Transparency & Texturing

- Barycentric Coordinates
- Texturing Surfaces
- Depth Testing
- Alpha Blending
- The Graphics Pipeline Revisited

#### The "Simpler" Graphics Pipeline



# Interpolating Values for Triangles

- **Goal:** interpolate triangle vertices for any point within triangle
- Coordinates ( $\phi_i, \phi_i, \phi_k$ ) should represent weighted average
  - $\phi_i + \phi_j + \phi_k = 1$
  - Similarly,  $1 \phi_i \phi_j = \phi_k$
  - Gives a 2D parameterization of triangle point  $(\phi_i, \phi_j)$ 
    - Known as barycentric coordinates
- If each point has some attribute  $(\alpha_i, \alpha_j, \alpha_k)$ , can linearly interpolate  $\alpha_i \phi_i + \alpha_j \phi_j + \alpha_k \phi_k$ 
  - **Example:**  $[black]\phi_i + [green]\phi_j + [red]\phi_k$



### **Barycentric Coordinates**



- Inversely proportional to the distance between the target point and a point within the triangle
- Can be computed as:

$$\phi_i(x) = d_i(x) / h_i$$

• How would you compute  $h_i$ ?  $d_i(x)$ ?



### Barycentric Coordinates [ Another Way ]



- Directly proportional to the area created by the triangle composed of the other two target points and a point within the triangle
- Can be computed as:

$$\phi_i(x) = \frac{\operatorname{area}(x, x_j, x_k)}{\operatorname{area}(x_i, x_j, x_k)}$$

\*\* Interesting read of barycentric coordinates for n-gons: https://www.inf.usi.ch/hormann/barycentric/

#### **Perspective-Incorrect Interpolation**



- Due to perspective projection (homogeneous divide), barycentric interpolation of values on a triangle with different depths is not an affine function of screen XY coordinates
- Want to interpolate attribute values linearly in **3D object space**, not image space.

#### **Perspective-Incorrect Interpolation**





If we compute barycentric coordinates using 2D (projected) coordinates, leads to (derivative) discontinuity in interpolation where quad was split

#### **Perspective-Correct Interpolation**

- **Goal:** interpolate some attribute *v* at vertices
  - Compute depth *z* at each vertex
  - Evaluate Z := 1/z and P := v/z at each vertex
  - Interpolate Z and P using standard (2D) barycentric coordinates
  - At each fragment, divide interpolated *P* by interpolated *Z* to get final value



#### **Perspective-Correct Interpolation**

$$\begin{split} \phi_{(0,0,1)} &= 0.2 & P_{(0,0,1)} &= (0,0,0)/1 & Z_{(0,0,1)} &= 1 \\ \phi_{(0,3,2)} &= 0.1 & P_{(0,3,2)} &= (1,0,0)/2 & Z_{(0,3,2)} &= 1/2 \\ \phi_{(0,5,4)} &= 0.7 & P_{(0,5,4)} &= (0,1,0)/4 & Z_{(0,5,4)} &= 1/4 \end{split}$$

$$\begin{split} P_{interp} &= 0.2 * \left[ (0,0,0)/1 \right] + 0.1 * \left[ (1,0,0)/2 \right] * 0.7 * \left[ (0,1,0)/4 \right] \\ P_{interp} &= (0.05, 0.175, 0) \end{split}$$

$$\begin{split} &Z_{interp} = 0.2 * [1/1] + 0.1 * [1/2] * 0.7 * [1/4] \\ &Z_{interp} = 0.425 \end{split}$$

q = (0.05, 0.175, 0)/0.425q = (0.12, 0.412, 0)



#### What if z is equal to 0?

Remember the near clipping plane!

