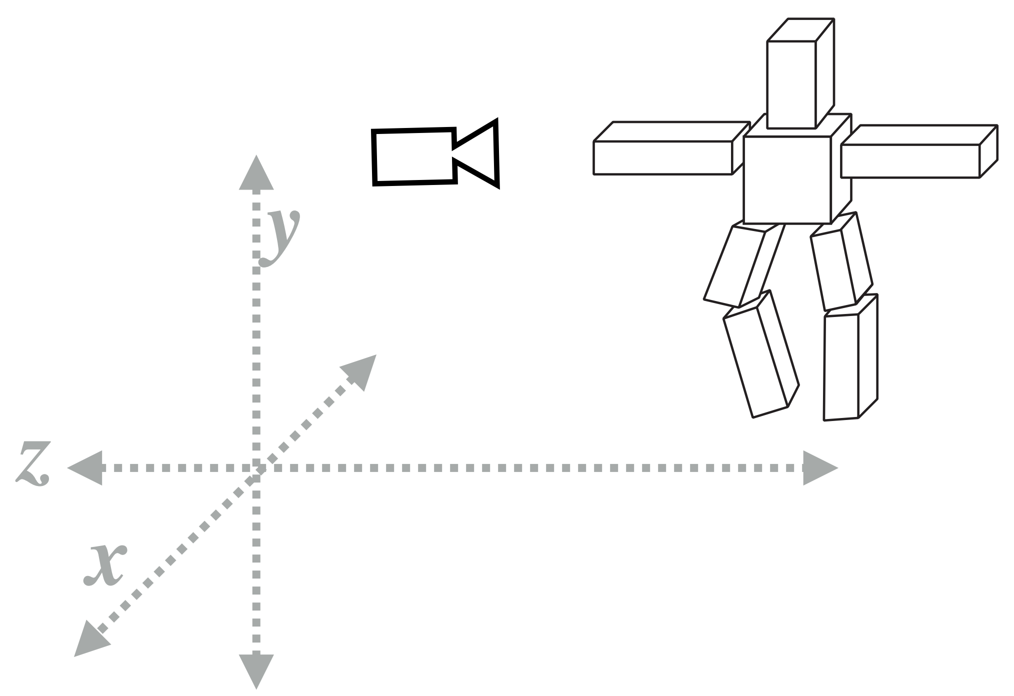
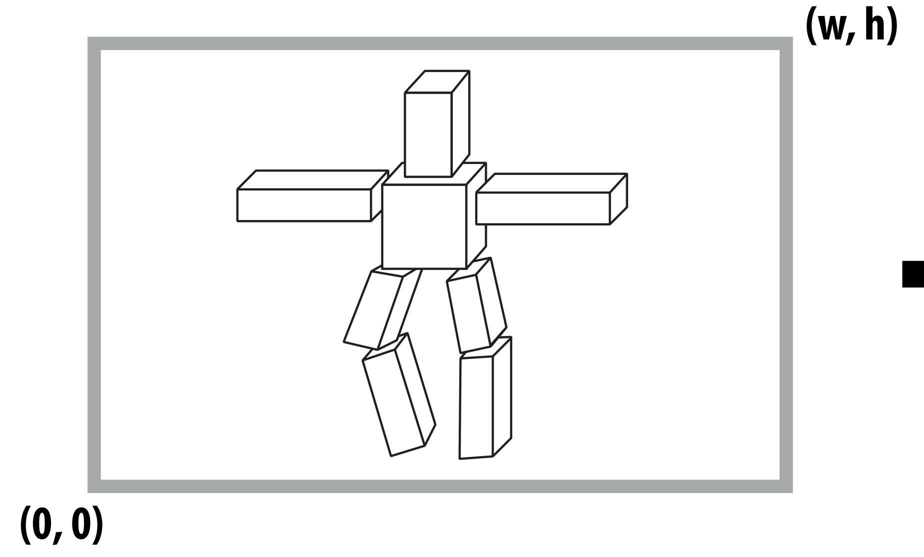
Depth and Transparency

Computer Graphics
CMU 15-462/15-662

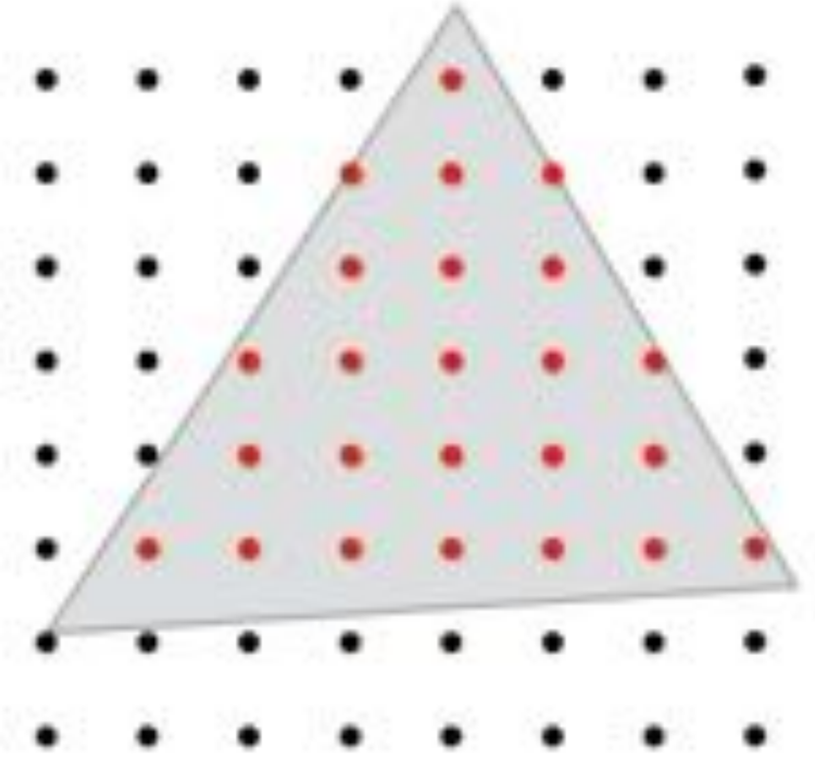
What we know how to do so far...



position objects in the world
(3D transformations)

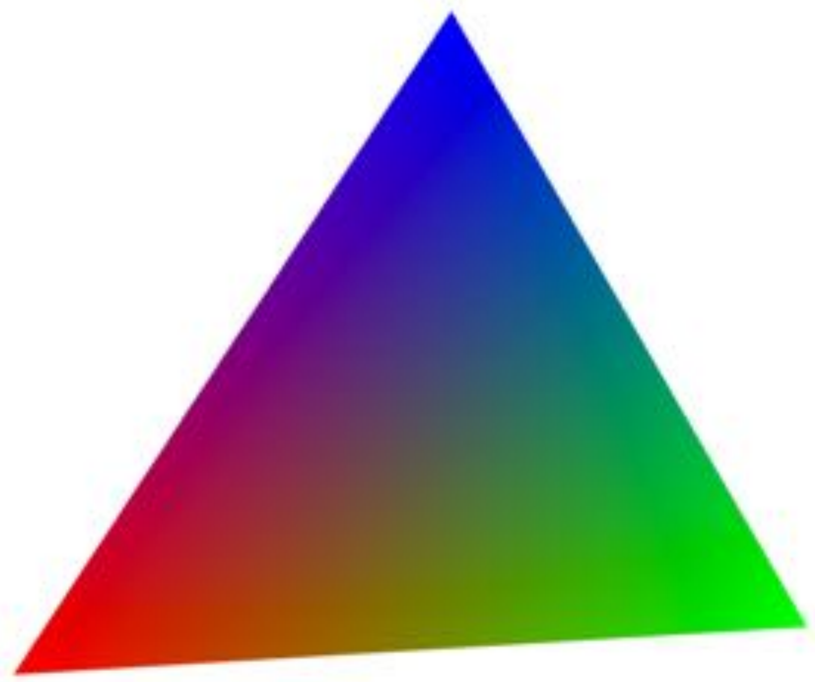


project objects onto the screen
(perspective projection)



sample triangle coverage
(rasterization)

Today



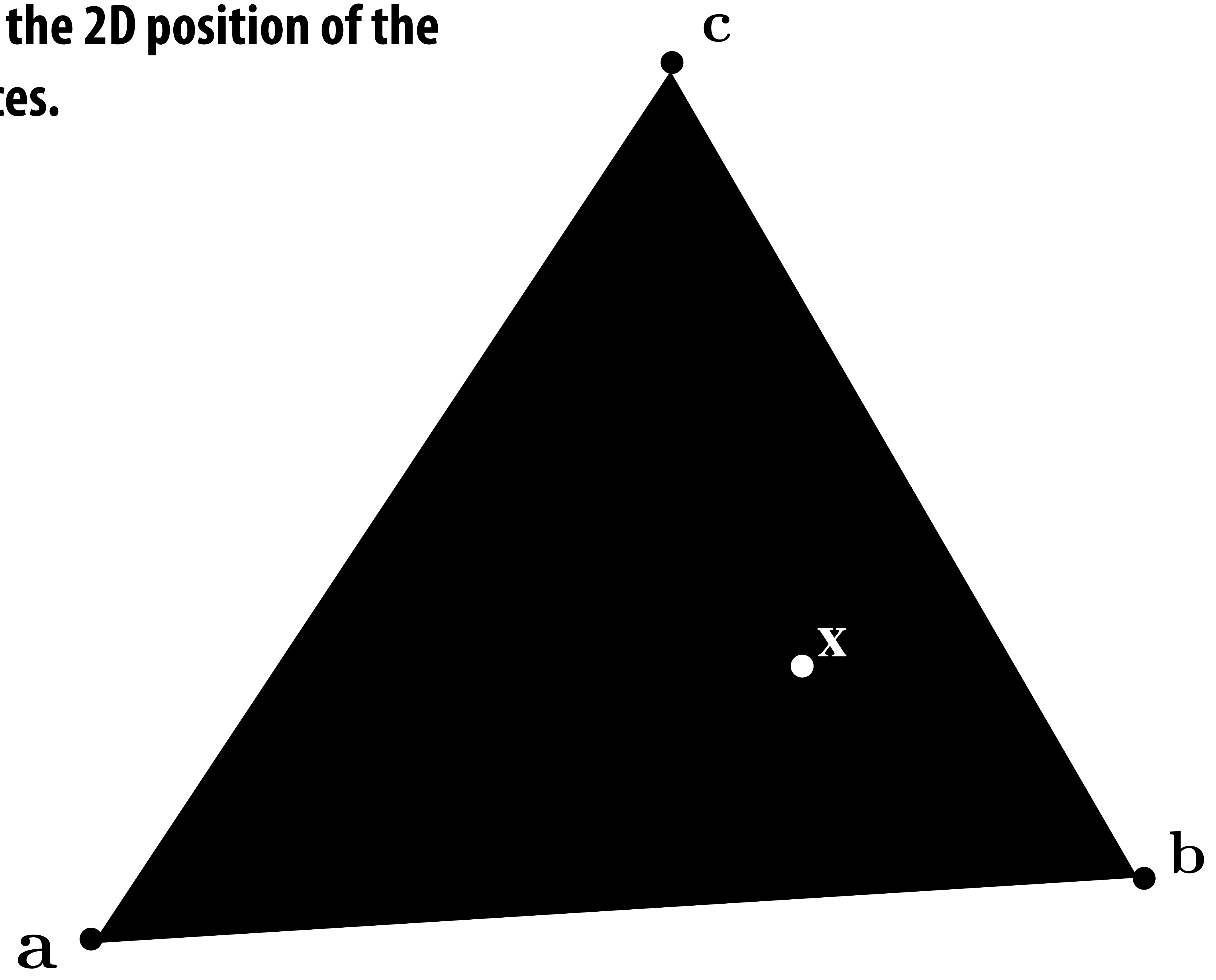
interpolate vertex attributes
(barycentric coodinates)

