## Transparency \& Texturing

- Barycentric Coordinates
- Texturing Surfaces
- Depth Testing
- Alpha Blending
- The Graphics Pipeline Revisited


## The "Simpler" Graphics Pipeline



## Interpolating Values for Triangles

- Goal: interpolate triangle vertices for any point within triangle
- Coordinates $\left(\phi_{i}, \phi_{j}, \phi_{k}\right)$ should represent weighted average
- $\phi_{i}+\phi_{j}+\phi_{k}=1$
- Similarly, $1-\phi_{i}-\phi_{j}=\phi_{k}$
- Gives a 2D parameterization of triangle point ( $\phi_{i}, \phi_{j}$ )
- Known as barycentric coordinates
- If each point has some attribute $\left(\alpha_{i}, \alpha_{j}, \alpha_{k}\right)$, can linearly interpolate $\alpha_{i} \phi_{i}+\alpha_{j} \phi_{j}+\alpha_{k} \phi_{k}$
- Example: [black] $\phi_{i}+[$ green $] \phi_{j}+[$ red $] \phi_{k}$


## Barycentric Coordinates



- Inversely proportional to the distance between the target point and a point within the triangle
- Can be computed as:

$$
\phi_{i}(x)=d_{i}(x) / h_{i}
$$

- How would you compute $h_{i}$ ? $d_{i}(x)$ ?



## Barycentric Coordinates [ Another Way ]



- Directly proportional to the area created by the triangle composed of the other two target points and a point within the triangle
- Can be computed as:

$$
\phi_{i}(x)=\frac{\operatorname{area}\left(x, x_{j}, x_{k}\right)}{\operatorname{area}\left(x_{i}, x_{j}, x_{k}\right)}
$$

## Perspective-Incorrect Interpolation



## Perspective-Incorrect Interpolation



If we compute barycentric coordinates using 2D (projected) coordinates, leads to (derivative) discontinuity in interpolation where quad was split

## Perspective-Correct Interpolation

- Goal: interpolate some attribute $v$ at vertices
- Compute depth $z$ at each vertex
- Evaluate $Z:=1 / z$ and $P:=v / z$ at each vertex
- Interpolate $Z$ and $P$ using standard (2D) barycentric coordinates
- At each fragment, divide interpolated $P$ by interpolated $Z$ to get final value



## Perspective-Correct Interpolation

$$
\begin{array}{lll}
\phi_{(0,0,1)}=0.2 & P_{(0,0,1)}=(0,0,0) / 1 & Z_{(0,0,1)}=1 \\
\phi_{(0,3,2)}=0.1 & P_{(0,3,2)}=(1,0,0) / 2 & Z_{(0,3,2)}=1 / 2 \\
\phi_{(0,5,4)}=0.7 & P_{(0,5,4)}=(0,1,0) / 4 & Z_{(0,5,4)}=1 / 4 \\
& \\
P_{\text {interp }}=0.2 *[(0,0,0) / 1]+0.1 *[(1,0,0) / 2] * 0.7 *[(0,1,0) / 4] \\
P_{\text {interp }}=(0.05,0.175,0) \\
\\
Z_{\text {interp }}=0.2 *[1 / 1]+0.1 *[1 / 2] * 0.7 *[1 / 4] \\
Z_{\text {interp }}=0.425
\end{array}
$$



What if $z$ is equal to $\mathbf{0}$ ?
Remember the near clipping plane!

