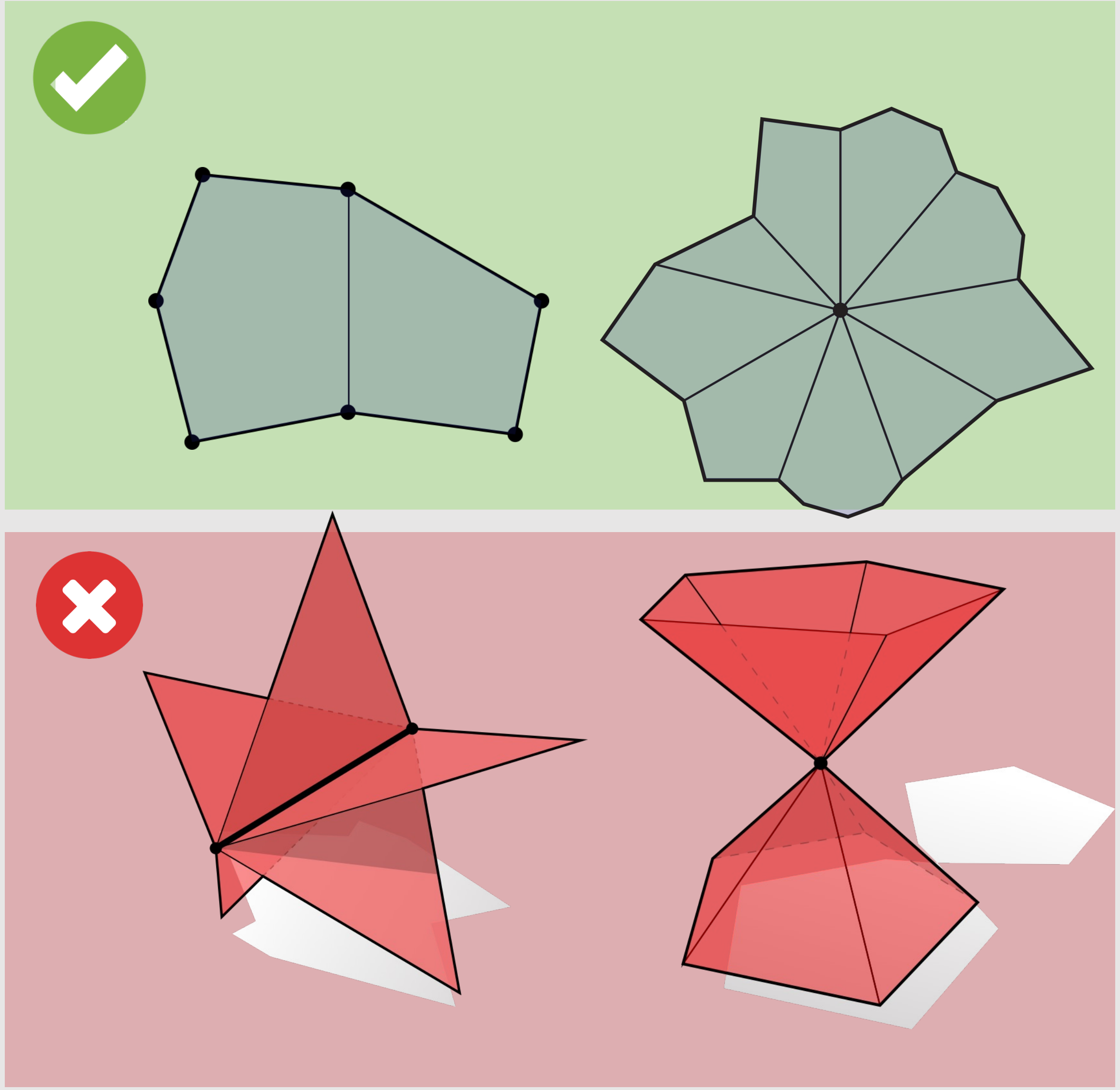
- ~~Alpha Blending~~
- ~~The Graphics Pipeline Revisited~~
- Introduction to Geometry
  - ~~Implicit & Explicit Geometry~~
  - Manifold Geometry
  - Local Geometric Operations