???

put samples into frame buffer
(depth & alpha)

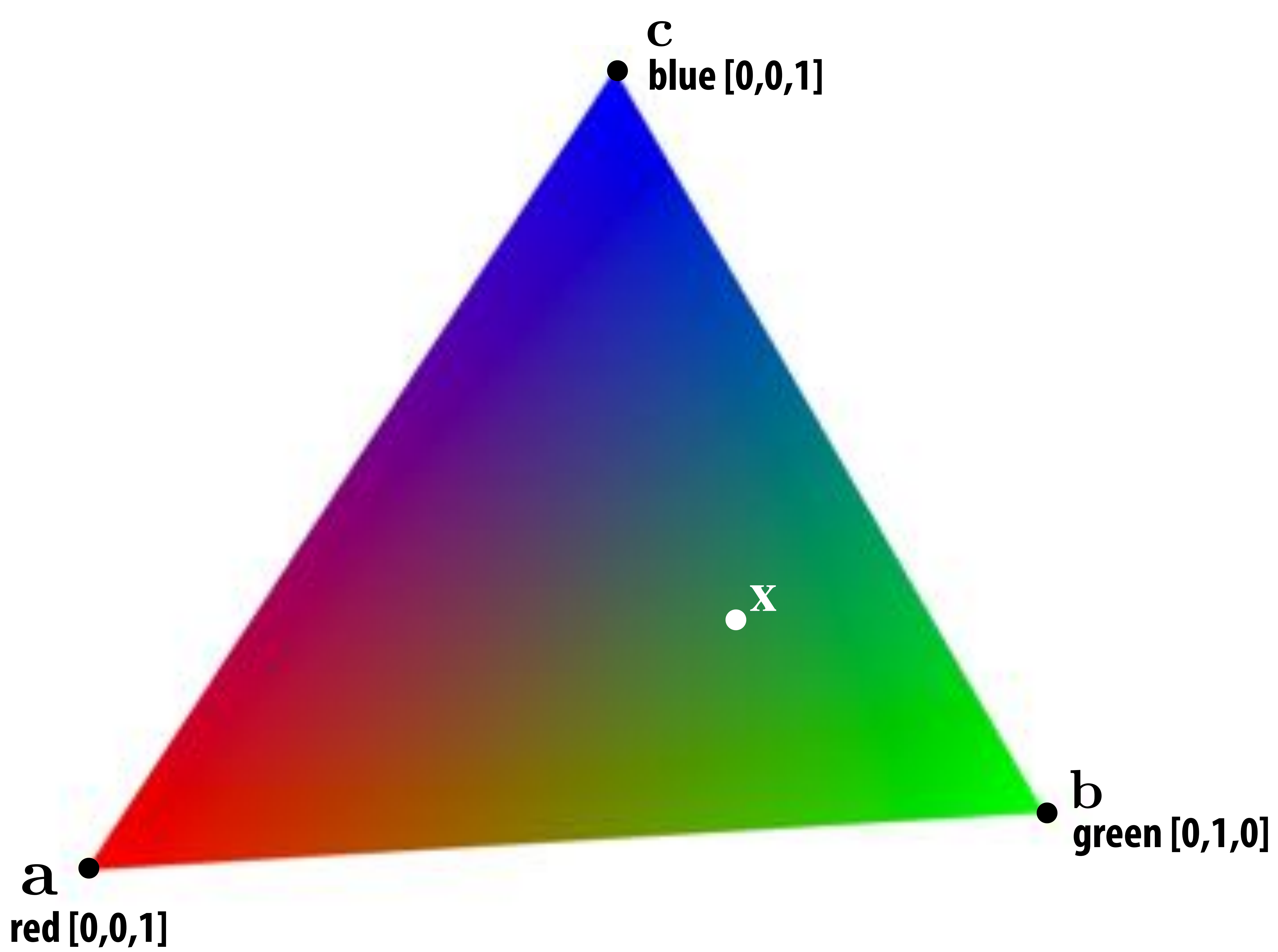
Coverage(x,y)

Previously discussed how to sample coverage given the 2D position of the triangle's vertices.



What if our triangle is not all the same color (or any other property)?

Consider sampling color(x,y)



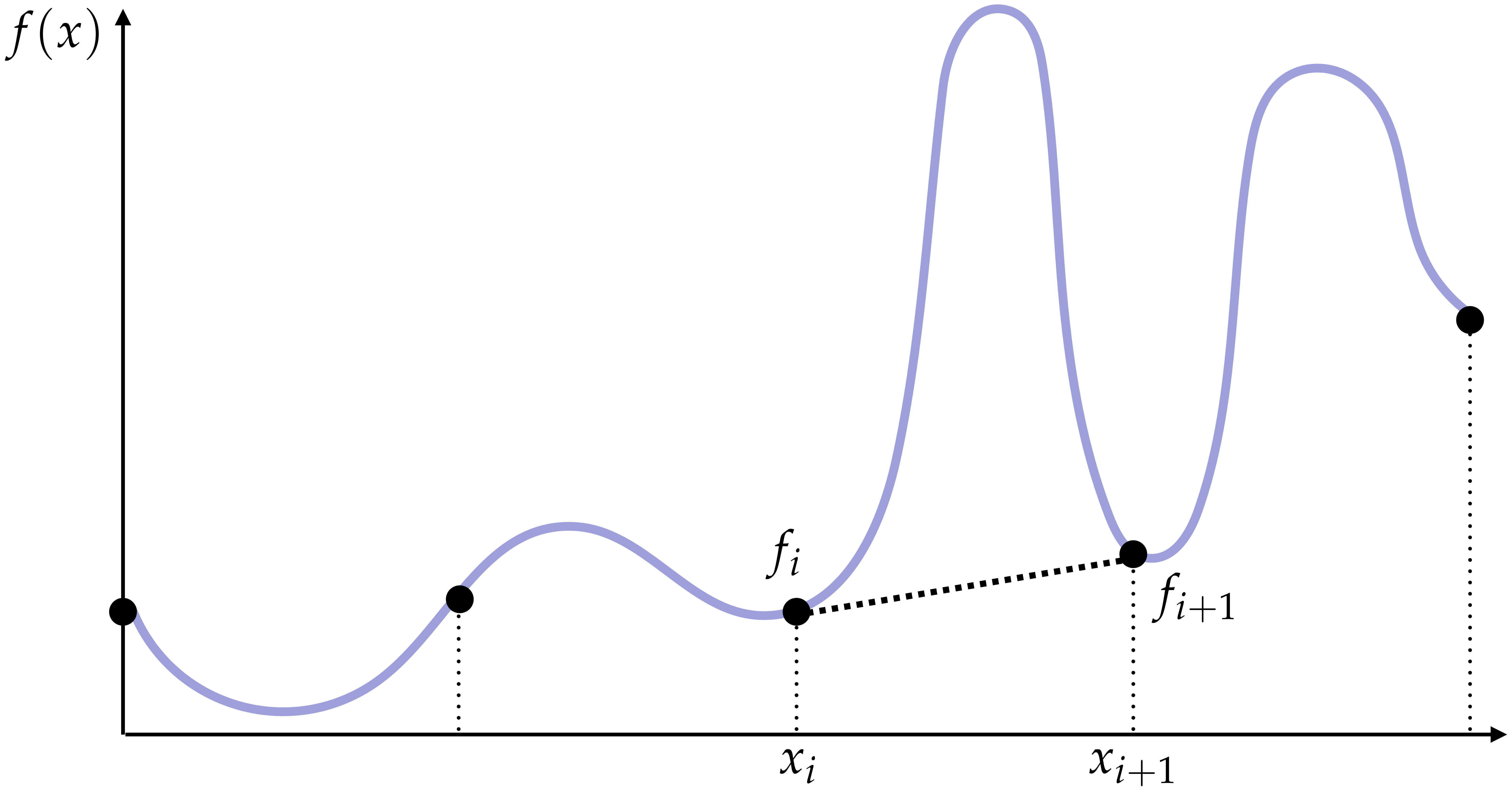
What is the triangle's color at the point $\mathbf{x}$?

Standard strategy: interpolate color values at vertices.

Linear interpolation in 1D

Suppose we've sampled values of a function f(x) at points x_i, i.e., $f_i := f(x_i)$

Q: How do we construct a function that "connects the dots" between x_i and x_{i+1}?