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## The "Simpler" Graphics Pipeline



#### **Textures in Graphics**

- Textures are buffers of data (images) that are read into the graphics pipeline and are used for:
  - Coloring mapping
  - Normal mapping
  - Displacement mapping
  - Roughness mapping
  - Occlusion mapping
  - Reflection mapping
    - Textures can also be written into
      - Think a scratch pad for data
- Useful for maximizing quality while minimizing the number of polygons
  - Rough surfaces can be approximated by smooth surfaces with rough textures
- A single pixel of a texture is known as a **texel**



The Last of Us Part II (2020) Naughty Dog

### **Textures in Graphics**



=

[ textured geometry ]



[monochrome texture]



[ fluffy geometry ]

+

#### **Texture Coordinates**

- **Goal:** map surface geometry coordinates to image coordinates
- Barycentric coordinates let us represent 3D geometry in 2D by their surface coordinates
  - Known as surface parameterization
- Not always a 1-to-1 map!
  - A surface only half the number of pixels of a texture may only use up half the texels\*\*



\*\*We will learn ways that surfaces may use more texels than there are pixels on the surface

#### Texture Example



Each vertex has a coordinate (u,v) in texture space

#### Texture Example



Each triangle "copies" a piece of the image back to the surface

#### **Periodic Texturing**



Why do you think texture coordinates might repeat over the surface?

## Periodic Texturing



Used for tiling textures

## How Texturing Is Done

- An artist goes into a program and drags/paints/stretches/warps textures onto surfaces
  - The resulting distortion of the texture on the surface is saved as the surface parameterization
- Computing the texture mapping function is never done by hand!
  - Always use an interactive program to do it
- Also known as **uv mapping** 
  - u and v are the two barycentric coordinates that we want to map onto texture space



Texturing (2017) Blender

Texture mapping maps a non-integer coordinate to another non-integer coordinate. But textures can only be accessed via integer...



How do we know what texel(s) to sample?

# Nearest Neighbor Sampling

- Idea: Grab texel nearest to requested location in texture
- Requires:
  - 1 memory lookup
  - 0 linear interpolations

$$x' \leftarrow round(x - 0.5), \quad y' \leftarrow round(y - 0.5)$$
  
 $t \leftarrow tex. lookup(x', y')$ 



x' and y' are half-integer coordinates Helps account for 0.5 offset from texture coordinate centers

## **Bilinear Interpolation Sampling**

- Idea: Grab nearest 4 texels and blend them together based on their inverse distance from the requested location
  - Blend two sets of pixels along one axis, then blend the remaining pixels
- Requires:
  - 4 memory lookup
  - 3 linear interpolations



$$x' \leftarrow floor(x - 0.5), \quad y' \leftarrow floor(y - 0.5)$$

$$\begin{array}{l} \Delta x \leftarrow x - x' \\ \Delta y \leftarrow y - y' \end{array}$$

$$\begin{split} t_{(x,y)} &\leftarrow tex. \, lookup(x',y') \\ t_{(x+1,y)} &\leftarrow tex. \, lookup(x'+1,y') \\ t_{(x,y+1)} &\leftarrow tex. \, lookup(x',y'+1) \\ t_{(x+1,y+1)} &\leftarrow tex. \, lookup(x',+1\,y'+1) \end{split}$$

$$t_x \leftarrow (1 - \Delta x) * t_{(x,y)} + \Delta x * t_{(x+1,y)} t_y \leftarrow (1 - \Delta x) * t_{(x,y+1)} + \Delta x * t_{(x+1,y+1)}$$

$$t \leftarrow (1 - \Delta y) * t_x + \Delta y * t_y$$

Minification vs. Magnification



- Magnification [ Nearest Neighbor, Bilinear ]:
  - Example: camera is very close to scene object
  - Single screen pixel maps to tiny region of texture
  - Can just interpolate value at screen pixel center
- Minification [ ??? ]
  - *Example:* scene object is very far away
  - Single screen pixel maps to large region of texture
  - Need to compute average texture value over pixel to avoid aliasing

# Aliasing Due To Minification



# Pre-Filtering Texture



## **Texture Pre-Filtering**

- **Texture aliasing** occurs because a single pixel on the screen covers many pixels of the texture
- Ideally, want to average a bunch of texels in a very large region (expensive!)
  - Instead, we can pre-compute the averages (once) and just look up these averages (many times) at run-time
- Q: Which averages to pre-compute
  - A: a lot of them!