Manifold Assumption

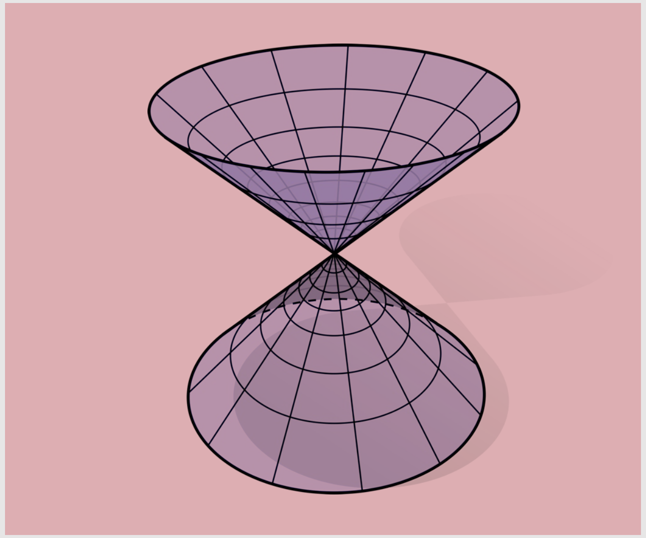
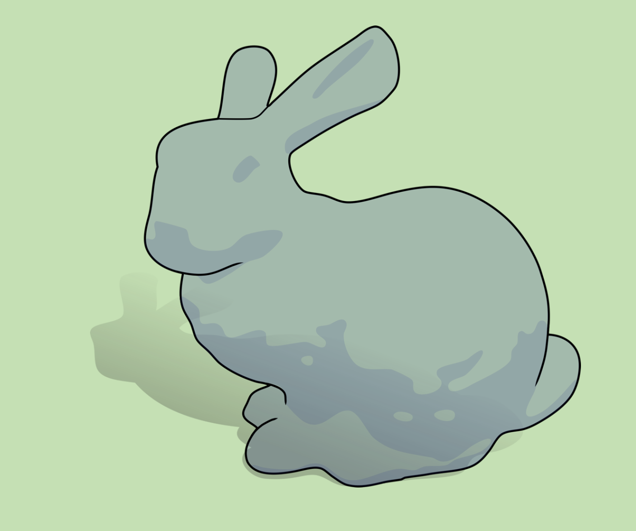
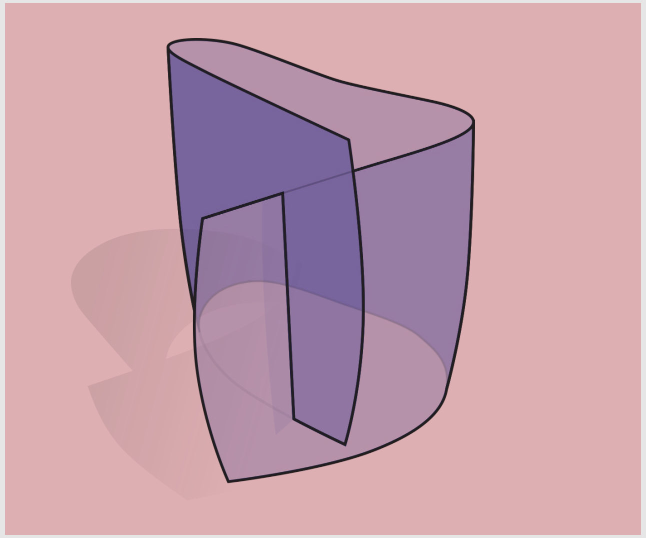
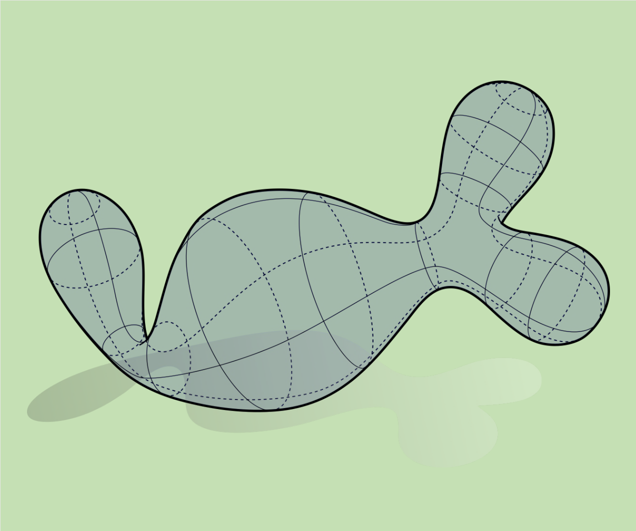
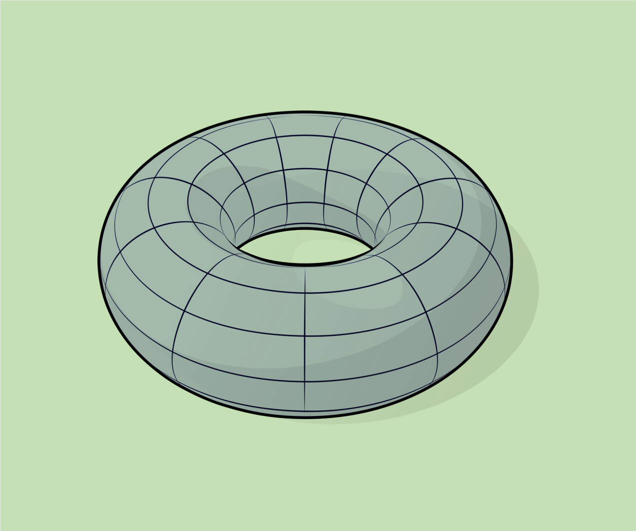
- A mesh is manifold if and only if it can exist in real life
  - Important for simulation/3D printing
- Everything in real life has volume to it
  - Likewise, every manifold surface has some volume it encases
  - Allows us to think of manifold surfaces as ‘shells’ to an inner volume
    - **Example:** M&Ms
- Everything in real life, when zoomed in far enough, should be able to have a rectangular coordinate grid
  - Likewise, every manifold surface should be planar when zoomed in far enough
    - **Example:** Planet Earth

Manifold Properties

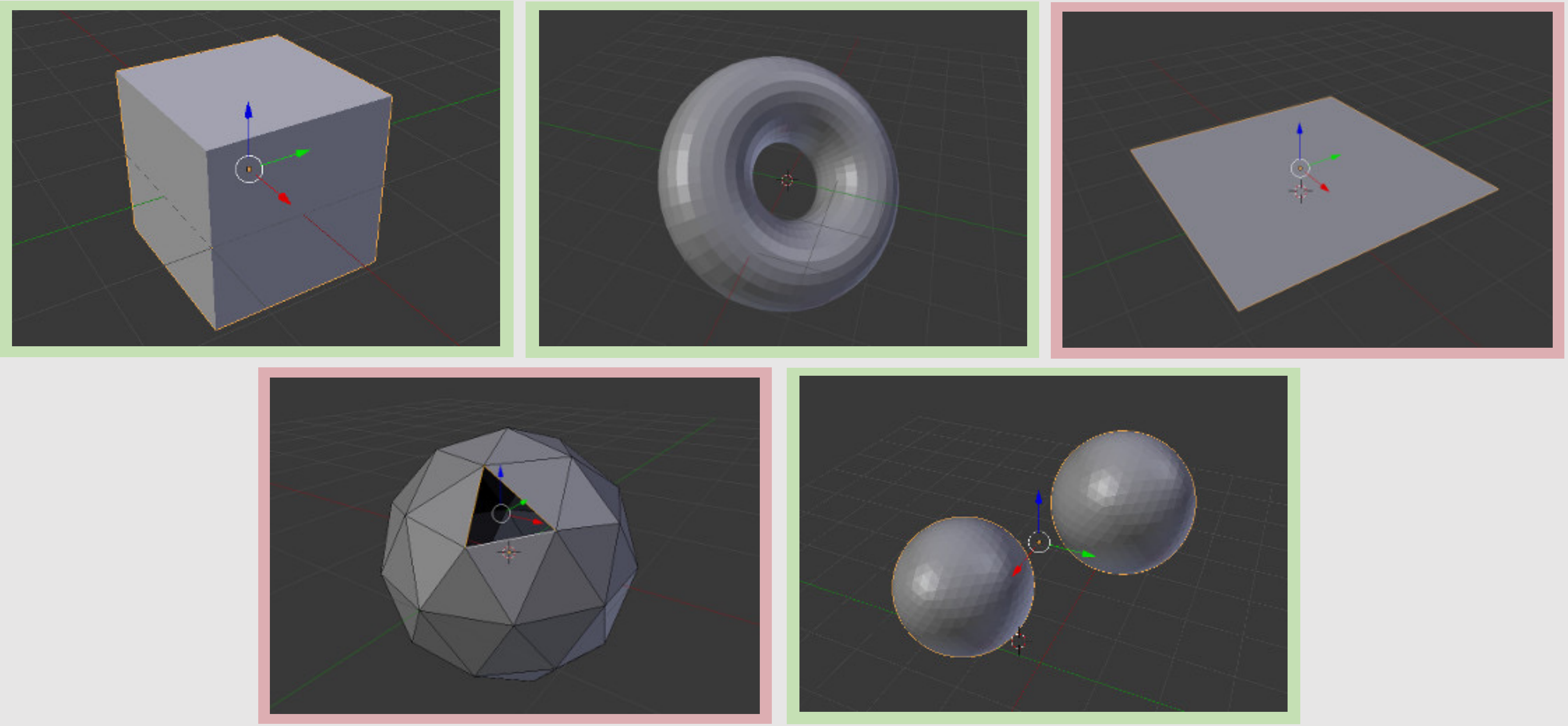
- For polygonal surfaces, check for **“fins”** and **”fans”**
- Every edge is contained in only two polygons (no **“fins”**)
  - The extra 3rd or 4th or 5th or so forth polygon is the fin of a fish
- The polygons containing each vertex make a single **“fan”**
  - We should be able to loop around the faces around a vertex in a clear way



