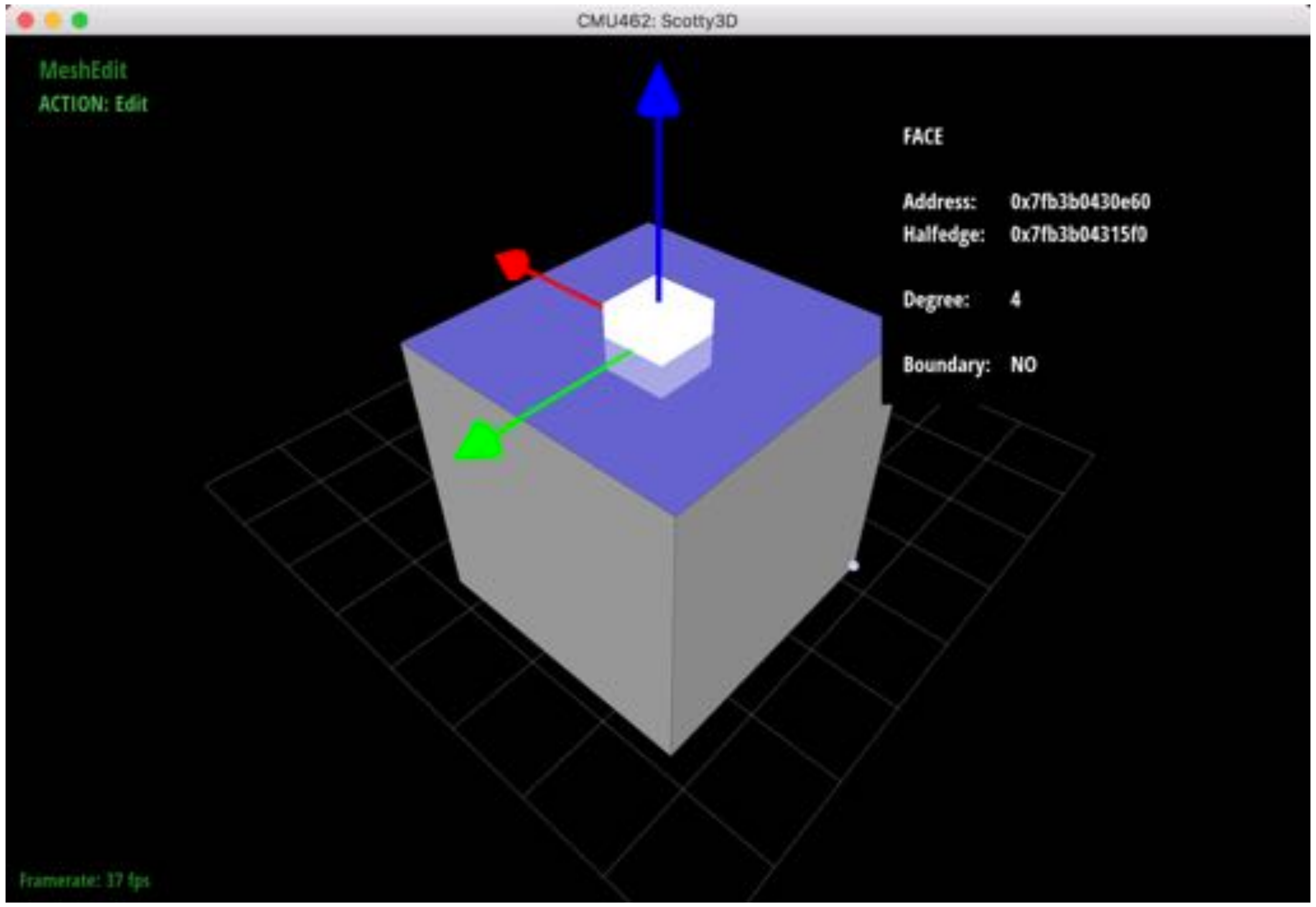
Introduction to Geometry

Computer Graphics
CMU 15-462/15-662

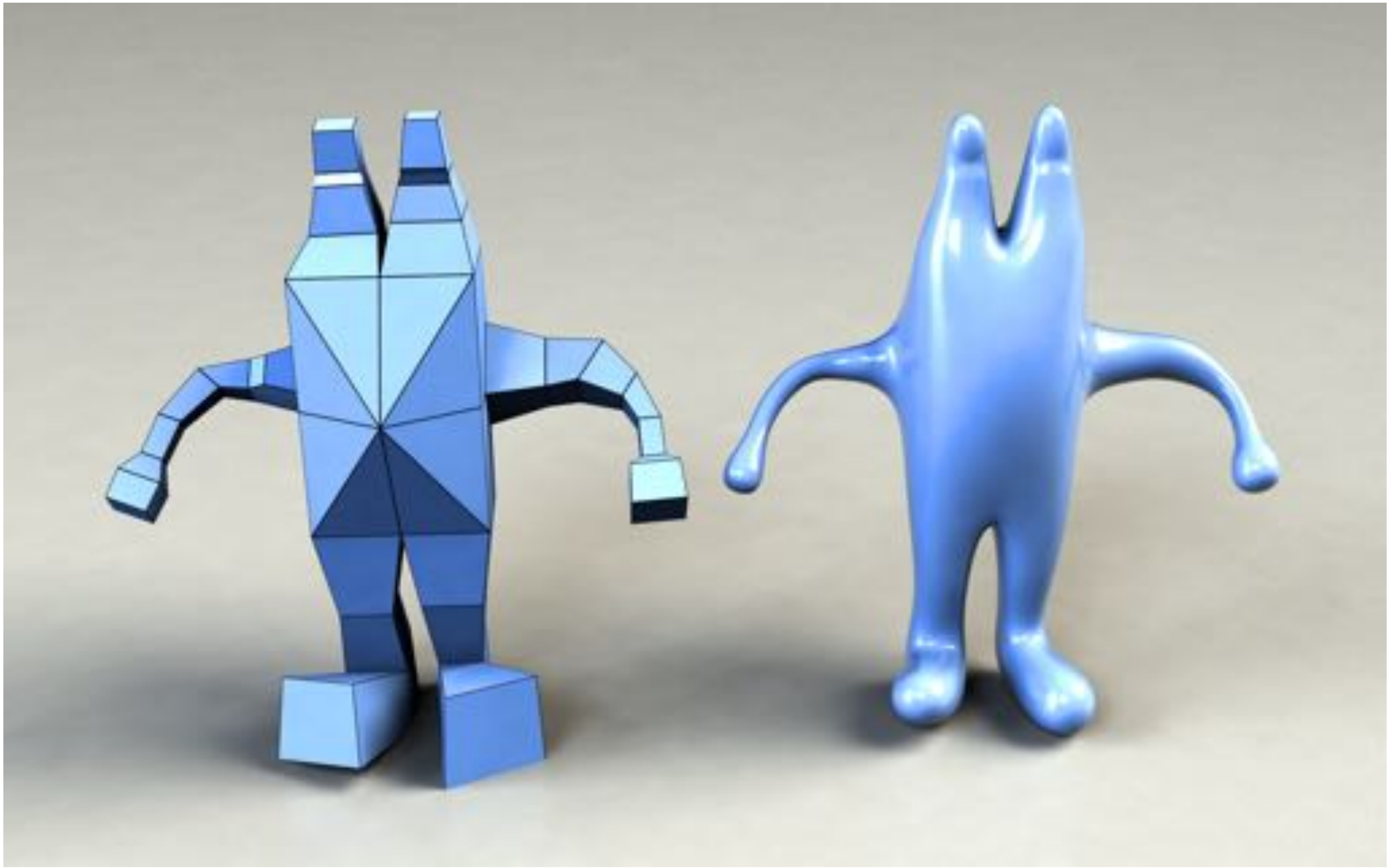
Assignment 2

- **Start building up "Scotty3D"; first part is 3D modeling**



3D Modeling

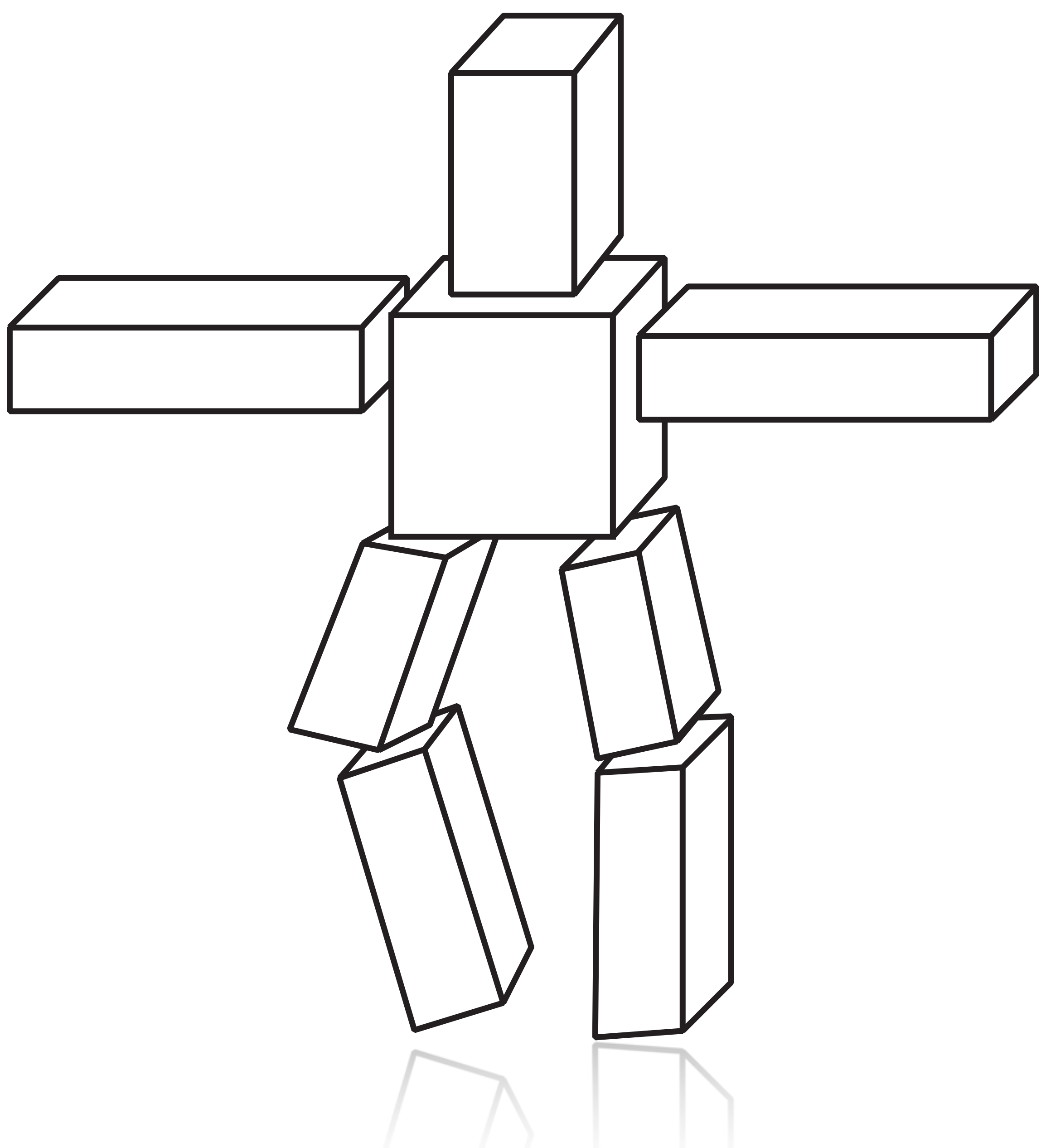
- **Don't just make great software… make great art! :-)**



(This mesh was created in Scotty3D in about 5 minutes... you can do much better!)

Increasing the complexity of our models

Transformations

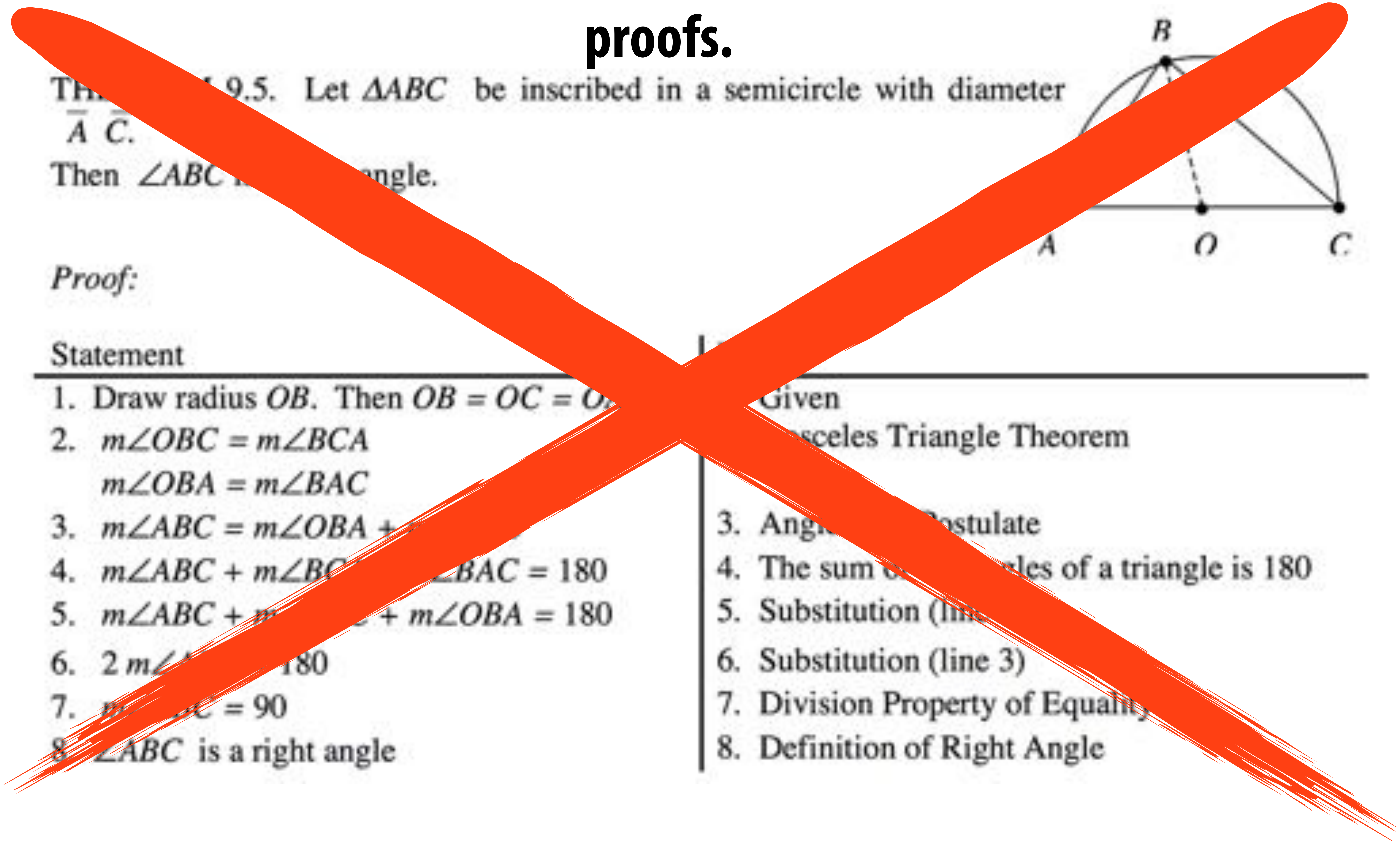


Geometry

Materials, lighting, ...

Q: What is geometry?

A: Geometry is the study of two-column proofs.



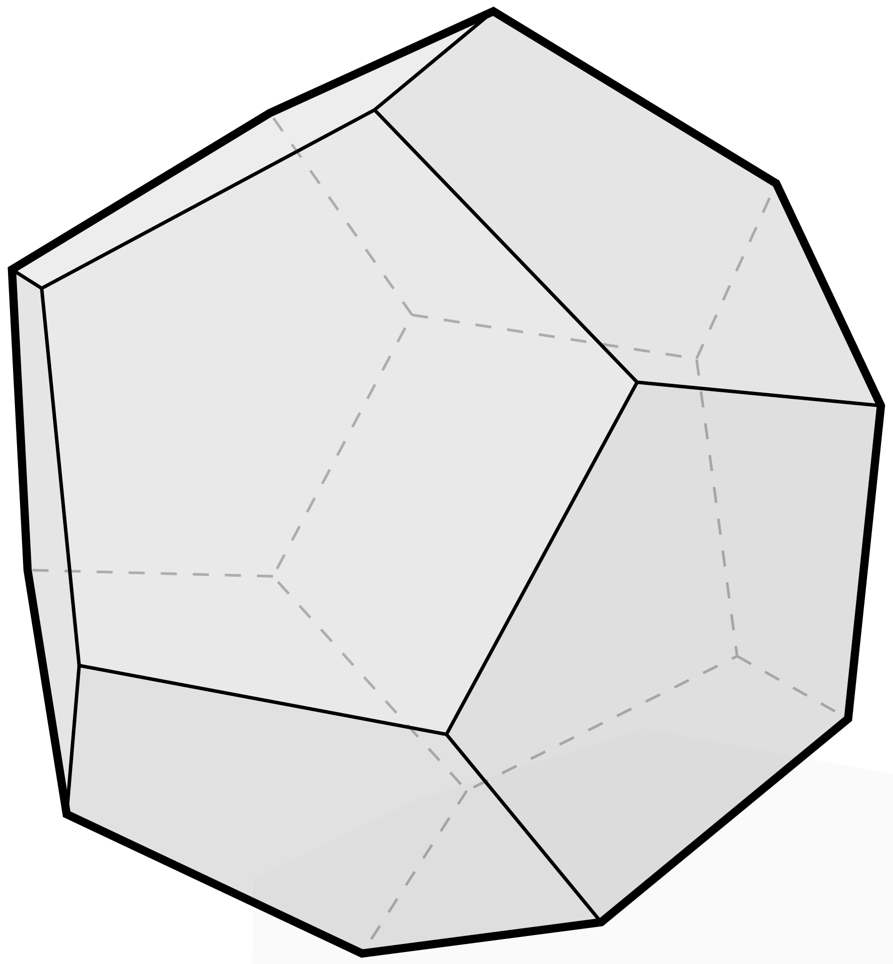
Ceci n'est pas géométrie.

What is geometry?

"Earth" "measure"

ge•om•et•ry /jēˈämətrē/ *n.*

1. The study of shapes, sizes, patterns, and positions.
2. The study of spaces where some quantity (lengths, angles, etc.) can be *measured*.



Plato: "...the earth is in appearance like one of those balls which have leather coverings in twelve pieces..."

How can we describe geometry?

IMPLICIT

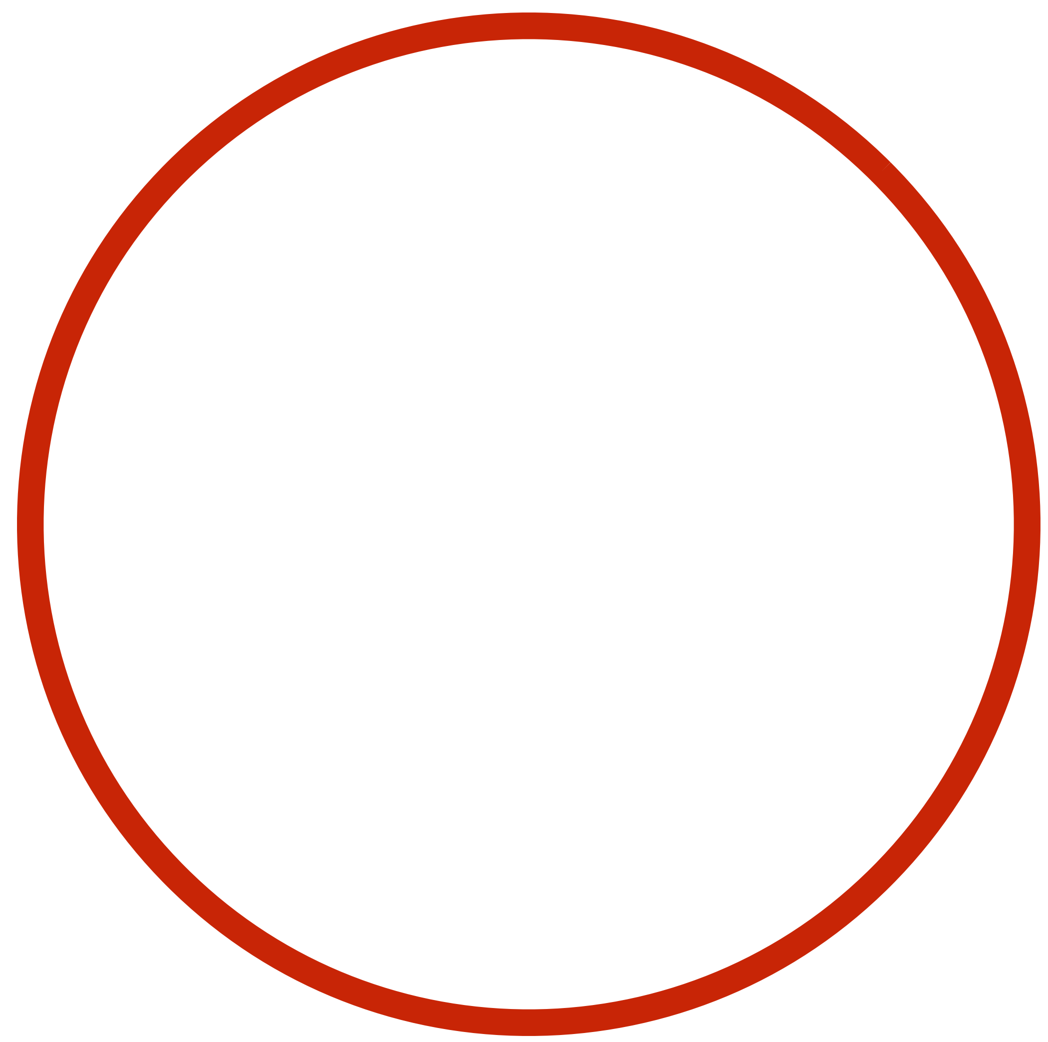
$$x^2 + y^2 = 1$$

LINGUISTIC

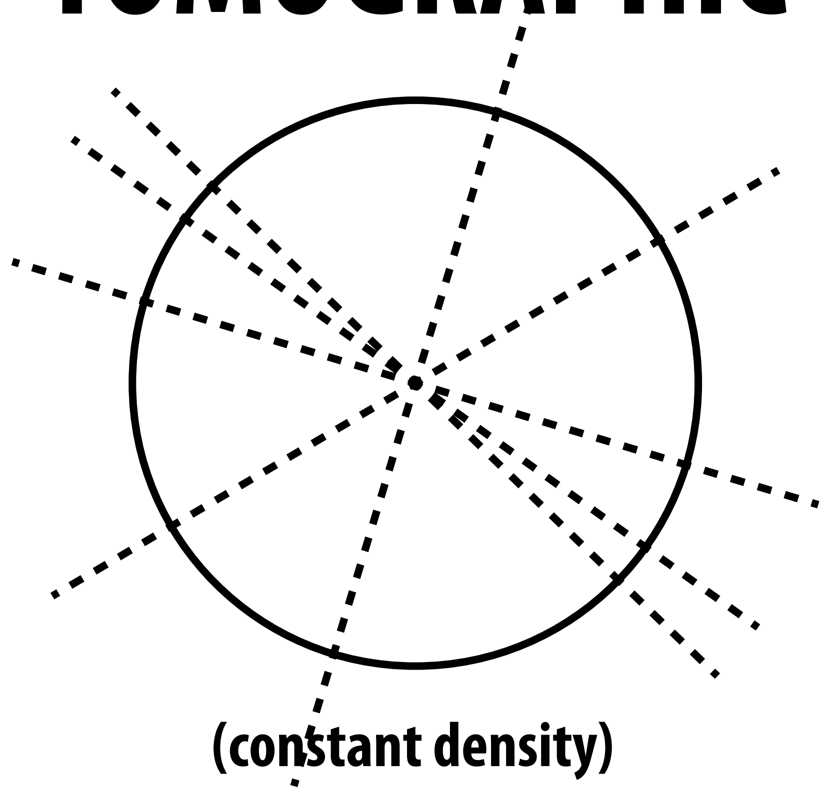
“unit circle”