#### Mip-Map [L. Williams '83]

- **Rough idea:** precompute a prefiltered image at every possible scale
  - The image at depth d is the result of applying a 2x2 avg filter on the image at depth d-1
    - The image at depth 0 is the base image
- Mip-Map generates  $log_2[min(wth, hgt)] + 1$  levels
  - Each level the width and height gets halved
- Memory overhead: (1+1/3)x original texture

• 
$$1 + \frac{1}{4} + \frac{1}{16} + \dots = \sum \frac{1}{4}^{j} = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$



#### Mip-Map [L. Williams '83]

- Storing an RGB Mip-Map can be fit into an image twice the width and twice the height of the original image
  - See diagram for proof : )
  - Does not work as nicely for RGBA!
- Issue: bad spatial locality
  - Requesting a texel requires lookup in 3 very different regions of an image





Which mip-map level do we use?

## Sponza Bilinear Interpolation [Level 0]



## Sponza Bilinear Interpolation [Level 2]



## Sponza Bilinear Interpolation [Level 4]



# Sponza Bilinear Interpolation [Varying Level]



# Sponza Visualization of Level



# Computing MipMap Depth

- Correlation between distance of surface to camera and level of mip-map accessed
  - More specifically, correlation between screenspace movement across the surface compared to texture movement and level of mip-map access
- If moving over a pixel in screen space is a big jump in texture space, then we call it **minification** 
  - Sample from a lower level of mip-map
- If moving over a pixel in screen space is a small jump in texture space, then we call it **magnification** 
  - Sample from a higher level of mip-map


### Computing MipMap Depth



Where dx and dy measure the change in screen space and du and dv measure the change in texture space

$$L_x^2 = \left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2 \qquad L_y^2 = \left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2$$
$$L = \sqrt{\max(L_x^2, L_y^2)}$$

*L* measures the Euclidean distance of the change. We take the max to get a single number.



The mipmap level is not an integer... Which level do we use?

#### **Trilinear Interpolation Sampling**

- Idea: Perform bilinear interpolation on two layers of the mip-map that represents proper minification/magnification, blending the results together
- Requires:
  - 8 memory lookup
  - 7 linear interpolations





$$L \leftarrow \sqrt{\max(L_x^2, L_y^2)}$$
$$d \leftarrow \log_2 L$$

$$d' \leftarrow floor(d)$$
  
$$\Delta d \leftarrow d - d'$$

$$\begin{split} t_d &\leftarrow tex[d']. \, bilinear(x, y) \\ t_{d+1} &\leftarrow tex[d'+1]. \, bilinear(x, y) \\ t &\leftarrow (1 - \Delta d) * t_d + \Delta d * t_{d+1} \end{split}$$

# **Trilinear Interpolation Sampling**

- **Idea:** Perform bilinear interpolation on two layers of the mip-map that represents proper minification/magnification, blending the results together
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 $dv^2$ 

 $\frac{dx}{dv^2}$ 

du<sup>2</sup>

### **Trilinear Assumption**

- Trilinear filtering assumes that samples shrink at the same rate along *u* and *v* 
  - Taking the max says we would rather overcompensate than undercompensate filtering
- Bilinear and Trilinear filtering are **isotropic** filtering methods
  - iso same, tropic direction
  - Values should be same regardless of viewing direction
- What does it mean for samples to shrink at very different rates along *u* and *v*?
  - Think of a plane rotated away from the camera
    - Changes in *v* larger than changes in *u*



- What does it mean for samples to shrink at very different rates a or d v?
  - Think of a plane rotated away from the camera
    - Changes in v larger than changes in u



#### **Anisotropic Filtering**

- Anisotropic filtering is dependent on direction
  - *an* not, iso same, *tropic* direction
- Idea: create a new texture map that downsamples the x and y axis by 2 separately
  - Instead of taking the max, use each coordinate to index into correct location in map

$$L = \int par(L_x^2, L_y^2)$$
$$(d_x, d_y) = (log_2 \sqrt{L_x^2}, log_2 \sqrt{L_y^2})$$