Manifold Check

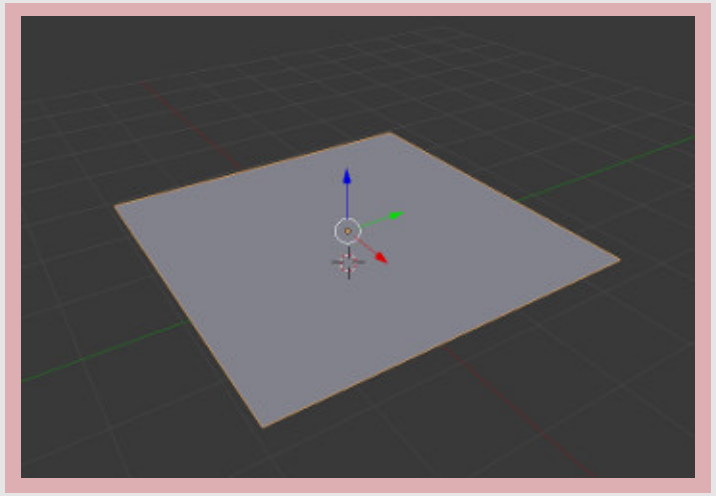
Manifold Check



**https://github.com/rlguy/Blender-FLIP-Fluids/wiki/Manifold-Meshes

Planes Are Not Manifold

- Each edge of a plane only touches 1 polygon
  - Breaks the "fin" constraint
- More intuitively: **no notion of thickness!**
  - Can not be represented in real life
  - Paper (best approximation of plane) still has thickness
- **How to make manifold:** add a second polygon that overlaps with the first plane, connecting all the edges
  - Messy, two polygon will overlap, but will fix the manifold issue
- **How to make manifold:** add a new type of edge denoting it as a boundary
  - The "boundary" edge

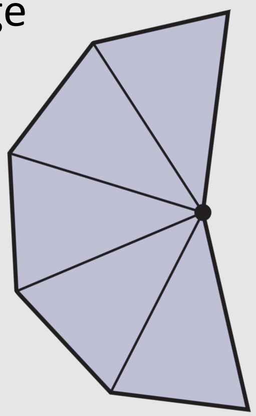


Boundary Edges

- Objects in real life (Ex: pants) have boundaries
  - Boundary geometry loops around to create the inner seams of the pants
  - The volume enclosed by pants are not where your legs go, but the physical thickness of the pants

- Representing both the inside and outside of pants is expensive!
  - Use boundary edges

- A boundary edge has 1 polygon per edge
  - **This does not mean planes are manifold!** This just gives us a way to represent complex manifold geometry as simpler non-manifold geometry



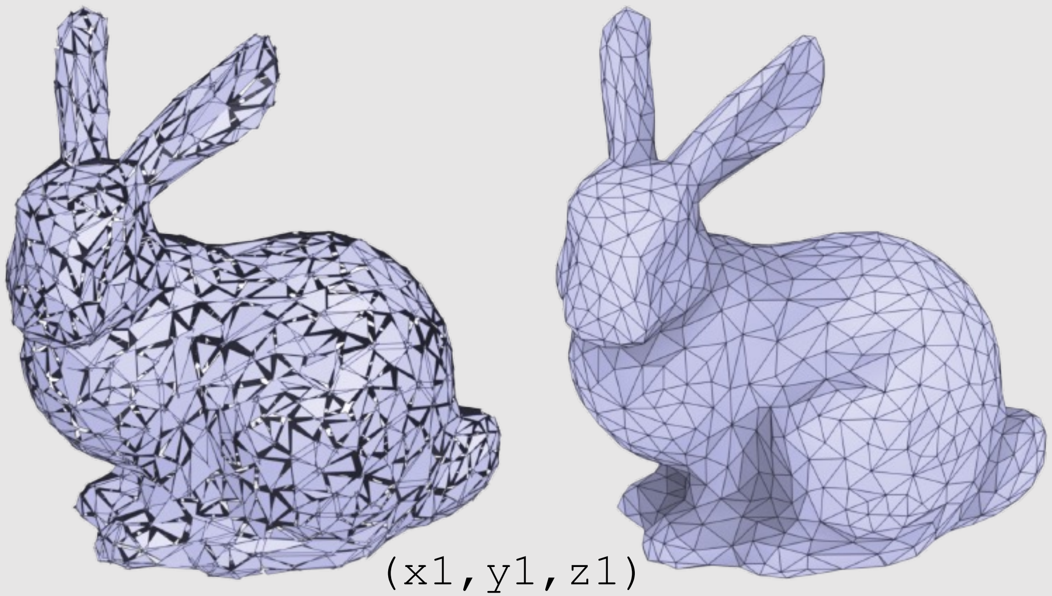
What are some ways to describe the connectivity of geometry?

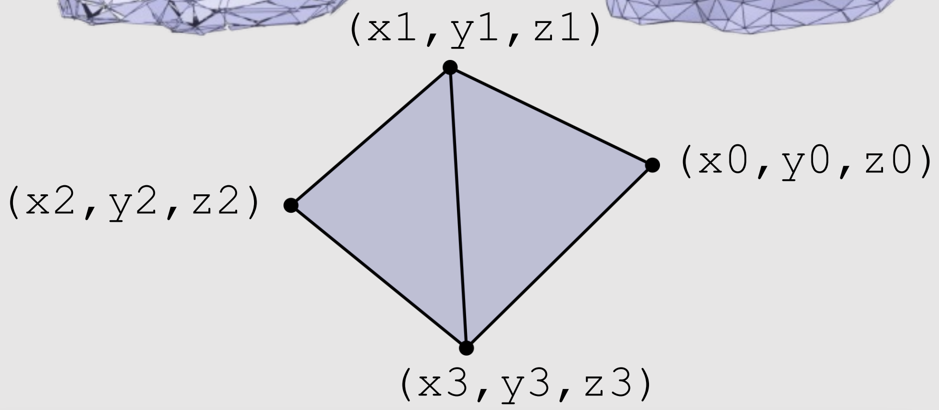
Polygon Soup

- Most basic idea imaginable:
  - For each triangle, just store three coordinates
  - No other information about connectivity
  - Not much different from point cloud
    - A “Triangle cloud”?