EXPLICIT

$$(\underbrace{\cos\theta}_{x}, \underbrace{\sin\theta}_{y})$$

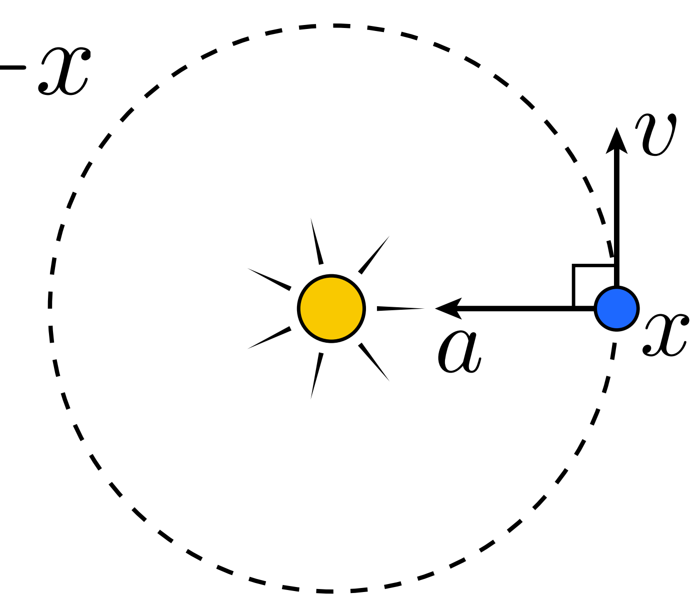


TOMOGRAPHIC



DYNAMIC

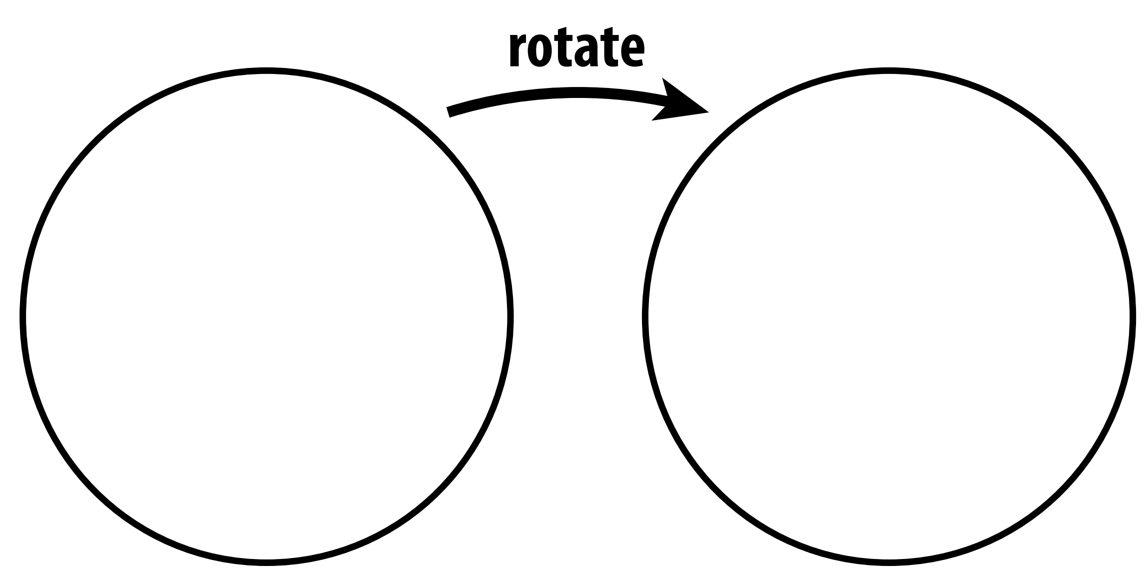
$$\frac{d^2}{dt^2}x = -x$$



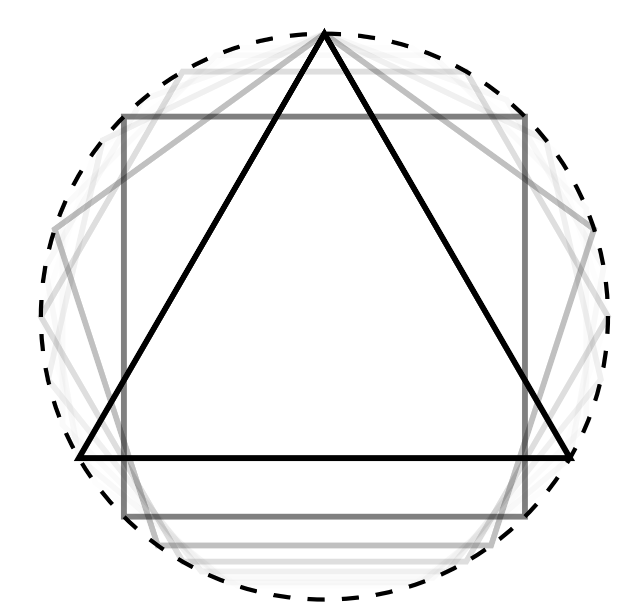
CURVATURE

$$\kappa = 1$$

SYMMETRIC



DISCRETE

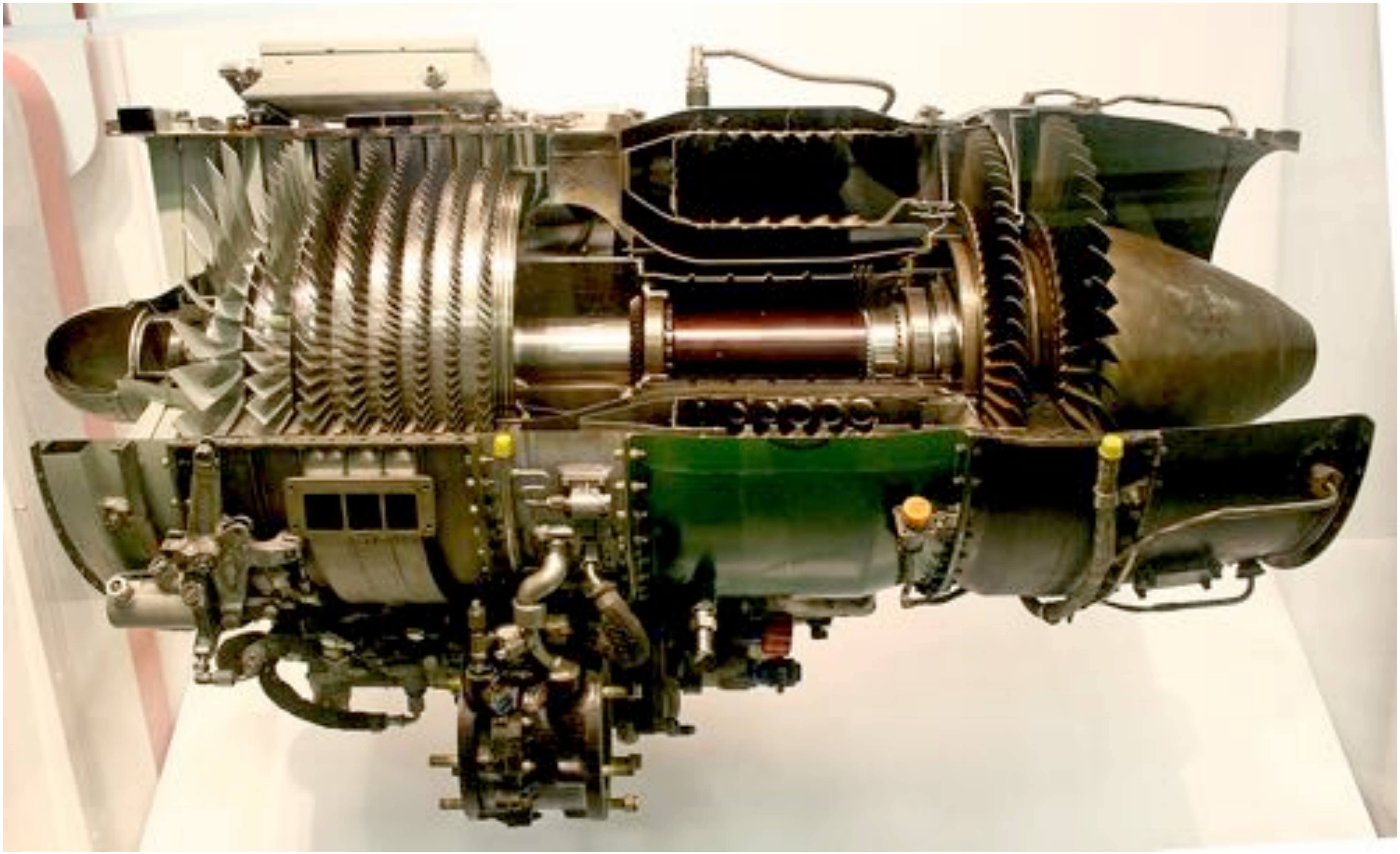


n → ∞

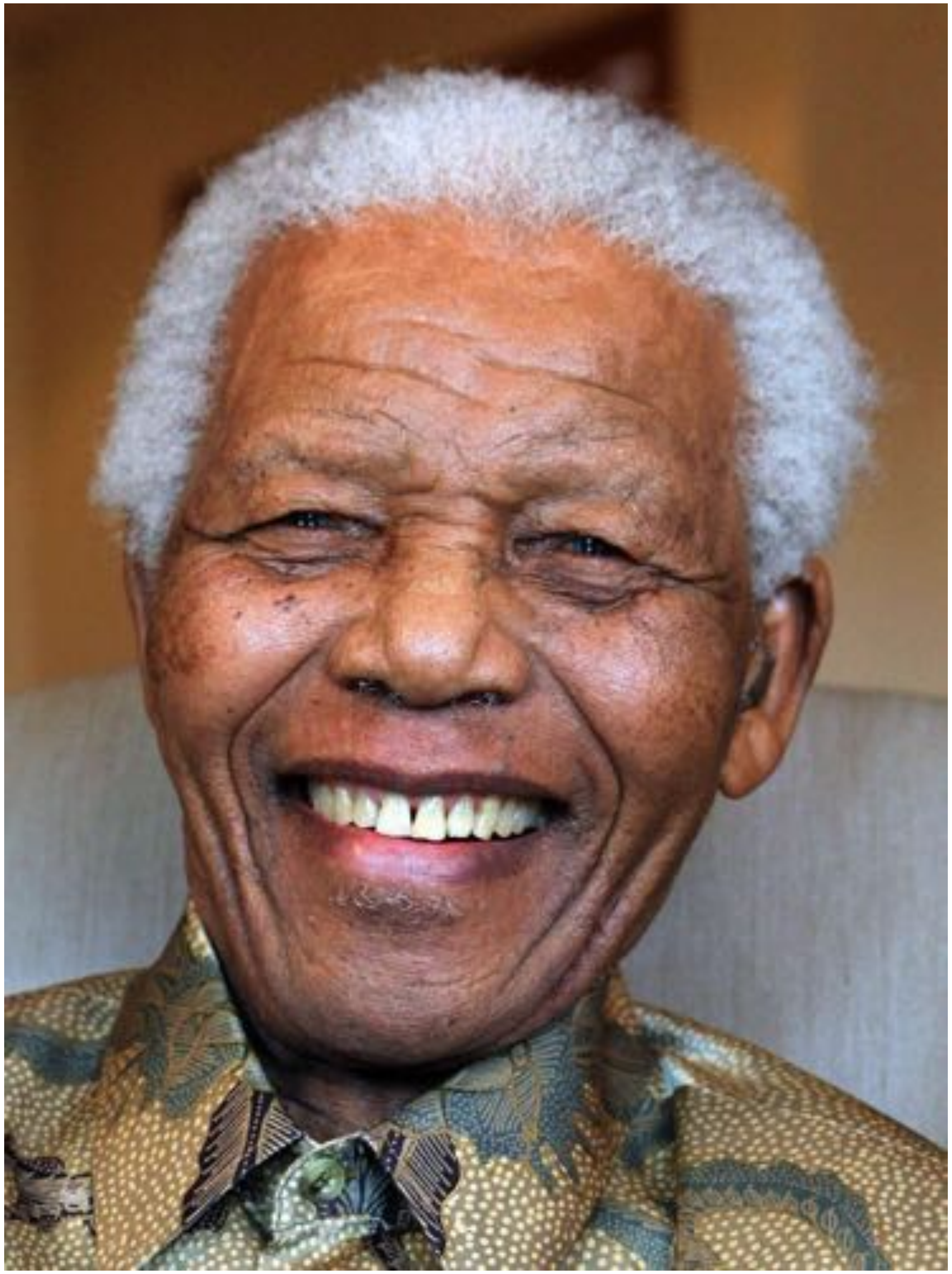
Given all these options, what's the best way to encode geometry on a computer?

Examples of geometry

Examples of geometry



Examples of geometry



Examples of geometry

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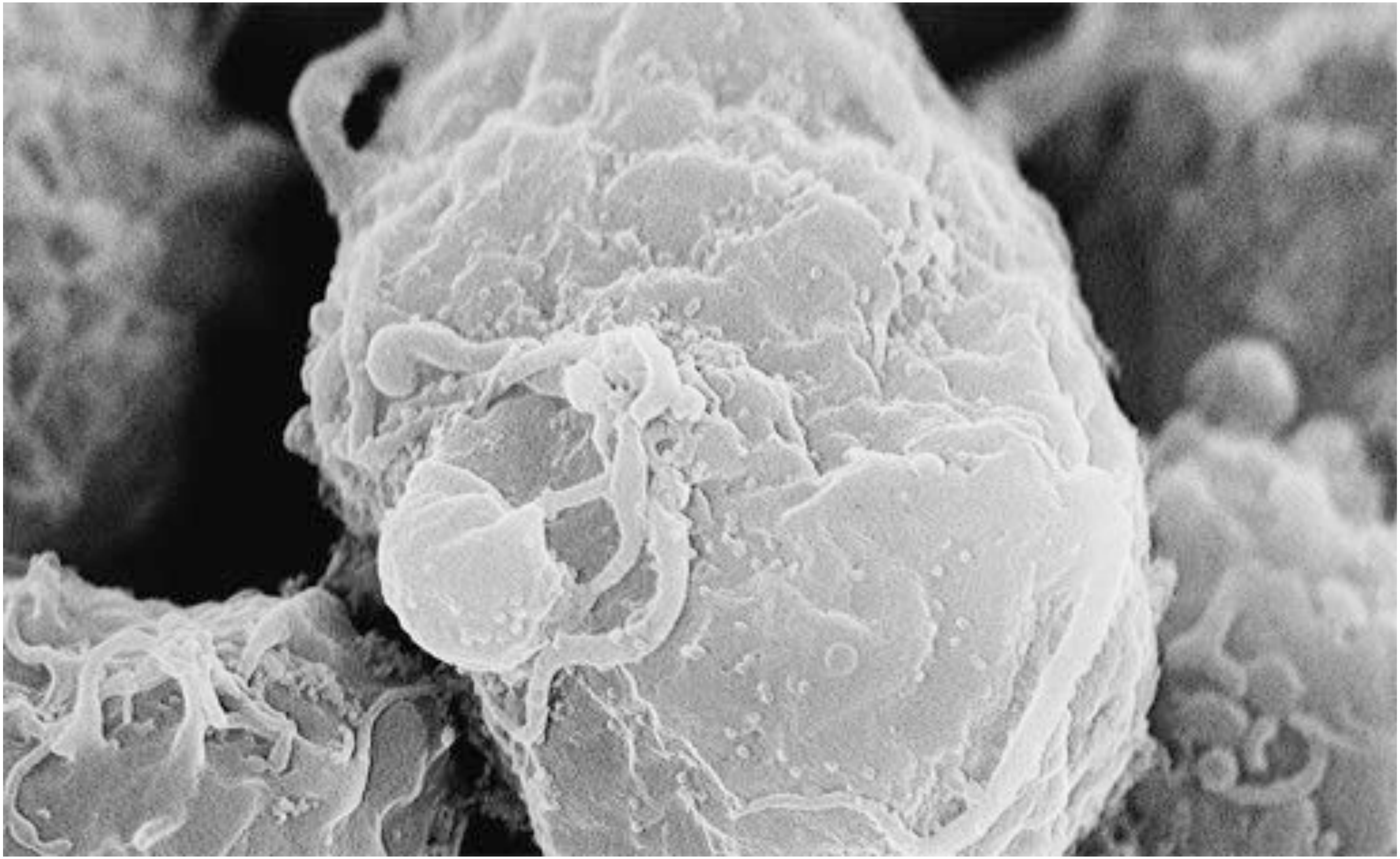
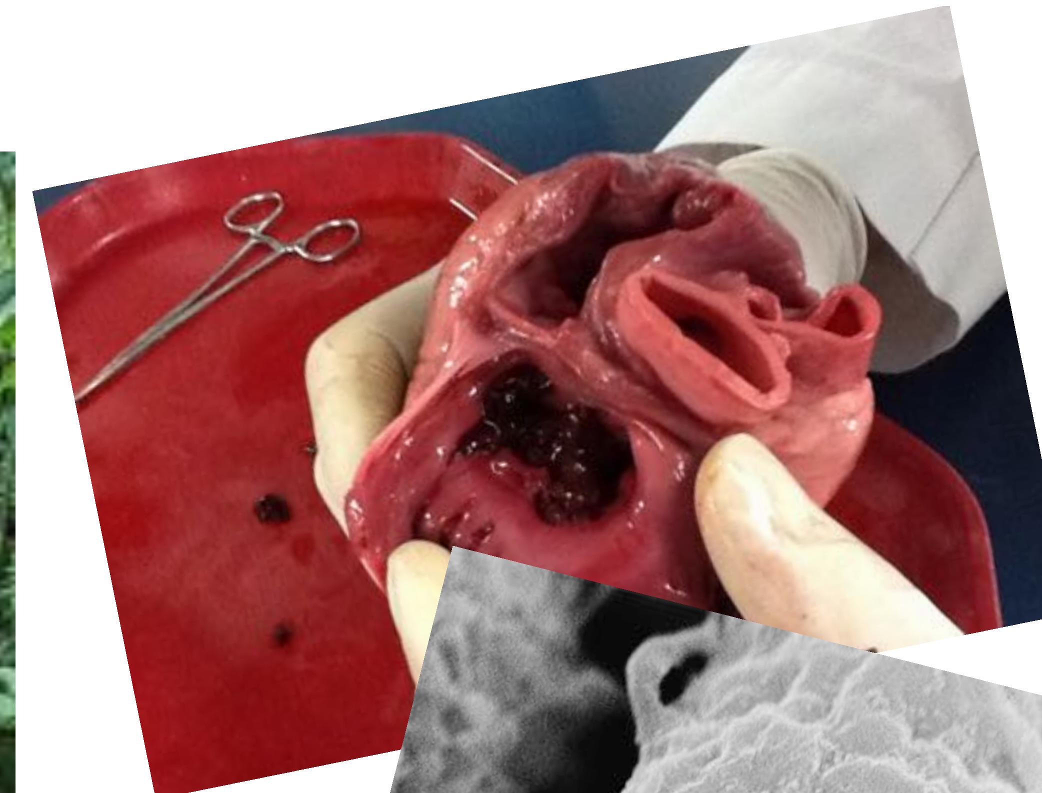


It’s a Jungle Out There!



No one "best" choice—geometry is hard!

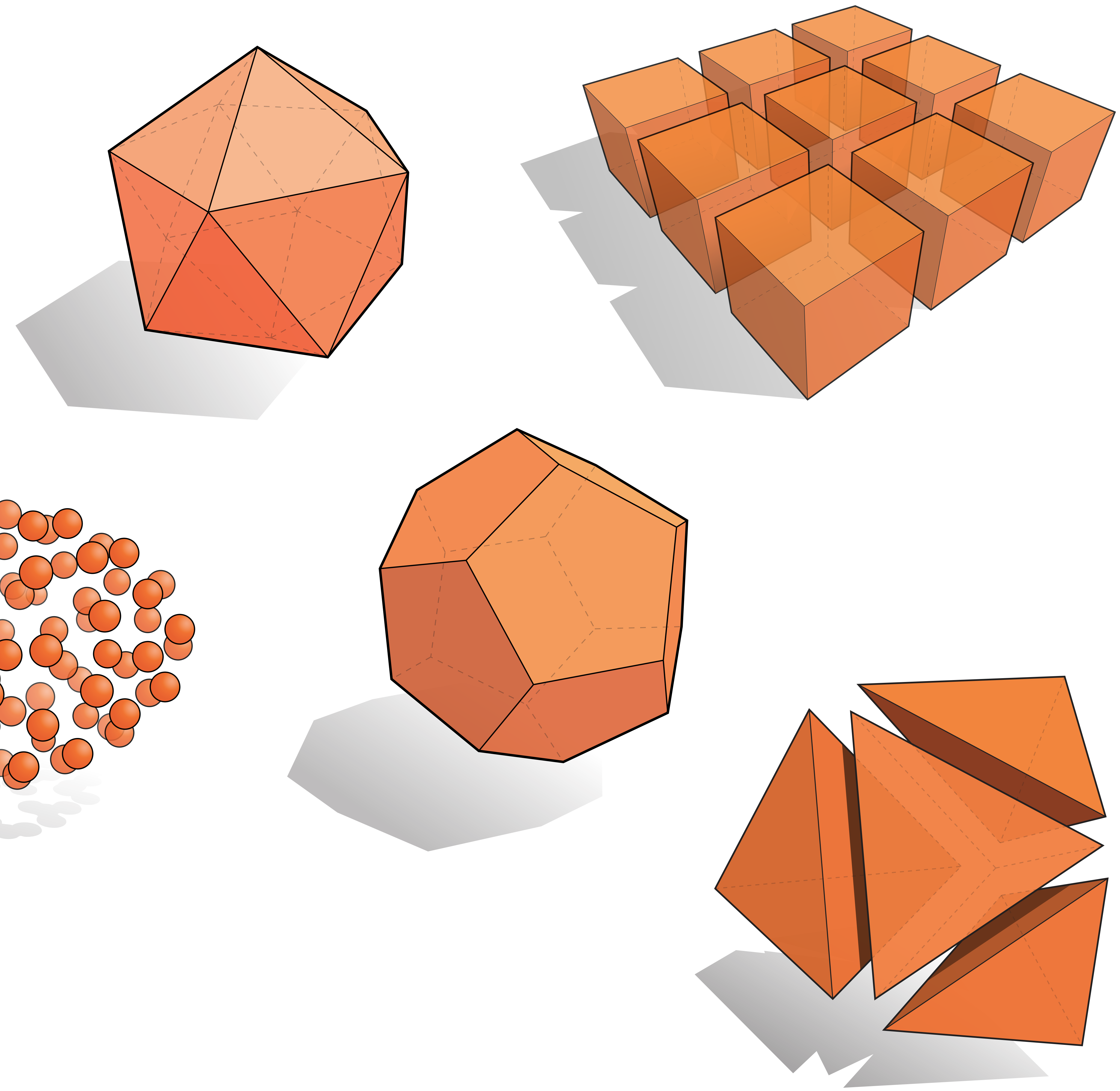
"I hate meshes.
I cannot believe how hard this is.
Geometry is hard."

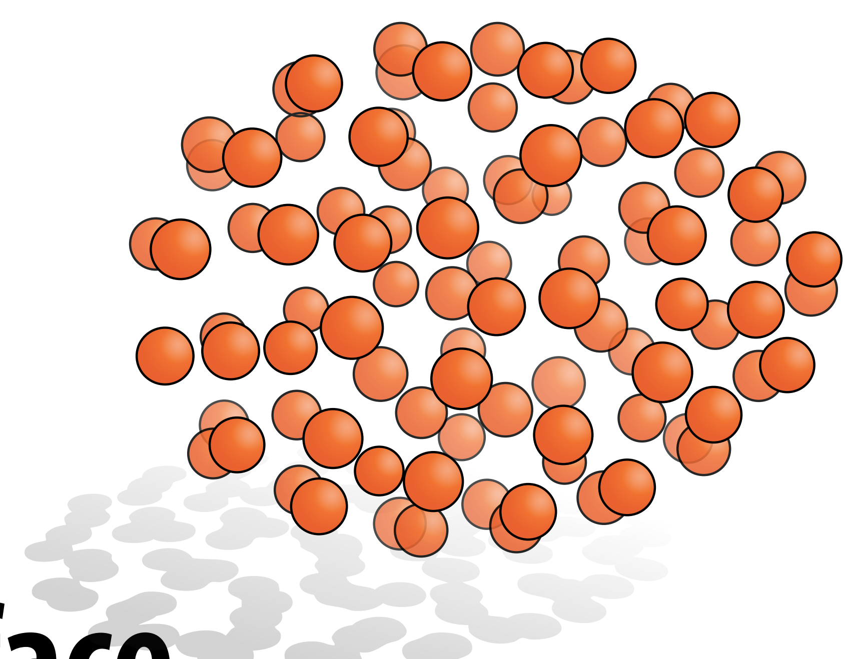
—David Baraff

Senior Research Scientist
Pixar Animation Studios

Many ways to digitally encode geometry

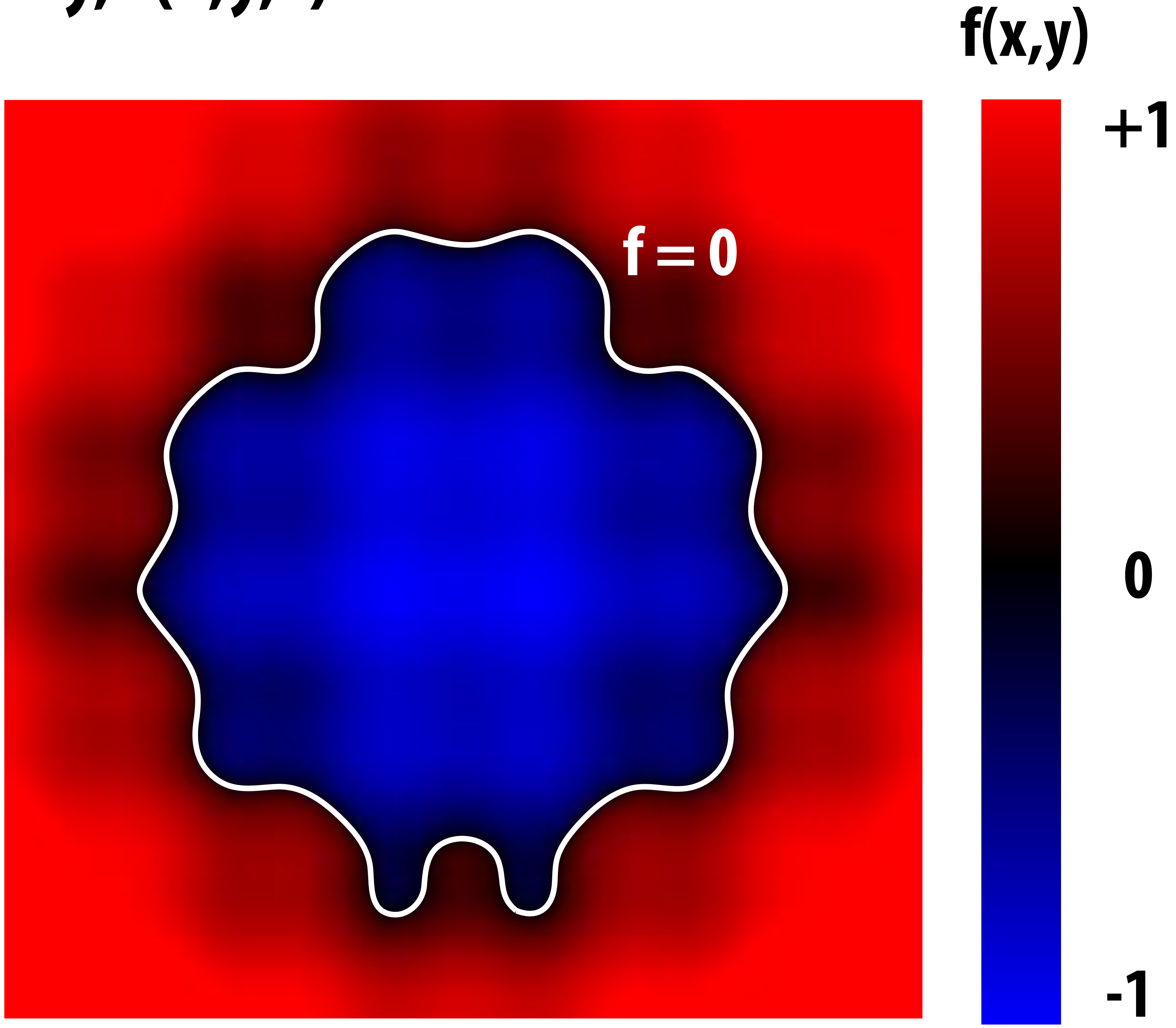
- **EXPLICIT**
  - point cloud
  - polygon mesh
  - subdivision, NURBS
  - ...
- **IMPLICIT**
  - level set
  - algebraic surface
  - L-systems
  - ...
- **Each choice best suited to a different task/type of geometry**





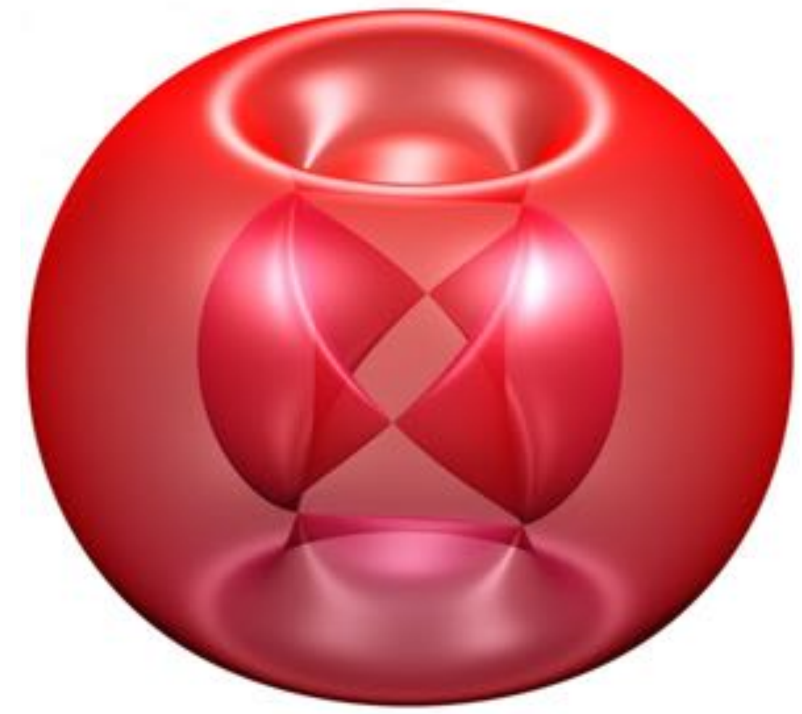
“Implicit” Representations of Geometry

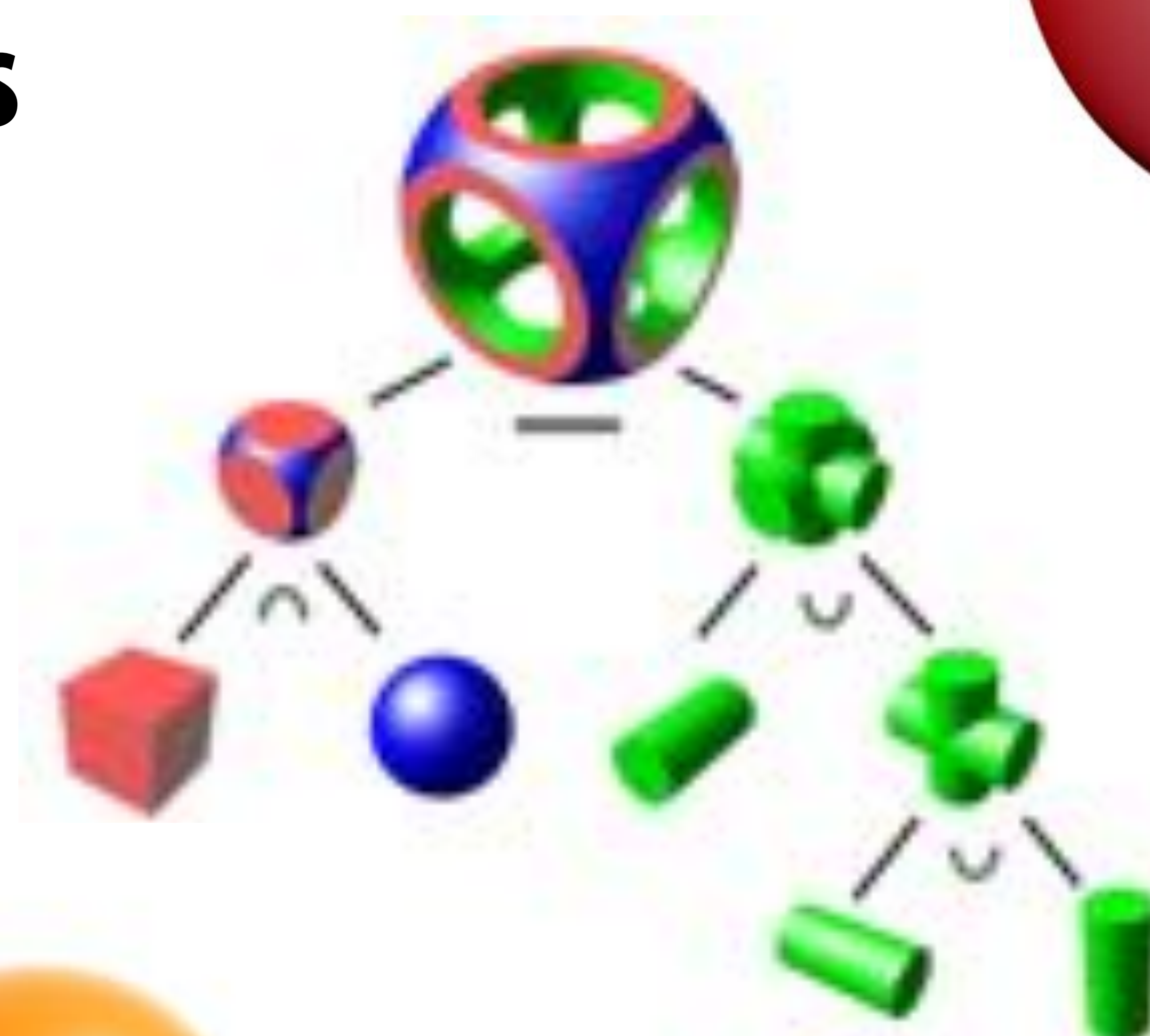
- Points aren’t known directly, but satisfy some relationship
- E.g., unit sphere is all points such that $x^2+y^2+z^2=1$
- More generally, $f(x,y,z) = 0$



Many implicit representations in graphics

- algebraic surfaces
- constructive solid geometry
- level set methods
- blobby surfaces
- fractals
- ...

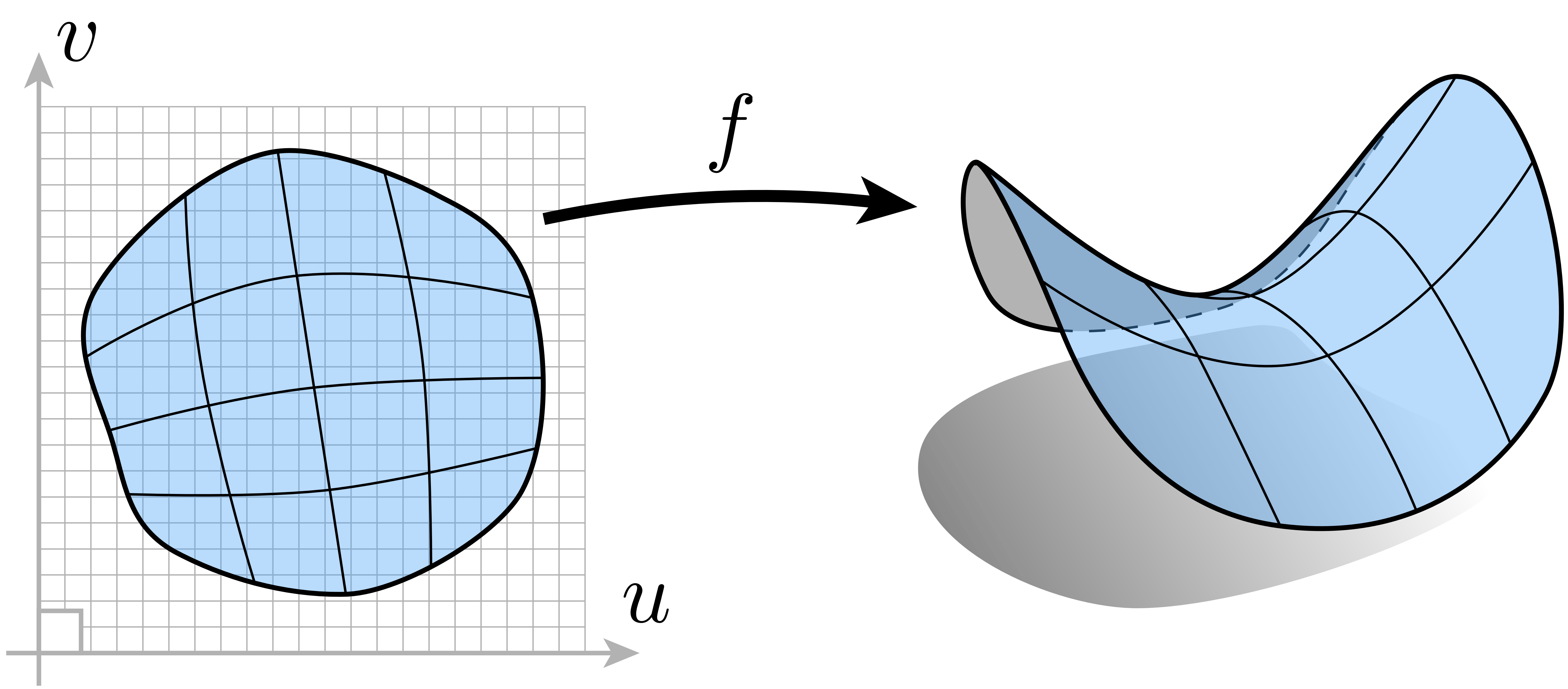




(Will see some of these a bit later.)