## - Barycentric Coordinates

- Texturing Surfaces
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## The "Simpler" Graphics Pipeline



## Textures in Graphics

- Textures are buffers of data (images) that are read into the graphics pipeline and are used for:
- Coloring mapping
- Normal mapping
- Displacement mapping
- Roughness mapping
- Occlusion mapping
- Reflection mapping
- Textures can also be written into
- Think a scratch pad for data
- Useful for maximizing quality while minimizing the number of polygons
- Rough surfaces can be approximated by smooth surfaces with rough textures
- A single pixel of a texture is known as a texel


## Textures in Graphics



## Texture Coordinates

- Goal: map surface geometry coordinates to image coordinates
- Barycentric coordinates let us represent 3D geometry in 2D by their surface coordinates
- Known as surface parameterization
- Not always a 1-to-1 map!
- A surface only half the number of pixels of

[ texture ]
 a texture may only use up half the texels**


## Texture Example

[ texture coordinates on surface ]

[ texture coordinates on texture ]


Each vertex has a coordinate ( $u, v$ ) in texture space

## Texture Example

[ rendered results ]

[ texture data]


Each triangle "copies" a piece of the image back to the surface

Periodic Texturing


Why do you think texture coordinates might repeat over the surface?

Periodic Texturing


Used for tiling textures

## How Texturing Is Done

- An artist goes into a program and drags/paints/stretches/warps textures onto surfaces
- The resulting distortion of the texture on the surface is saved as the surface parameterization
- Computing the texture mapping function is never done by hand!
- Always use an interactive program to do it
- Also known as uv mapping
- $u$ and $v$ are the two barycentric coordinates that we want to map onto texture space


Texture mapping maps a non-integer coordinate to another non-integer coordinate.
But textures can only be accessed via integer...

How do we know what texel(s) to sample?


## Nearest Neighbor Sampling

- Idea: Grab texel nearest to requested location in texture
- Requires:

- 1 memory lookup
- O linear interpolations

$x^{\prime}$ and $y^{\prime}$ are half-integer coordinates
Helps account for 0.5 offset from texture coordinate centers


## Bilinear Interpolation Sampling

- Idea: Grab nearest 4 texels and blend them together based on their inverse distance from the requested location
- Blend two sets of pixels along one axis, then blend the remaining pixels
- Requires:
- 4 memory lookup
- 3 linear interpolations

$x^{\prime} \leftarrow$ floor $(x-0.5), \quad y^{\prime} \leftarrow \operatorname{floor}(y-0.5)$
$\Delta x \leftarrow x-x^{\prime}$
$\Delta y \leftarrow y-y^{\prime}$
$t_{(x, y)} \leftarrow$ tex.lookup $\left(x^{\prime}, y^{\prime}\right)$
$t_{(x+1, y)} \leftarrow$ tex. lookup $\left(x^{\prime}+1, y^{\prime}\right)$
$t_{(x, y+1)} \leftarrow$ tex. lookup $\left(x^{\prime}, y^{\prime}+1\right)$
$t_{(x+1, y+1)} \leftarrow$ tex. $\operatorname{lookup}\left(x^{\prime},+1 y^{\prime}+1\right)$
$t_{x} \leftarrow(1-\Delta x) * t_{(x, y)}+\Delta x * t_{(x+1, y)}$
$t_{y} \leftarrow(1-\Delta x) * t_{(x, y+1)}+\Delta x * t_{(x+1, y+1)}$
$t \leftarrow(1-\Delta y) * t_{x}+\Delta y * t_{y}$


## Minification vs. Magnification



- Magnification [ Nearest Neighbor, Bilinear ]:
- Example: camera is very close to scene object
- Single screen pixel maps to tiny region of texture
- Can just interpolate value at screen pixel center
- Minification [ ??? ]
- Example: scene object is very far away
- Single screen pixel maps to large region of texture
- Need to compute average texture value over pixel to avoid aliasing

Aliasing Due To Minification


## Pre-Filtering Texture



## Texture Pre-Filtering

- Texture aliasing occurs because a single pixel on the screen covers many pixels of the texture
- Ideally, want to average a bunch of texels in a very large region (expensive!)
- Instead, we can pre-compute the averages (once) and just look up these averages (many times) at run-time
- Q: Which averages to pre-compute
- A: a lot of them!