- **Pros:**
  - [+] Really stupid simple

- **Cons:**
  - [-] Really stupid
  - [-] Redundant storage of vertices
  - [-] Very difficult to find neighboring polygons





x0,y0,z0	x1,y1,z1	x3,y3,z3
x1,y1,z1	x2,y2,z2	x3,y3,z3

Adjacency List

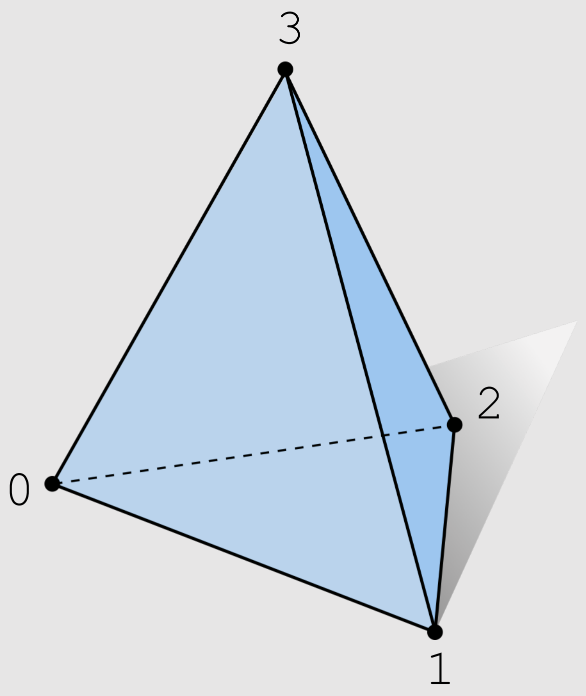
- A little more complicated:
  - Store triples of coordinates (x,y,z)
  - Store tuples of indices referencing the coordinates needed to build each triangle
- Pros:
  - [+] No duplicate coordinates
  - [+] Lower memory footprint
  - [+] Easy to keep geometry manifold
  - [+] Supports nonmanifold geometry
  - [+] Easy to change connectivity of geometry
- Cons:
  - [-] Very difficult to find neighboring polygons
  - [-] Difficult to add/remove mesh elements

VERTICES

	x	y	z
0:	-1	-1	-1
1:	1	-1	1
2:	1	1	-1
3:	-1	1	1

POLYGONS

i	j	k
0	2	1
0	3	2
3	0	1
3	1	2



Incidence Matrices

- If we want to know our neighbors, let's store them:
  - Store triples of coordinates (x,y,z) Store incidence matrix between vertices + edges, and edges + faces
    - 1 means touch, 0 means no touch
    - Store as sparse matrix

- **Pros:**
  - [+] No duplicate coordinates
  - [+] Finding neighbors is O(1)
  - [+] Easy to keep geometry manifold
  - [+] Supports nonmanifold geometry

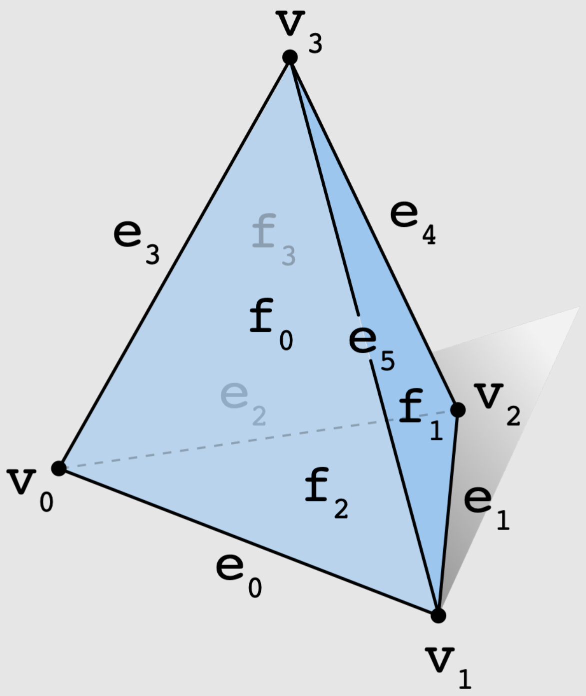
- **Cons:**
  - [-] Larger memory footprint
  - [-] Hard to change connectivity with fixed indices
  - [-] Difficult to add/remove mesh elements

VERTEX↔EDGE

	v0	v1	v2	v3
e0	1	1	0	0
e1	0	1	1	0
e2	1	0	1	0
e3	1	0	0	1
e4	0	0	1	1
e5	0	1	0	1

EDGE↔FACE

	e0	e1	e2	e3	e4	e5
f0	1	0	0	1	0	1
f1	0	1	0	0	1	1
f2	1	1	1	0	0	0
f3	0	0	1	1	1	0



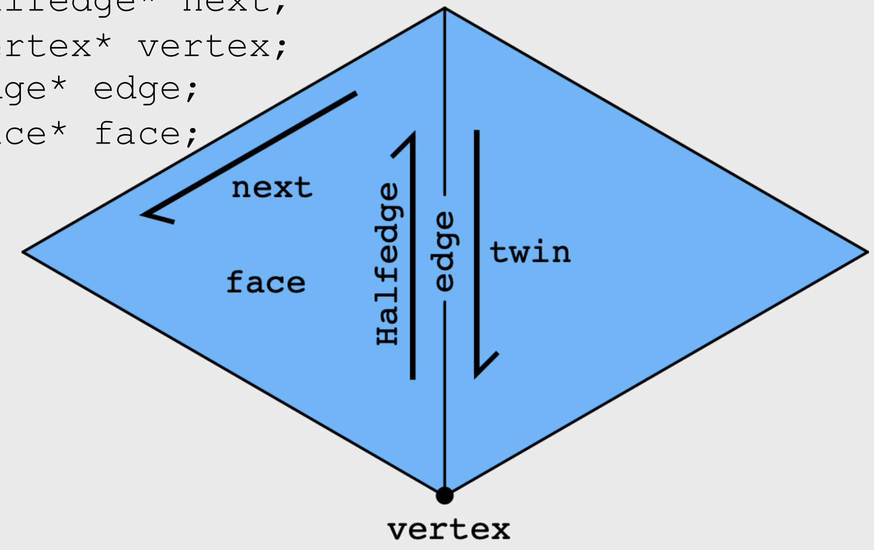
Halfedge Data Structure

- Let's store a little, but not a lot, about our neighbors:
  - Halfedge data structure added to our geometry
  - Each edge gets 2 halfedges
    - Each halfedge ”glues” an edge to a face

- **Pros:**
  - [+] No duplicate coordinates
  - [+] Finding neighbors is O(1)
  - [+] Easy to traverse geometry
  - [+] Easy to change mesh connectivity
  - [+] Easy to add/remove mesh elements
  - [+] Easy to keep geometry manifold

- **Cons:**
  - [-] Does not support nonmanifold geometry

```
struct Halfedge
{
    Halfedge* twin;
    Halfedge* next;
    Vertex* vertex;
    Edge* edge;
    Face* face;
};
```