"Explicit" Representations of Geometry

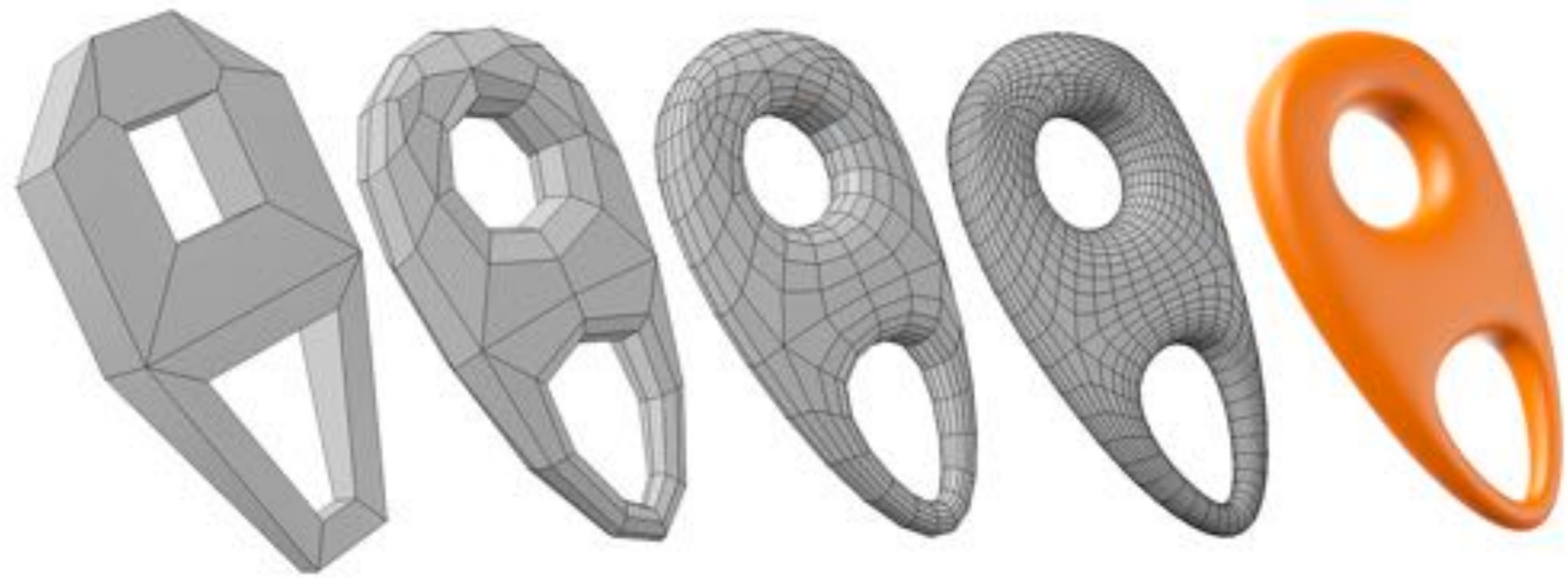
- **All points are given directly**
- **E.g., points on sphere are** $(\cos(u)\sin(v), \sin(u)\sin(v), \cos(v)),$
  $\text{for } 0 \leq u < 2\pi \text{ and } 0 \leq v \leq \pi$
- **More generally:** $f : \mathbb{R}^2 \to \mathbb{R}^3; (u,v) \mapsto (x,y,z)$

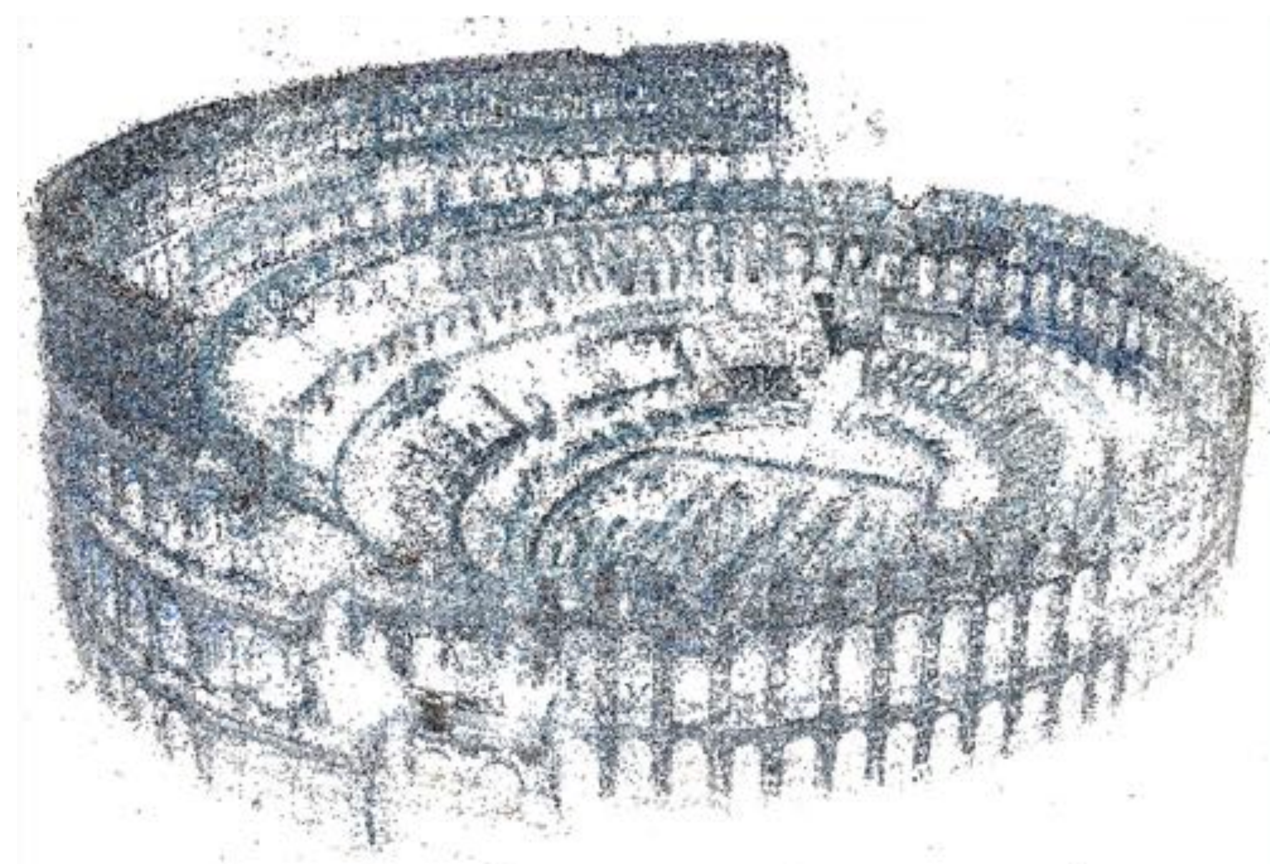


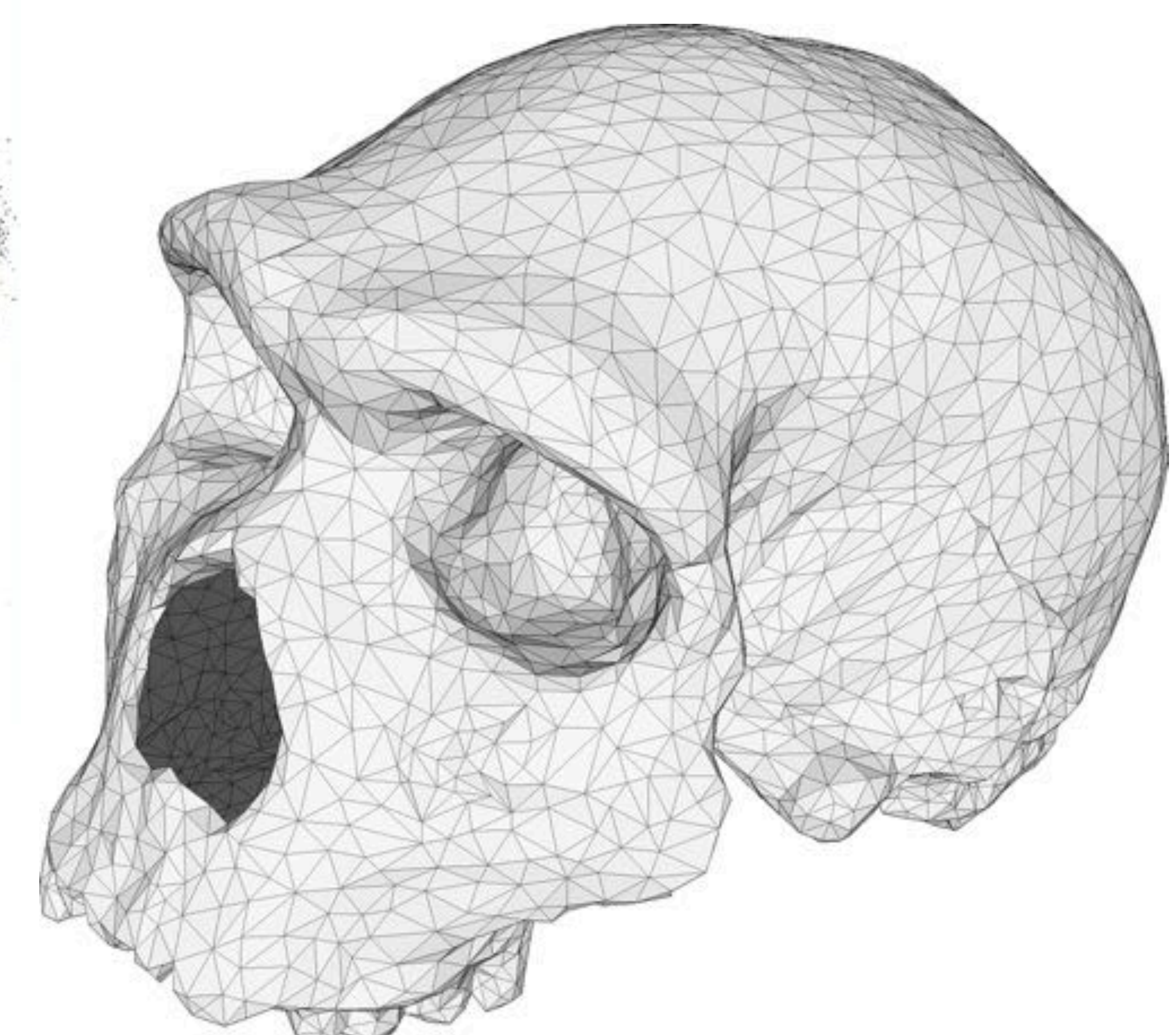
- **(Might have a bunch of these maps, e.g., one per triangle!)**

Many explicit representations in graphics

- triangle meshes
- polygon meshes
- subdivision surfaces
- NURBS
- point clouds
- ...







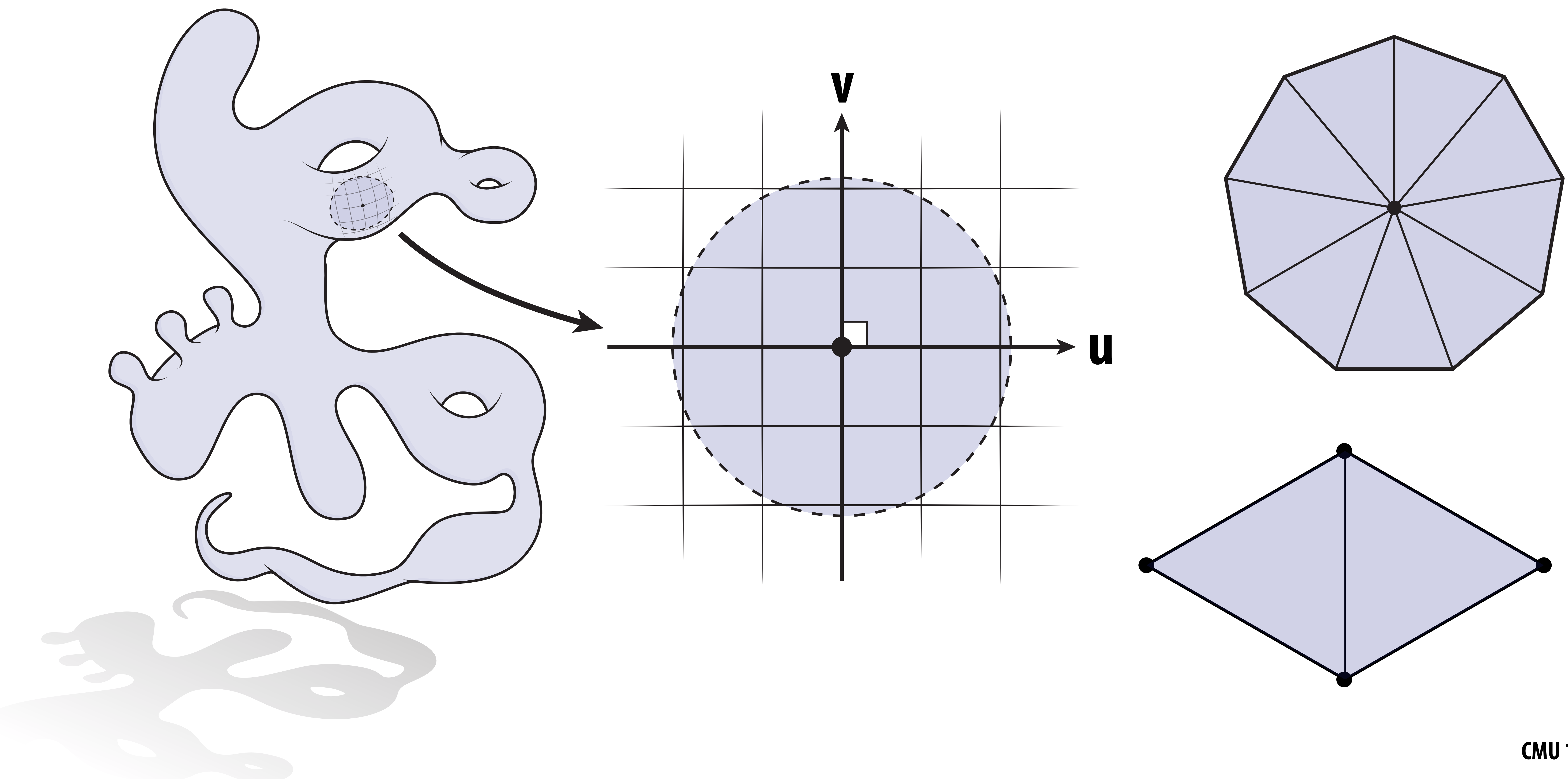
(Will see some of these a bit later.)

Ok, so we have many ways to represent surfaces.

But what is a surface anyway?

Manifold Assumption

- First, let's define manifold geometry
- Can be hard to understand motivation at first!
- Let's revisit a more familiar example...

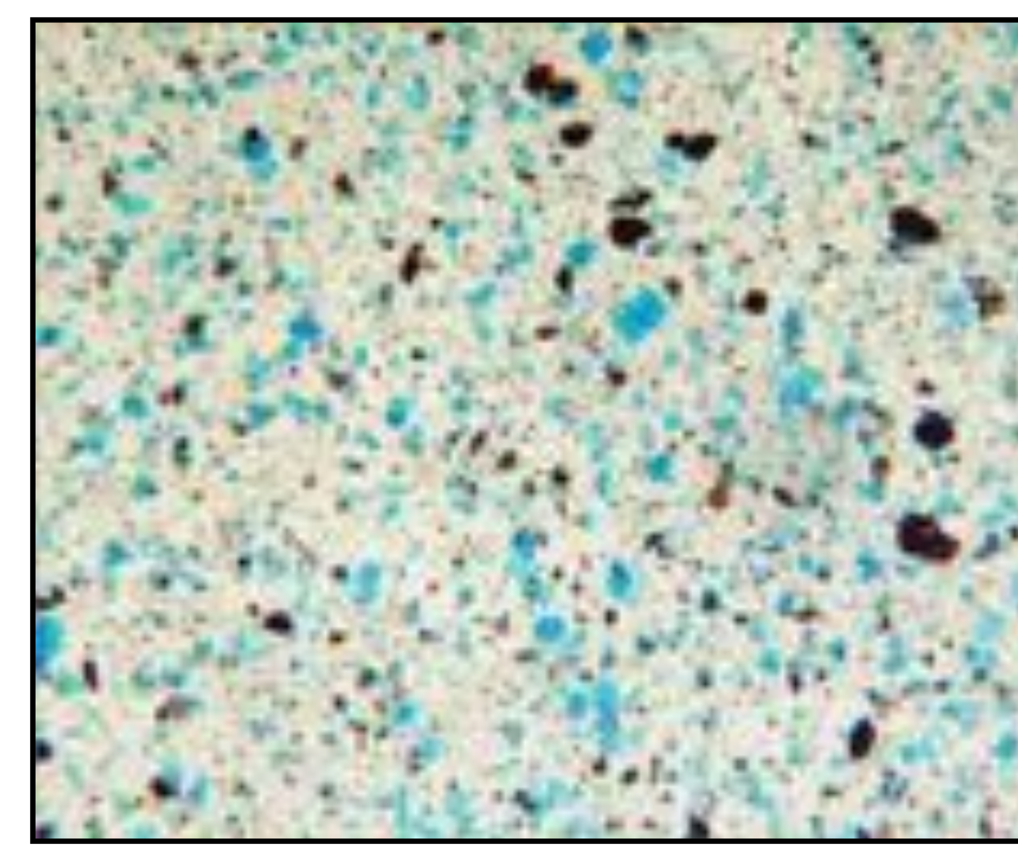


Bitmap Images, Revisited

To encode images, we used a regular grid of pixels:

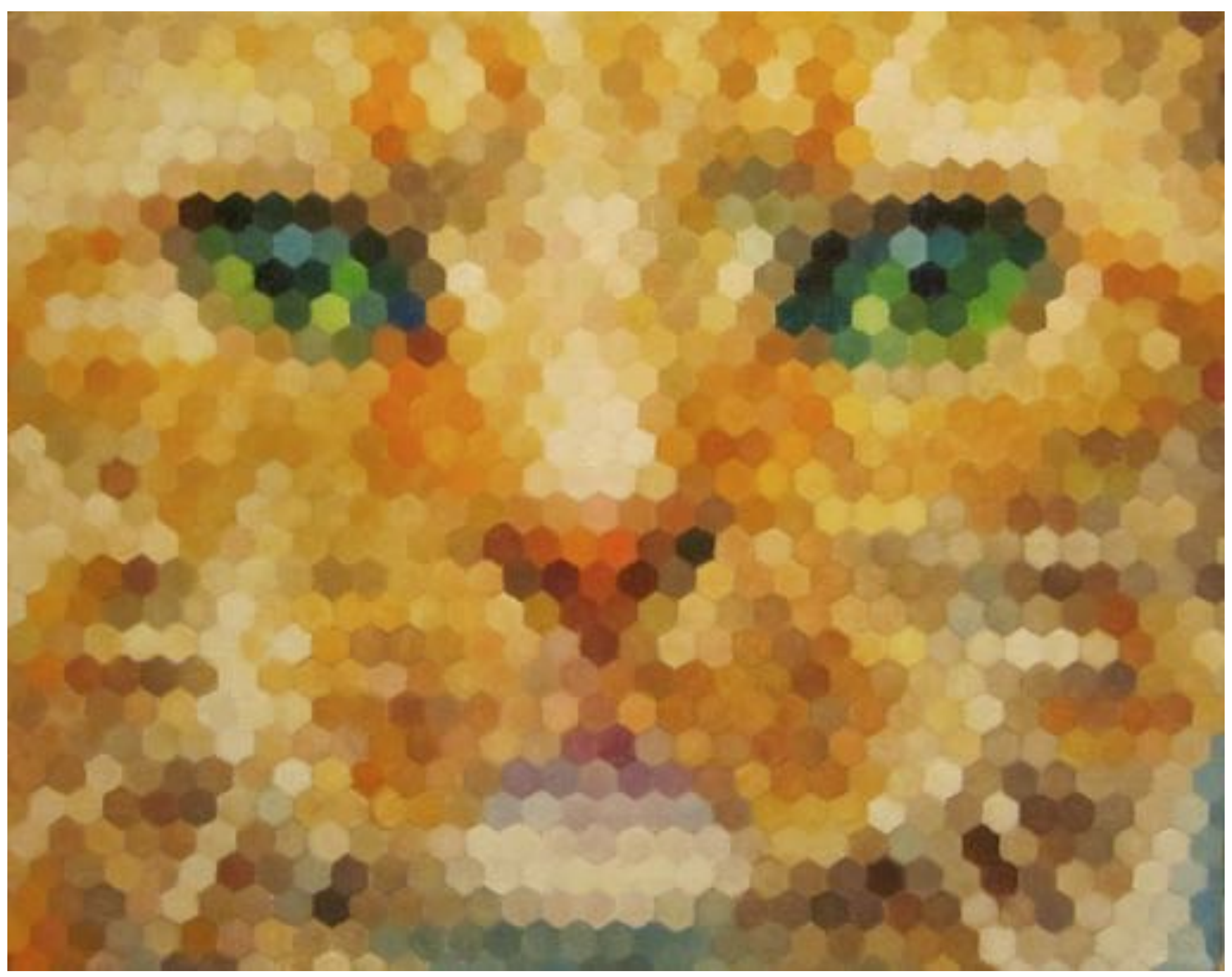
But images are not fundamentally made of little squares:

Goyō Hashiguchi, *Kamisuki* (ca 1920)



photomicrograph of paint

So why did we choose a square grid?



...rather than dozens of possible alternatives?

Regular grids make life easy

- One reason: SIMPLICITY / EFFICIENCY
  - E.g., always have four neighbors
  - Easy to index, easy to filter…
  - Storage is just a list of numbers
- Another reason: GENERALITY
  - Can encode basically any image
- Are regular grids always the best choice for bitmap images?
  - No! E.g., suffer from anisotropy, don’t capture edges, ...
  - But more often than not are a pretty good choice
- Will see a similar story with geometry...

	`(i,j-1)`	
`(i-1,j)`	`(i,j)`	`(i+1,j)`
	`(i,j+1)`	

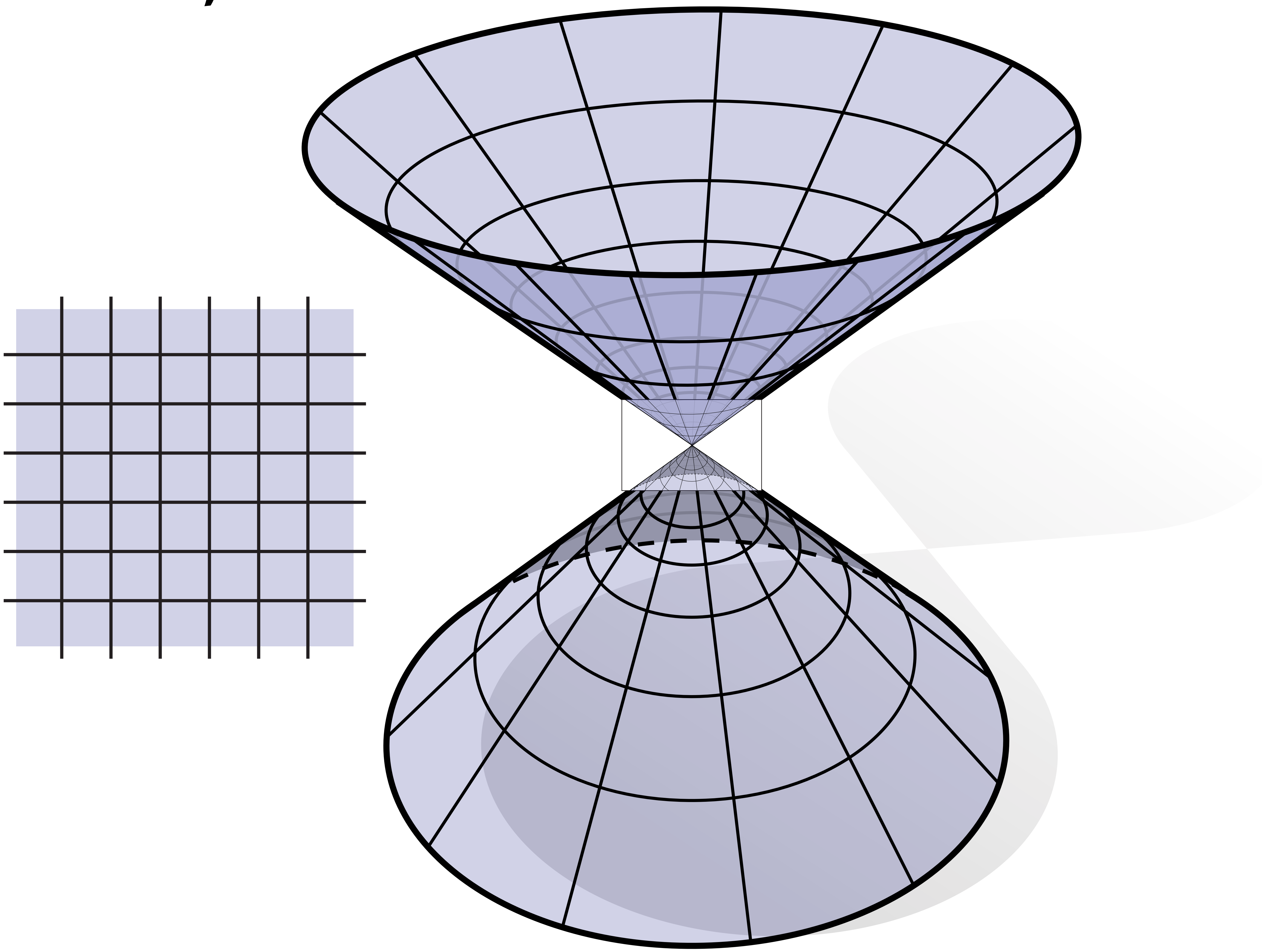
So, how should we encode surfaces?

Smooth Surfaces

- Intuitively, a <u>surface</u> is the boundary or "shell" of an object
- (Think about the candy shell, not the chocolate.)
- Surfaces are manifold:
  - If you zoom in far enough, can draw a regular coordinate grid
  - E.g., the Earth from space vs. from the ground

Isn't every shape manifold?

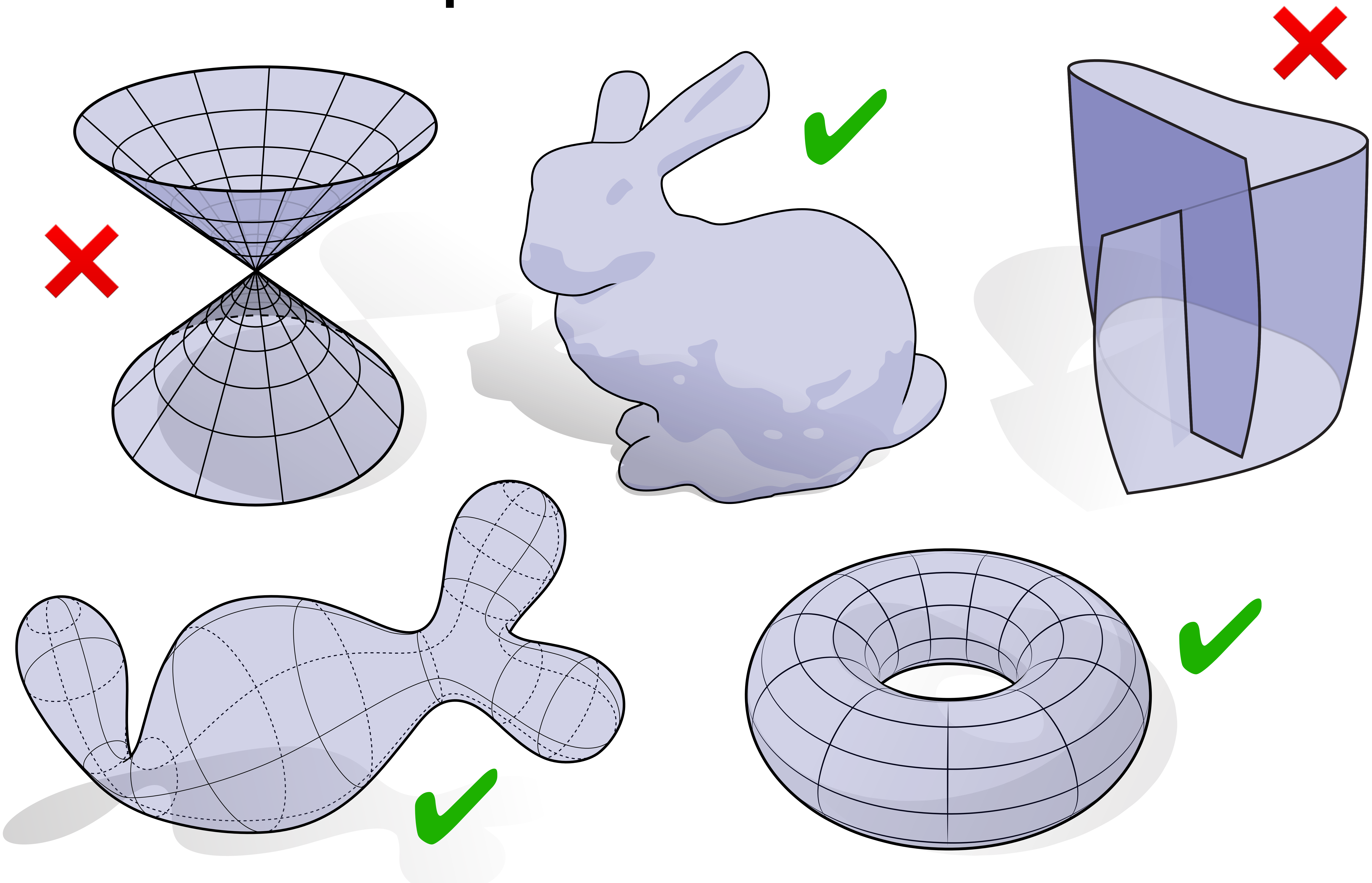
- No, for instance:



Can't draw ordinary 2D grid at center, no matter how close we get.

Examples—Manifold vs. Nonmanifold

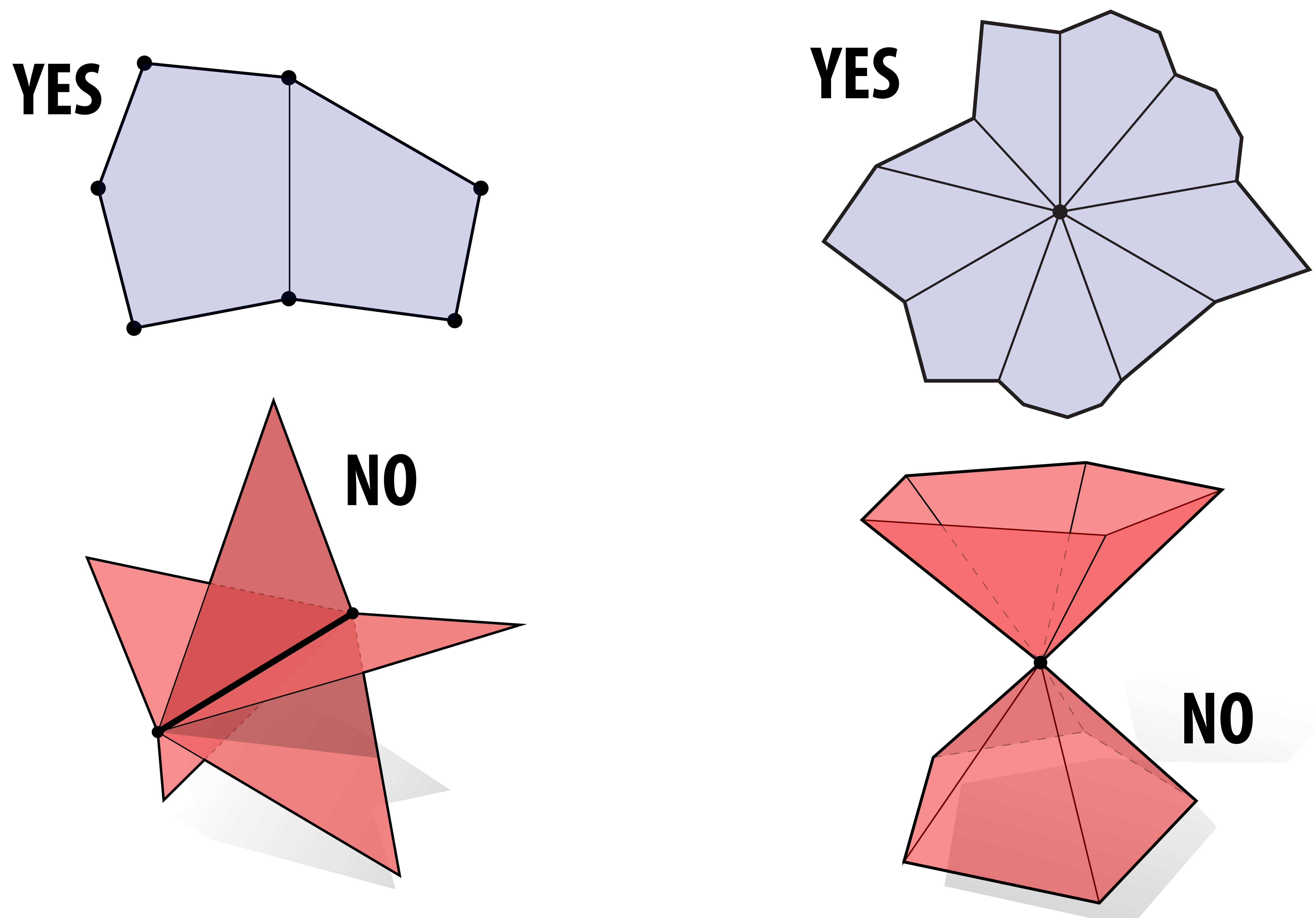
- Which of these shapes are manifold?