## Mip-Map [L. Willims ‘83]

- Rough idea: precompute a prefiltered image at every possible scale
- The image at depth $d$ is the result of applying a $2 \times 2$ avg filter on the image at depth d-1
- The image at depth 0 is the base image
- Mip-Map generates $\log _{2}[\min (w t h, h g t)]+1$ levels
- Each level the width and height gets halved
- Memory overhead: $(1+1 / 3) x$ original texture
- $1+\frac{1}{4}+\frac{1}{16}+\cdots=\sum \frac{1}{4}^{j}=\frac{1}{1-\frac{1}{4}}=\frac{4}{3}$


Mip-Map [L. williams '83]

- Storing an RGB Mip-Map can be fit into an image twice the width and twice the height of the original image
- See diagram for proof:)
- Does not work as nicely for RGBA!
- Issue: bad spatial locality
- Requesting a texel requires lookup in 3 very different regions of an image


Which mip-map level do we use?

Sponza Bilinear Interpolation [ Level 0]


Sponza Bilinear Interpolation [ Level 2 ]


Sponza Bilinear Interpolation [ Level 4]


## Sponza Bilinear Interpolation [ Varying Level ]



Sponza Visualization of Level


## Computing MipMap Depth

- Correlation between distance of surface to camera and level of mip-map accessed
- More specifically, correlation between screenspace movement across the surface compared to texture movement and level of mip-map access
- If moving over a pixel in screen space is a big jump in texture space, then we call it minification
- Sample from a lower level of mip-map
- If moving over a pixel in screen space is a small jump in texture space, then we call it magnification
- Sample from a higher level of mip-map

u



## Computing MipMap Depth

More formally:

$$
\begin{array}{ll}
\frac{d u}{d x}=u_{10}-u_{00} & \frac{d u}{d y}=u_{01}-u_{00} \\
\frac{d v}{d x}=v_{10}-v_{00} & \frac{d v}{d y}=v_{01}-v_{00}
\end{array}
$$

Where $d x$ and $d y$ measure the change in screen space and $d u$ and $d v$ measure the change in texture space

$$
\begin{gathered}
L_{x}^{2}=\left(\frac{d u}{d x}\right)^{2}+\left(\frac{d v}{d x}\right)^{2} \quad L_{y}^{2}=\left(\frac{d u}{d y}\right)^{2}+\left(\frac{d v}{d y}\right)^{2} \\
L=\sqrt{\max \left(L_{x}^{2}, L_{y}^{2}\right)}
\end{gathered}
$$

$L$ measures the Euclidean distance of the change. We take the max to get a single number.


The mipmap level is not an integer...
Which level do we use?

## Trilinear Interpolation Sampling

- Idea: Perform bilinear interpolation on two layers of the mip-map that represents proper minification/magnification, blending the results together
- Requires:
- 8 memory lookup
- 7 linear interpolations

$$
L \leftarrow \sqrt{\max \left(L_{x}{ }^{2}, L_{y}{ }^{2}\right)}
$$



$$
\begin{aligned}
& L_{x}{ }^{2} \leftarrow \frac{d u^{2}}{d x}+\frac{d v^{2}}{d x} \\
& L_{y}{ }^{2} \leftarrow \frac{d u^{2}}{d y}+\frac{d v^{2}}{d y}
\end{aligned}
$$

$$
d \leftarrow \log _{2} L
$$

$$
d^{\prime} \leftarrow \operatorname{floor}(d)
$$

$$
\Delta d \leftarrow d-d^{\prime}
$$

$\xrightarrow{(1 \text { Lerp) }}$
$t_{d} \leftarrow$ tex $\left[d^{\prime}\right]$. bilinear $(x, y)$
$t_{d+1} \leftarrow$ tex $\left[d^{\prime}+1\right]$. bilinear $(x, y)$
$t \leftarrow(1-\Delta d) * t_{d}+\Delta d * t_{d+1}$

## Trilinear Interpolation Sampling

- Idea: Perform bilinear interpolation on two layers of the mip-map that represents proper minification/magnification, blending the results together
- Requires:
- 8 memory lookup
- 7 linear interpolations

Level ceil(d)

ing the $\operatorname{maxi}^{\frac{d x}{2}}+\frac{d x}{2}$
$L_{x}{ }^{2} \leftarrow \frac{d u^{2}}{d x}+\frac{d v^{2}}{d x}$