- Texture map is now a grid of downsampled textures
  - Known as a RipMap



# Rip Map

- Same idea as MipMap, but for anisotropic filtering
  - 4x memory footprint
  - New width:  $w' = w + \frac{w}{2} + \frac{w}{4} + \dots = 2w$
  - New height:  $h' = h + \frac{h}{2} + \frac{h}{4} + \dots = 2h$ 
    - New area: w'h' = 4wh
- Fun fact: a MipMap is just the diagonal of a RipMap
  - If  $d_x = d_y$ , then we have trilinear interpolation



#### Isotropic vs Anisotropic Filtering



overbluring in u direction

[ anisotropic ]

[isotropic (trilinear)]

# Sampling Comparisons

	[ Nearest ]	[ Bilinear ]	[ Trilinear ]	[ Anisotropic ]
No. samples	1	4	8	16
No. interps	0	3	7	15
No. operations	~3	~19	>54	>54
Texture locality	good	good	bad	very bad
Memory overhead	1x	1x	4/3x	4x
Anti-aliasing	bad	normal	good	great

# **Texture Sampling Pipeline**

- 1. Compute u and v from screen sample (x,y) via barycentric interpolation
- 2. Approximate du/dx, du/dy, dv/dx, dv/dy by taking differences of screen-adjacent samples
- 3. Compute mip map level *d*
- 4. Convert normalized [0,1] texture coordinate (u,v) to pixel locations  $(U,V) \in [W,H]$  in texture image
- 5. Determine addresses of texels needed for filter (e.g., eight neighbors for trilinear)
- 6. Load texels into local registers
- 7. Perform tri-linear interpolation according to (U,V,d)
- 8. (...even more work for anisotropic filtering...)

Lot of repetitive work every time we want to shade a pixel!

GPUs instead implement these instructions on fixed-function hardware.

This is why we have texture caches and texture filtering units.

- Barycentric Coordinates
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#### The "Simpler" Graphics Pipeline



- For each **sample**, the depth buffer stores the depth of the closest triangle seen so far
  - Done at the sample granularity, not pixel granularity



# Depth of a Triangle



- A triangle is composed of 3 different 3D points, each with a depth value *z*
- To get the depth at any point (*x*, *y*) inside the triangle, interpolate depth at vertices with barycentric coordinates















— sample passed depth test



# Depth Buffer (Z-buffer) Per Sample



#### Depth Buffer (Z-buffer) Per Sample



Able to capture triangle intersections by performing tests per sample

### Depth Buffer (Z-buffer) Sample Code

```
draw_sample(x, y, d, c) //new depth d & color c at (x,y)
{
    if(d < zbuffer[x][y])
    {
        // triangle is closest object seen so far at this
        // sample point. Update depth and color buffers.
        zbuffer[x][y] = d; // update zbuffer
        color[x][y] = c; // update color buffer
    }
    // otherwise, we've seen something closer already;
    // don't update color or depth
}</pre>
```

Why is it that we first shade the pixel and then assign the resulting color after depth check? **Deferred shading** (advanced algorithm) fixes this issue.

- Barycentric Coordinates
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# Alpha Values



- Another common image format: RGBA
  - Alpha channel specifies 'opacity' of object
  - Basically how transparent it is
  - Most common encoding is 8-bits per channel (0-255)
- Compositing A over B != B over A
  - Consider the extreme case of two opaque objects...

### Non-Premultiplied Alpha

• **Goal:** Composite image *B* with alpha  $\alpha_B$  over image *A* with alpha  $\alpha_A$ 

$$A = (A_r, A_g, A_b)$$
$$B = (B_r, B_g, B_b)$$

• <u>Composite RGB:</u> what B lets through • <u>Composite Alpha:</u>  $C = \alpha_B B + (1 - \alpha_B) \alpha_A A$   $\alpha_C$ appearance of semitransparent B appearance of semitransparent A





$$\alpha_C = \alpha_B + (1 - \alpha_B)\alpha_A$$



#### **Premultiplied Alpha**

• **Goal:** Composite image *B* with alpha  $\alpha_B$  over image *A* with alpha  $\alpha_A$ 

 $A' = (\alpha_A A_r, \alpha_A A_g, \alpha_A A_b, \alpha_A)$  $B' = (\alpha_B B_r, \alpha_B B_g, \alpha_B B_b, \alpha_B)$  B A B over A