Suppose we have a polygon mesh
(an explicit representation)

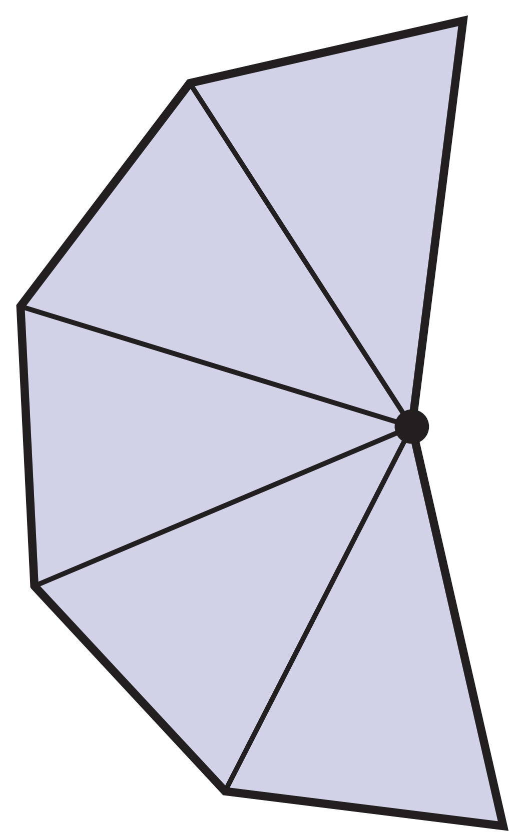
A manifold polygon mesh has fans, not fins

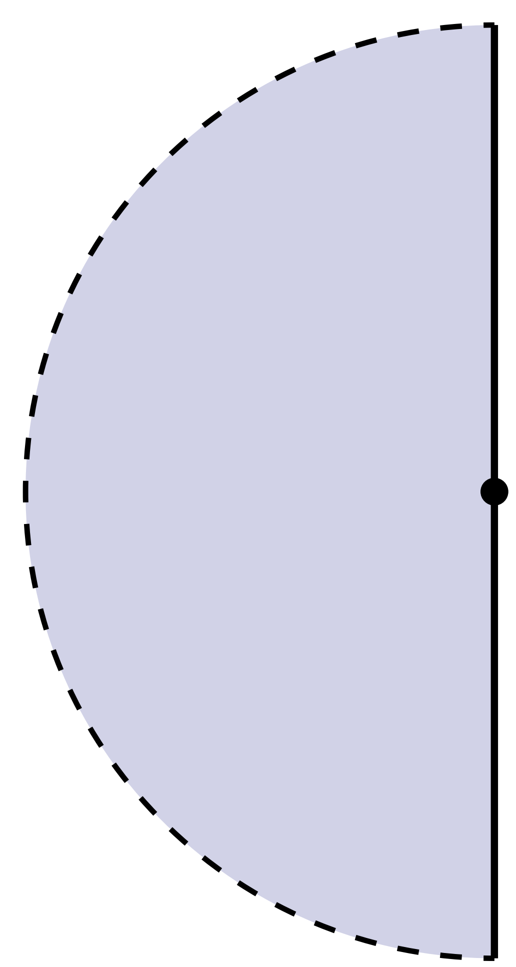
- For polygonal surfaces just two easy conditions to check:
  1. Every edge is contained in only two polygons (no “fins”)
  2. The polygons containing each vertex make a single “fan”



What about boundary?

- The boundary is where the surface “ends.”
- E.g., waist & ankles on a pair of pants.
- Locally, looks like a half disk
- Globally, each boundary forms a loop



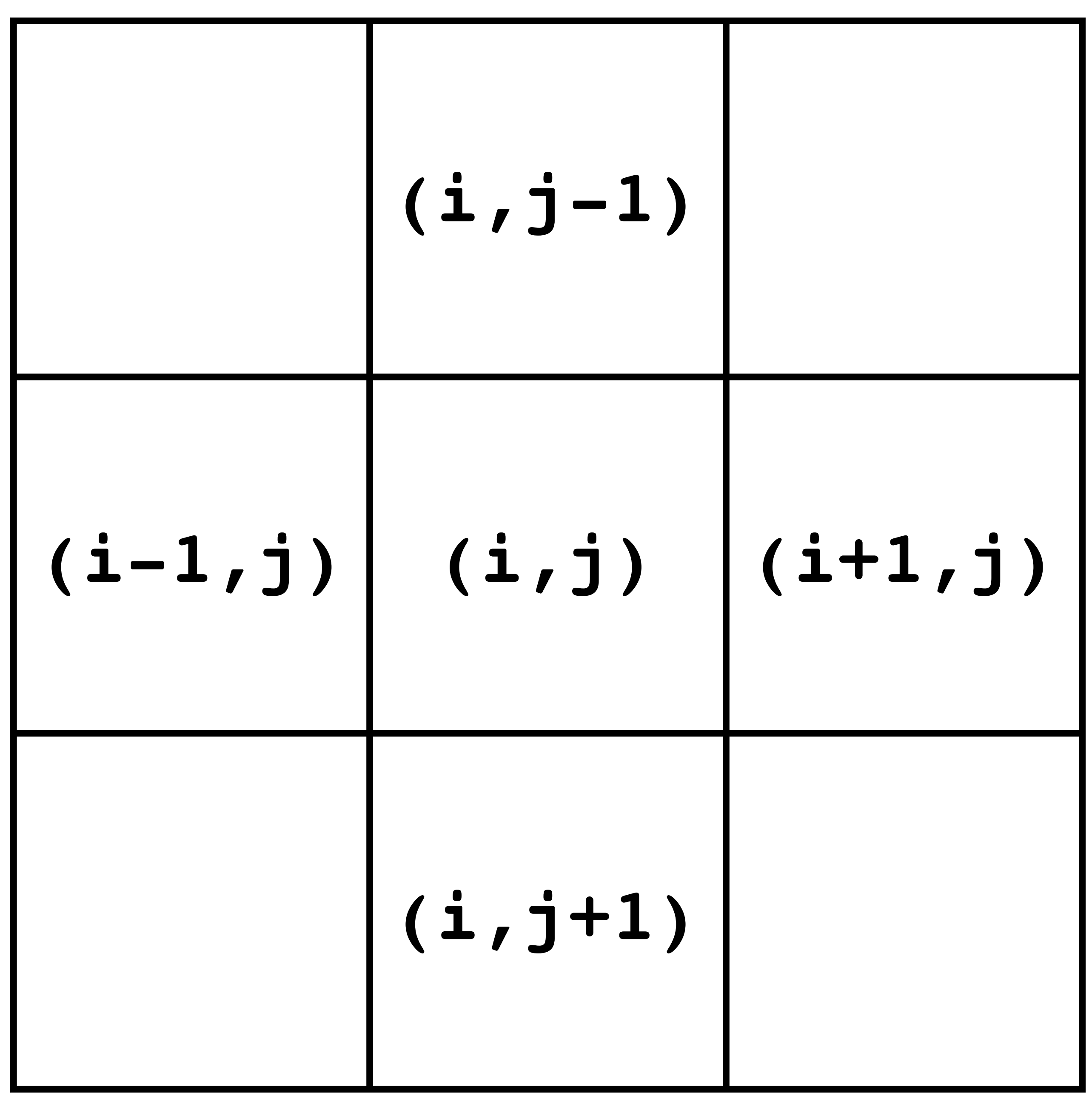


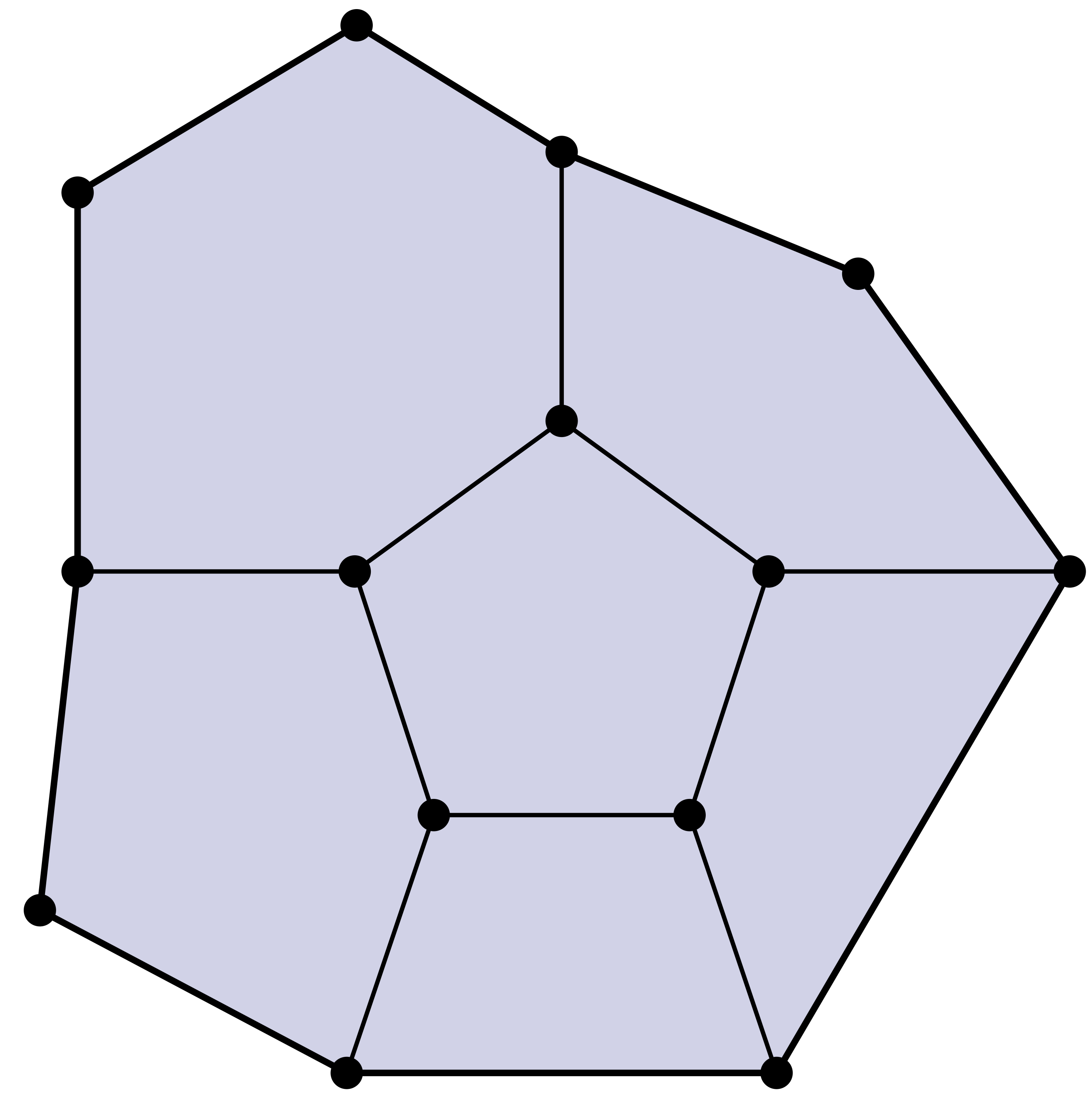
- Polygon mesh:
  - one polygon per boundary edge
  - boundary vertex looks like “pacman”

Ok, but why is the manifold assumption useful?

Keep it Simple!

- **Same motivation as for images:**
  - make some assumptions about our geometry to keep data structures/algorithms simple and efficient
  - in many common cases, doesn't fundamentally limit what we can do with geometry





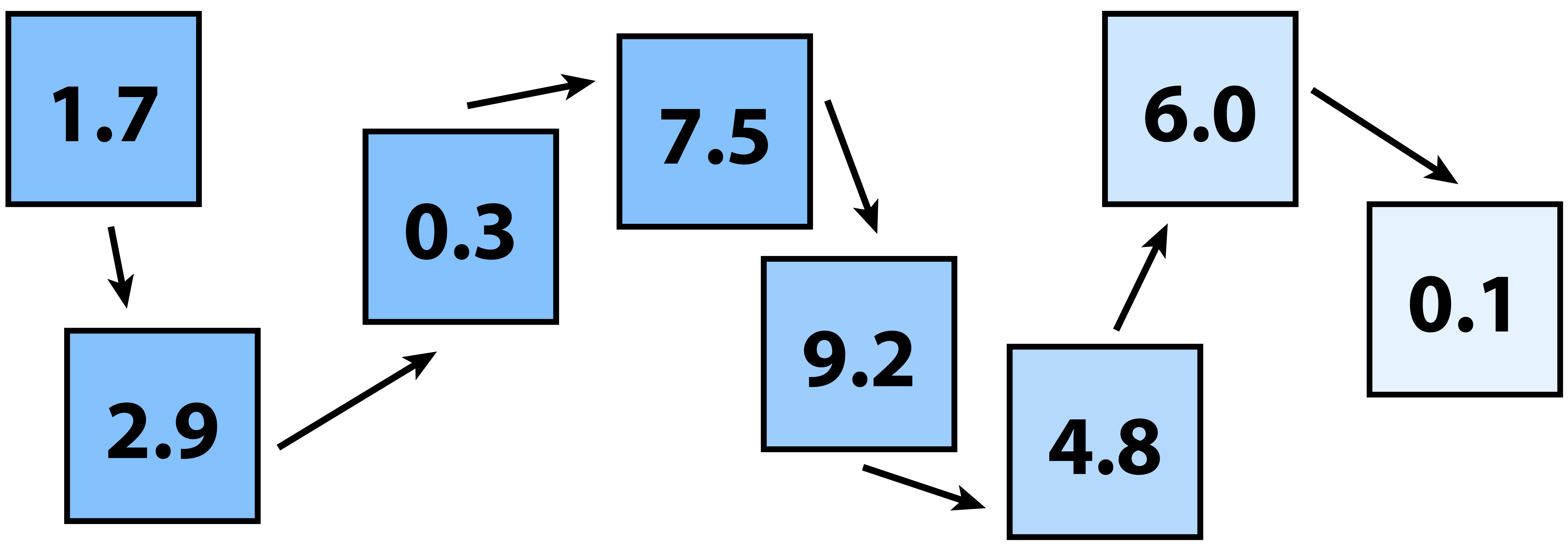
Let’s talk about how to encode all this data

Warm up: storing numbers

- Q: What data structures can we use to store a list of numbers?
- One idea: use an array (constant time lookup, coherent access)

1.7	2.9	0.3	7.5	9.2	4.8	6.0	0.1

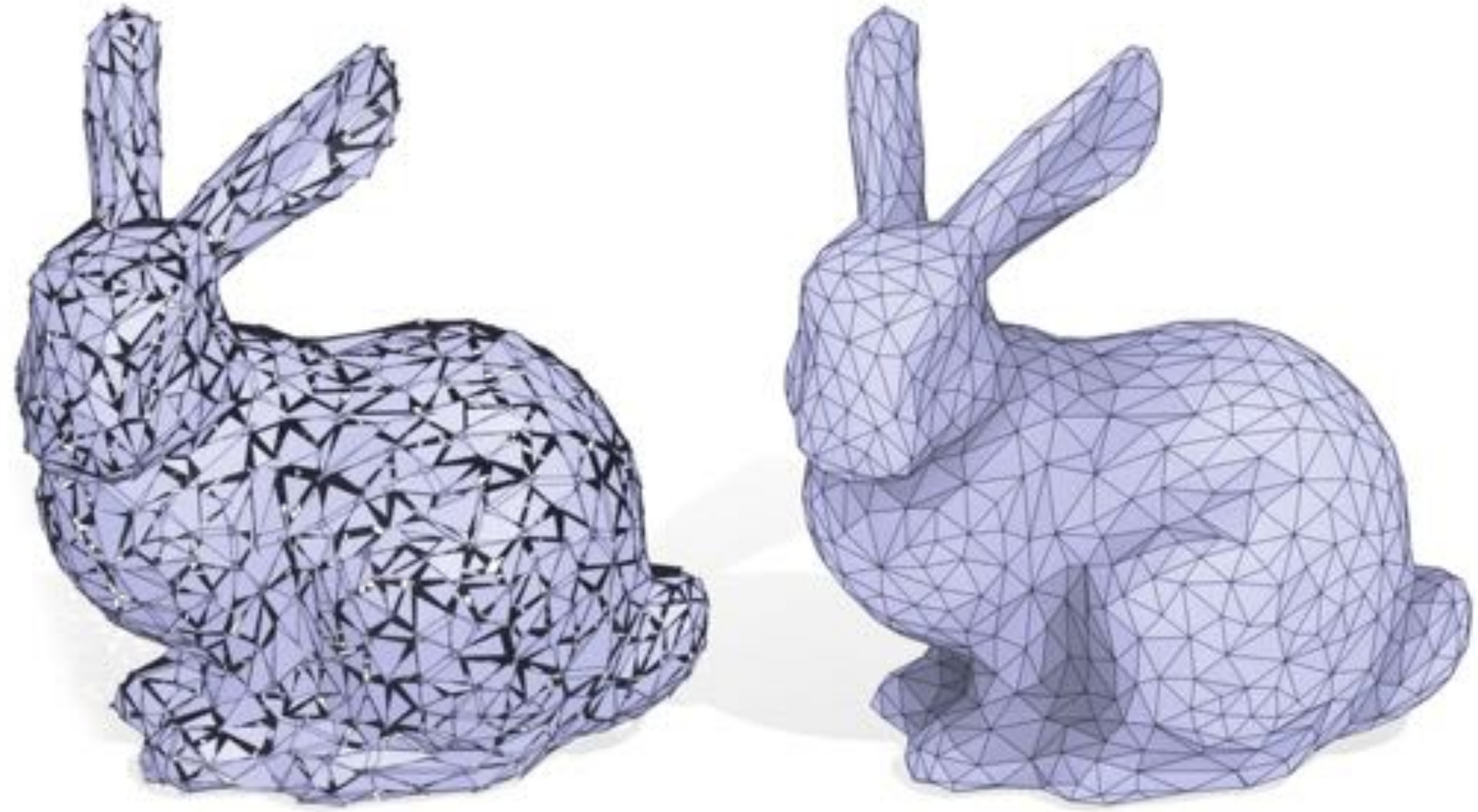
- Alternative: use a linked list (linear lookup, incoherent access)

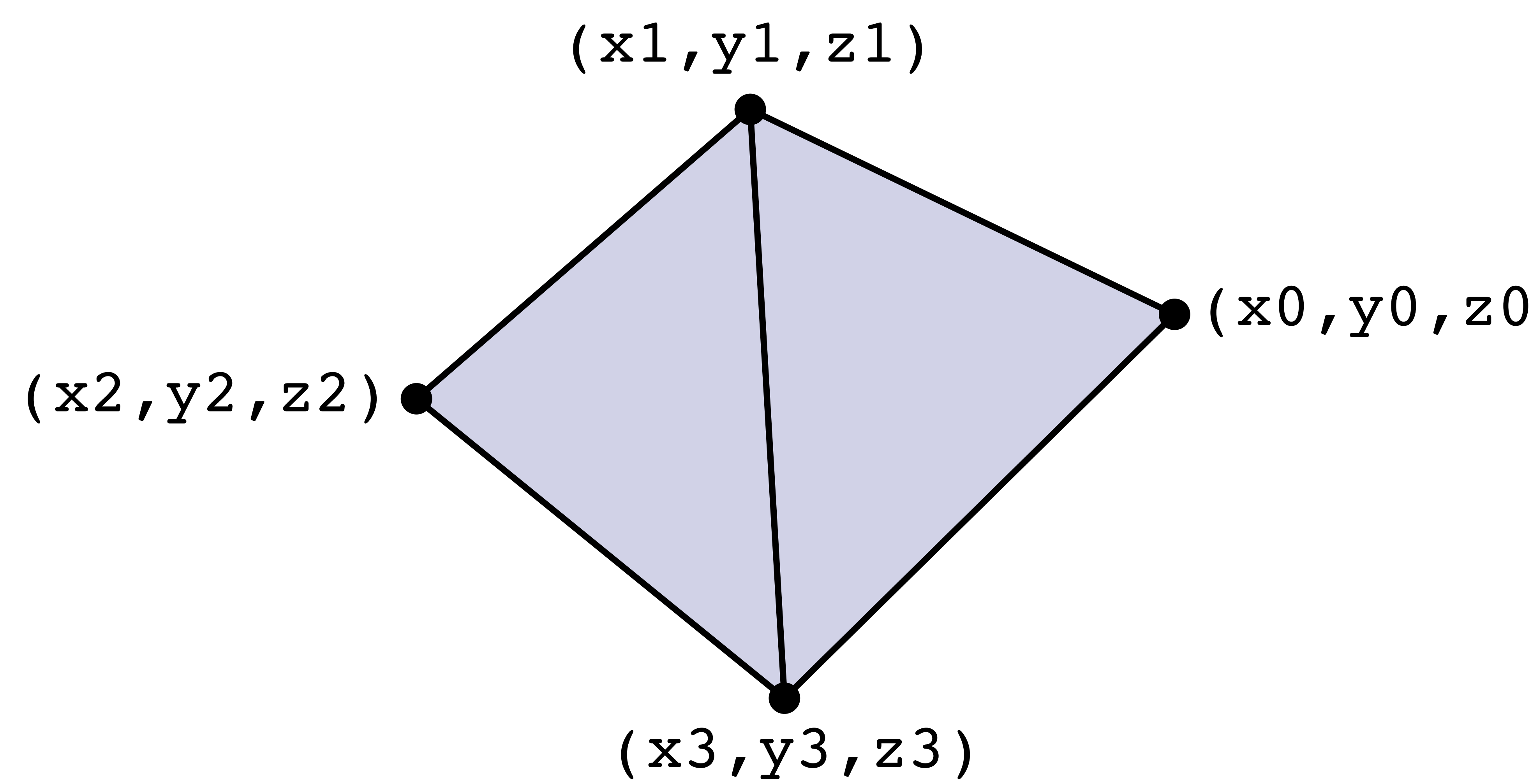


- Q: Why bother with the linked list?
- A: For one, we can easily insert numbers wherever we like...

Polygon Soup

- **Most basic idea:**
  - For each triangle, just store three coordinates
  - No other information about connectivity
  - Not much different from point cloud! (“Triangle cloud?”)
- **Pros:**
  - Really stupidly simple
- **Cons:**
  - Redundant storage
  - Hard to do much beyond simply drawing the mesh on screen
  - Need spatial data structures (later) to find neighbors





x0,y0,z0	x1,y1,z1	x3,y3,z3
x1,y1,z1	x2,y2,z2	x3,y3,z3

Adjacency List (Array-like)

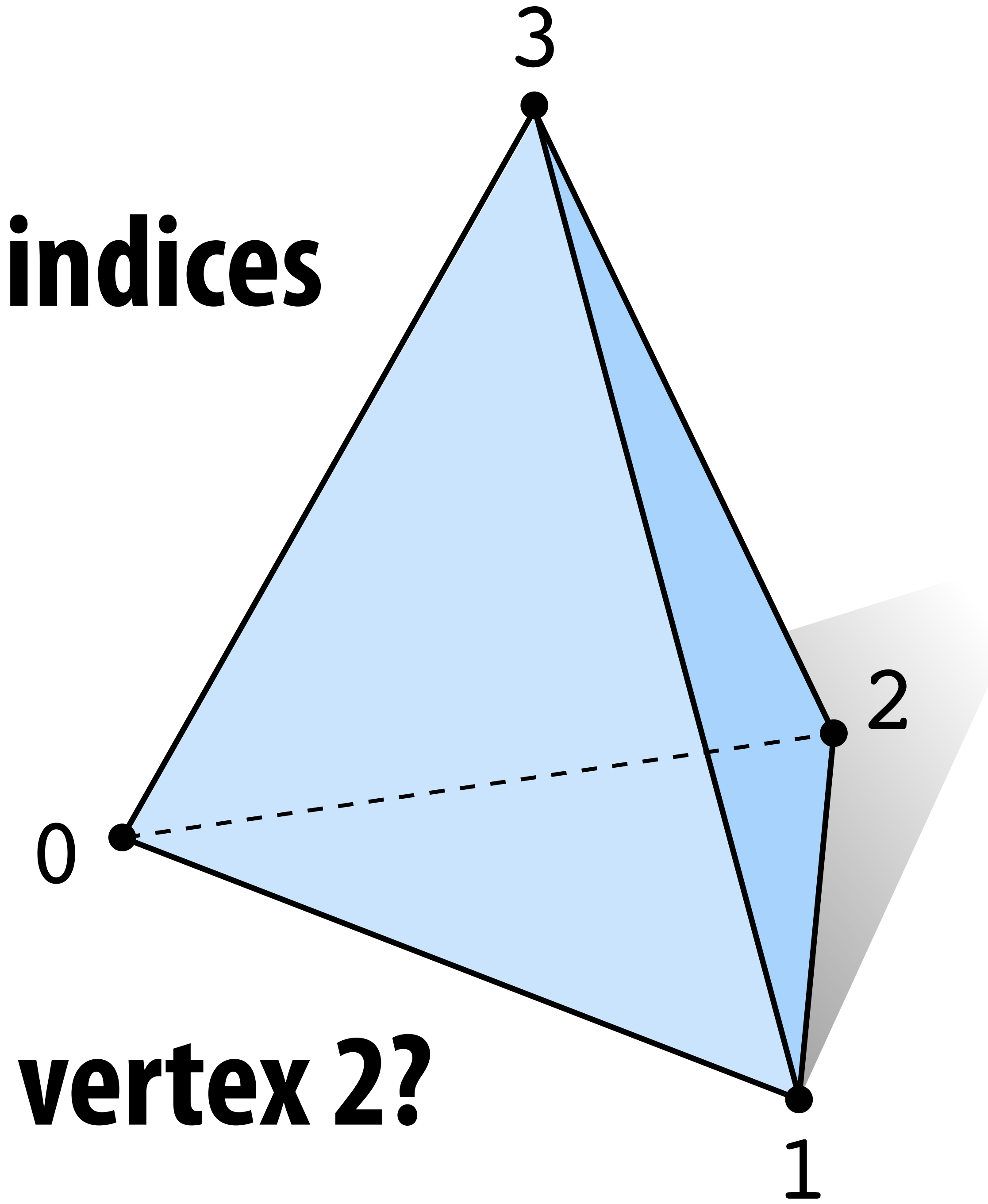
- Store triples of coordinates (x,y,z), tuples of indices
- E.g., tetrahedron:

VERTICES

	x	y	z
0:	−1	−1	−1
1:	1	−1	1
2:	1	1	−1
3:	−1	1	1

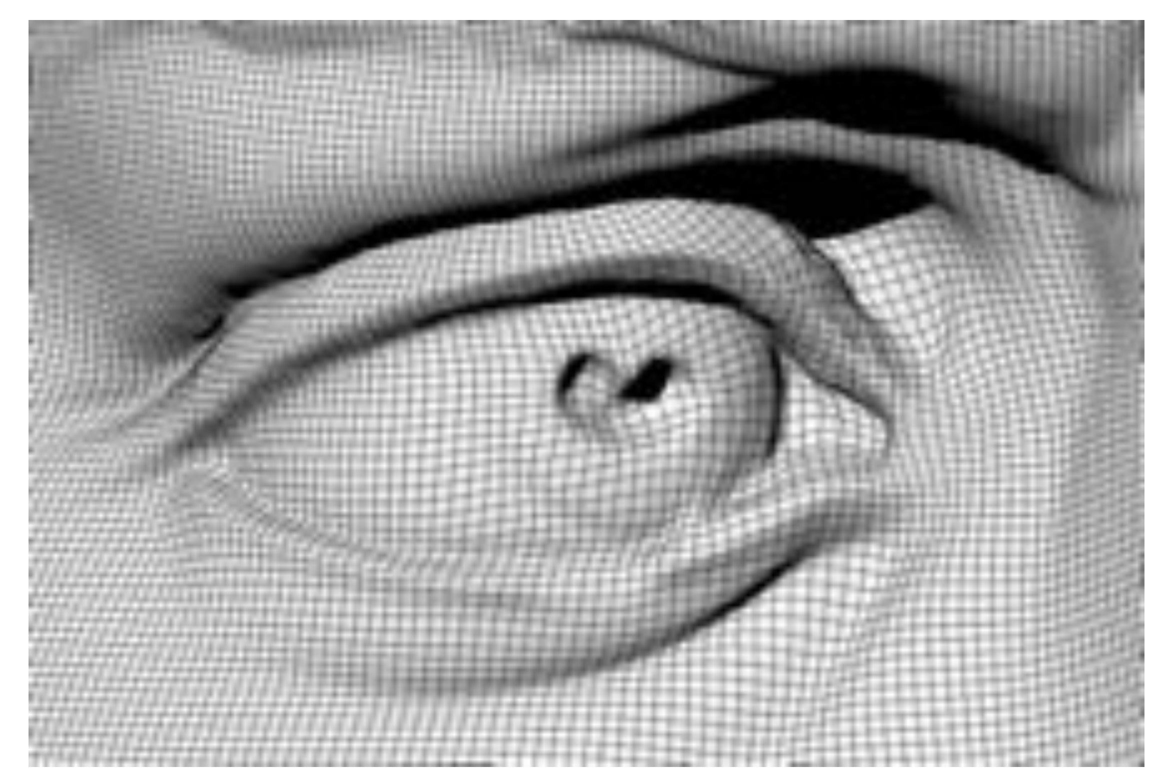
POLYGONS

i	j	k
0	2	1
0	3	2
3	0	1
3	1	2



- Q: How do we find all the polygons touching vertex 2?
- Ok, now consider a more complicated mesh:

~1 billion polygons



Very expensive to find the neighboring polygons! (What's the cost?)