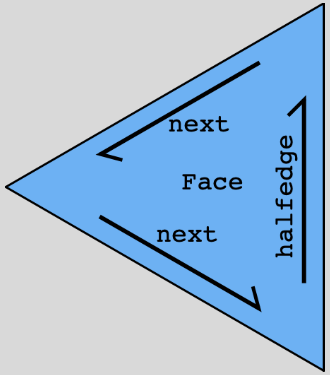
Halfedge Data Structure

```
struct Halfedge
{
   Halfedge* twin;
   Halfedge* next;
   Vertex* vertex;
   Edge* edge;
   Face* face;
};
```

- Makes mesh traversal easy
  - Use "twin" and "next" pointers to move around the mesh
  - Use "vertex", "edge", and "face" pointers to grab element

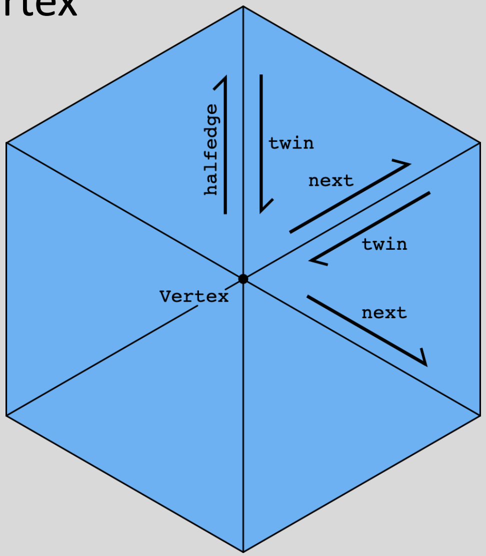
Example: visit all vertices in a face

```
Halfedge* h = f->halfedge;
do {
   h = h->next;
   // do something w/ h->vertex
}
while( h != f->halfedge );
```



Example: visit all neighbors of a vertex

```
Halfedge* h = v->halfedge;
do {
   h = h->twin->next;
}
while( h != v->halfedge );
```



Note: only makes sense if mesh is manifold!

Halfedge Data Structure

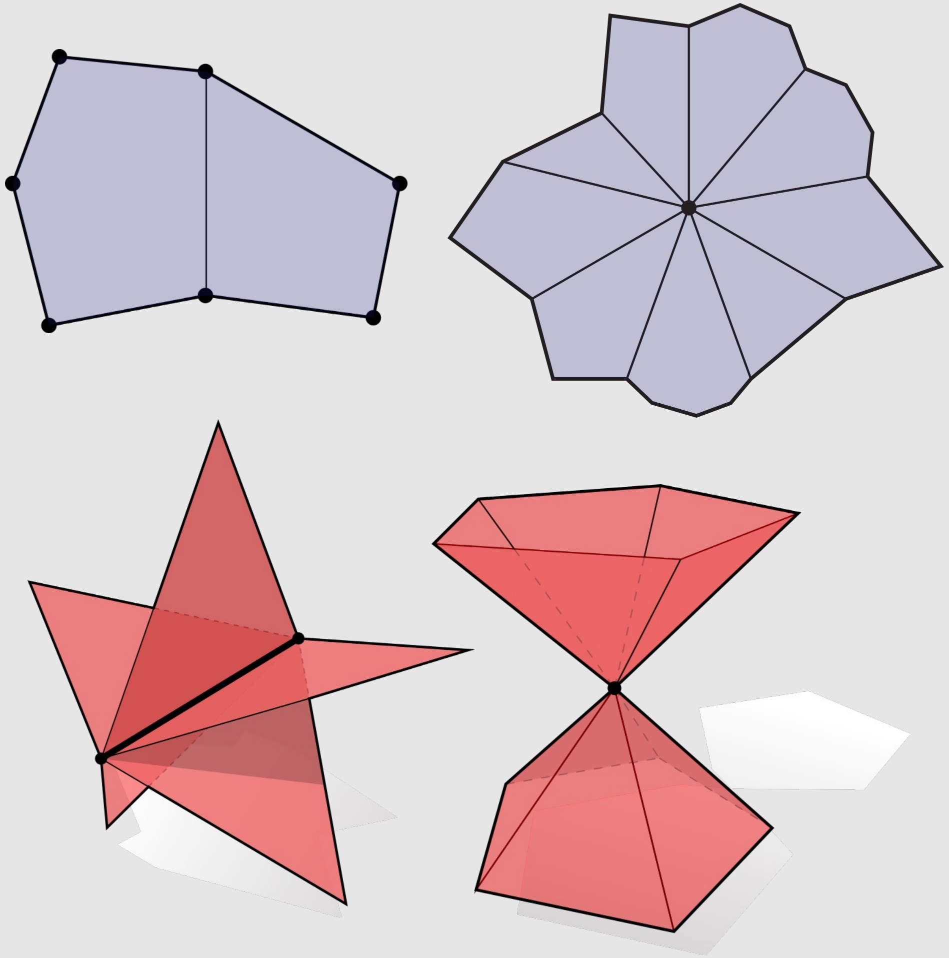
- Halfedge meshes are always manifold!

- Halfedge data structures have the following constraints:

```
h->twin->twin == h // my twin's twin is me
h->twin != h // I am not my own twin
h2->next = h //every h's is someone's "next"
```

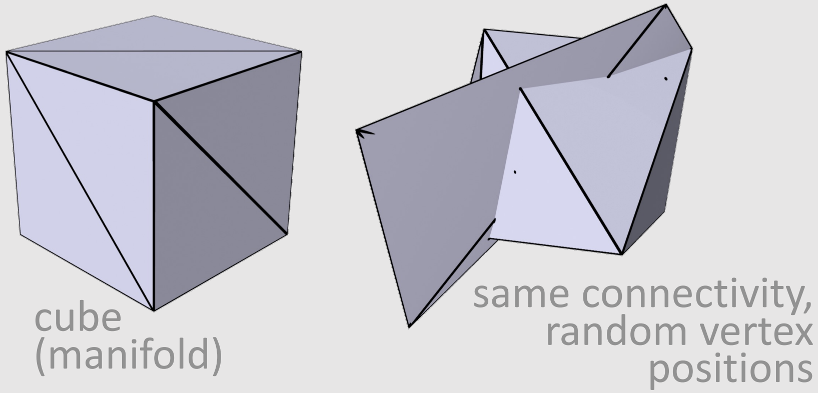
- Keep following **next** and you'll traverse a face
- Keep following **twin** and you'll traverse an edge
- Keep following **next->twin** and you'll traverse a vertex