$d^{\prime} \leftarrow f \operatorname{loor}(d)$
$\Delta d \leftarrow d-d^{\prime}$
$t_{d} \leftarrow$ tex $\left[d^{\prime}\right]$. . ilinear $(x, y)$
$t_{d+1} \leftarrow$ tex $\left[d^{\prime}+1\right]$. bilinear $(x, y)$
$t \leftarrow(1-\Delta d) * t_{d}+\Delta d * t_{d+1}$

## Trilinear Assumption

- Trilinear filtering assumes that samples shrink at the same rate along $u$ and $v$
- Taking the max says we would rather overcompensate than undercompensate filtering
- Bilinear and Trilinear filtering are isotropic filtering methods
- iso - same, tropic - direction
- Values should be same regardless of viewing direction
- What does it mean for samples to shrink at very different rates along $u$ and $v$ ?
- Think of a plane rotated away from the camera
- Changes in $v$ larger than changes in $u$
- Trilinear filtering assumes that samples shrink at

- Values should be same regardless of viewing direction
- What does it mean for samples to shrink at very different rates 2 or 5 d $v$ ?
- Think of a plane rotated away from the camera
- Changes in $v$ larger than changes in $u$



## Anisotropic Filtering

- Anisotropic filtering is dependent on direction
- an - not, iso - same, tropic - direction
- Idea: create a new texture map that downsamples the $x$ and $y$ axis by 2 separately
- Instead of taking the max, use each coordinate to index into correct location in map

$$
\begin{aligned}
L & =\sqrt{\left.\sqrt{2}, L_{x}, L_{y}^{2}\right)} \\
\left(d_{x}, d_{y}\right) & =\left(\log _{2} \sqrt{L_{x}^{2}}, \log _{2} \sqrt{L_{y}^{2}}\right)
\end{aligned}
$$

- Texture map is now a grid of downsampled textures
- Known as a RipMap



## Rip Map

- Same idea as MipMap, but for anisotropic filtering
- 4x memory footprint
- New width: $w^{\prime}=w+\frac{w}{2}+\frac{w}{4}+\cdots=2 w$
- New height: $h^{\prime}=h+\frac{h}{2}+\frac{h}{4}+\cdots=2 h$
- New area: $w^{\prime} h^{\prime}=4 w h$
- Fun fact: a MipMap is just the diagonal of a RipMap
- If $d_{x}=d_{y}$, then we have trilinear interpolation


Isotropic vs Anisotropic Filtering
overbluring in $u$ direction

[ isotropic (trilinear) ]
[ anisotropic ]

## Sampling Comparisons

| No. samples | [ Nearest ] | [ Bilinear ] | [ Trilinear ] |
| :---: | :---: | :---: | :---: | | [ Anisotropic ] |
| :---: |
| No. interps |
| No. operations |

## Texture Sampling Pipeline

1. Compute $u$ and $v$ from screen sample $(x, y)$ via barycentric interpolation
2. Approximate $d u / d x, d u / d y, d v / d x, d v / d y$ by taking differences of screen-adjacent samples
3. Compute mip map level $d$
4. Convert normalized $[0,1]$ texture coordinate $(u, v)$ to pixel locations $(U, V) \in[W, H]$ in texture image
5. Determine addresses of texels needed for filter (e.g., eight neighbors for trilinear)
6. Load texels into local registers
7. Perform tri-linear interpolation according to ( $U, V, d$ )
8. (...even more work for anisotropic filtering...)

Lot of repetitive work every time we want to shade a pixel!

GPUs instead implement these instructions on fixed-function hardware.

This is why we have texture caches and texture filtering units.