Incidence Matrices

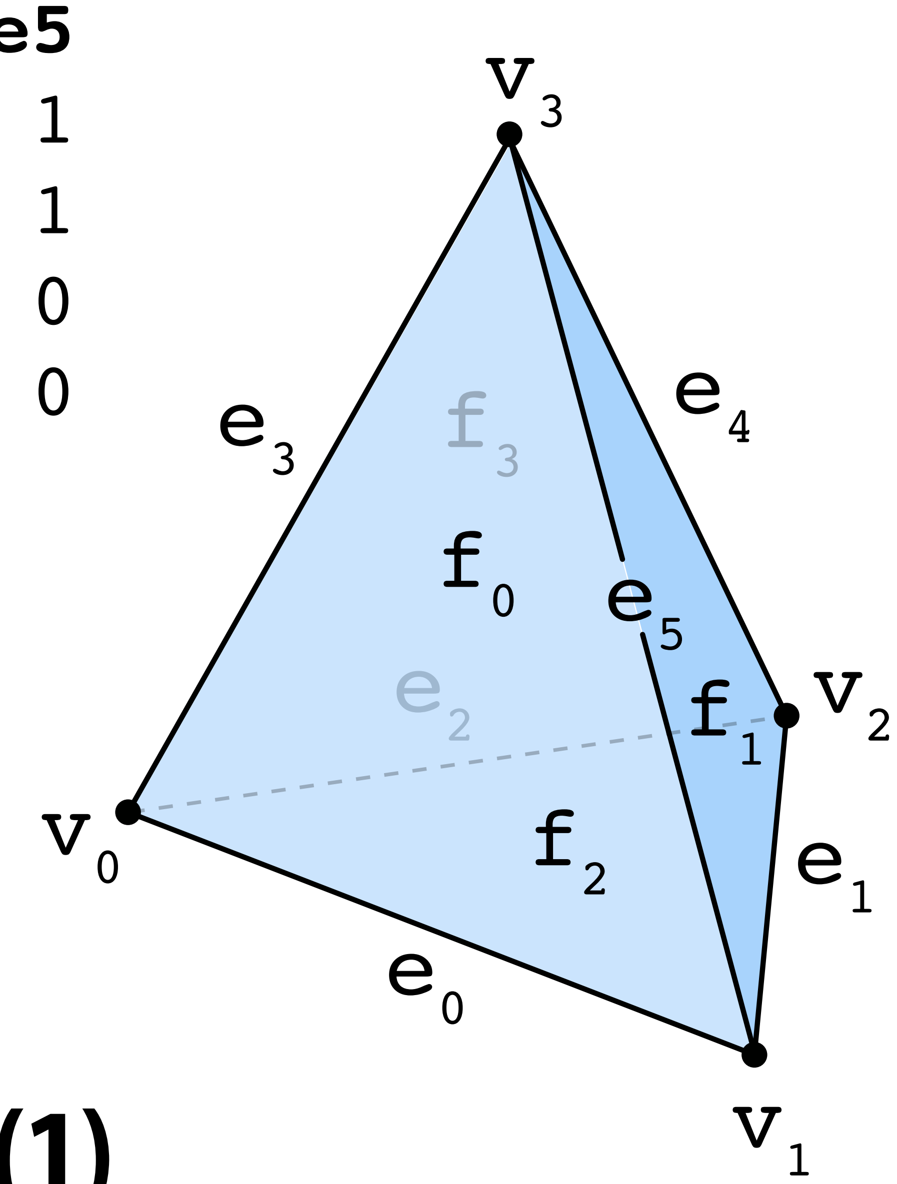
- If we want to know who our neighbors are, why not just store a list of neighbors?
- Can encode all neighbor information via incidence matrices
- E.g., tetrahedron:

VERTEX↔EDGE

	v0	v1	v2	v3
e0	1	1	0	0
e1	0	1	1	0
e2	1	0	1	0
e3	1	0	0	1
e4	0	0	1	1
e5	0	1	0	1

EDGE↔FACE

	e0	e1	e2	e3	e4	e5
f0	1	0	0	1	0	1
f1	0	1	0	0	1	1
f2	1	1	1	0	0	0
f3	0	0	1	1	1	0

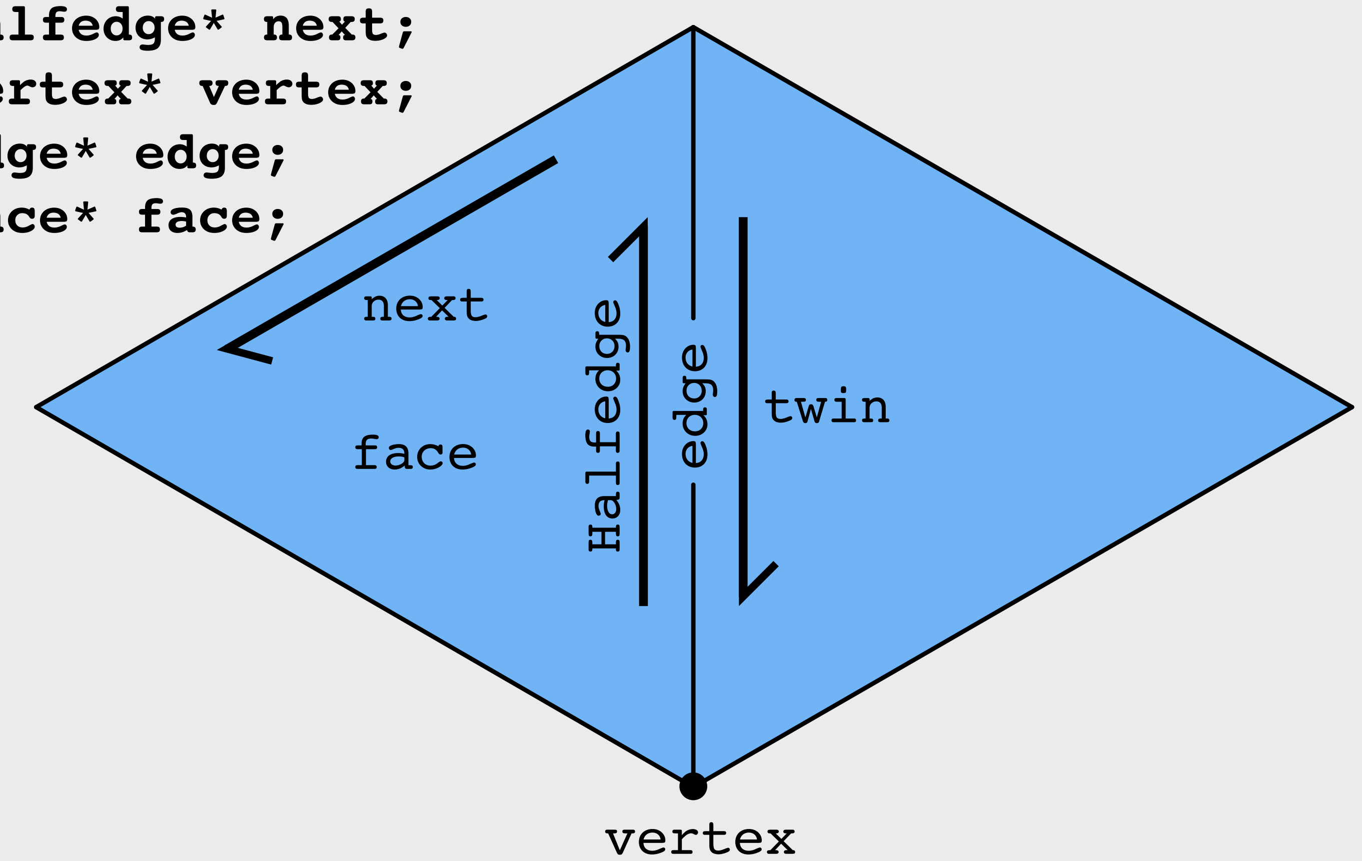


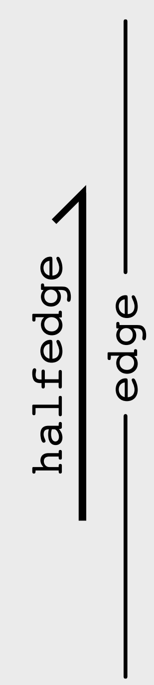
- 1 means "touches"; 0 means "does not touch"
- Instead of storing lots of 0's, use sparse matrices
- Still large storage cost, but finding neighbors is now O(1)
- Hard to change connectivity, since we used fixed indices
- Bonus feature: mesh does not have to be manifold

Halfedge Data Structure (Linked-list-like)

- Store some information about neighbors
- Don't need an exhaustive list; just a few key pointers
- Key idea: two halfedges act as "glue" between mesh elements:

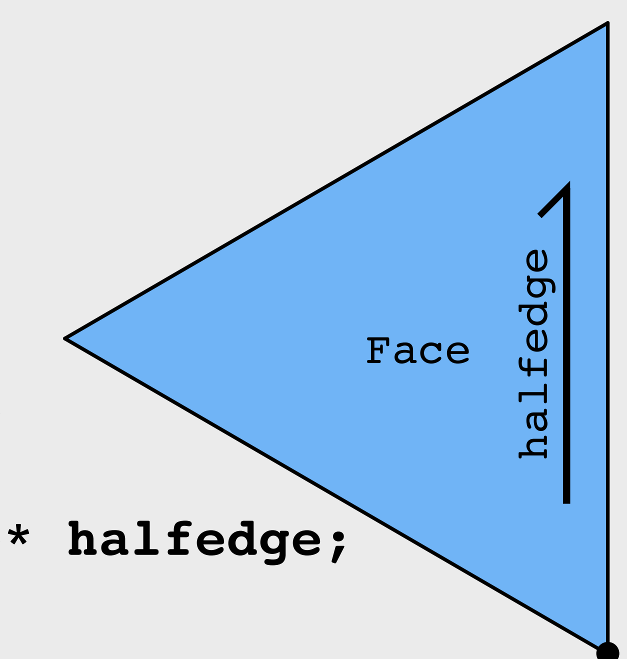
```
struct Halfedge
{
   Halfedge* twin;
   Halfedge* next;
   Vertex* vertex;
   Edge* edge;
   Face* face;
};
```

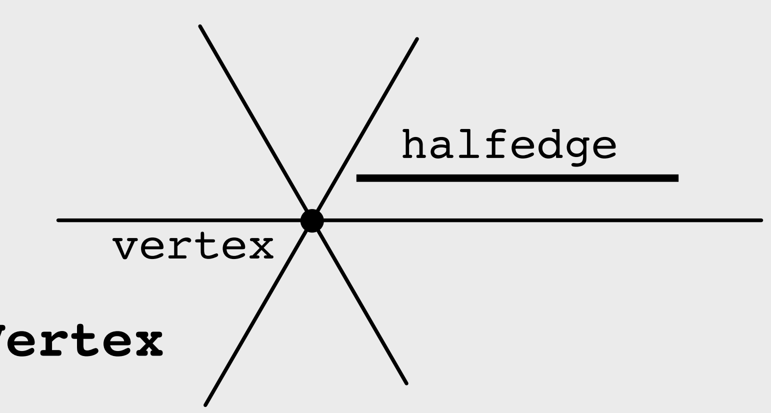





```
struct Edge
{
   Halfedge* halfedge;
};
```

```
struct Face
{
   Halfedge* halfedge;
};
```





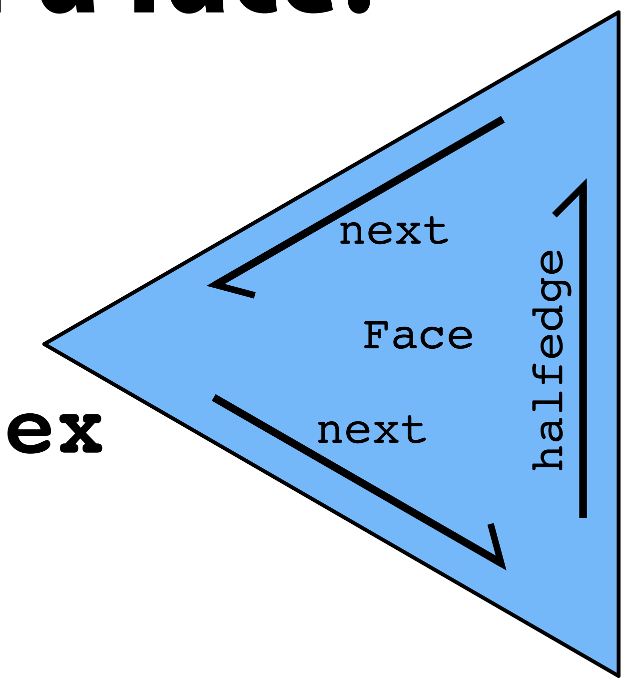

```
struct Vertex
{
   Halfedge* halfedge;
};
```

- Each vertex, edge face points to just one of its halfedges.

Halfedge makes mesh traversal easy

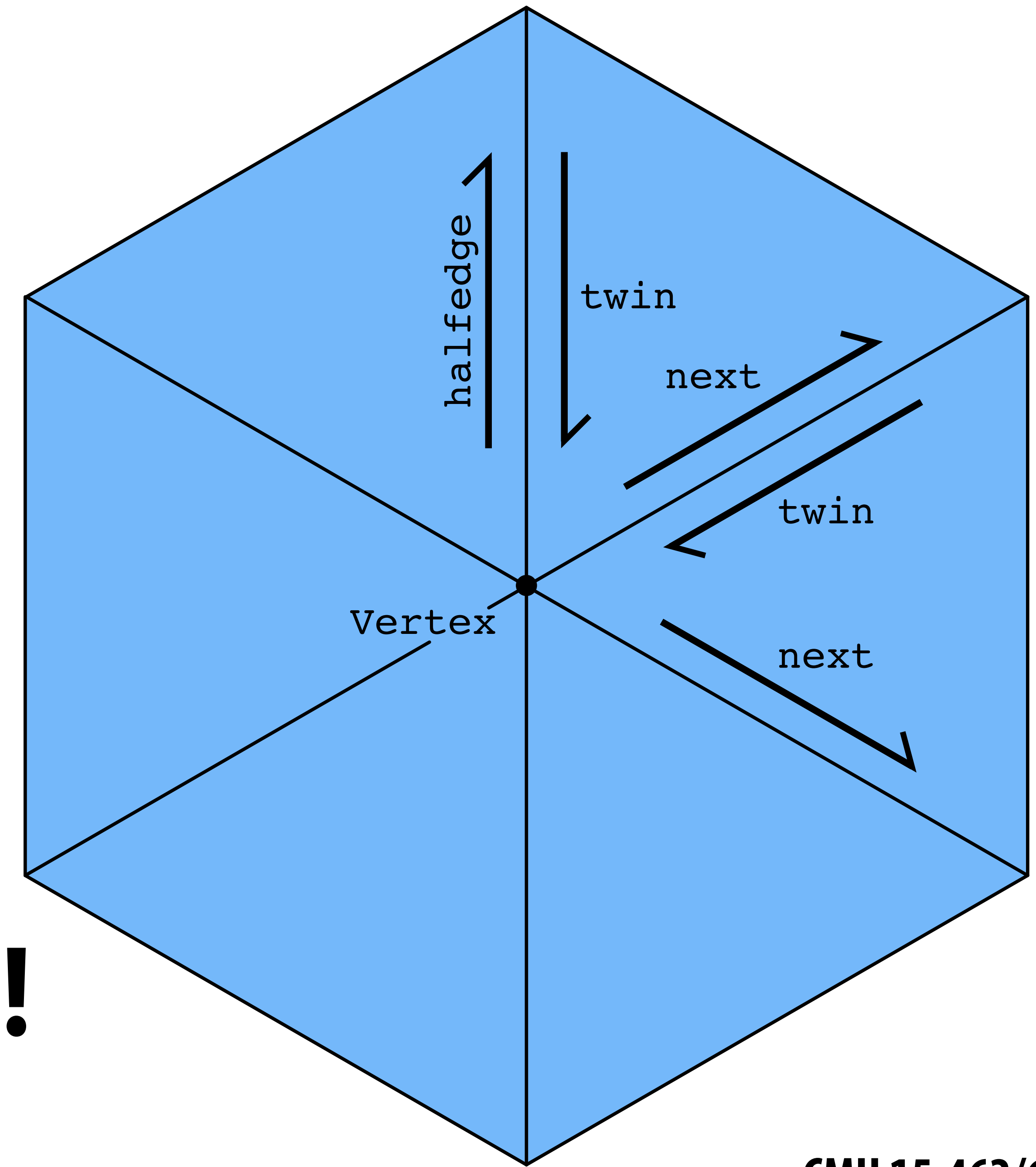
- Use "twin" and "next" pointers to move around mesh
- Use "vertex", "edge", and "face" pointers to grab element
- Example: visit all vertices of a face:

```
Halfedge* h = f->halfedge;
do {
   h = h->next;
   // do something w/ h->vertex
}
while( h != f->halfedge );
```



- Example: visit all neighbors of a vertex:

```
Halfedge* h = v->halfedge;
do {
   h = h->twin->next;
}
while( h != v->halfedge );
```



- Note: only makes sense if mesh is manifold!

Halfedge connectivity is always manifold

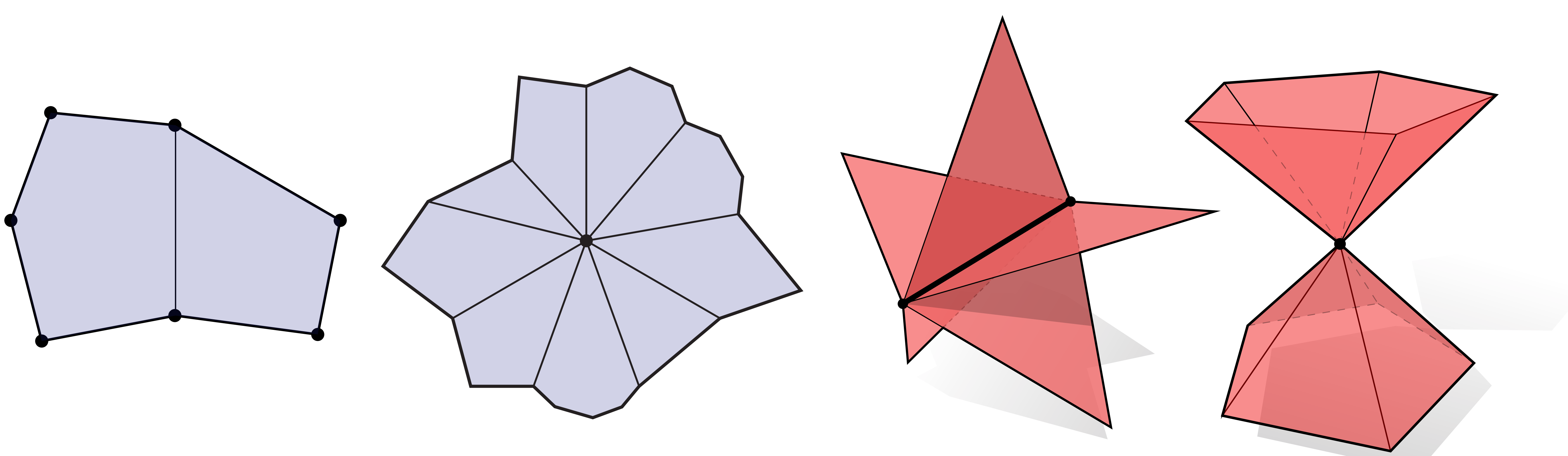
- Consider simplified halfedge data structure
- Require only "common-sense" conditions

```
struct Halfedge {
   Halfedge *next, *twin;
};
```

(pointer to yourself!)

```
twin->twin == this
twin != this
every he is someone's "next"
```

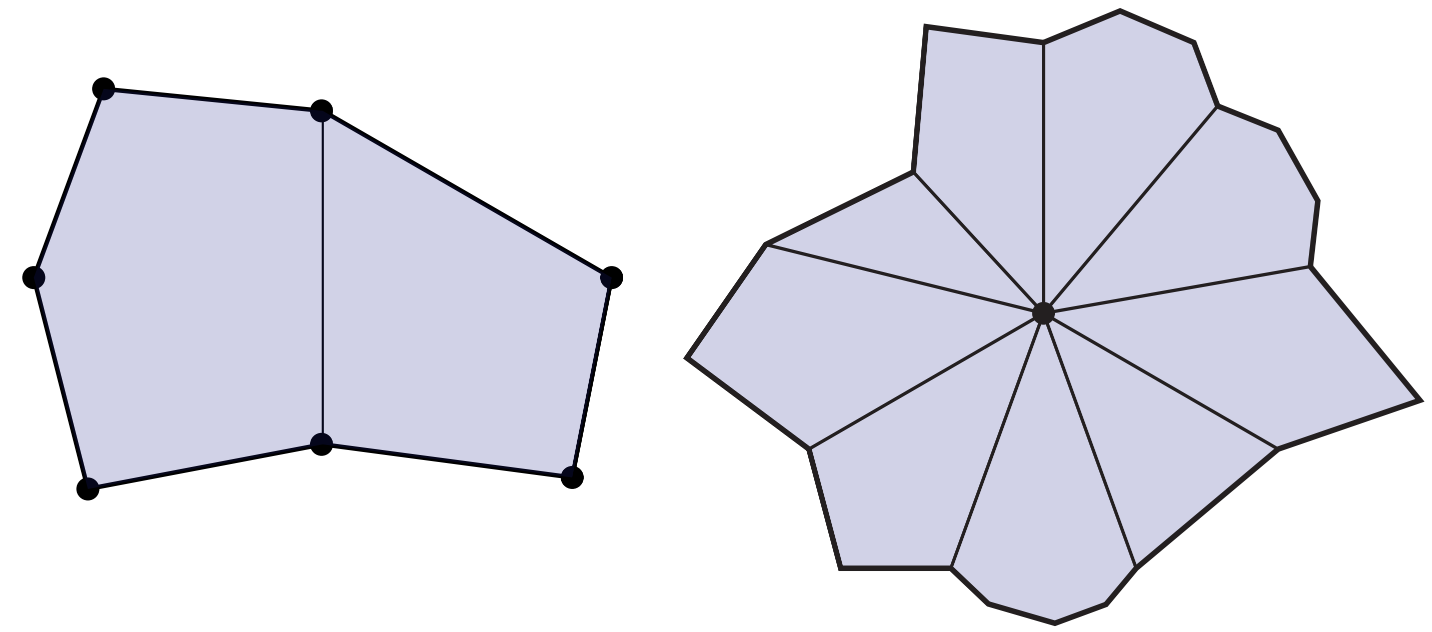
- Keep following `next`, and you'll get faces.
- Keep following `twin` and you'll get edges.
- Keep following `next->twin` and you'll get vertices.

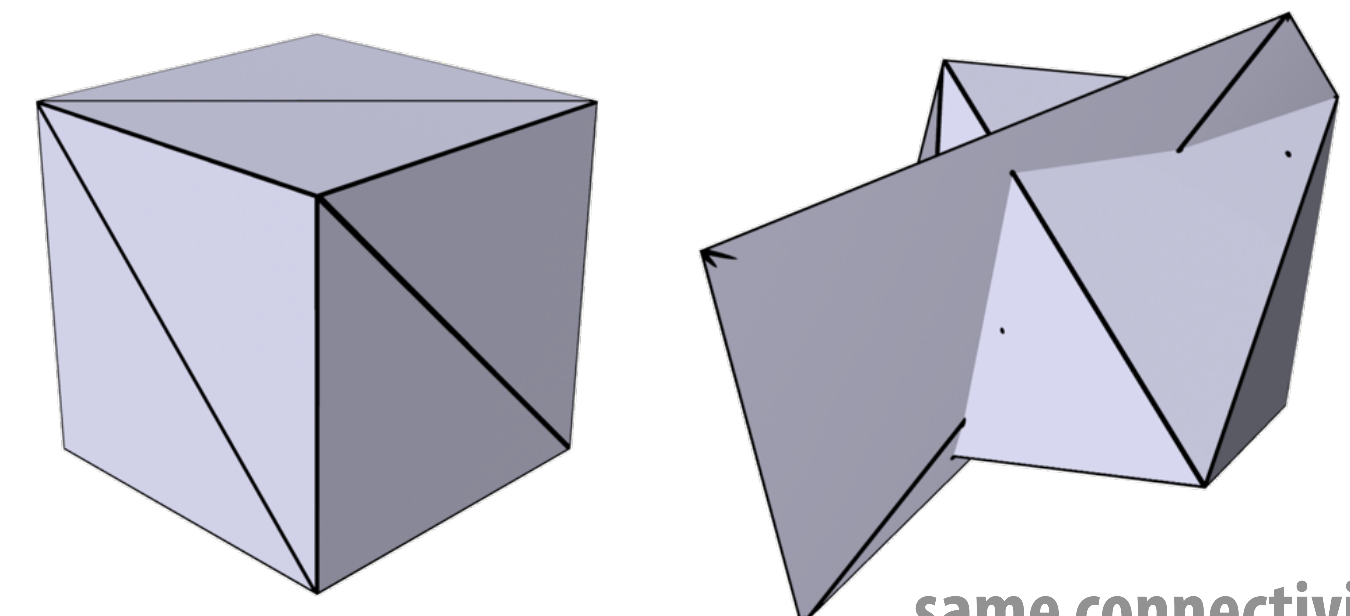


Q: Why, therefore, is it impossible to encode the red figures?

Connectivity vs. Geometry

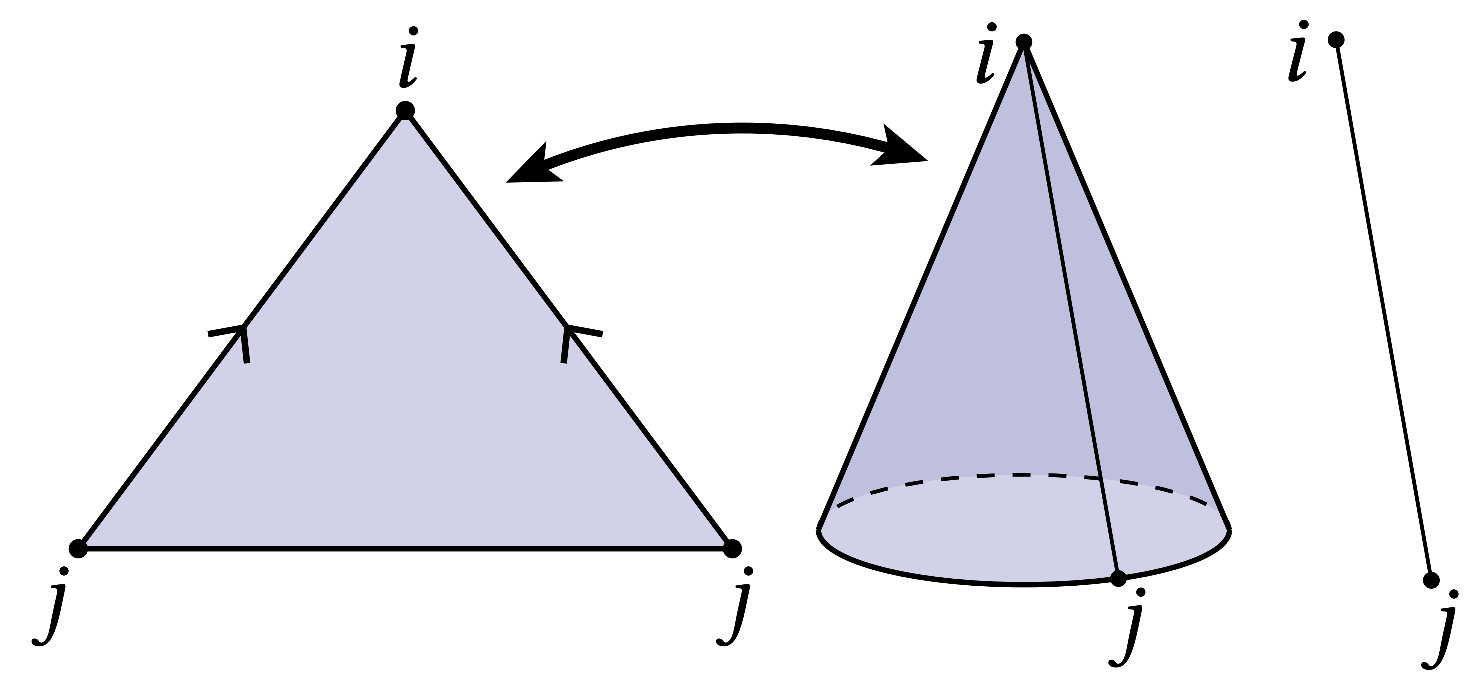
- **Recall manifold conditions (fans not fins):**
  - **every edge contained in two faces**
  - **every vertex contained in one fan**
- **These conditions say <u>nothing</u> about vertex positions! Just connectivity**
- **Hence, can have perfectly good (manifold) connectivity, even if geometry is awful**
- **In fact, sometimes you can have perfectly good manifold connectivity for which <u>any</u> vertex positions give "bad" geometry!**
- **Can lead to confusion when debugging: mesh looks "bad", even though connectivity is fine**

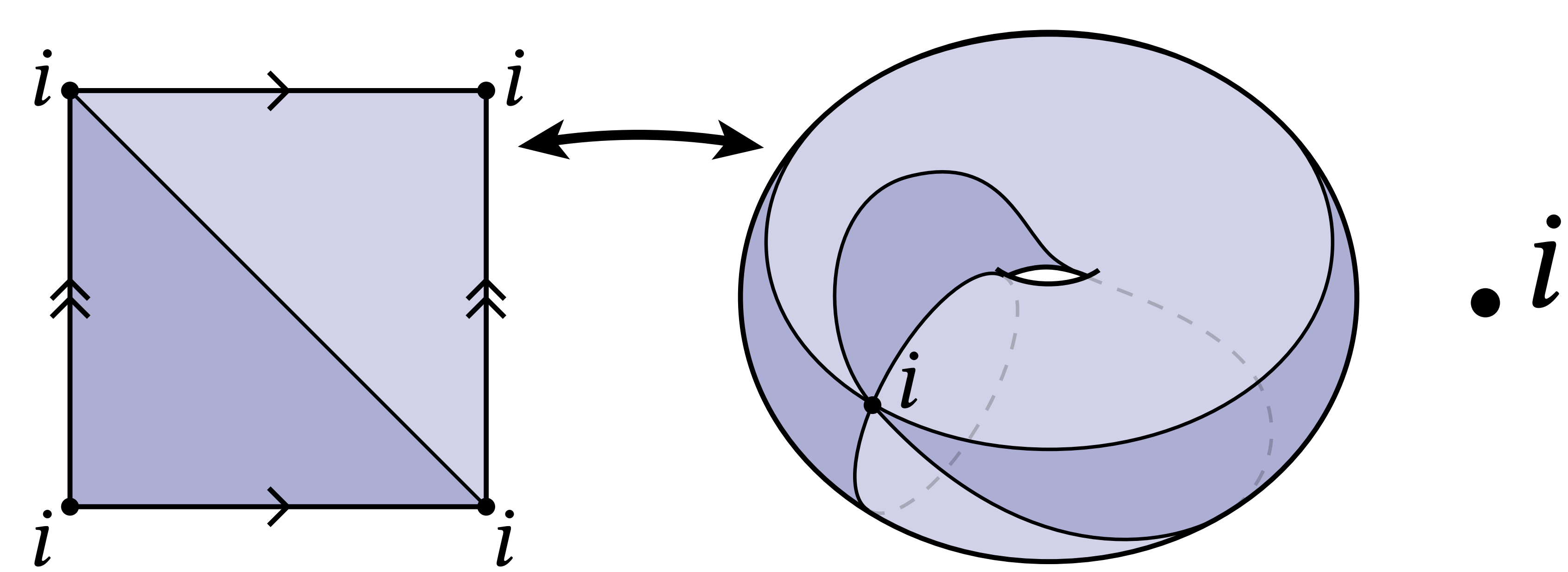


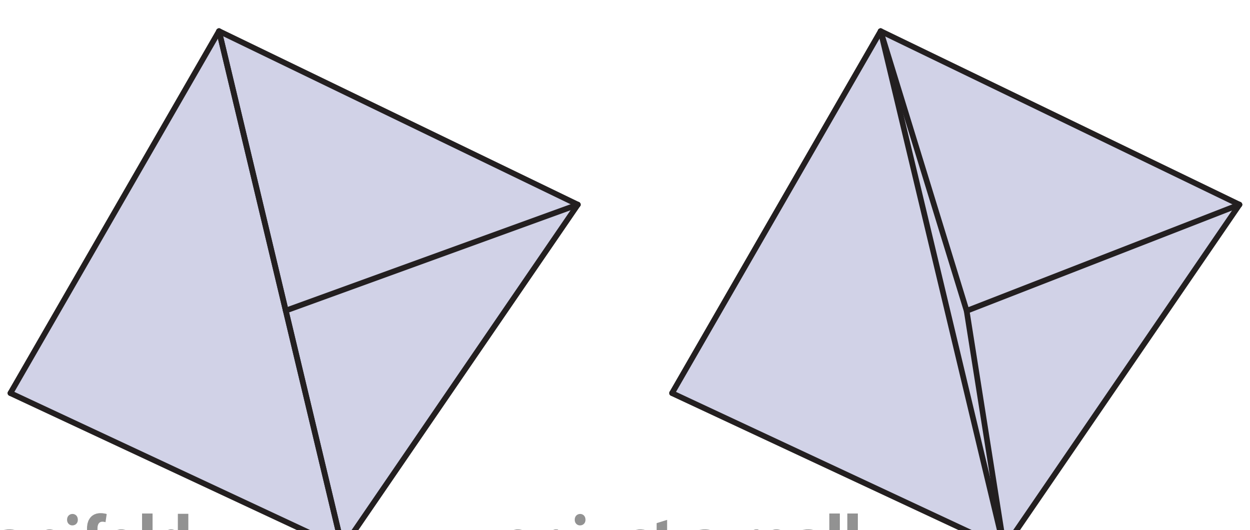


cube (manifold)

same connectivity, random vertex positions





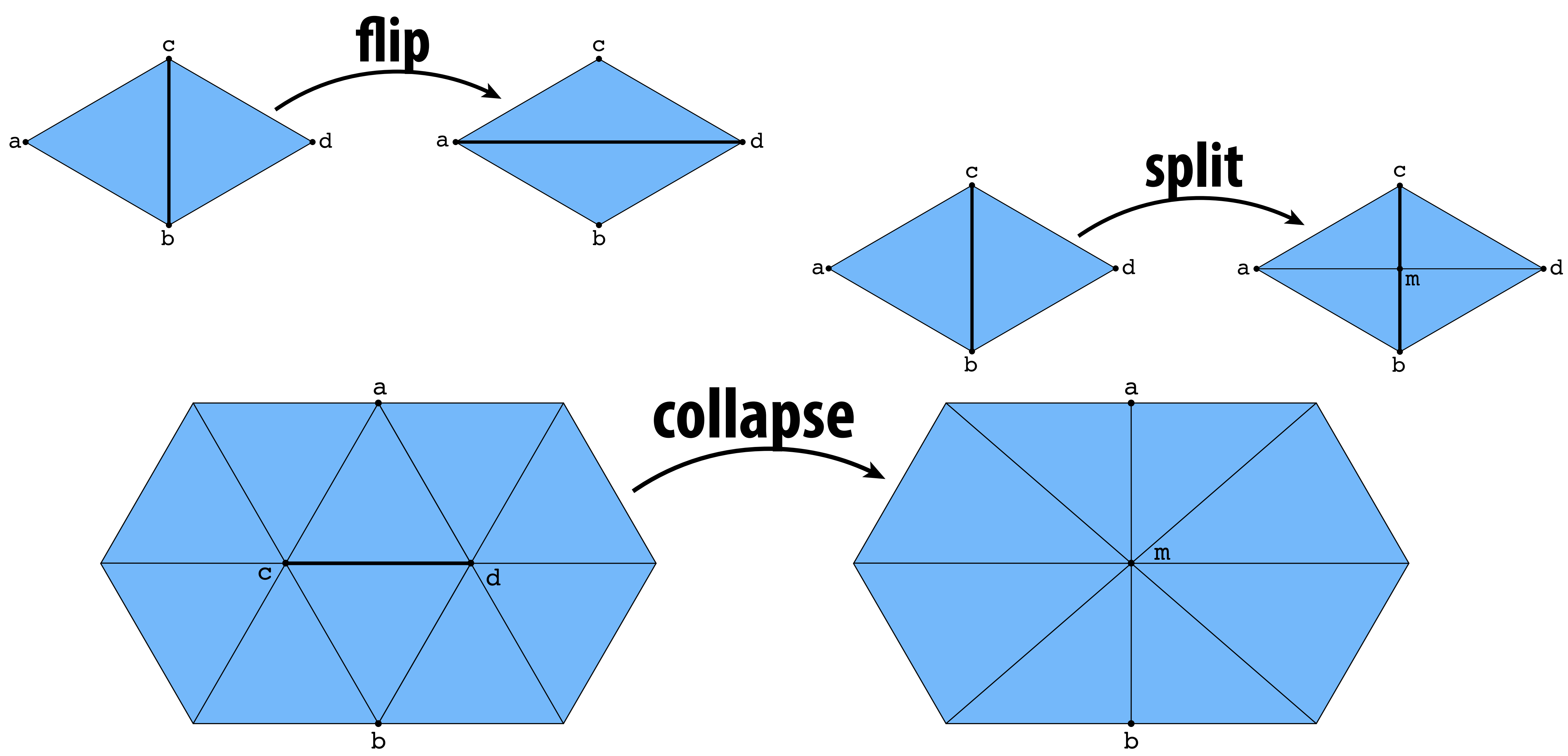


non manifold connectivity?