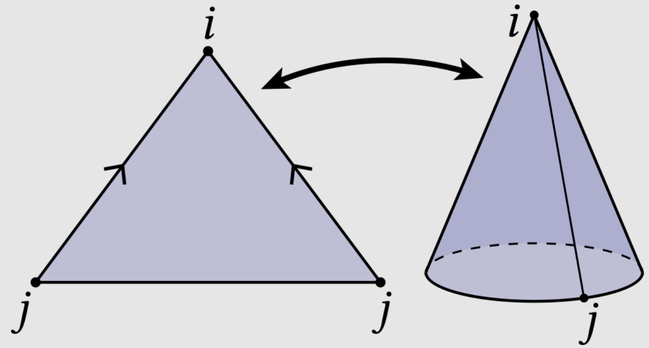
- **Q: Why, therefore, is it impossible to encode the red figures?**
  - First shape violates first 2 conditions
  - Second shape violates 3rd condition

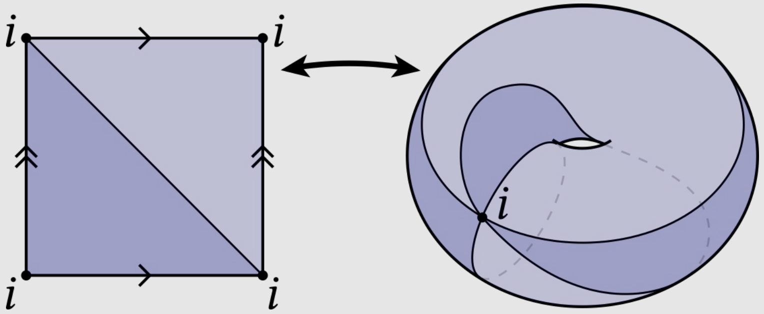


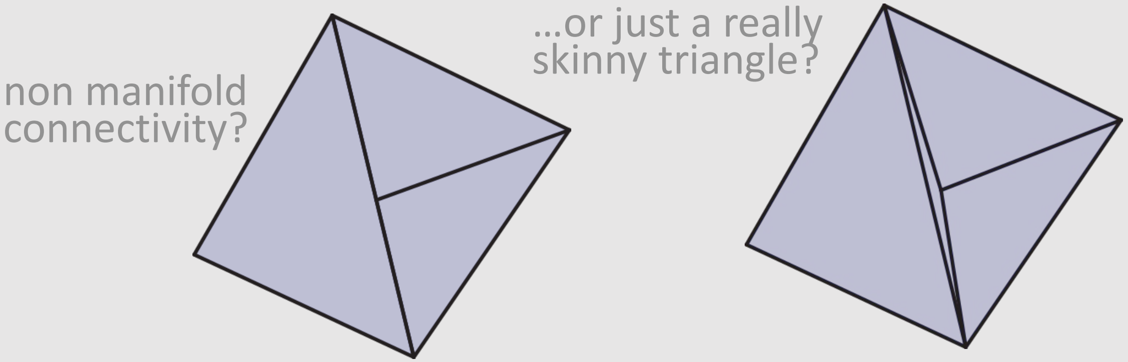
Connectivity vs Geometry

- Recall manifold conditions (fans not fins):
  - These conditions say nothing about vertex positions! Just connectivity
- Can have perfectly good (manifold) connectivity, even if geometry is awful
  - Can have perfectly good manifold connectivity for which any vertex positions give “bad” geometry!
- Leads to confusion when debugging:
  - Mesh looks “bad”, even though connectivity is fine









- ~~Alpha Blending~~
- ~~The Graphics Pipeline Revisited~~
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    - ~~Manifold Geometry~~
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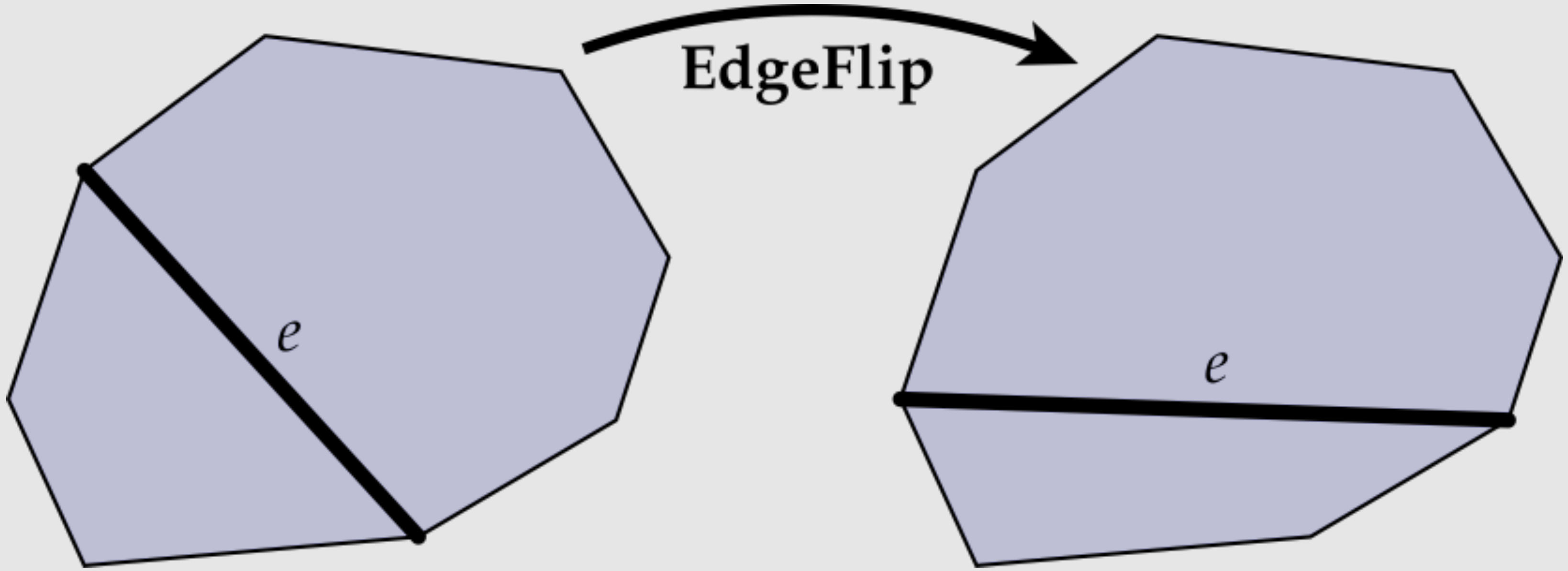
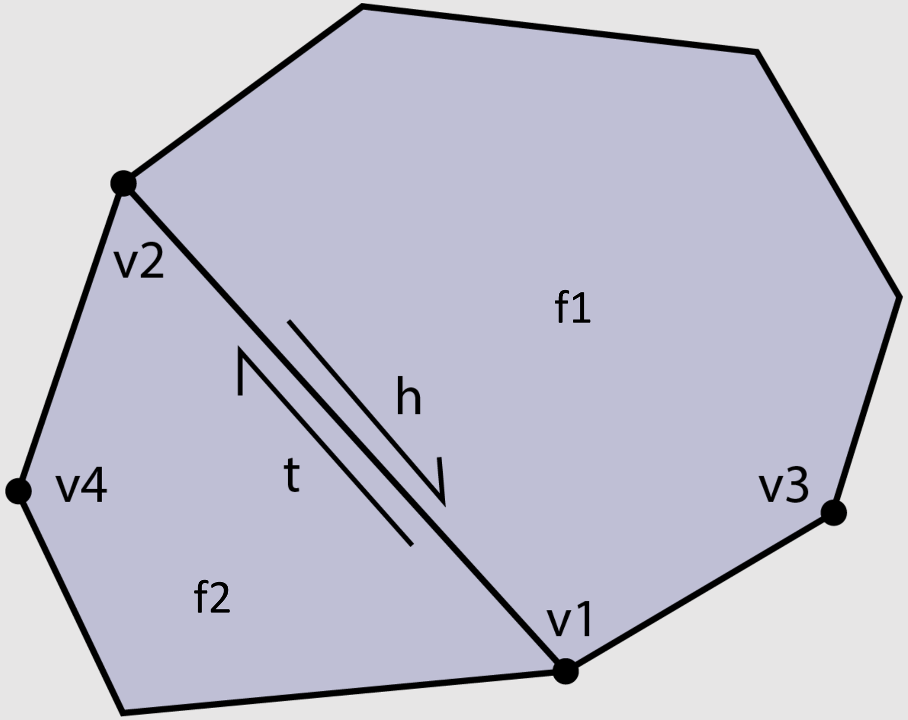
Edge Flip