## - Barycentric Coordinates

- Texturing Surfaces
- Depth Testing
- Alpha Blending
- The Graphics Pipeline Revisited


## The "Simpler" Graphics Pipeline



## Depth Buffer ( Z-buffer )

- For each sample, the depth buffer stores the depth of the closest triangle seen so far
- Done at the sample granularity, not pixel granularity



## Depth of a Triangle



- A triangle is composed of 3 different 3D points, each with a depth value $z$
- To get the depth at any point $(x, y)$ inside the triangle, interpolate depth at vertices with barycentric coordinates


## Depth Buffer ( Z-buffer )



## Depth Buffer ( Z-buffer )

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| [ color buffer ] |  |  |  |  |  |  |  |  | [ depth buffer ] |  |  |  |  |  |  |  |  |
| near |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## Depth Buffer ( Z-buffer )



## Depth Buffer ( Z-buffer )

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| [ color buffer ] |  |  |  |  |  |  |  |  | [ depth buffer ] |  |  |  |  |  |  |  |  |
| near |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Depth Buffer ( Z-buffer ) Per Sample


## Depth Buffer ( Z-buffer ) Per Sample

Able to capture triangle intersections by performing tests per sample

## Depth Buffer ( Z-buffer ) Sample Code

```
draw_sample(x, y, d, c) //new depth d & color c at (x,y)
{
    if(d < zbuffer[x][y])
    {
        // triangle is closest object seen so far at this
        // sample point. Update depth and color buffers.
        zbuffer[x][y] = d; // update zbuffer
        color[x][y] = c; // update color buffer
    }
    // otherwise, we've seen something closer already;
    // don't update color or depth
}
```

Why is it that we first shade the pixel and then assign the resulting color after depth check?
Deferred shading (advanced algorithm) fixes this issue.

## - Barycentric Coordinates

- Texturing Surfaces
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- The Graphics Pipeline Revisited


## Alpha Values

- Another common image format: RGBA
- Alpha channel specifies 'opacity' of object
- Basically how transparent it is
- Most common encoding is 8 -bits per channel (0-255)
- Compositing A over B != B over A
- Consider the extreme case of two opaque objects...


$$
\alpha=3 / 4
$$

$$
\alpha=1 / 2
$$

$$
\alpha=1 / 4
$$



## Non-Premultiplied Alpha

- Goal: Composite image $B$ with alpha $\alpha_{B}$ over image $A$ with alpha $\alpha_{A}$

$$
\begin{aligned}
& A=\left(A_{r}, A_{g}, A_{b}\right) \\
& B=\left(B_{r}, B_{g}, B_{b}\right)
\end{aligned}
$$

B over A

- Composite RGB: what B lets through

$$
C=\alpha_{B} B+\left(1-\alpha_{B}\right) \alpha_{A} A
$$

- Composite Alpha:

$$
\alpha_{C}=\alpha_{B}+\left(1-\alpha_{B}\right) \alpha_{A}
$$

Two different equations is inefficient!!

## Premultiplied Alpha

- Goal: Composite image $B$ with alpha $\alpha_{B}$ over image $A$ with alpha $\alpha_{A}$

$$
\begin{aligned}
& A^{\prime}=\left(\alpha_{A} A_{r}, \alpha_{A} A_{g}, \alpha_{A} A_{b}, \alpha_{A}\right) \\
& B^{\prime}=\left(\alpha_{B} B_{r}, \alpha_{B} B_{g}, \alpha_{B} B_{b}, \alpha_{B}\right)
\end{aligned}
$$

B over A

- Composite RGBA:

$$
C^{\prime}=B^{\prime}+\left(1-\alpha_{B}\right) A^{\prime}
$$

- Un-Premultiply for Final Color:

$$
\left(C_{r}, C_{g}, C_{b}, \alpha_{C}\right) \Longrightarrow\left(C_{r} / \alpha_{C}, C_{g} / \alpha_{C}, C_{b} / \alpha_{C}\right)
$$

## Why Premultiplied Matters [Upsample]


known as fringing

$B$ over $A$

upsampled color

$B$ over $A$ (premultiplied)


Something isn't right...