…or just a really skinny triangle?

Halfedge meshes are easy to edit

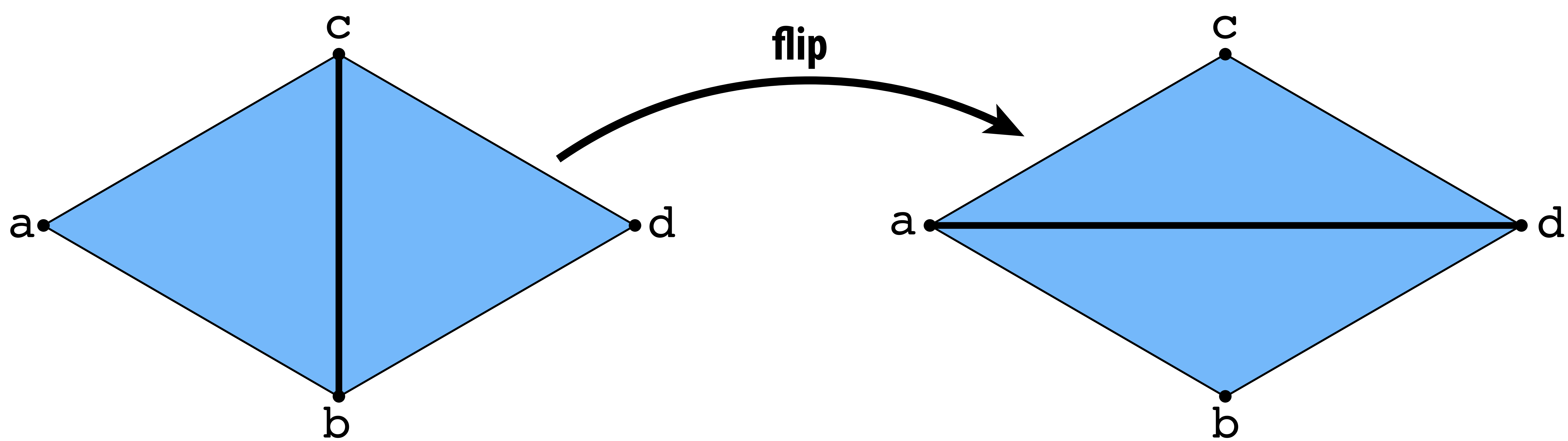
- Remember key feature of linked list: insert/delete elements
- Same story with halfedge mesh ("linked list on steroids")
- E.g., for triangle meshes, several atomic operations:



- How? Allocate/delete elements; reassigning pointers.
- Must be careful to preserve manifoldness!

Edge Flip (Triangles)

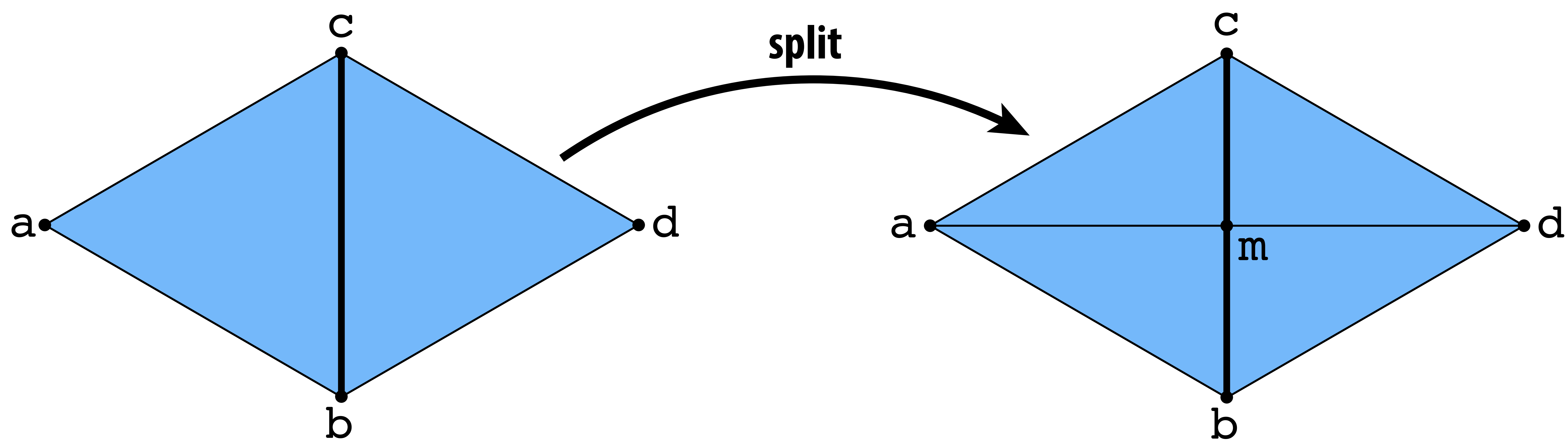
- Triangles (a,b,c), (b,d,c) become (a,d,c), (a,b,d):



- Long list of pointer reassignments (`edge->halfedge = ...`)
- However, no elements created/destroyed.
- Q: What happens if we flip twice?
- Challenge: can you implement edge flip such that pointers are unchanged after two flips?

Edge Split (Triangles)

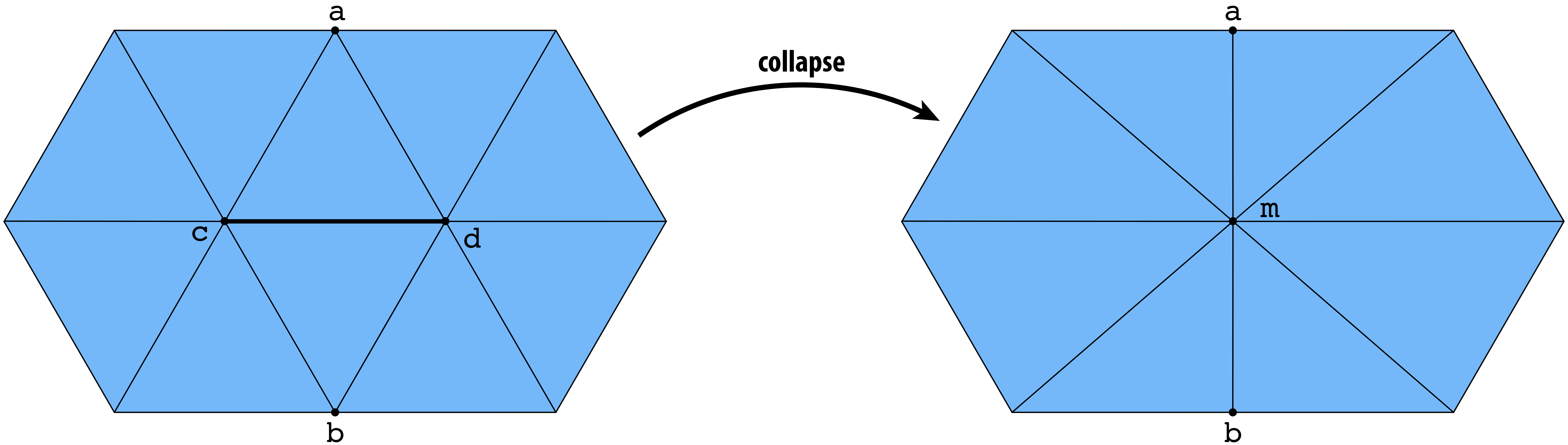
- **Insert midpoint m of edge (c,b), connect to get four triangles:**



- **This time, have to add new elements.**
- **Lots of pointer reassignments.**
- **Q: Can we "reverse" this operation?**

Edge Collapse (Triangles)

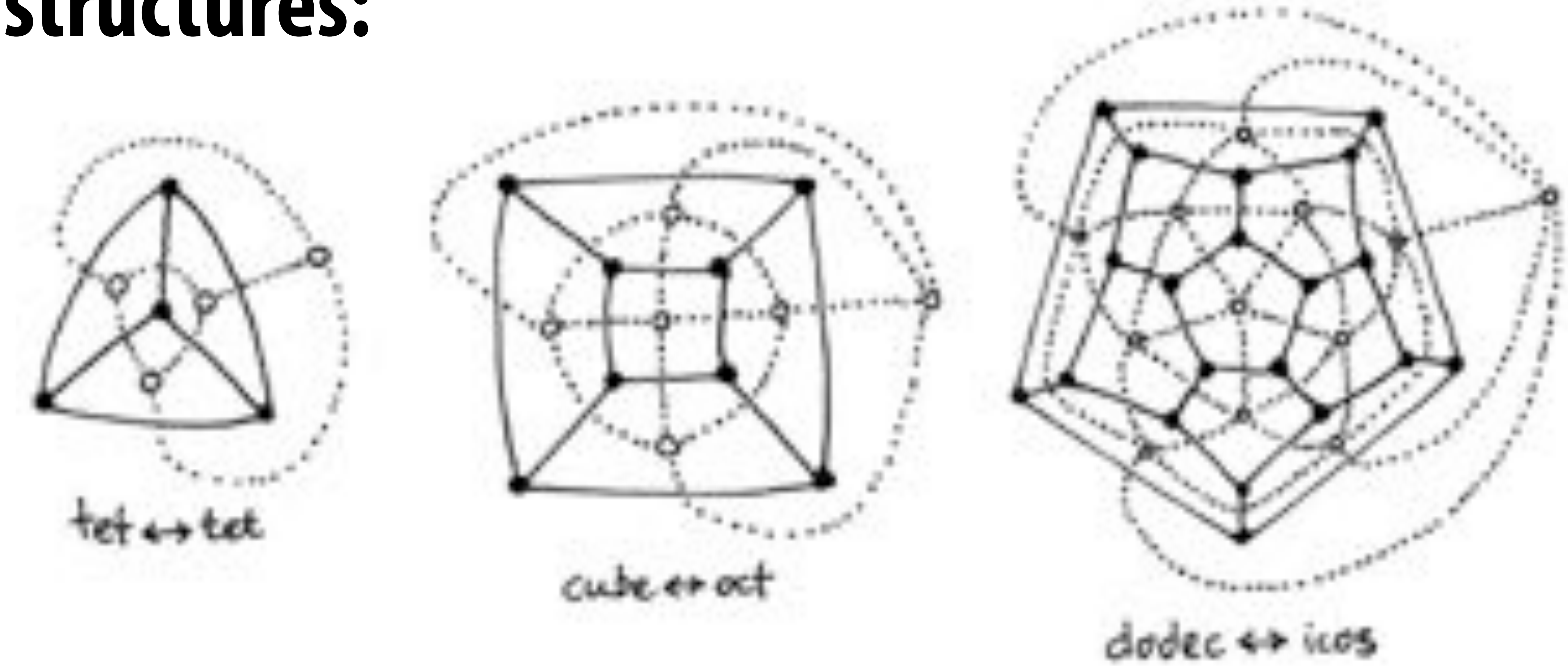
- Replace edge (b,c) with a single vertex m:



- Now have to delete elements.
- Still lots of pointer assignments!
- Q: How would we implement this with an adjacency list?
- Any other good way to do it? (E.g., different data structure?)

Alternatives to Halfedge

Paul Heckbert (former CMU prof.)
quadedge code - http://bit.ly/1QZLHos



- Many very similar data structures:
  - winged edge
  - corner table
  - quadedge
  - ...
- Each stores local neighborhood information
- Similar tradeoffs relative to simple polygon list:
  - CONS: additional storage, incoherent memory access
  - PROS: better access time for individual elements, intuitive traversal of local neighborhoods
- With some thought*, can design halfedge-type data structures with coherent data storage, support for non manifold connectivity, etc.

*see for instance http://geometry-central.net/

Comparison of Polygon Mesh Data Strucutres

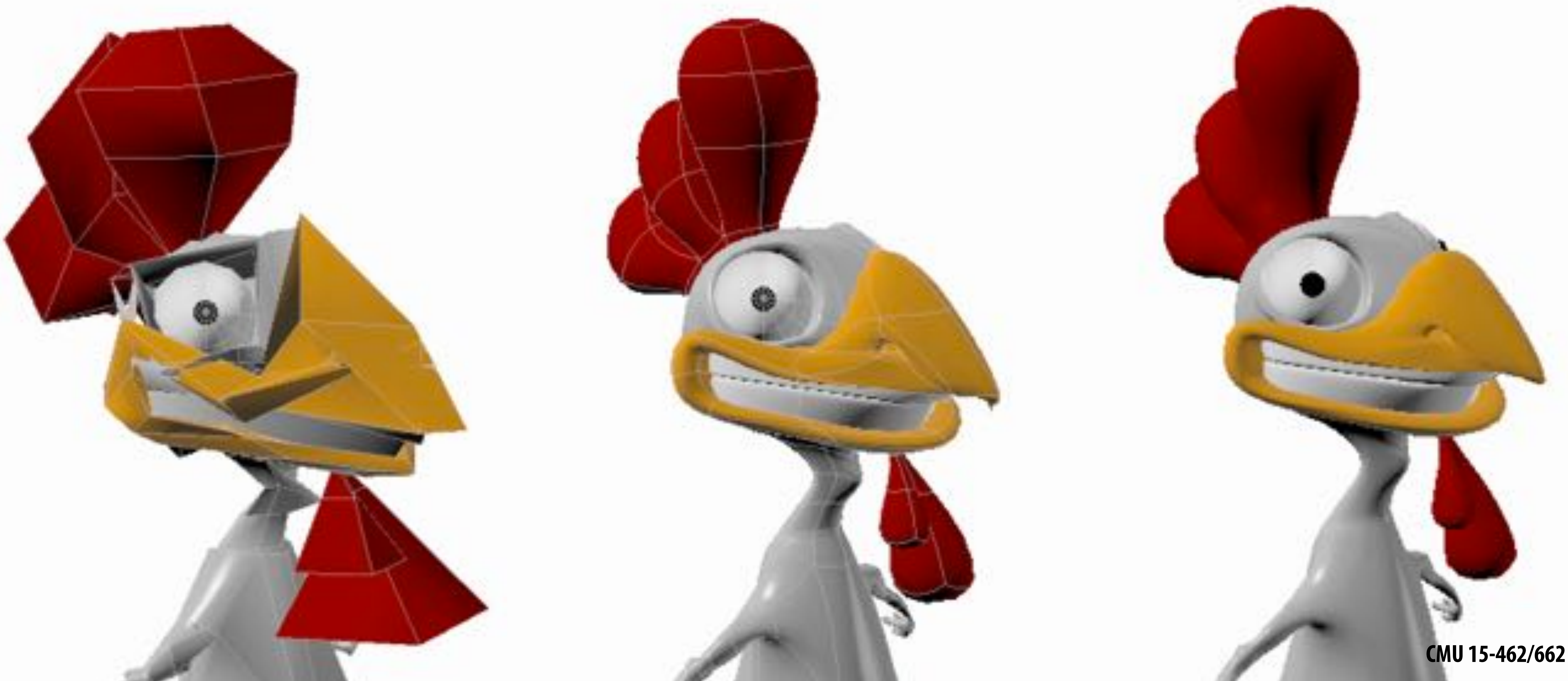
	Adjacency List	Incidence Matrices	Halfedge Mesh
constant-time neighborhood access?	NO	YES	YES
easy to add/remove mesh elements?	NO	NO	YES
nonmanifold geometry?	YES	YES	NO

Conclusion: pick the right data structure for the job!

Ok, but what can we actually do with our fancy new data structures?

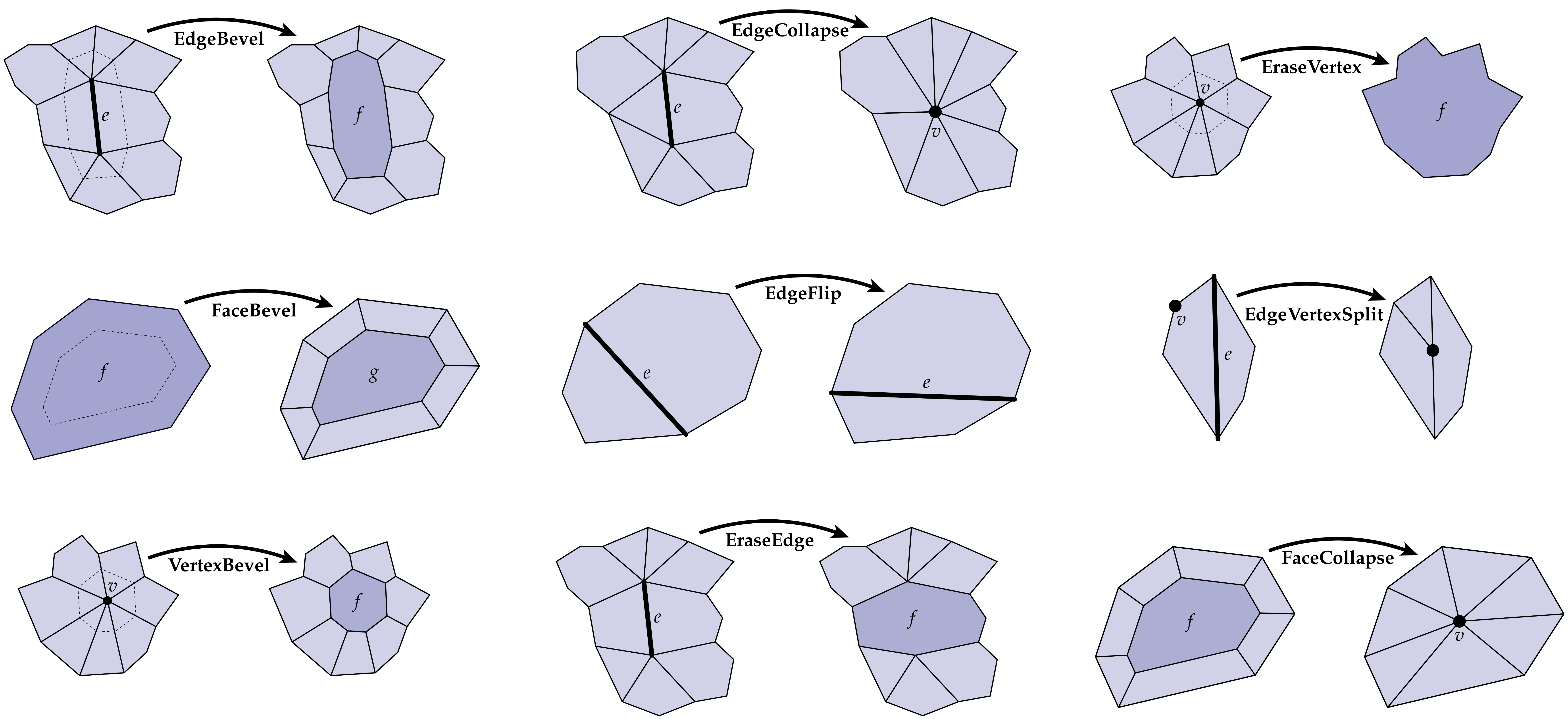
Subdivision Modeling

- **Common modeling paradigm in modern 3D tools:**
  - **Coarse "control cage"**
  - **Perform local operations to control/edit shape**
  - **Global subdivision process determines final surface**



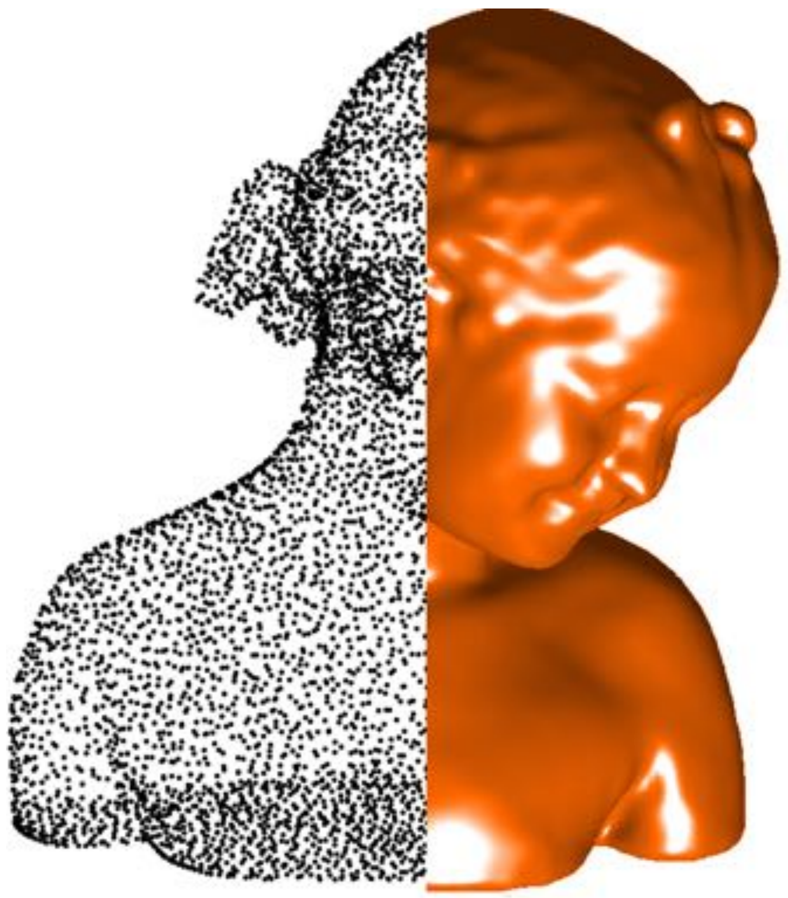
Subdivision Modeling—Local Operations

- **For general polygon meshes, we can dream up lots of local mesh operations that might be useful for modeling:**



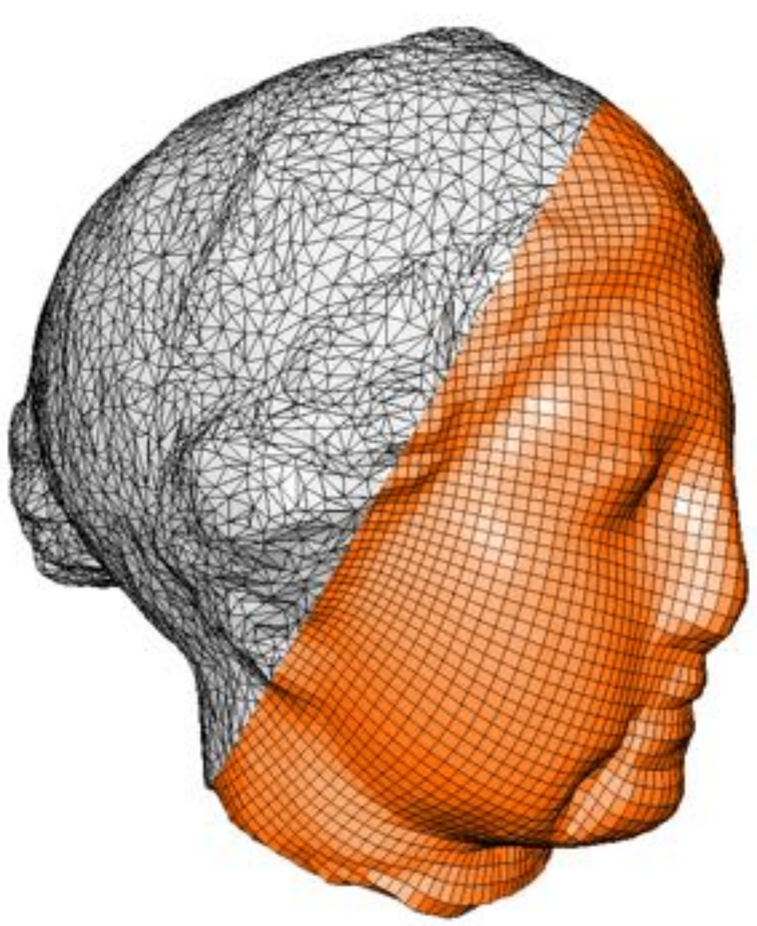
. . .and many, many more!