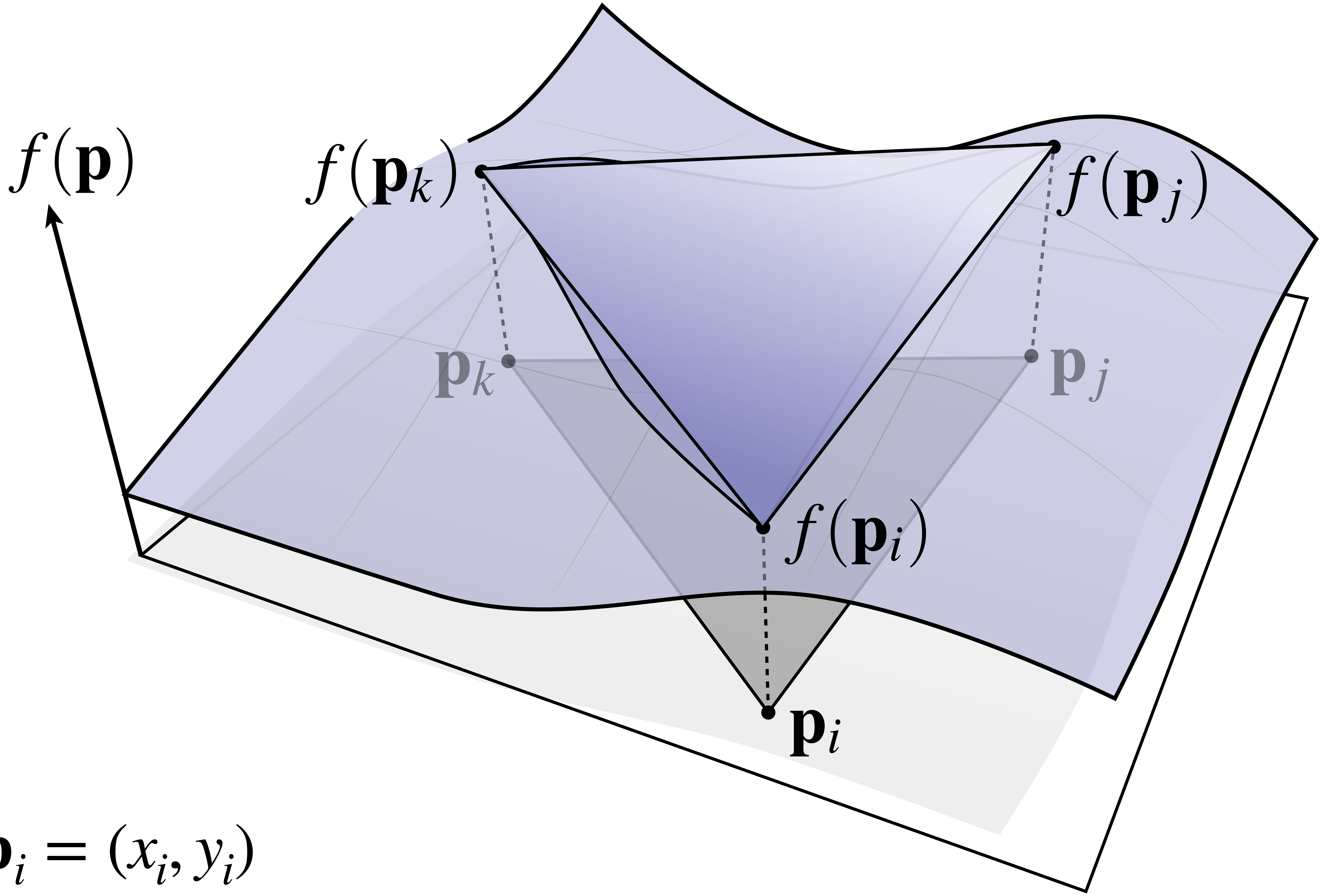
$$t := (x - x_i)/(x_{i+1} - x_i) \in [0,1]$$

$$\hat{f}(t) = f_i + t(f_{i+1} - f_i) = (1-t)f_i + t f_{i+1}$$

Linear interpolation in 2D

Suppose we've likewise sampled values of a function $f(\mathbf{p})$ at points $\mathbf{p}_i$, $\mathbf{p}_j$, $\mathbf{p}_k$ in 2D

Q: How do we "connect the dots" this time? E.g., how do we fit a plane?



$\mathbf{p}_i = (x_i, y_i)$

Linear interpolation in 2D

- **Want to fit a linear (really, affine) function to three values**
- **Any such function has three unknown coefficients a, b, and c:**

$$\hat{f}(x,y) = ax + by + c$$

- **To interpolate, we need to find coefficients such that the function matches the sample values at the sample points:**

$$\hat{f}(x_n, y_n) = f_n, \; n \in \{i, j, k\}$$

- **Yields three linear equations in three unknowns. Solution?**

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{(x_j y_i - x_i y_j) + (x_k y_j - x_j y_k) + (x_i y_k - x_k y_i)} \begin{bmatrix} f_i(y_k - y_j) + f_j(y_i - y_k) + f_k(y_j - y_i) \\ f_i(x_j - x_k) + f_j(x_k - x_i) + f_k(x_i - x_j) \\ f_i(x_k y_j - x_j y_k) + f_j(x_i y_k - x_k y_i) + f_k(x_j y_i - x_i y_j) \end{bmatrix}$$

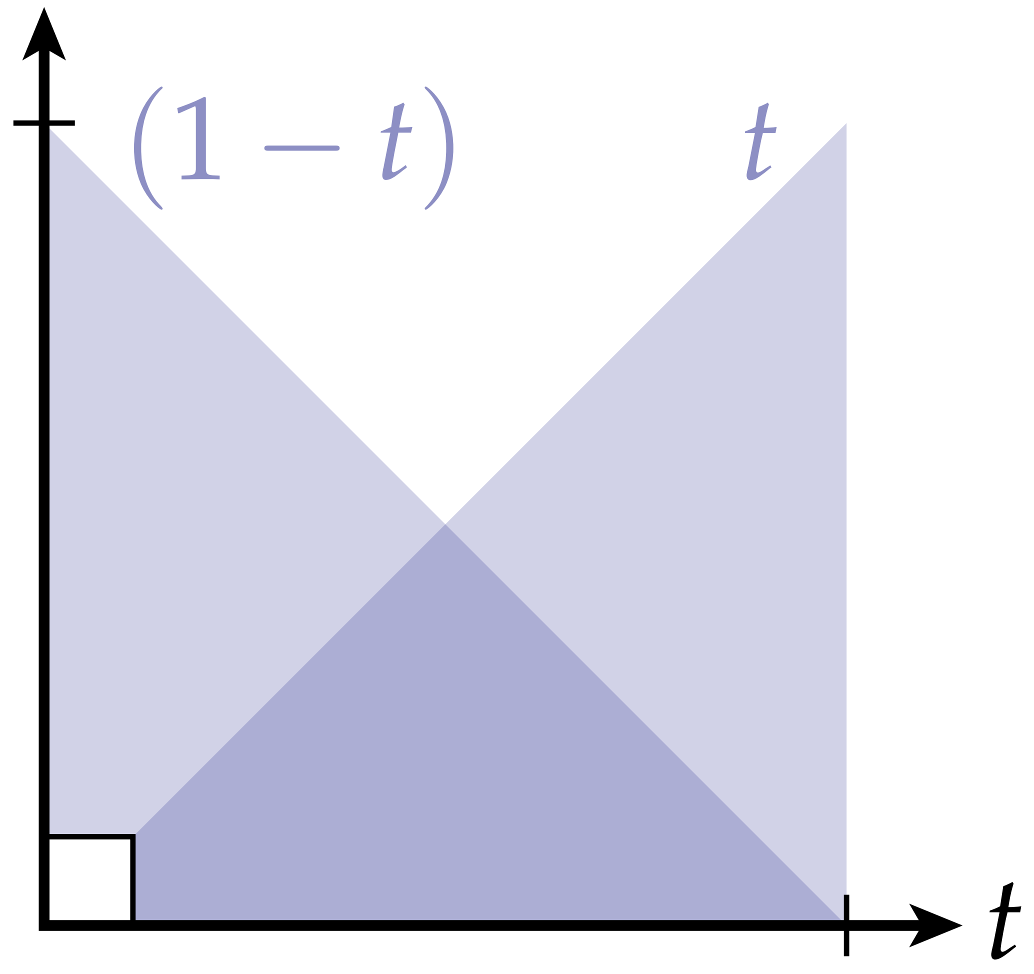
This is ugly. There <u>has</u> to be a better way to think about this…

1D Linear Interpolation, revisited

- **Let's think about how we did linear interpolation in 1D:**

$$\hat{f}(t) = (1-t)f_i + t f_j$$

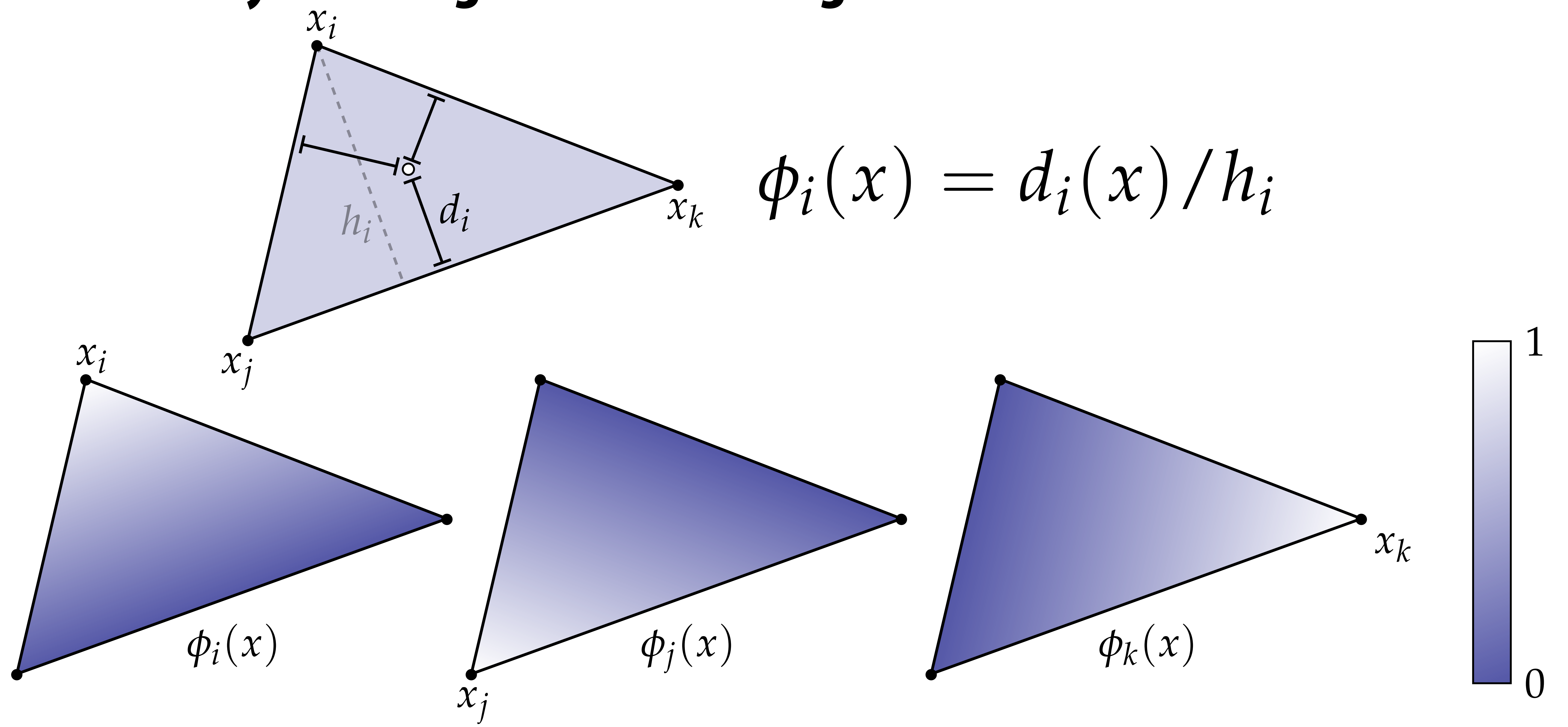
- **Can think of this as a linear combination of two functions:**



- **As we move closer to t=0, we approach the value of f at x_i**
- **As we move closer to t=1, we approach the value of f at x_j**

2D Linear Interpolation, revisited

- We can construct analogous functions for a triangle
- For a given point x, measure the distance to each edge; then divide by the height of the triangle:

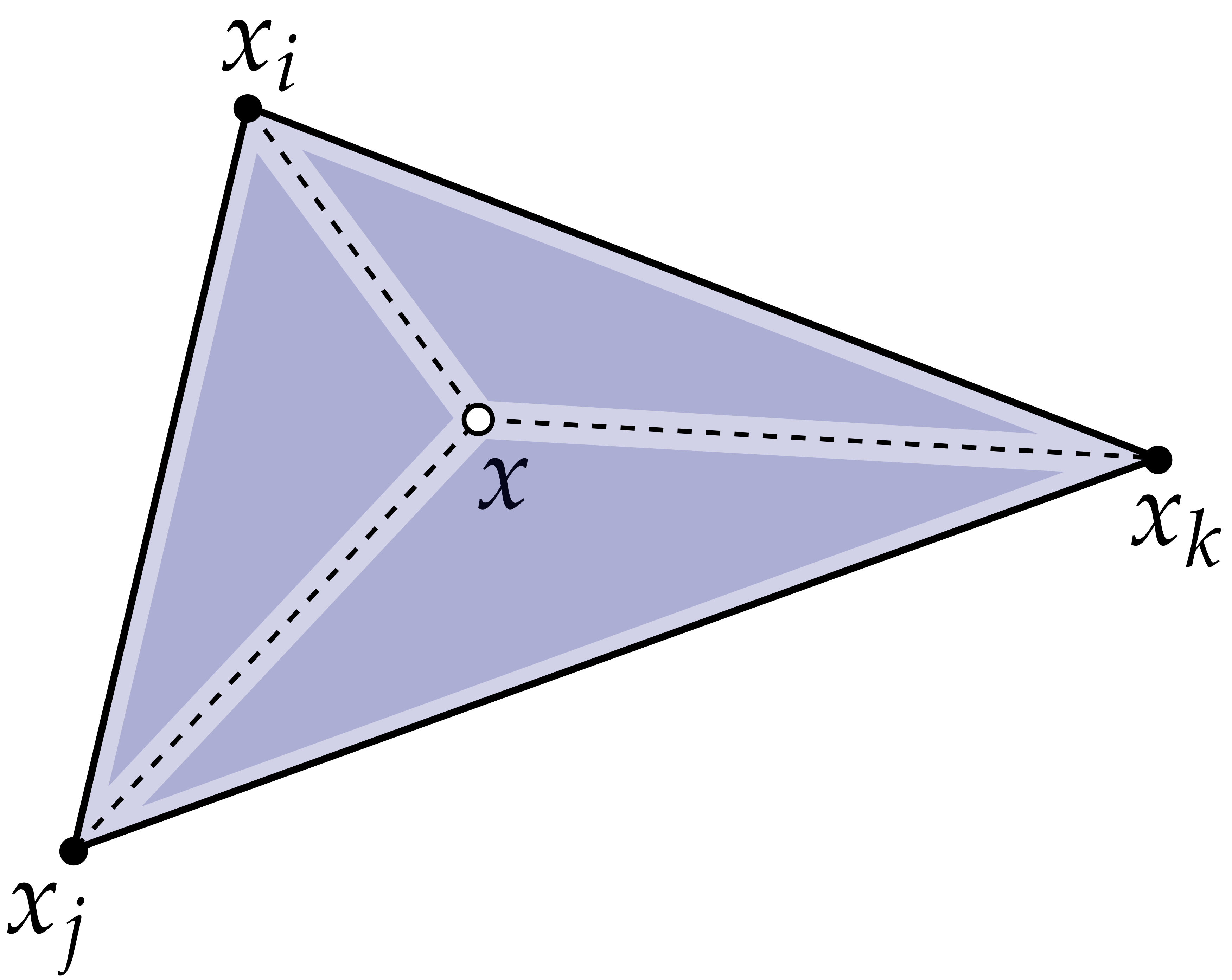


Interpolate by taking linear combination: $\hat{f}(x) = f_i\phi_i + f_j\phi_j + f_k\phi_k$

Q: Is this the same as the (ugly) function we found before?

2D Interpolation, another way

- I claim we can also get the same three basis functions as a ratio of triangle areas:



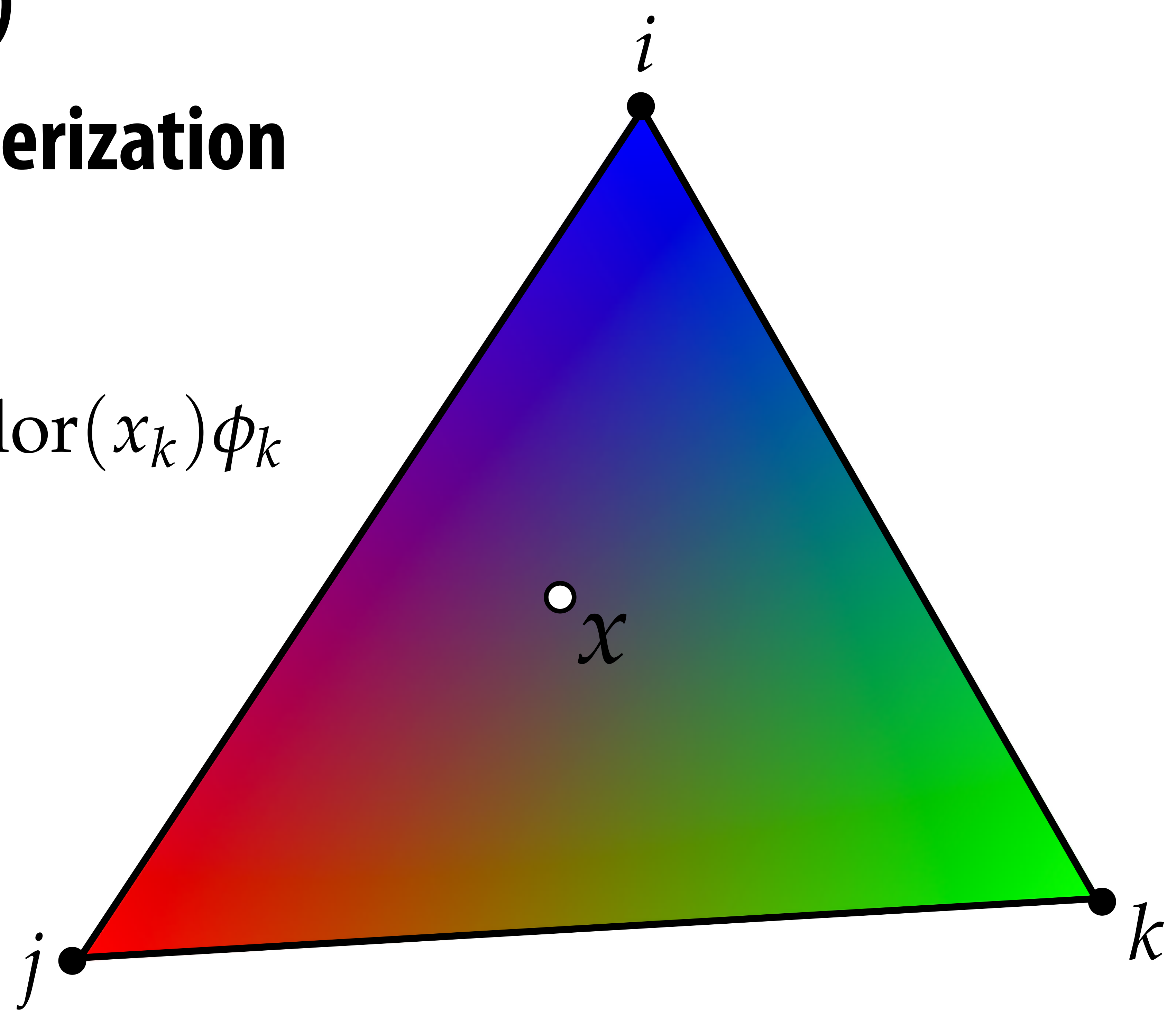
$$\phi_i(x) = \frac{\text{area}(x, x_j, x_k)}{\text{area}(x_i, x_j, x_k)}$$

Q: Do you buy it? (Why or why not?)

Barycentric Coordinates

- No matter how you compute them, the values of the three functions $\phi_i(\mathbf{x}), \phi_j(\mathbf{x}), \phi_k(\mathbf{x})$ for a given point are called barycentric coordinates
- Can be used to interpolate any attribute associated with vertices. (color*, texture coordinates, etc.)
- Importantly, these same three values fall out of the half-plane tests used for triangle rasterization! (Why?)
- Hence, get them for "free" during rasterization

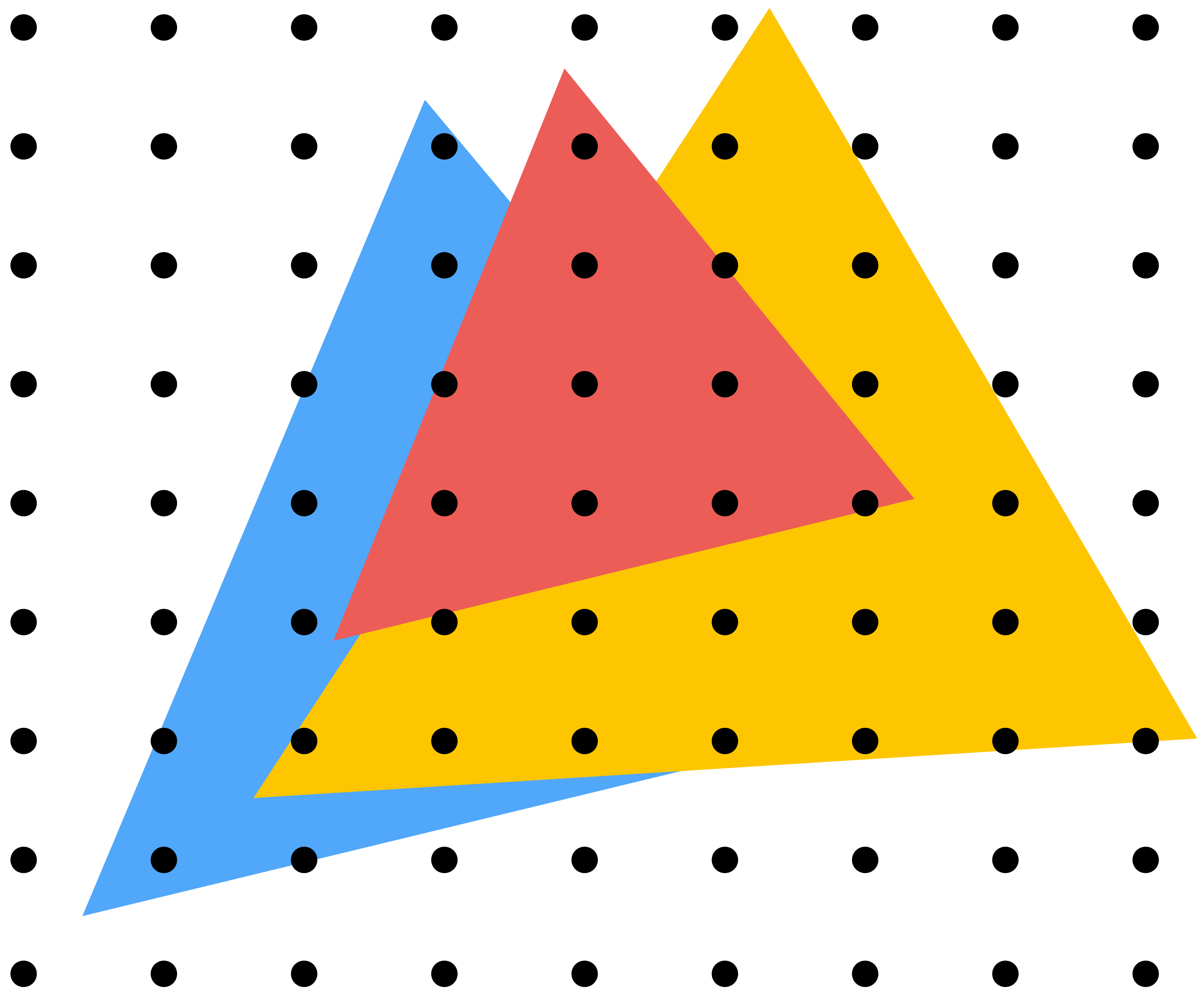
$$\text{color}(x) = \text{color}(x_i)\phi_i + \text{color}(x_j)\phi_j + \text{color}(x_k)\phi_k$$