Why Premultiplied Matters [Downsample]


## Closed Under Composition

- Goal: Composite bright red image $B$ with alpha 0.5 over bright red image $A$ with alpha 0.5

$$
\begin{aligned}
& A=(1,0,0,0.5) \\
& B=(1,0,0,0.5)
\end{aligned}
$$



- Non-Premultiplied:

- Premultiplied:


## Blend Methods

When writing to color buffer, can use any blend method

$$
\begin{aligned}
& D_{R G B A}=S_{R G B A}+D_{R G B A} \\
& D_{R G B A}=S_{R G B A}-D_{R G B A} \\
& D_{R G B A}=-S_{R G B A}+D_{R G B A} \\
& D_{R G B A}=\min \left(S_{R G B A}, D_{R G B A}\right) \\
& D_{R G B A}=\max \left(S_{R G B A}, D_{R G B A}\right) \\
& D_{R G B A}=S_{R G B A}+D_{R G B A} *\left(1-S_{A}\right)
\end{aligned}
$$

Blend Add
Blend Subtract
Blend Reverse Subtract
Blend Min
Blend Max
Blend Over
$S_{R G B A}$ and $D_{R G B A}$ are pre-multiplied

## Updated Depth Buffer ( Z-buffer ) Sample Code

```
    draw_sample(x, y, d, c) //new depth d & color c at (x,y)
    { we still be
iufldaNe zbitesfor x ] [y])
    alpha primitives?
        zouffer[x][y]= d;
        color[x][y] c.rgba + (1-c.a) * color[x][y];
        }
        // otherwise, we've seen something closer already;
        // don't update color or depth
    }
```

Assumes color $[x][y]$ and c are both premultiplied.
Triangles must be rendered back to front!
A over B != B over A

## Blend Render Order

- For mixtures of opaque and transparent triangles:
- Step 1: render opaque primitives (in any order) using depth-buffered occlusion
- If pass depth test, triangle overwrites value in color buffer at sample
- Depth READ and WRITE
- Step 2: disable depth buffer update, render semitransparent surfaces in back-to-front order.
- If pass depth test, triangle is composited OVER contents of color buffer at sample
- Depth READ only



## - Barycentric Coordinates

- Texturing Surfaces
- Depth Testing
- Alpha-Blending
- The Graphics Pipeline Revisited


## The "Simpler" Graphics Pipeline

$\cdots$ ,



Sample triangle coverage


Interpolate triangle attributes at covered samples

## The Inputs

```
positions = { texcoords ={
    v0x, v0y, v0z, v0u, v0v,
    v1x, v1y, v1x, v1u, v1v,
    v2x, v2y, v2z, v2u, v2v,
    v3x, v3y, v3x, v3u, v3v,
    v4x, v4y, v4z, v4u, v4v,
    v5x, v5y, v5x v5u, v5v
};
```

[ vertices ]

Object-to-camera-space transform $T \in \mathbb{R}^{4 \times 4}$
Perspective projection transform $P \in \mathbb{R}^{4 \times 4}$
Output image ( $W, H$ )
[ camera properties ]

[ textures ]

[ machine]

## Step 1: Transform

Transform triangle vertices into camera space


## Step 2: Perspective Projection

## Apply perspective projection transform to transform

 triangle vertices into normalized coordinate space
[ 3D camera space position ]

[ normalized space position ]

Step 3: Clipping

Discard triangles completely outside cube. Clip triangles partially in cube.

[ pre-clipping ]

[ post-clipping ]

## Step 4: Transform To Screen Coordinates

Perform homogeneous divide.
Transform vertex xy positions from normalized coordinates into screen coordinates (based on screen [w, h]).
( $w, h$ )

(0, 0)

## Step 5: Sample Coverage

Check if samples lie inside triangle.
Evaluate depth and barycentric coordinates at all passing samples.


## Step 6: Compute Color

Texture lookups, color interpolation, etc.


## Step 7: Depth Test

Check depth and update depth if closer primitive found.
(can be disabled)


## Step 8: Color Blending

Update color buffer with correct blending operation.

## The "Real" Graphics Pipeline



Doesn't look much different than what we discussed..