Geometry Processing

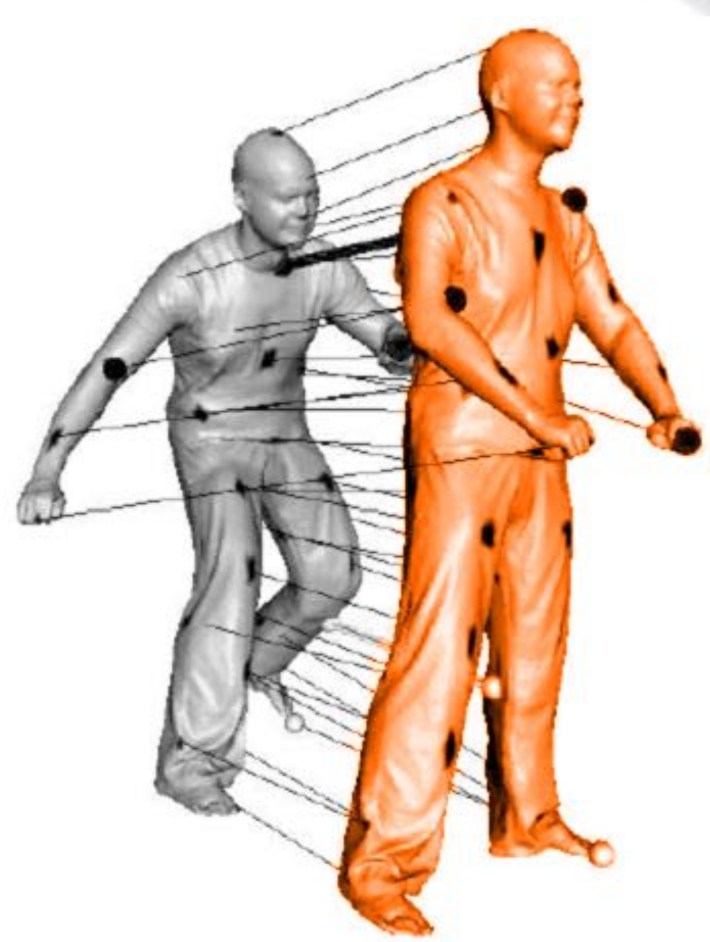


reconstruction

filtering

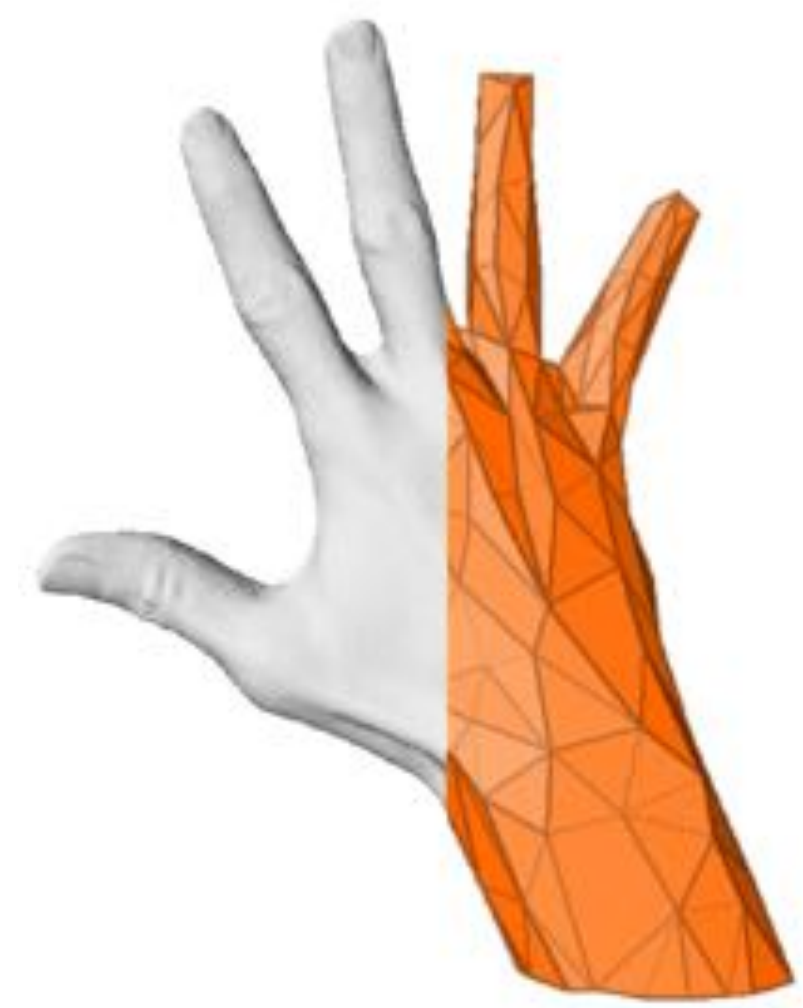


remeshing



shape analysis

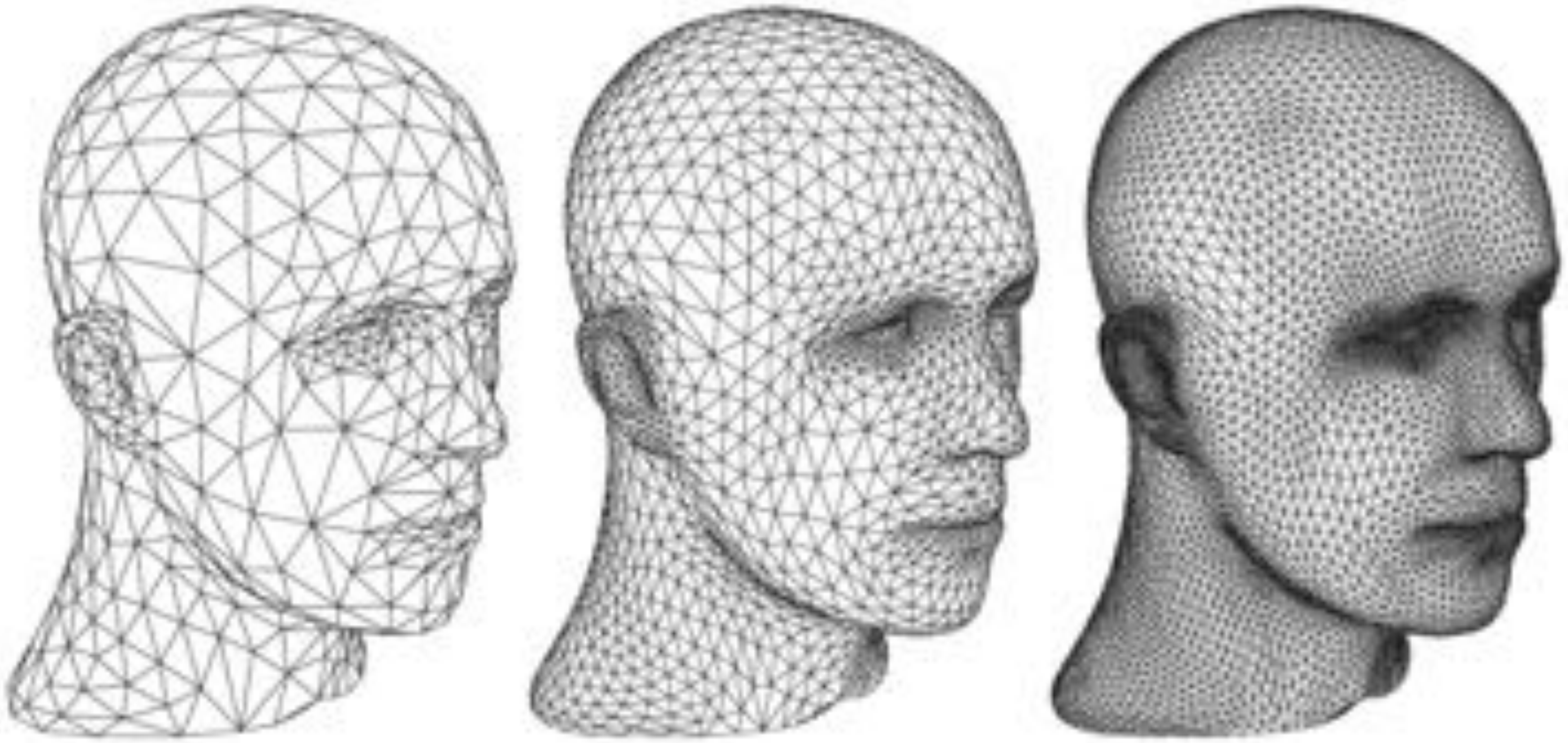
parameterization



compression

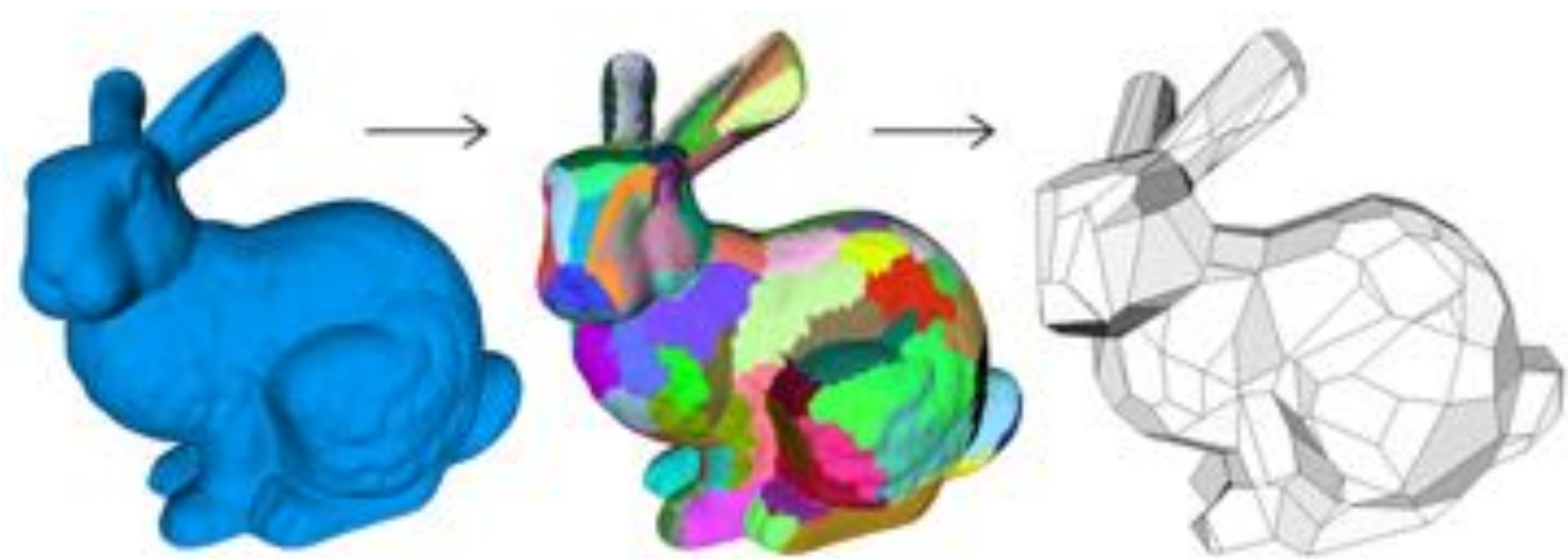
Geometry Processing: Upsampling

- Increase resolution via interpolation
- Images: e.g., bilinear, bicubic interpolation
- Polygon meshes:
  - subdivision
  - bilateral upsampling
  - ...



Geometry Processing: Downsampling

- Decrease resolution; try to preserve shape/appearance
- Images: nearest-neighbor, bilinear, bicubic interpolation
- Point clouds: subsampling (just take fewer points!)
- Polygon meshes:
  - iterative decimation, variational shape approximation, ...

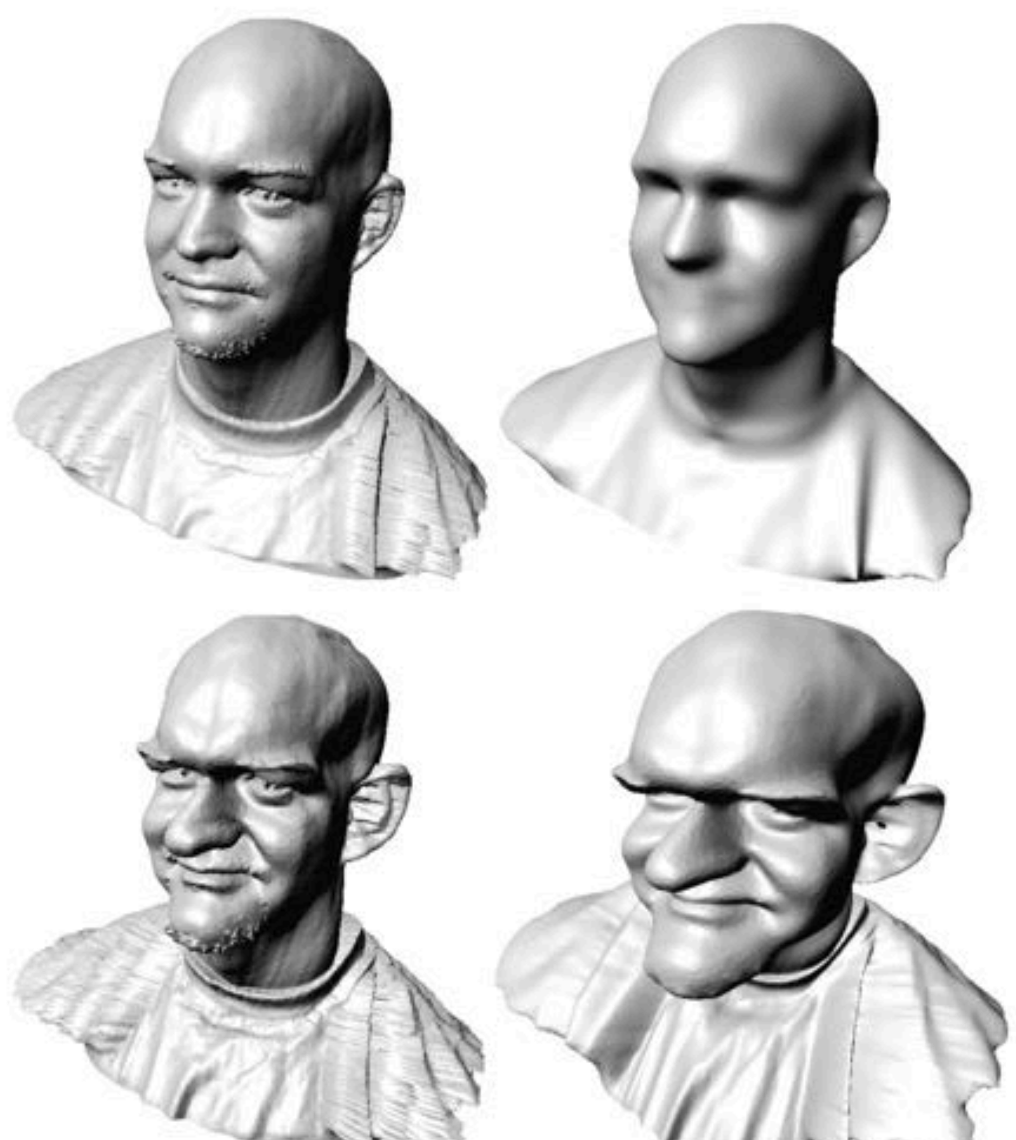


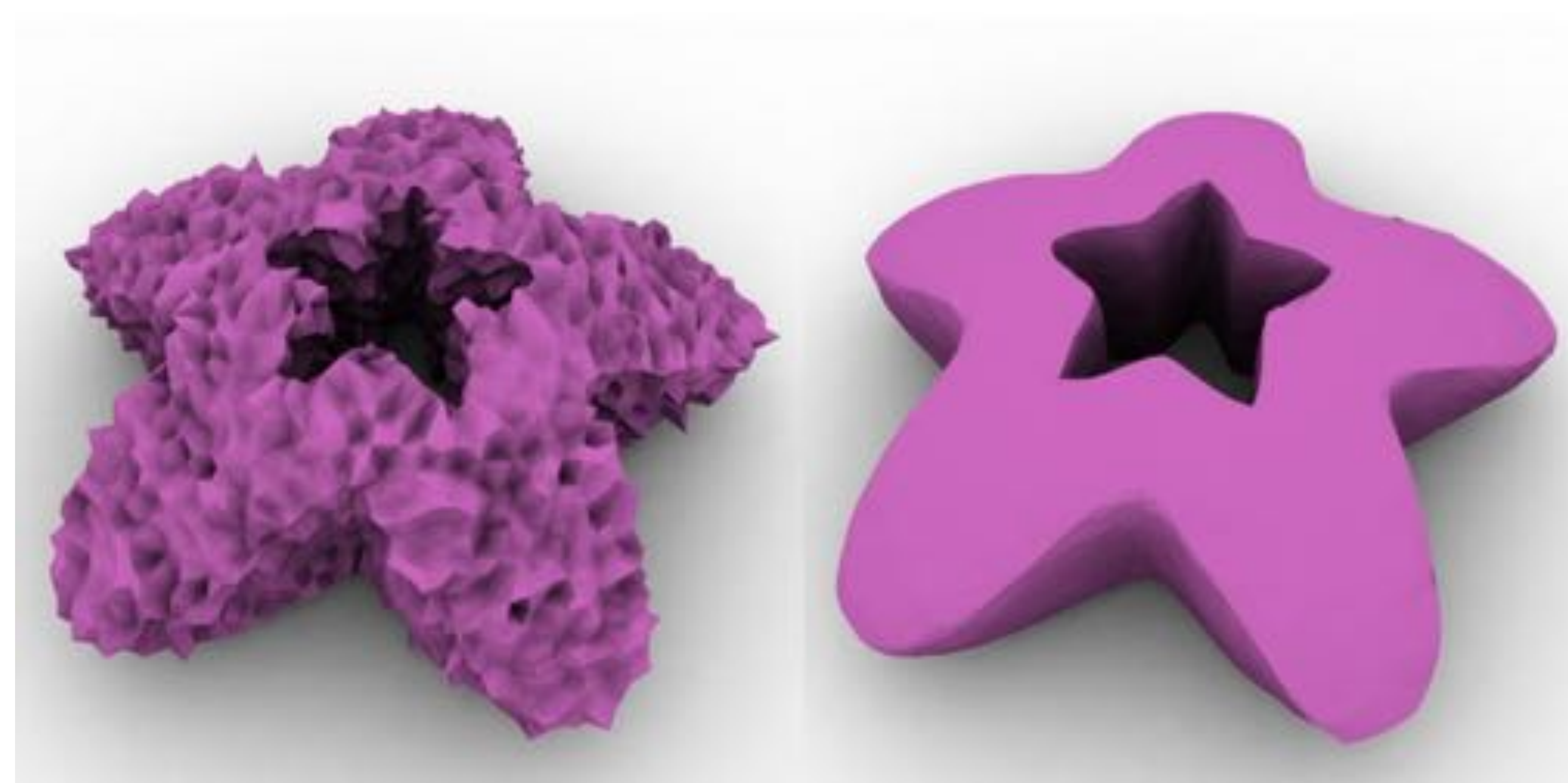
Geometry Processing: Resampling

- Modify sample distribution to improve quality
- Images: not an issue! (Pixels always stored on a regular grid)
- Meshes: shape of polygons is extremely important!
    - different notion of “quality” depending on task
    - e.g., visualization vs. solving equations

Geometry Processing: Filtering

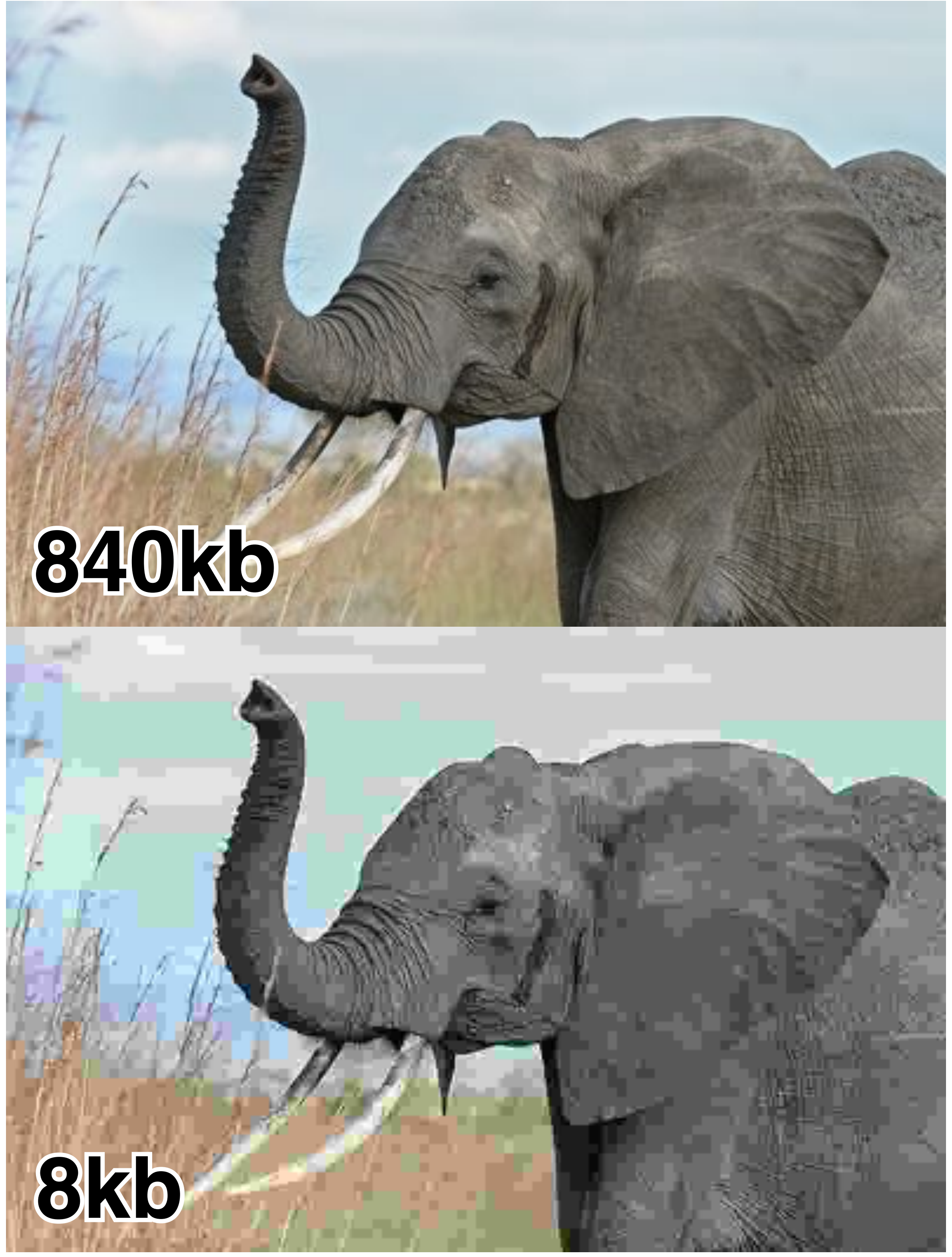
- Remove noise, or emphasize important features (e.g., edges)
- Images: blurring, bilateral filter, edge detection, ...
- Polygon meshes:
  - curvature flow
  - bilateral filter
  - spectral filter

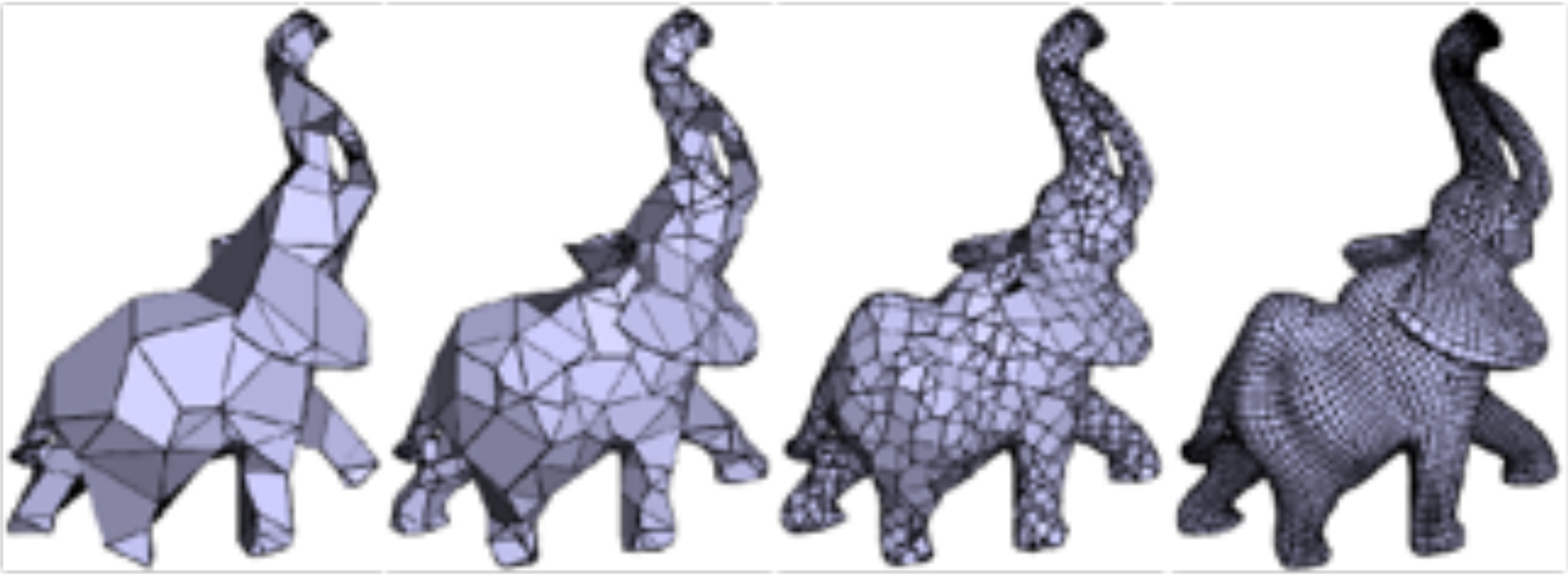




Geometry Processing: Compression

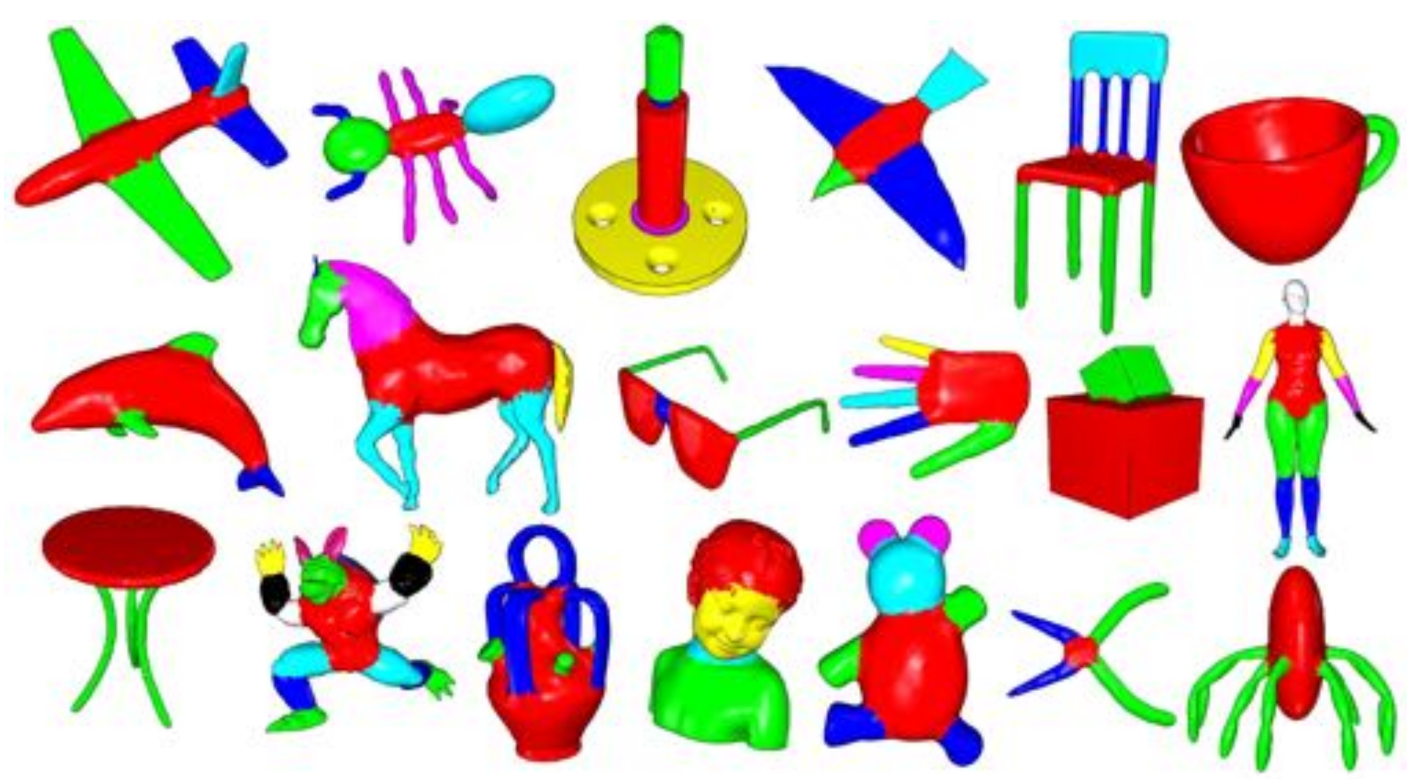
- Reduce storage size by eliminating redundant data/ approximating unimportant data
- Images:
  - run-length, Huffman coding - lossless
  - cosine/wavelet (JPEG/MPEG) - lossy
- Polygon meshes:
  - compress geometry and connectivity
  - many techniques (lossy & lossless)

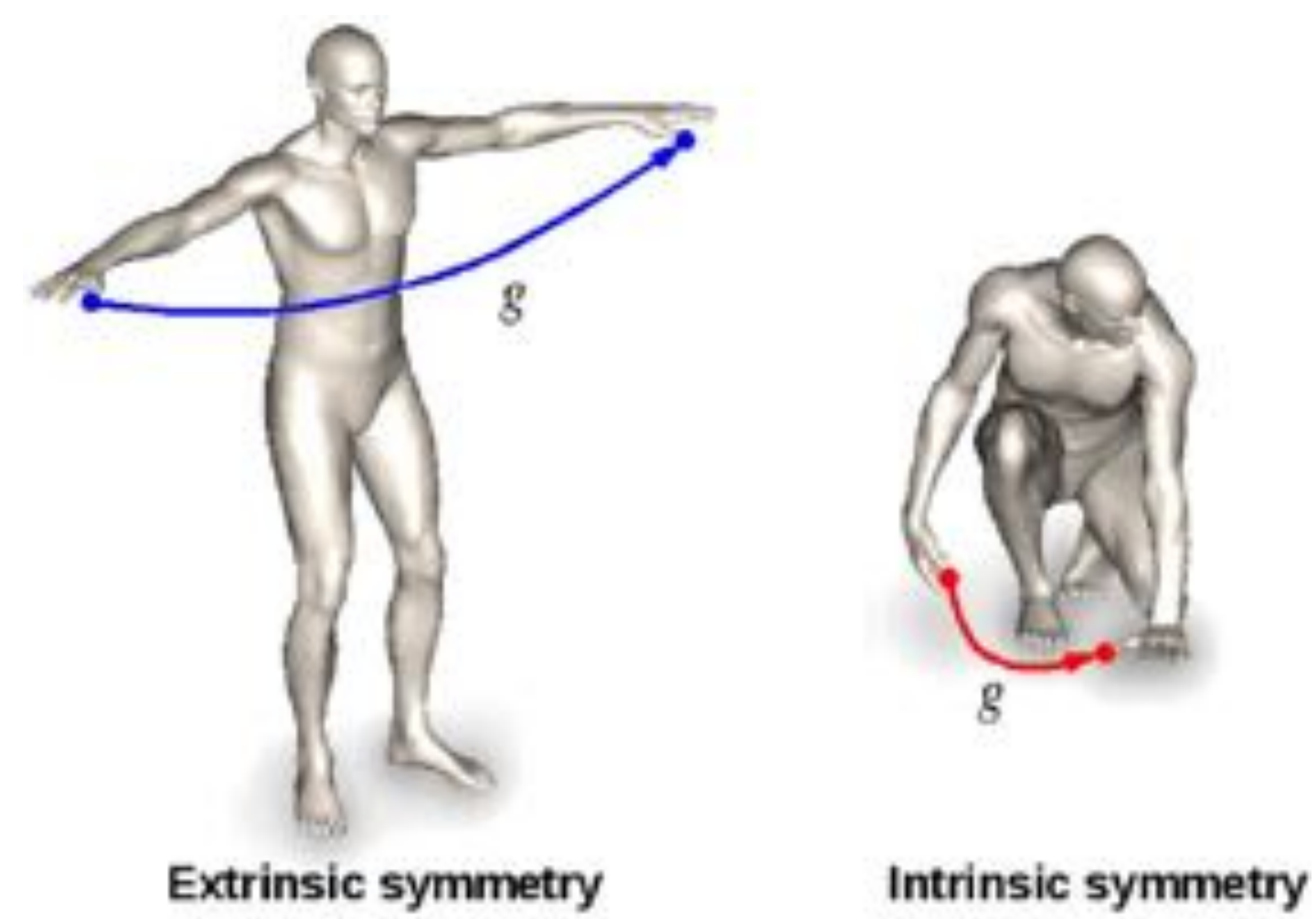


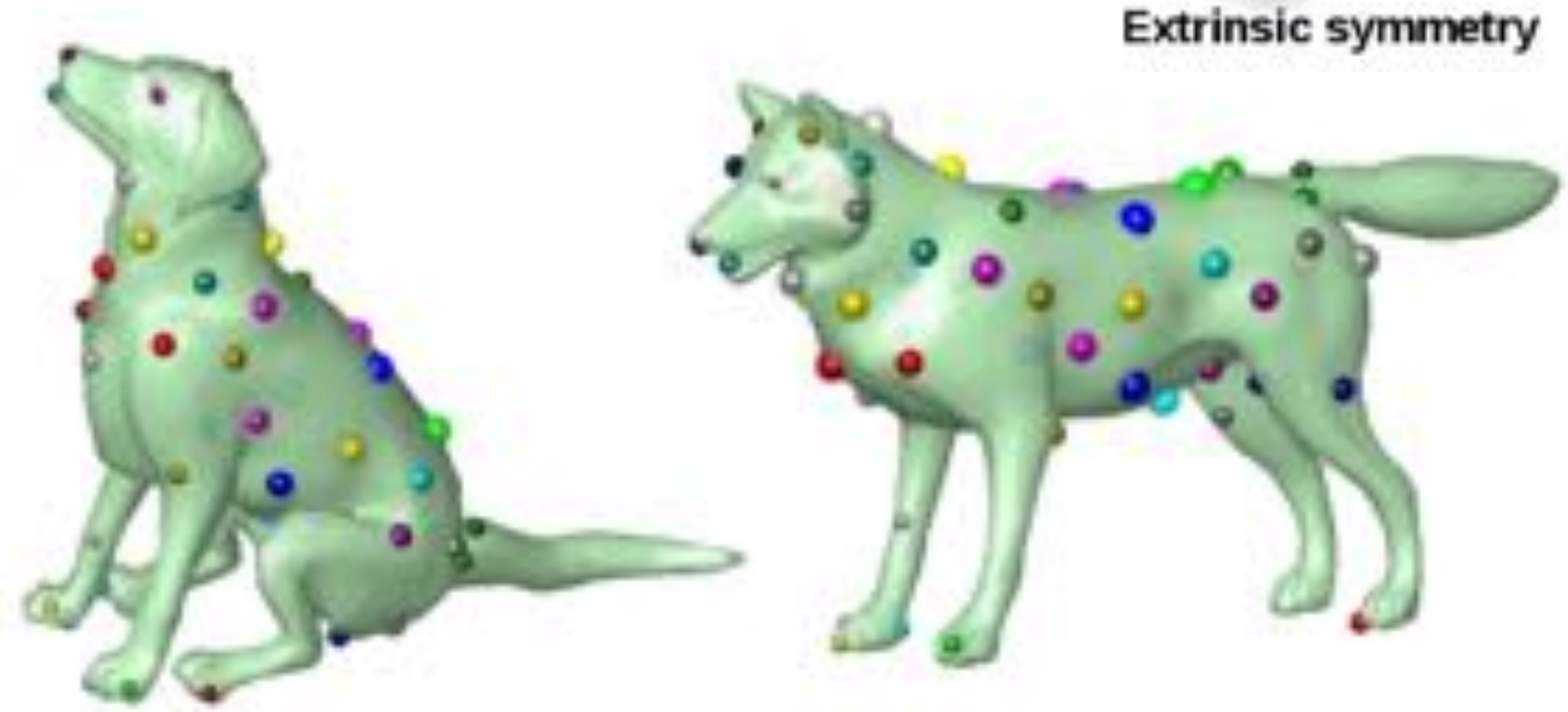


Geometry Processing: Shape Analysis

- Identify/understand important semantic features
- Images: computer vision, segmentation, face detection, ...
- Polygon meshes:
  - segmentation, correspondence, symmetry detection, ...