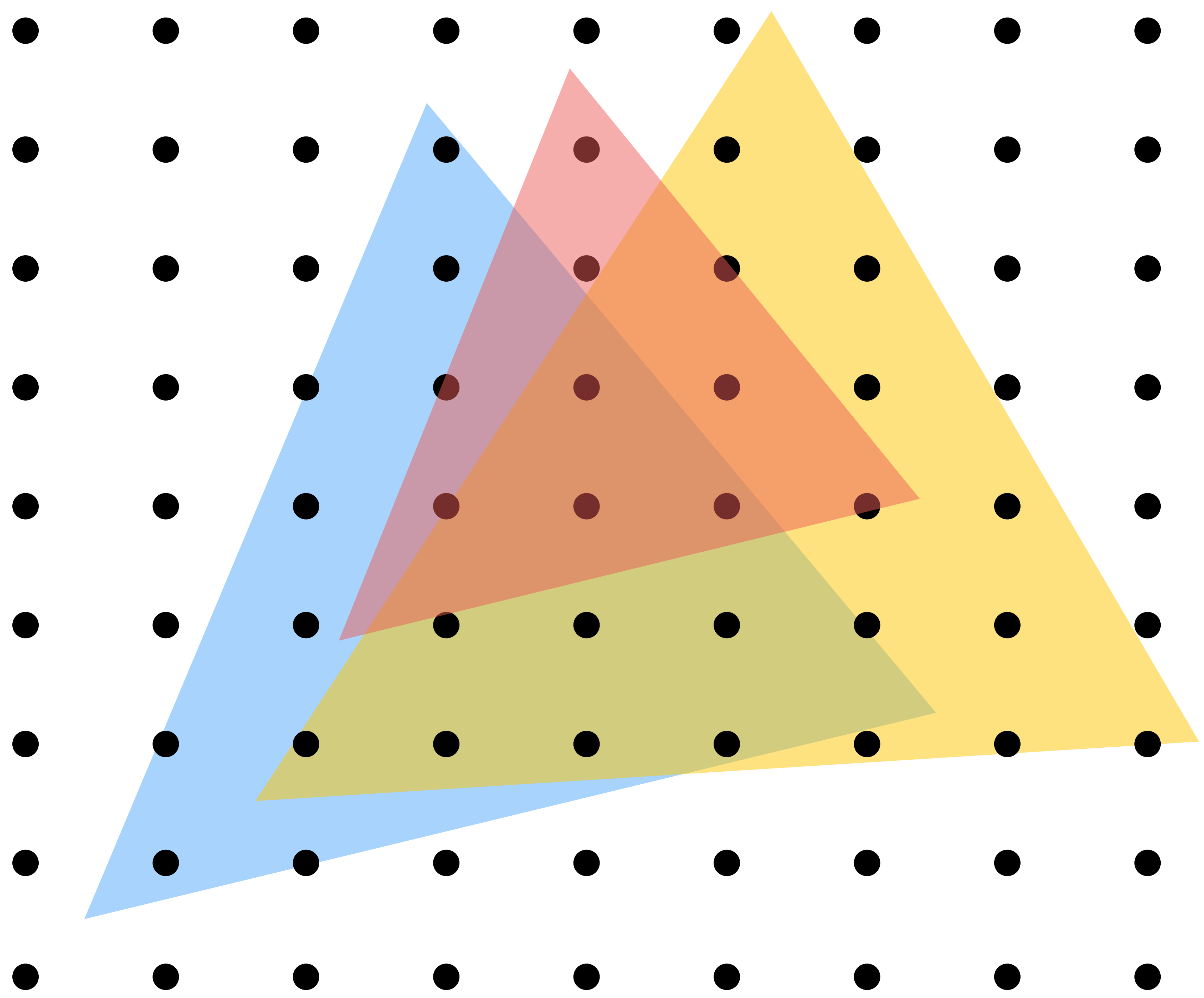
*Note: we haven't explained yet how to encode colors as numbers! We'll talk about that in a later lecture…

Occlusion

Occlusion: which triangle is visible at each covered sample point?