• Composite RGBA:

• Un-Premultiply for Final Color:

 $C' = B' + (1 - \alpha_B)A'$ 

 $(C_r, C_g, C_b, \alpha_c) \Longrightarrow (C_r/\alpha_c, C_g/\alpha_c, C_b/\alpha_c)$ 

# Why Premultiplied Matters [Upsample]



# Why Premultiplied Matters [Downsample]



#### **Closed Under Composition**

• **Goal:** Composite bright red image *B* with alpha 0.5 over bright red image *A* with alpha 0.5

A = (1, 0, 0, 0.5)B = (1, 0, 0, 0.5)



B over A

 Non-Premultiplied:
 Premultiplied:

 0.5 \* (1,0,0) + (1-0.5) \* 0.5 \* (1,0,0) 0.5 \* (0.5,0,0,0.5) + (1-0.5) \* (0.5,0,0,0.5) 

 (0.75, 0, 0) (0.75, 0, 0, 0.75) 

 (0.5 + (1-0.5) \* 0.5 = 0.75) (1,0,0) 

#### **Blend Methods**

When writing to color buffer, can use any blend method

 $D_{RGBA} = S_{RGBA} + D_{RGBA}$   $D_{RGBA} = S_{RGBA} - D_{RGBA}$   $D_{RGBA} = -S_{RGBA} + D_{RGBA}$   $D_{RGBA} = \min(S_{RGBA}, D_{RGBA})$   $D_{RGBA} = \max(S_{RGBA}, D_{RGBA})$  $D_{RGBA} = S_{RGBA} + D_{RGBA} * (1 - S_A)$ 

Blend Add Blend Subtract Blend Reverse Subtract Blend Min Blend Max Blend Over

 $S_{RGBA}$  and  $D_{RGBA}$  are pre-multiplied

#### Updated Depth Buffer (Z-buffer) Sample Code



Assumes color[x][y] and c are both premultiplied.

Triangles must be rendered back to front! A over B != B over A

### **Blend Render Order**

- For mixtures of opaque and transparent triangles:
  - **Step 1:** render opaque primitives (in any order) using depth-buffered occlusion
    - If pass depth test, triangle overwrites value in color buffer at sample
    - Depth **READ** and **WRITE**
  - **Step 2:** disable depth buffer update, render semitransparent surfaces in back-to-front order.
    - If pass depth test, triangle is composited OVER contents of color buffer at sample
    - Depth **READ** only

- Barycentric Coordinates
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### The "Simpler" Graphics Pipeline



# The Inputs

positions = {				<pre>texcoords ={</pre>		
vOx	, v0y	, v02	Ζ,	v0u,	v0v,	
vlx	, vly	, v1:	Χ,	vlu,	vlv,	
v2x	, v2y	, v2z	Ζ,	v2u,	v2v,	
v3x	, v3y	, v32	Χ,	v3u,	v3v,	
v4x	, v4y	, v42	Ζ,	v4u,	v4v,	
v5x	, v5y	, v5z	X	v5u,	v5v	
};				};		

[vertices]

[textures]

Object-to-camera-space transform  $T \in \mathbb{R}^{4 \times 4}$ Perspective projection transform  $P \in \mathbb{R}^{4 \times 4}$ Output image (W, H)

[ camera properties ]



# Step 1: Transform

Transform triangle vertices into camera space



#### **Step 2: Perspective Projection**

Apply perspective projection transform to transform triangle vertices into normalized coordinate space


# Step 3: Clipping



### Step 4: Transform To Screen Coordinates

Perform homogeneous divide. Transform vertex xy positions from normalized coordinates into screen coordinates (based on screen [w, h]).







### Step 5: Sample Coverage

Check if samples lie inside triangle. Evaluate depth and barycentric coordinates at all passing samples.



### Step 6: Compute Color

Texture lookups, color interpolation, etc.



### Step 7: Depth Test

#### Check depth and update depth if closer primitive found. (can be disabled)



# **Step 8: Color Blending**

Update color buffer with correct blending operation.



# The "Real" Graphics Pipeline