Goal: Move edge e around faces adjacent to it:



- No elements created/destroyed, just pointer reassignment
- Flipping the same edge multiple times yields original results

Edge Flip



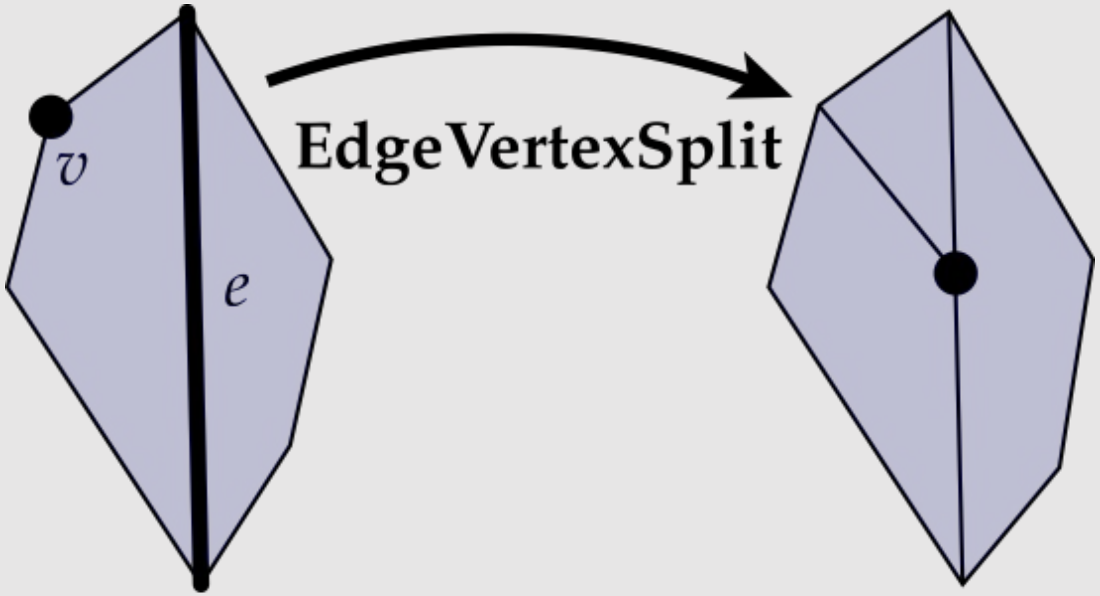

```
// collect
h = e->halfedge;
t = h->twin;
v1 = h->next->vertex;
v2 = t->next->vertex;
v3 = h->next->next->vertex;
v4 = t->next->next->vertex;
f1 = h->face;
f2 = t->face;

// disconnect
v1->halfedge = h->next;
v2->halfedge = t->next;
f1->halfedge = h;
f2->halfedge = t;

// connect
t->vertex = v3;
h->vertex = v4;
f1->halfedge = h;
f2->halfedge = t;
```

Edge Vertex Split

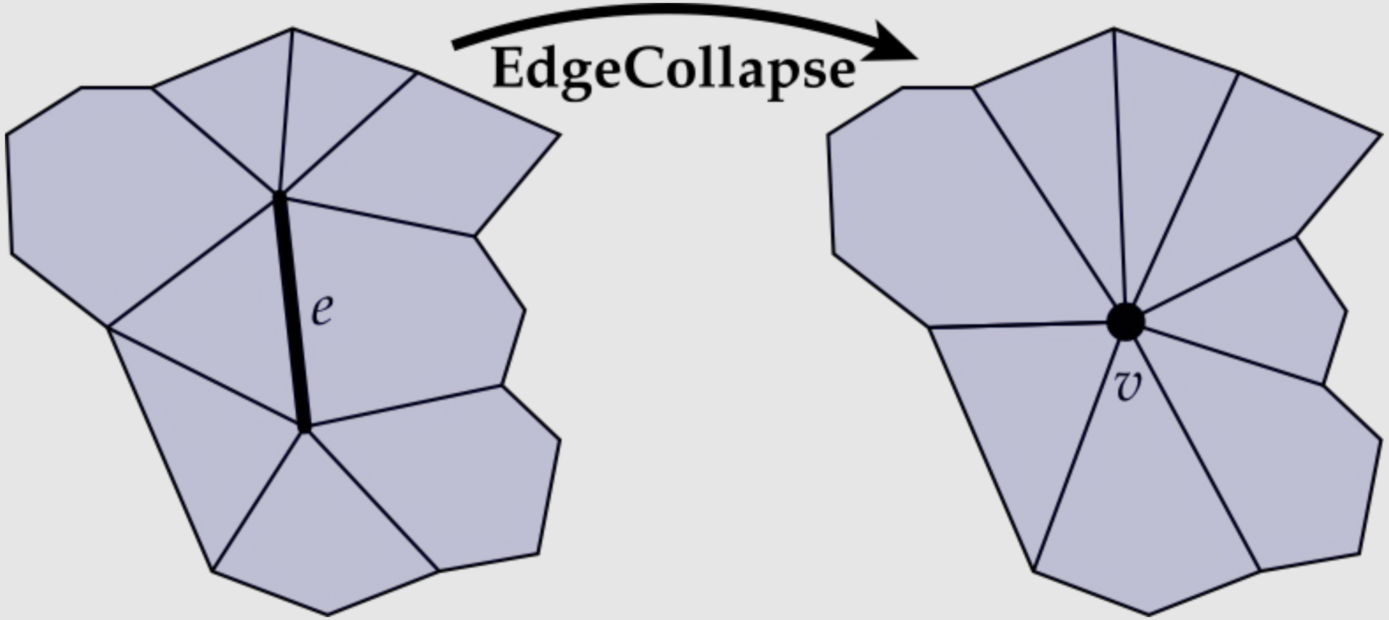
Goal: Insert edge between vertex v and midpoint of edge e:



- Creates a new vertex, new edge, and new face
- Involves much more pointer reassignments

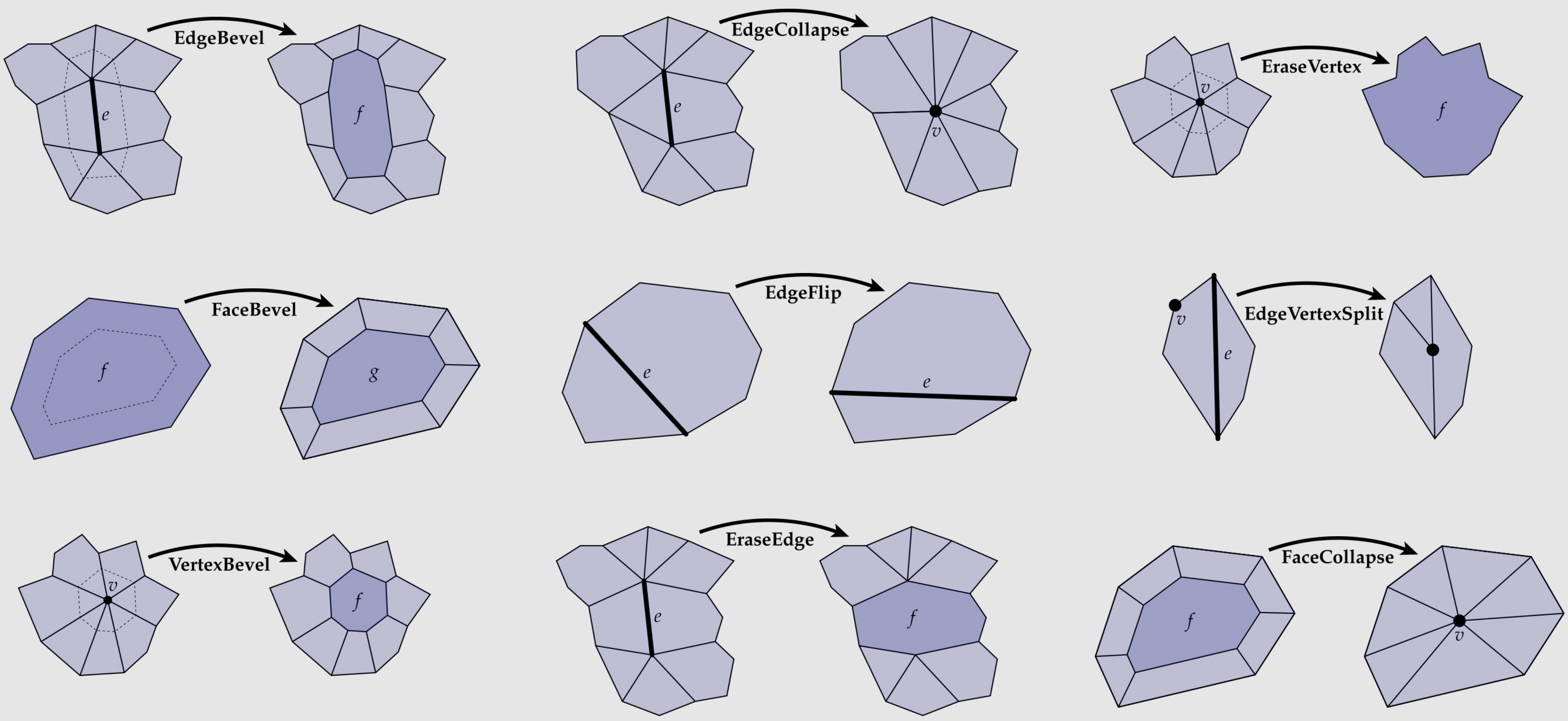
Edge Collapse

Goal: Replace edge (c,d) with a single vertex m:



- Deletes a vertex, (up to) 3 edges, and (up to) 2 faces
    - Depends on the degree of the original faces

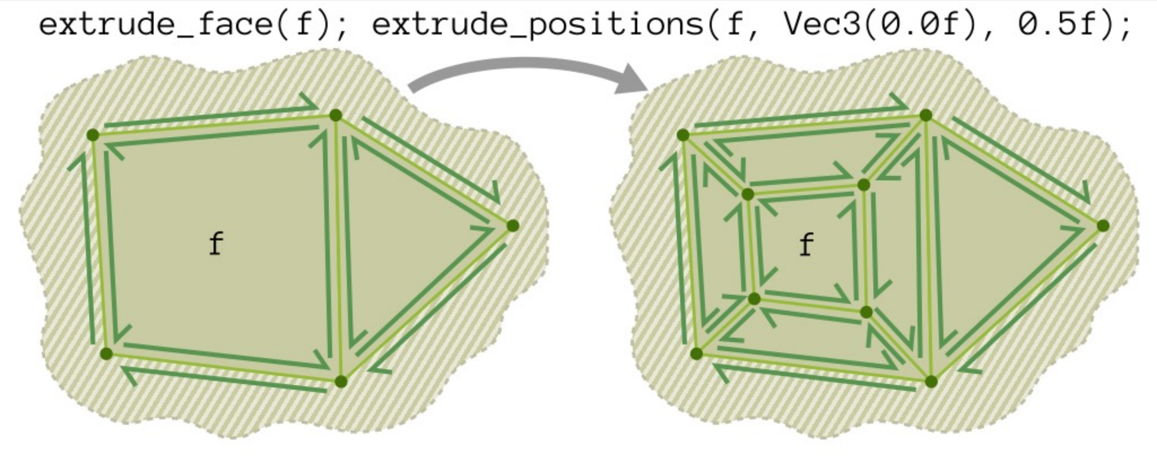
Local Operations



Many other local operations you will explore in your homework…

Local Operation Tips

- Always draw out a diagram
  - We've given you some unlabeled diagrams
  - With pen + paper, label the elements you'll need to collect/create



- Stage your code in the following way:
  - Create
  - Collect
  - Disconnect
  - Connect
  - Delete

- Write asserts around your code
  - Check if elements that should be deleted were deleted
  - Make sure there are no dangling references to anything that has been deleted
  - Make sure every element that you disconnected or reconnected is still valid
    - What it means for a vertex to be valid is not the same as what it means for an edge to be valid, etc.