Opaque Triangles

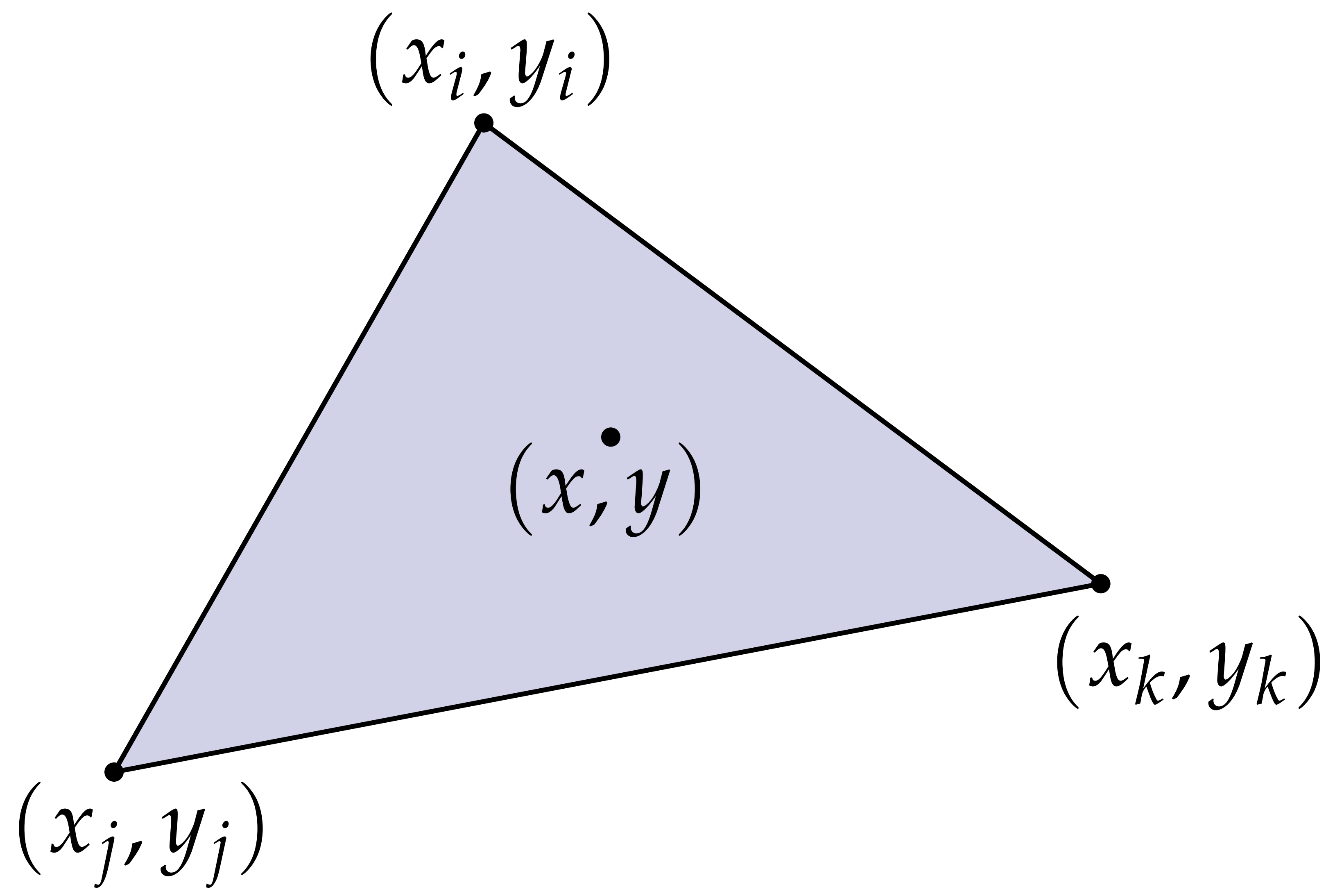


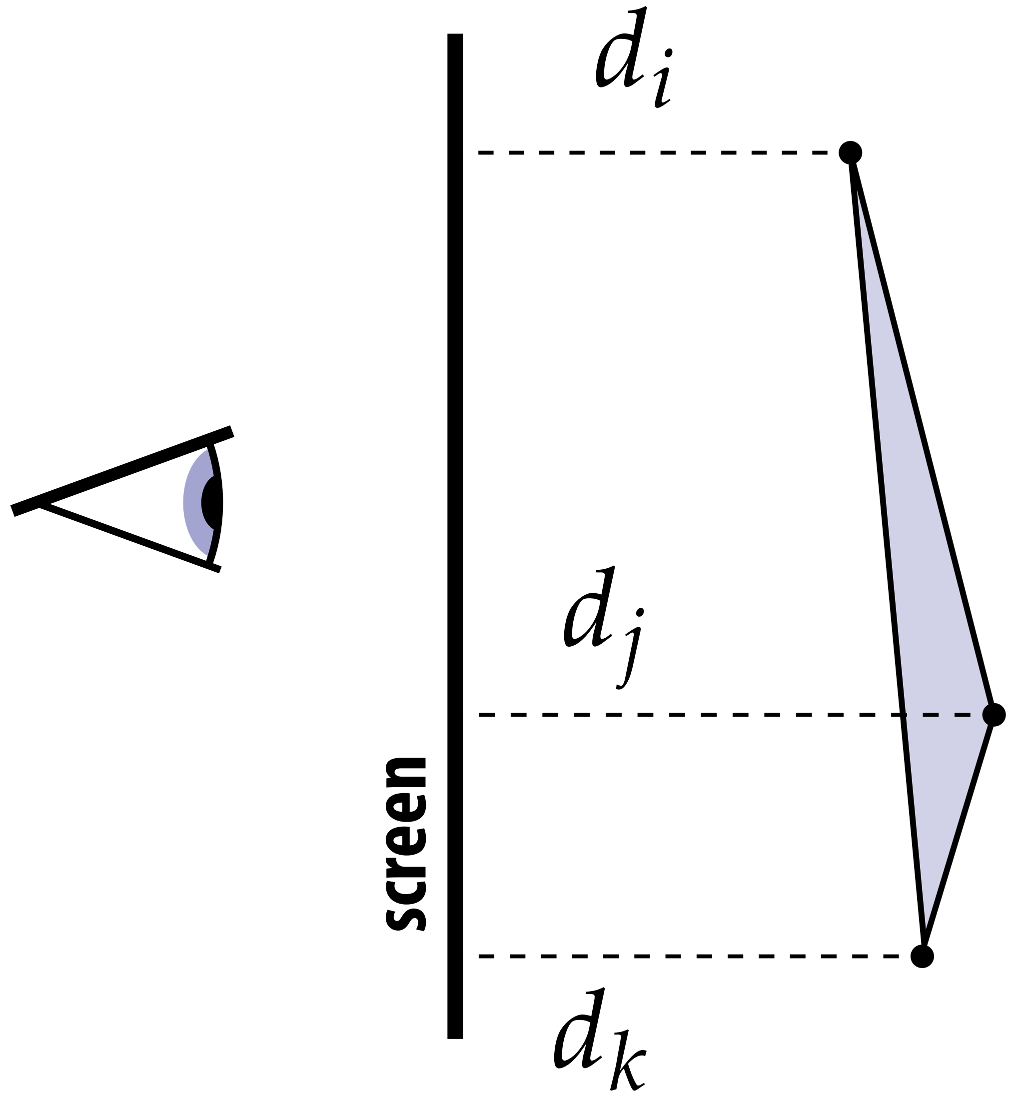
50% transparent triangles

Sampling Depth

Assume we have a triangle given by:

- **the projected 2D coordinates (x_i, y_i) of each vertex**
- **the "depth" d_i of each vertex (i.e., distance from the viewer)**



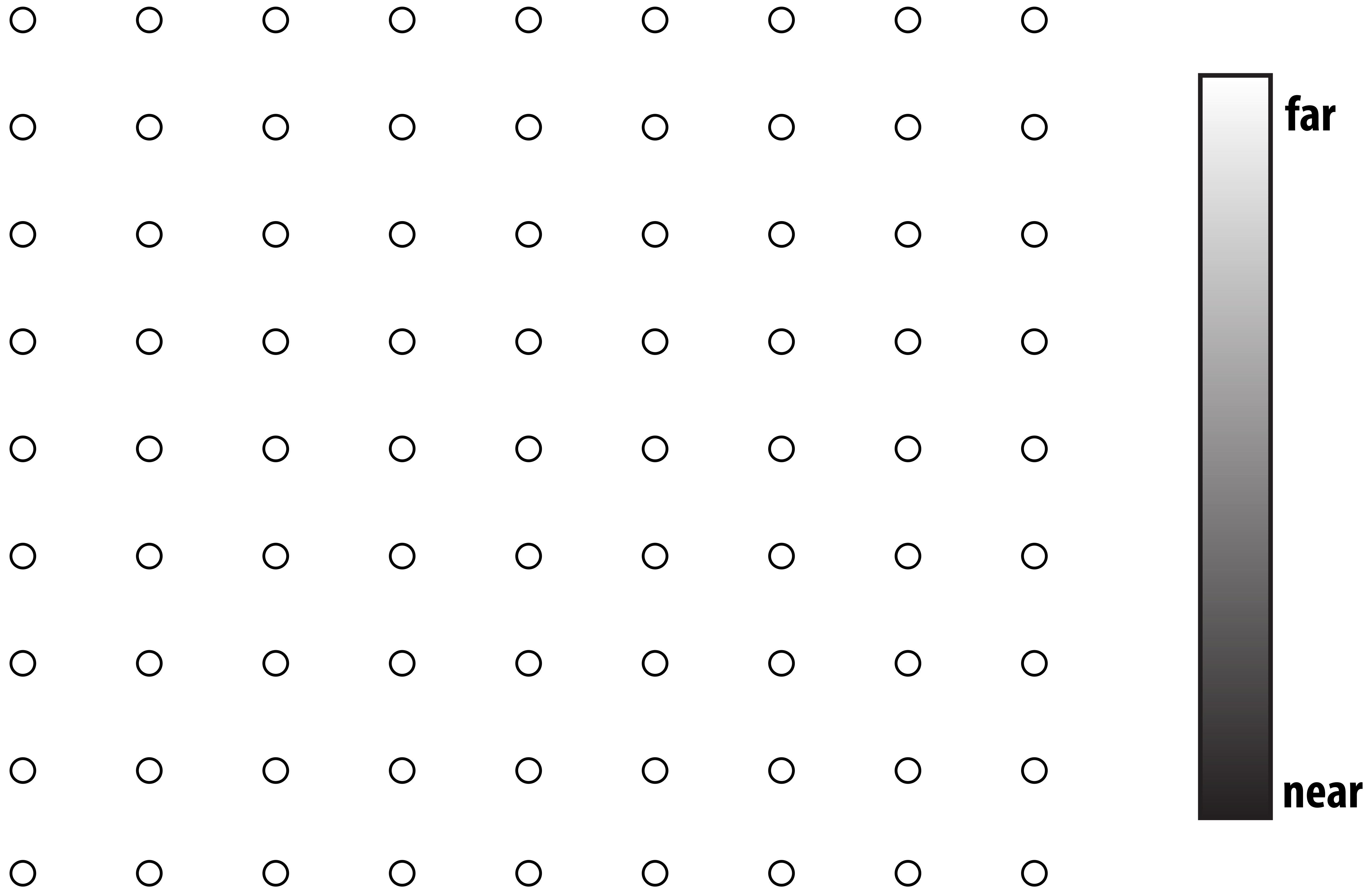


Q: How do we compute the depth d at a given sample point (x, y)?

A: Interpolate it using barycentric coordinates—just like any other attribute that varies linearly over the triangle

The depth-buffer (Z-buffer)

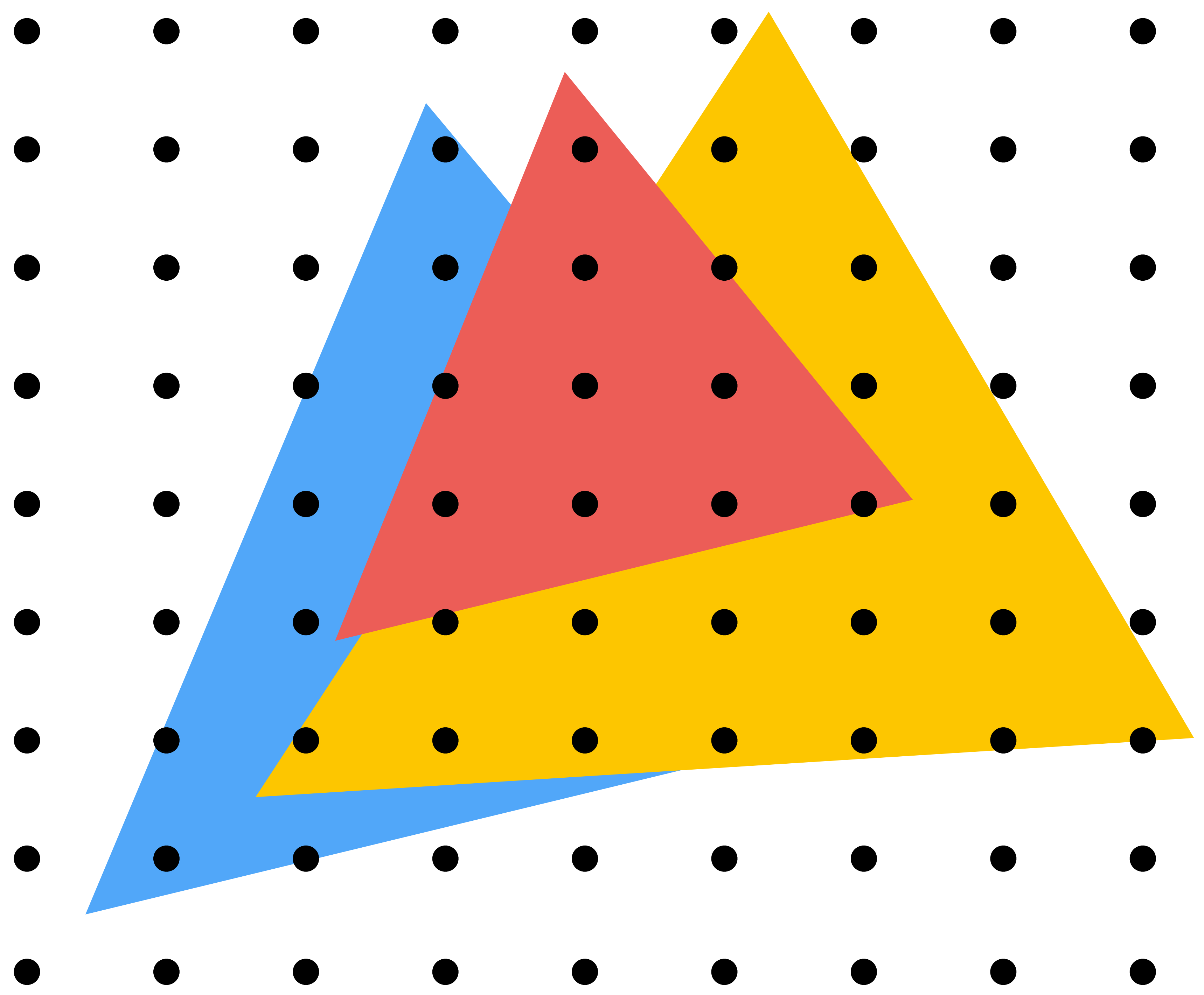
For each sample, depth-buffer stores the depth of the closest triangle seen so far



Initialize all depth buffer values to "infinity" (max value)