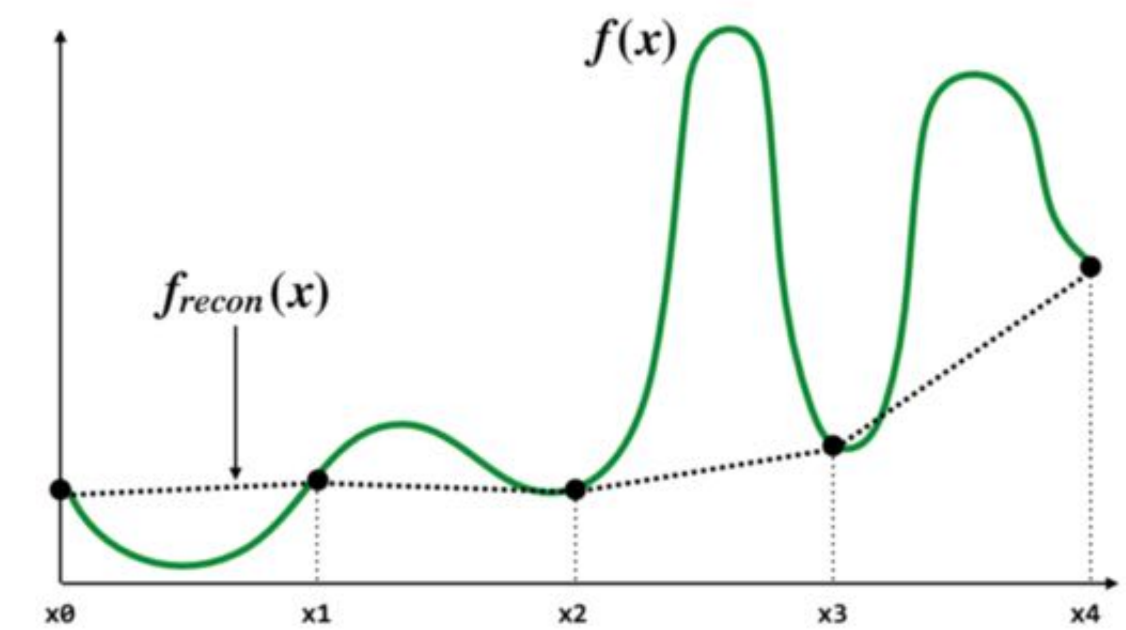






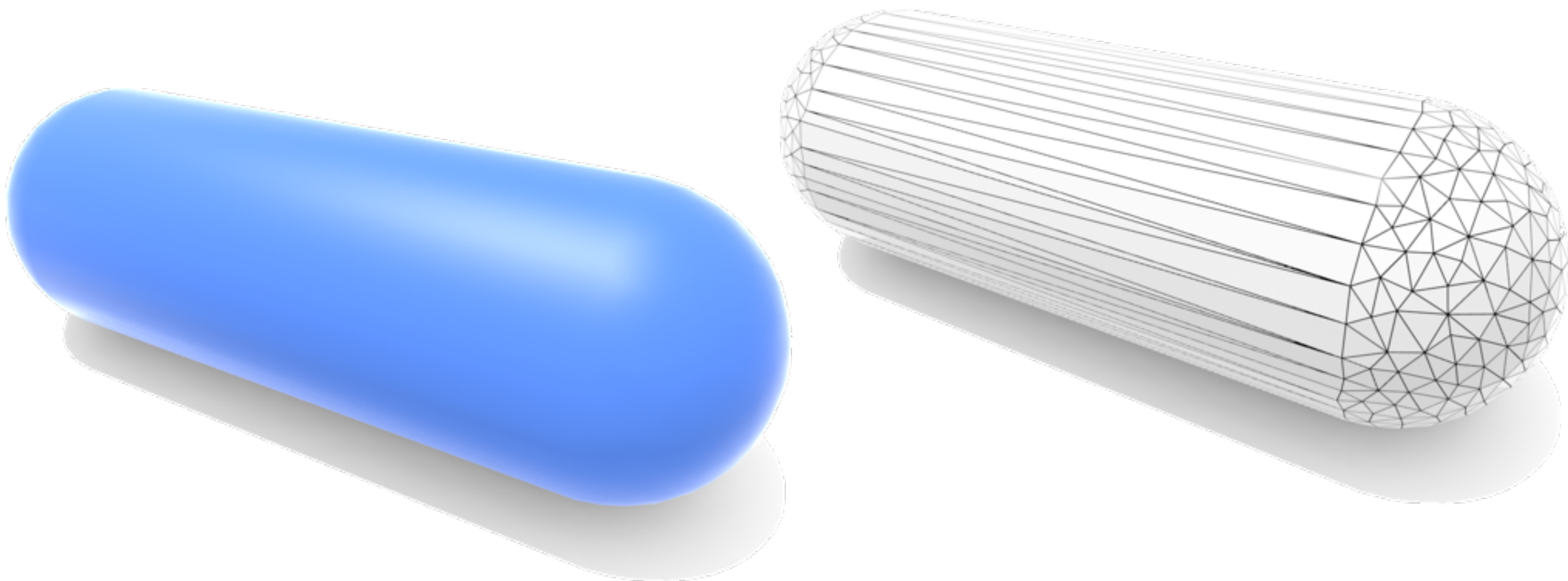
Remeshing is resampling

- Remember our discussion of aliasing
- Bad sampling makes signal appear different than it really is
- E.g., undersampled curve looks flat
- Geometry is no different!
    - undersampling destroys features
    - oversampling bad for performance



What makes a "good" mesh?

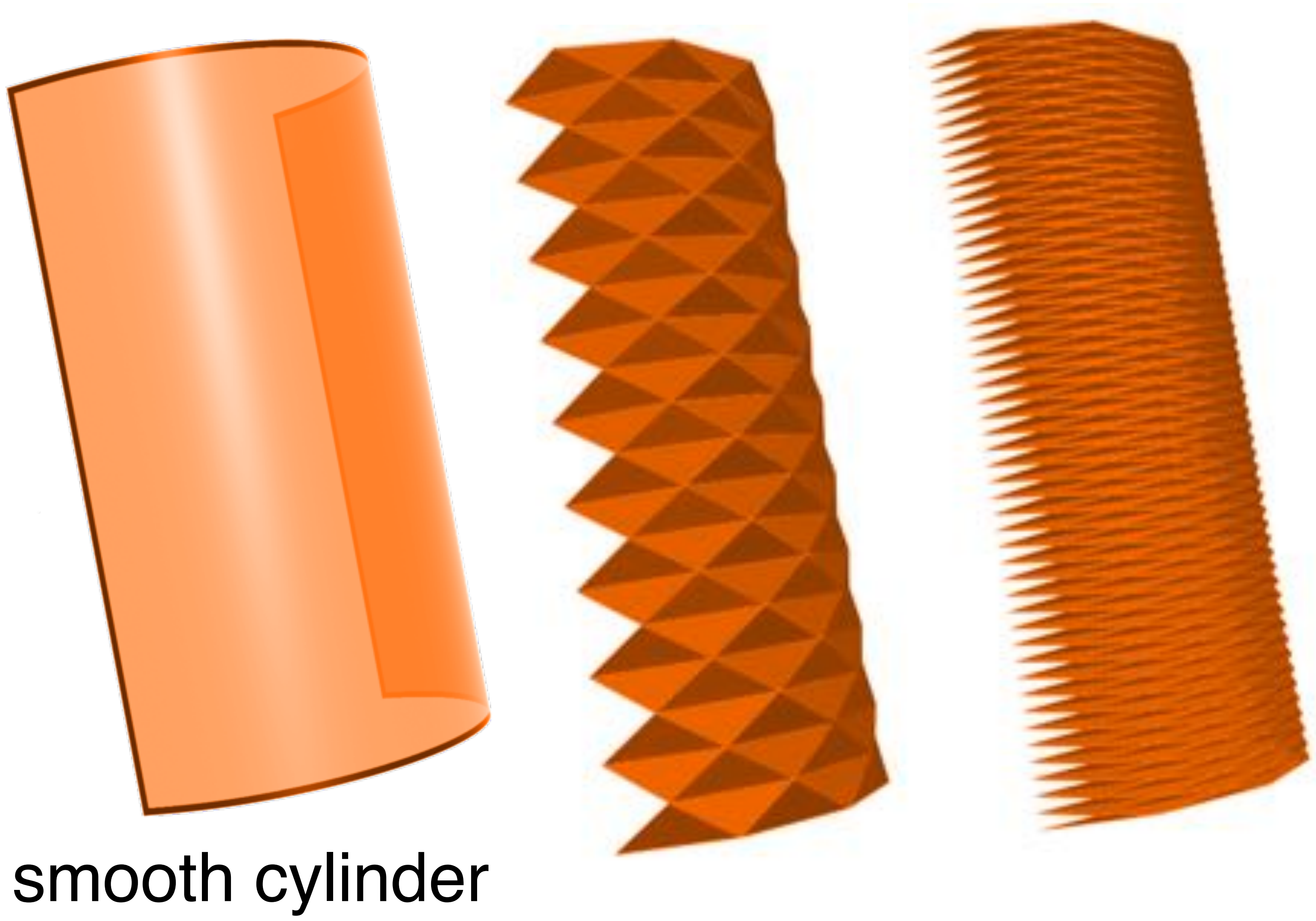
- One idea: good approximation of original shape!
- Keep only elements that contribute information about shape
- Add additional information where, e.g., curvature is large



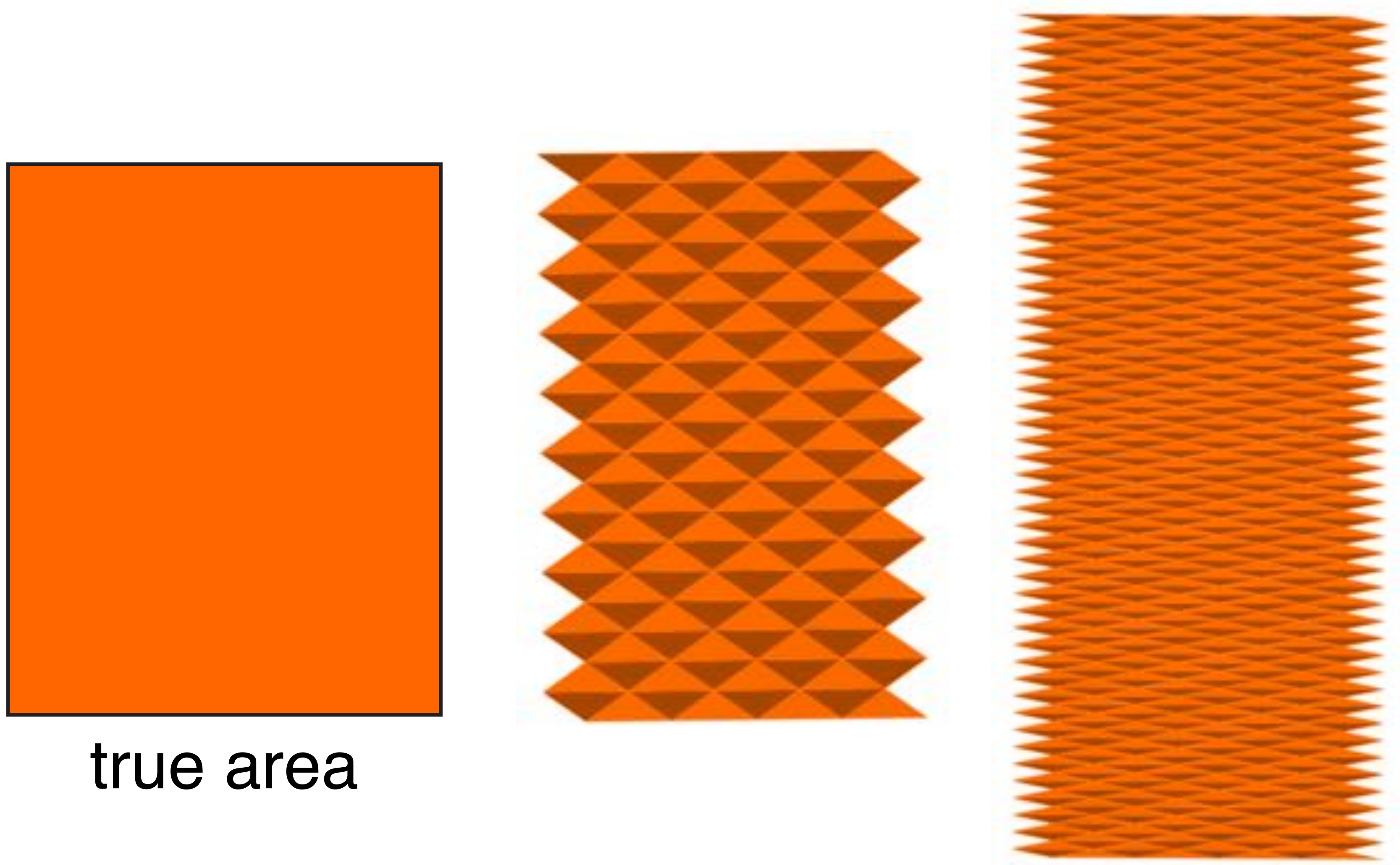
Approximation of position is not enough!

- Just because the vertices of a mesh are close to the surface it approximates does not mean it's a good approximation!
- Can still have wrong appearance, wrong area, wrong…
- Need to consider other factors*, e.g., close approximation of surface normals

vertices exactly on smooth cylinder



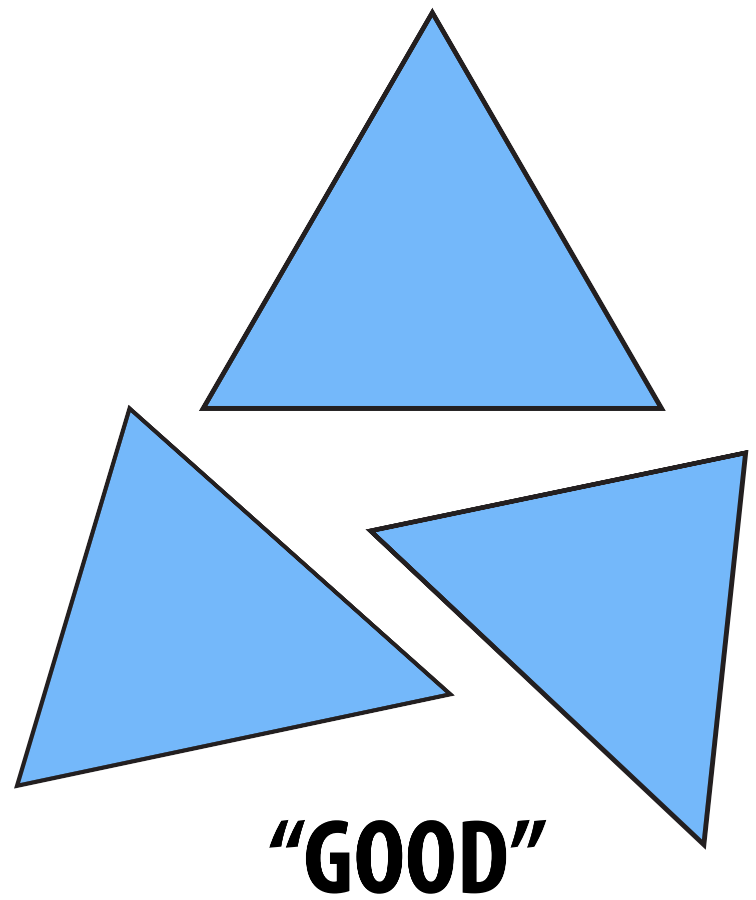
flattening of smooth cylinder & meshes

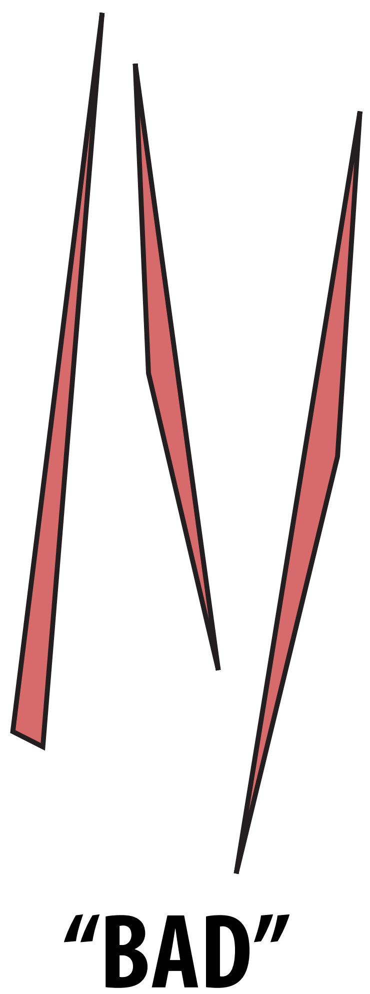


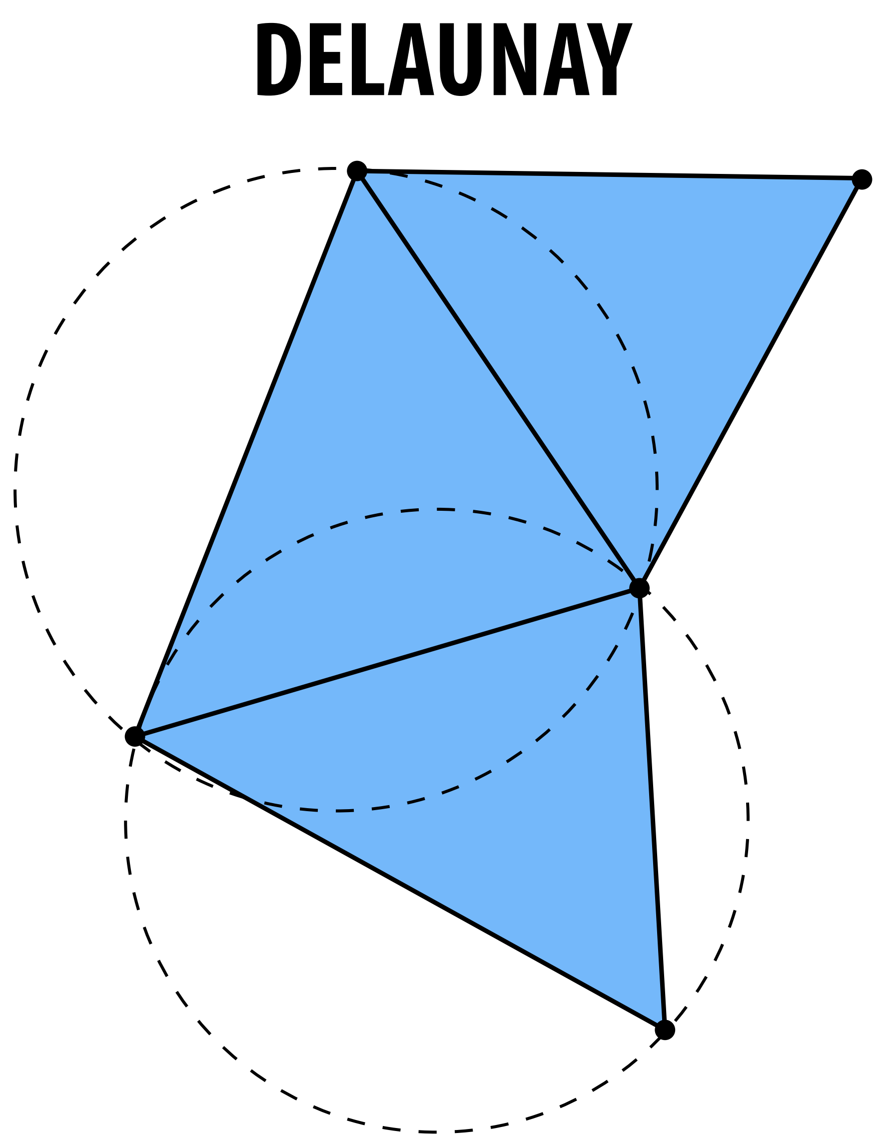
*See Hildebrandt et al (2007), "On the convergence of metric and geometric properties of polyhedral surfaces"

What else makes a "good" triangle mesh?

- Another rule of thumb: triangle





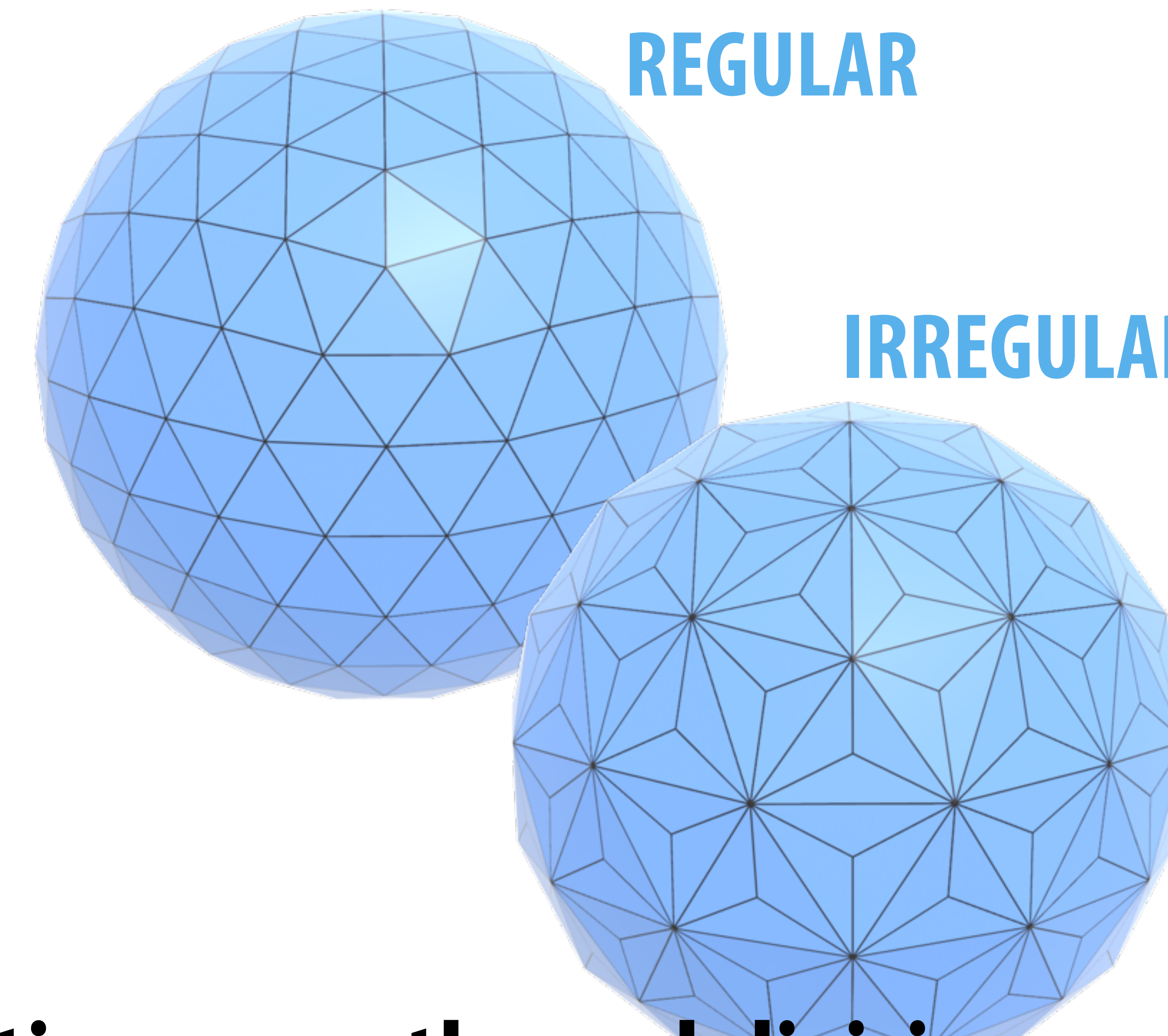


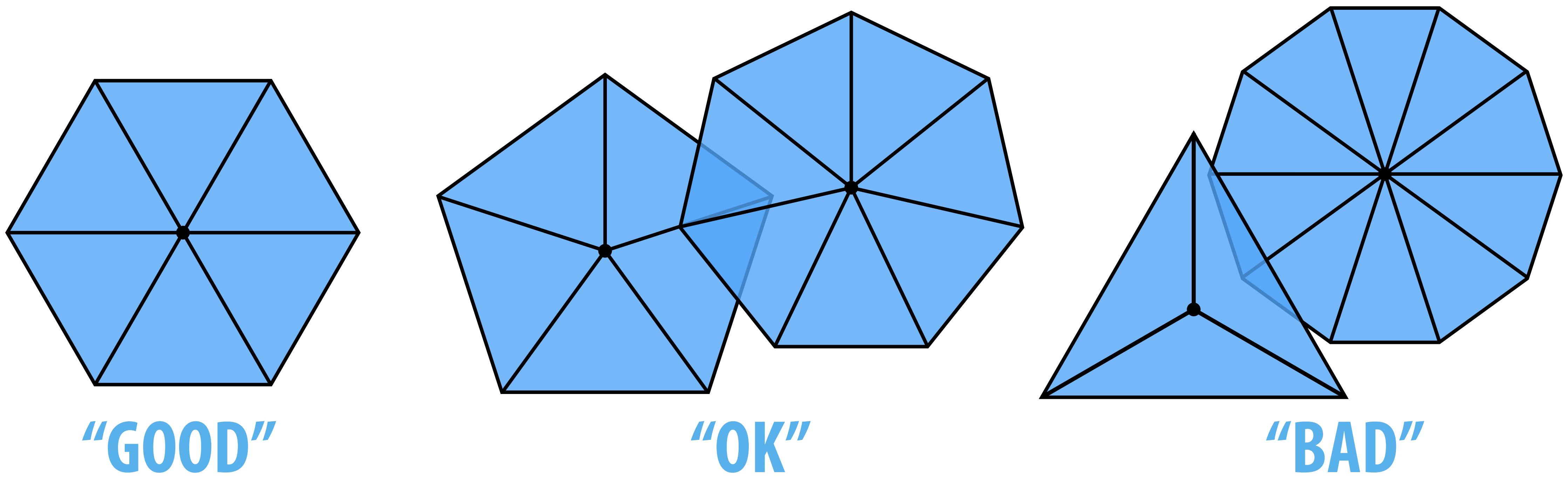
- E.g., all angles close to 60 degrees
- More sophisticated condition: Delaunay (empty circumcircles)
  – often helps with numerical accuracy/stability
  – coincides with <u>shockingly</u> many other desirable properties (maximizes minimum angle, provides smoothest interpolation, guarantees maximum principle…)
- Tradeoffs w/ good geometric approximation*
  –e.g., long & skinny might be "more efficient"

*see Shewchuk, "What is a Good Linear Element"

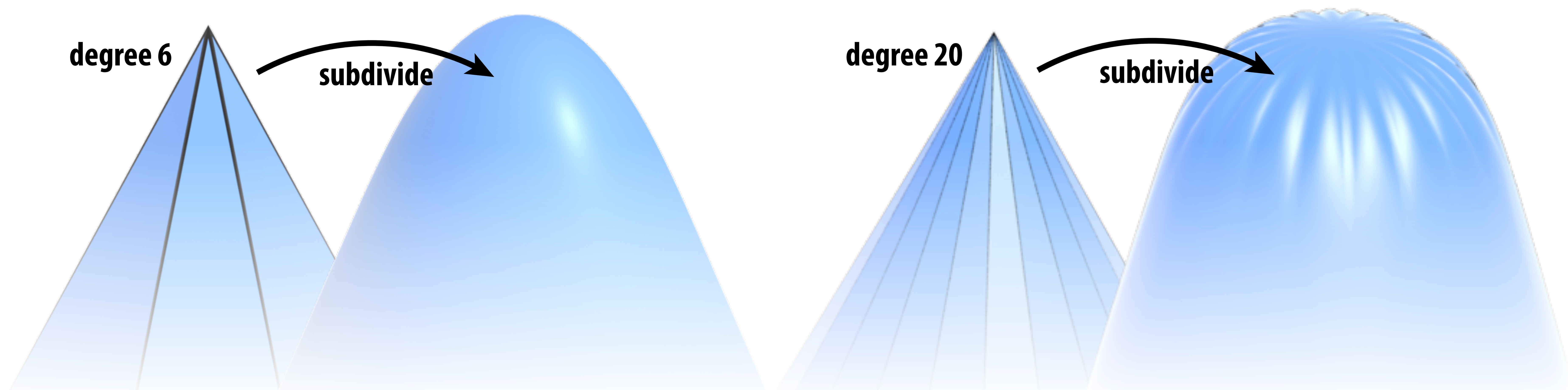
What else constitutes a “good” mesh?

- Another rule of thumb: regular vertex degree
- Degree 6 for triangle mesh, 4 for quad mesh





Why? Better polygon shape; more regular computation; smoother subdivision:



<u>Fact</u>: in general, can’t have regular vertex degree everywhere!

Next class sessions

- Subdivision + quadric error
- Geometric queries
- Many different ways to represent geometry (a late intro)

Feb 8	3D Rotations
Feb 13	Intro to Geometry / Halfedge Data Structure Assignment 1.5 DUE Assignment 2.0 OUT
Feb 15	Subdivision and Simplification
Feb 20	Geometric Queries Assignment 2.0 DUE Assignment 2.5 OUT
Feb 22	Midterm Review
Feb 27	MIDTERM
Mar 1	Other Geometric Representations
Mar 6	SPRING BREAK
Mar 8	SPRING BREAK
Mar 13	Spatial Data Structures Assignment 3.0 OUT
Mar 15	Color Assignment 2.5 DUE