Depth buffer example

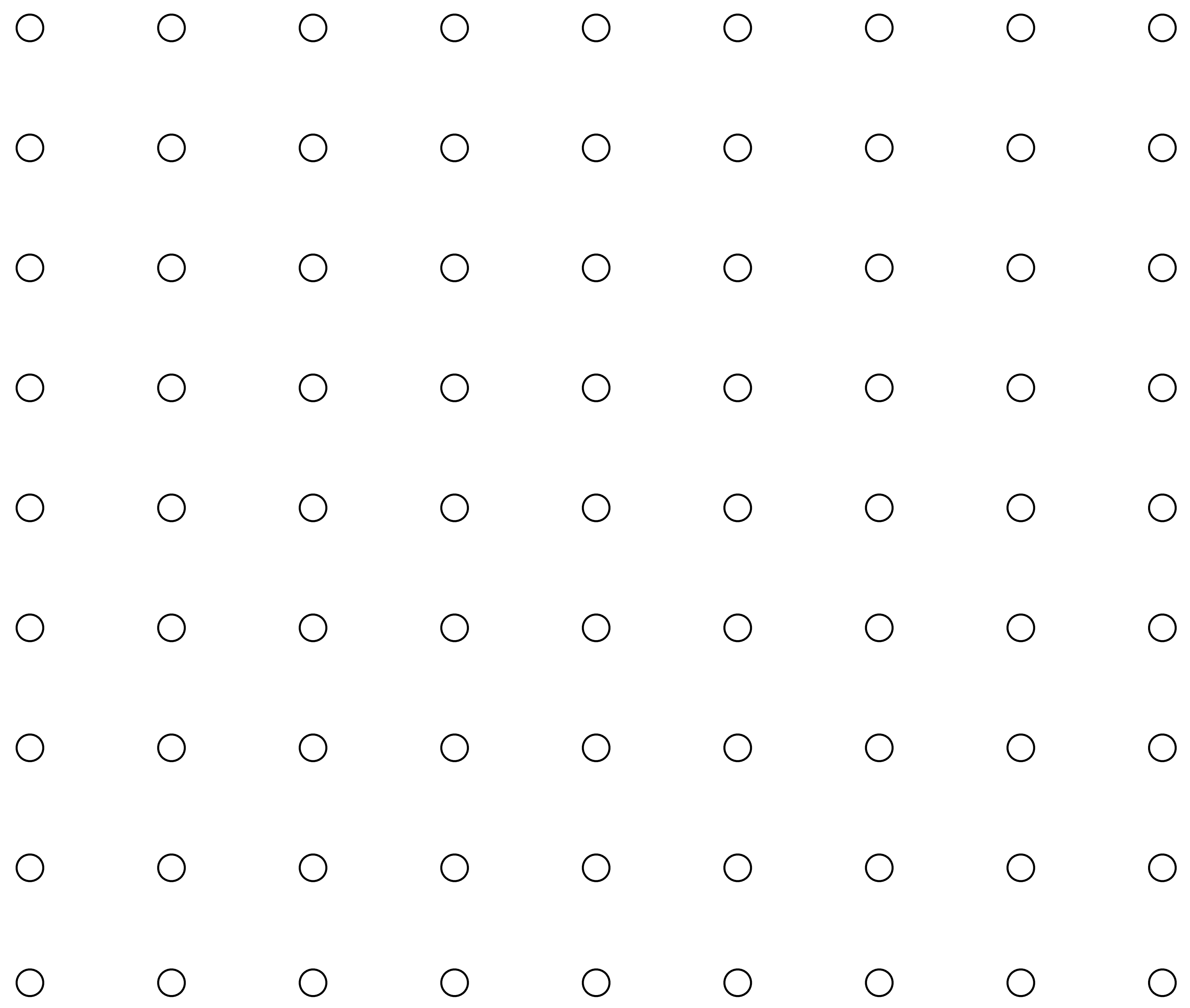
Example: rendering three opaque triangles



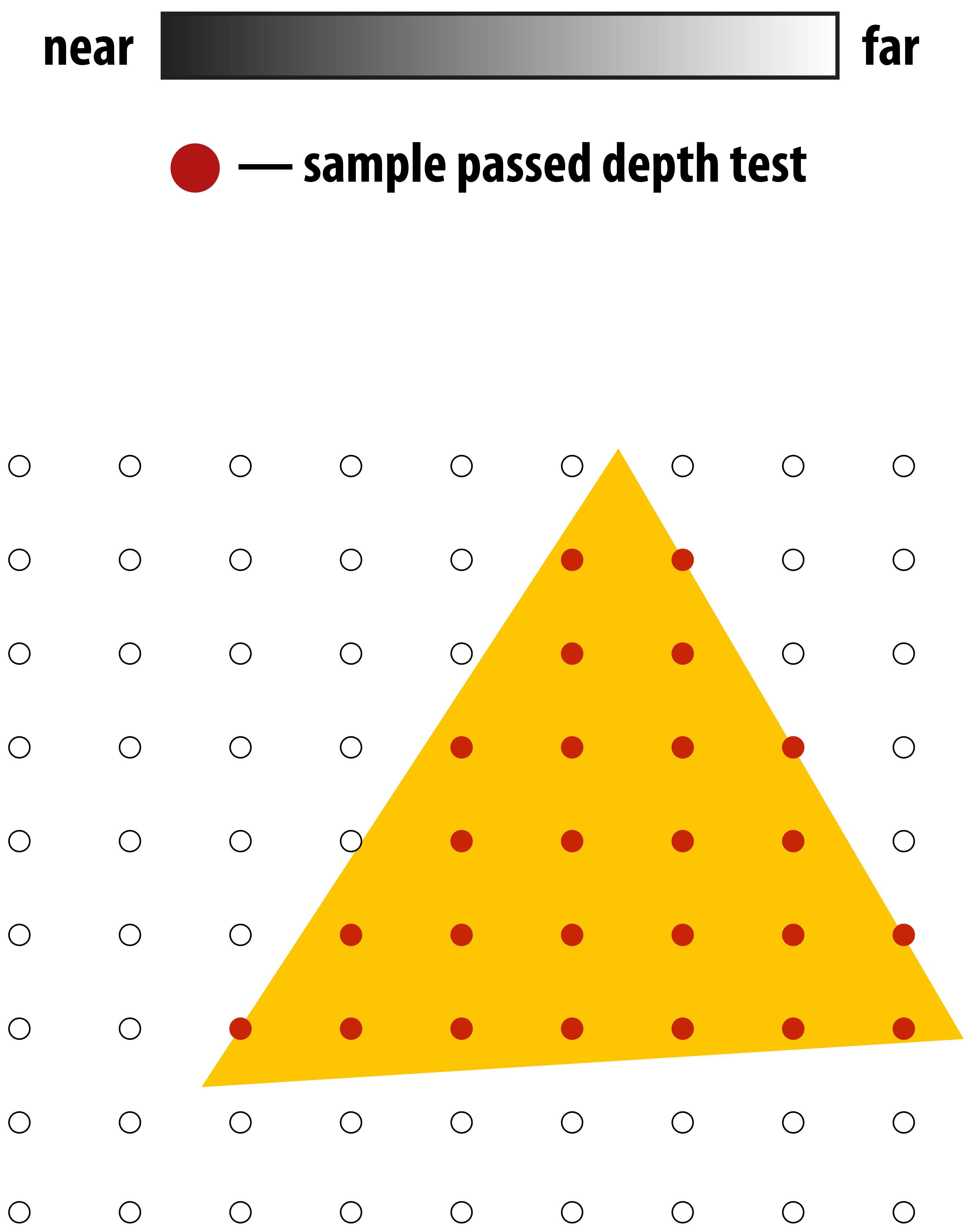
Occlusion using the depth-buffer (Z-buffer)

Processing yellow triangle:

depth = 0.5



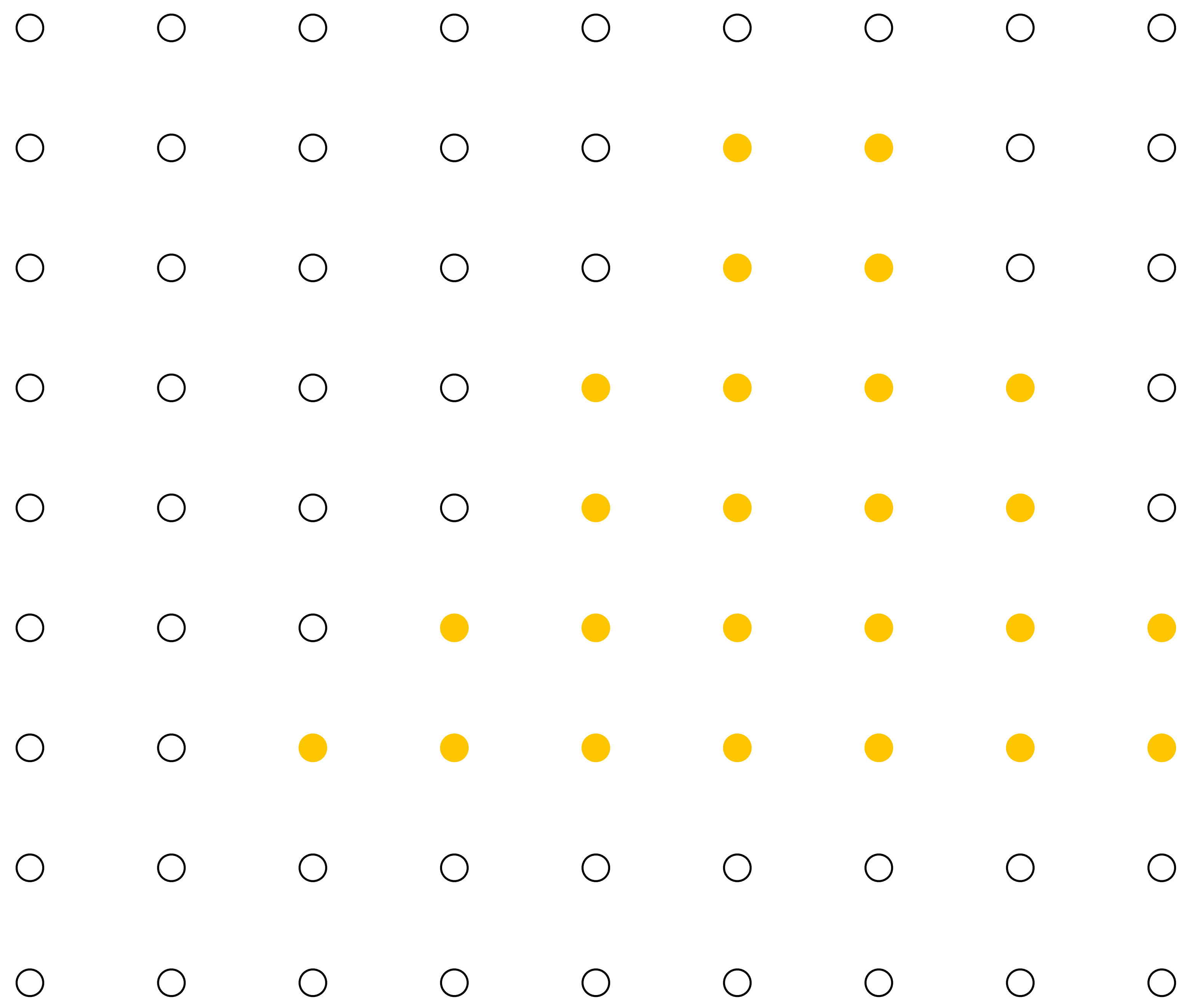
Color buffer contents



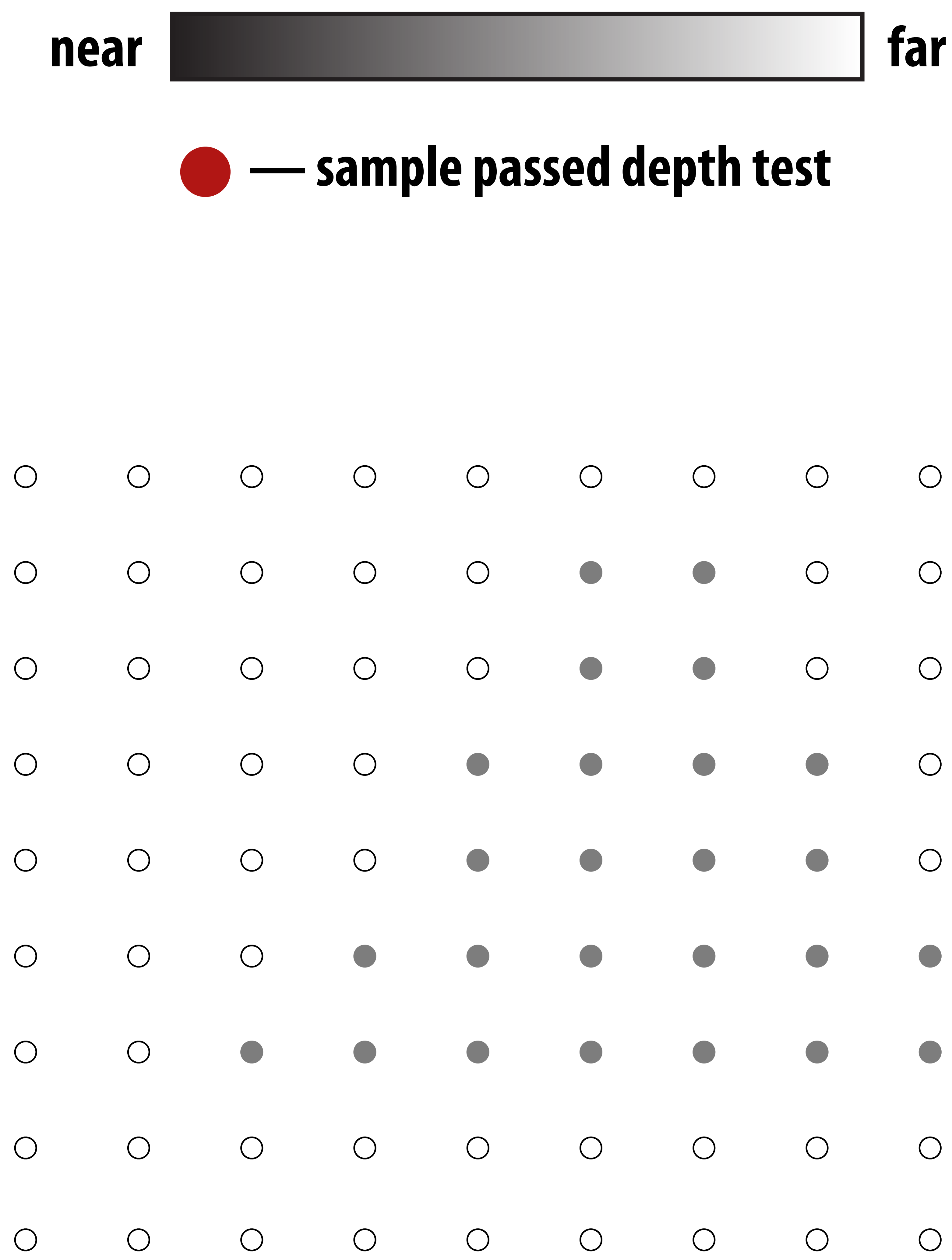
Depth buffer contents

Occlusion using the depth-buffer (Z-buffer)

After processing yellow triangle:



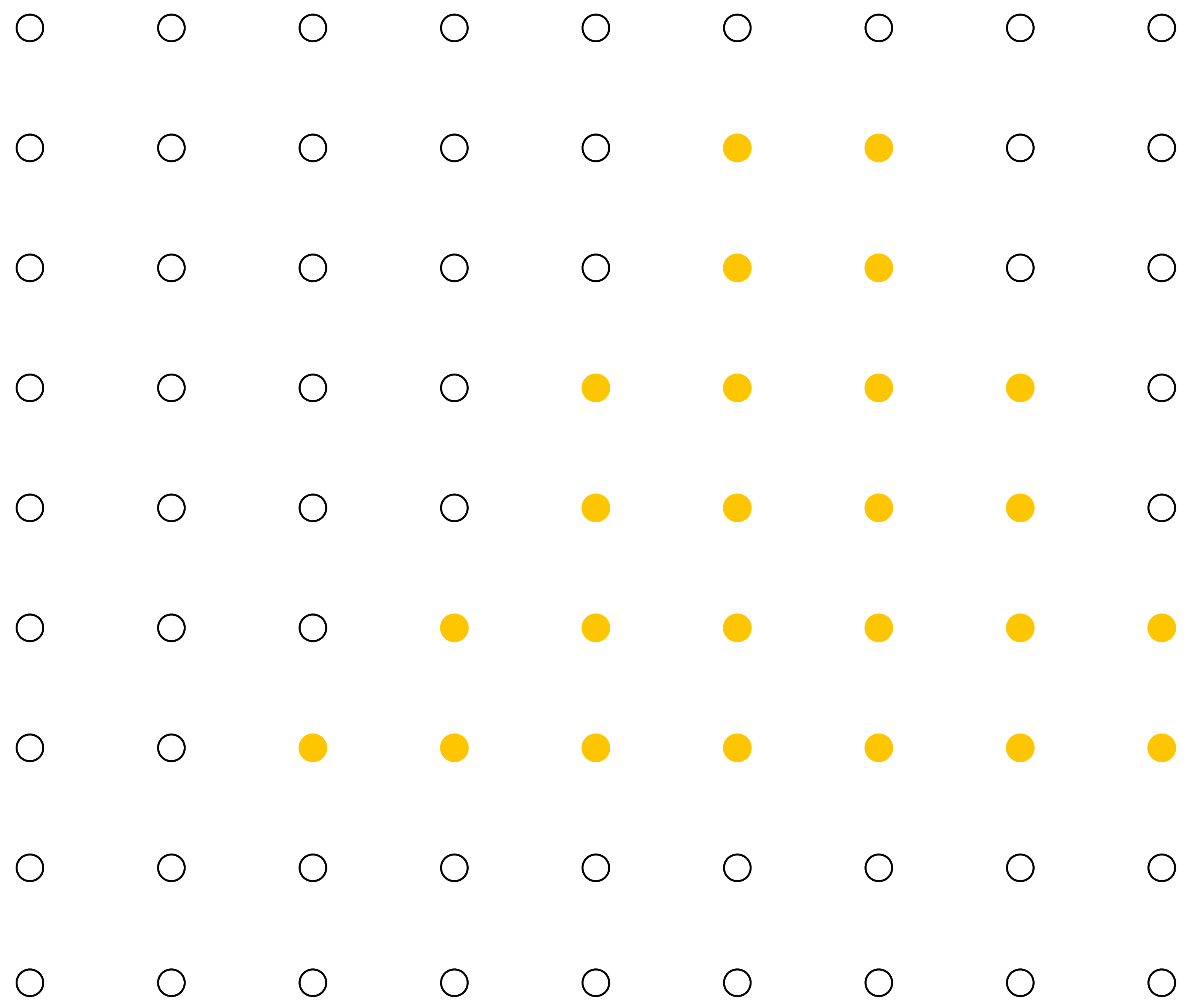
Color buffer contents



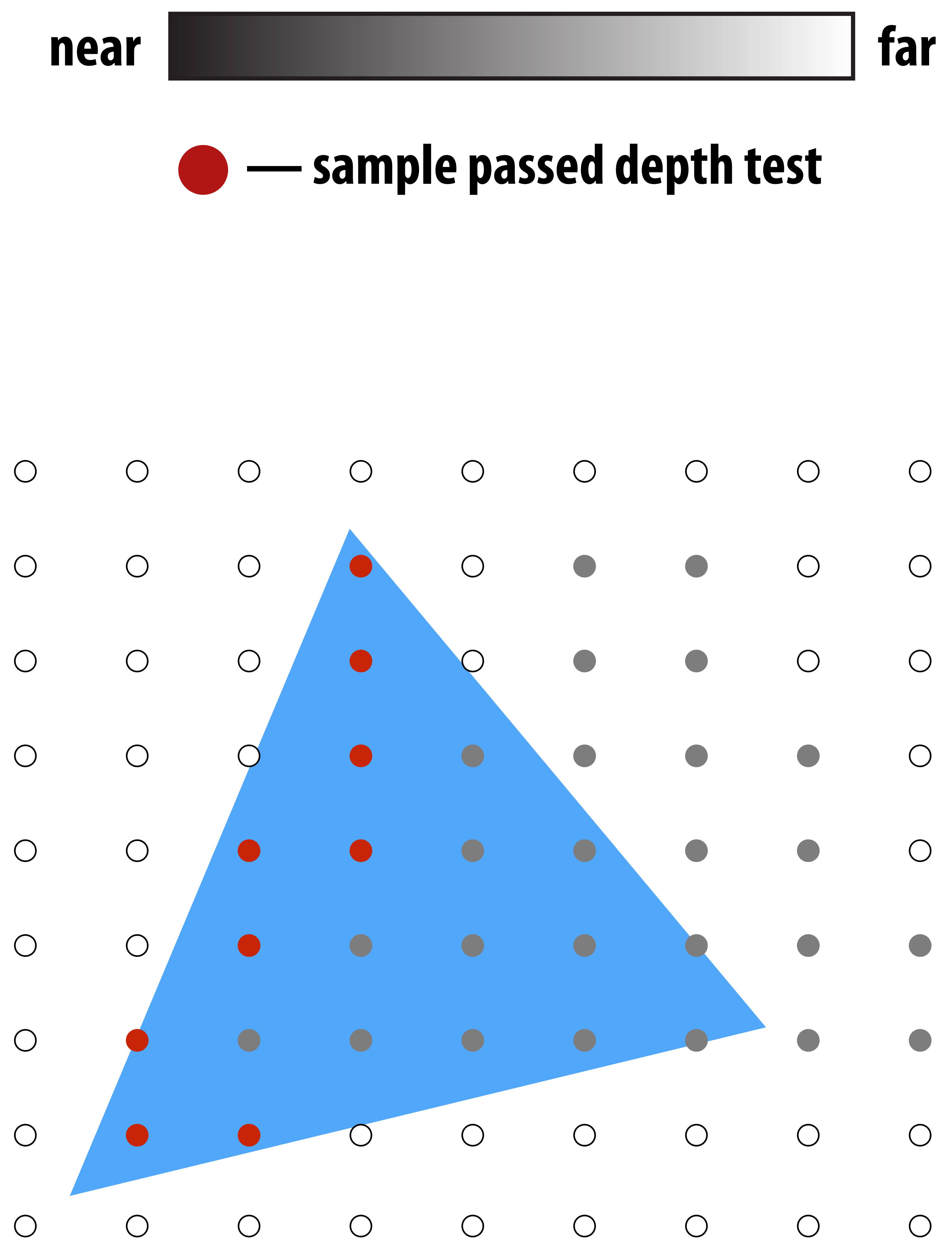
Depth buffer contents

Occlusion using the depth-buffer (Z-buffer)

Processing blue triangle:
depth = 0.75



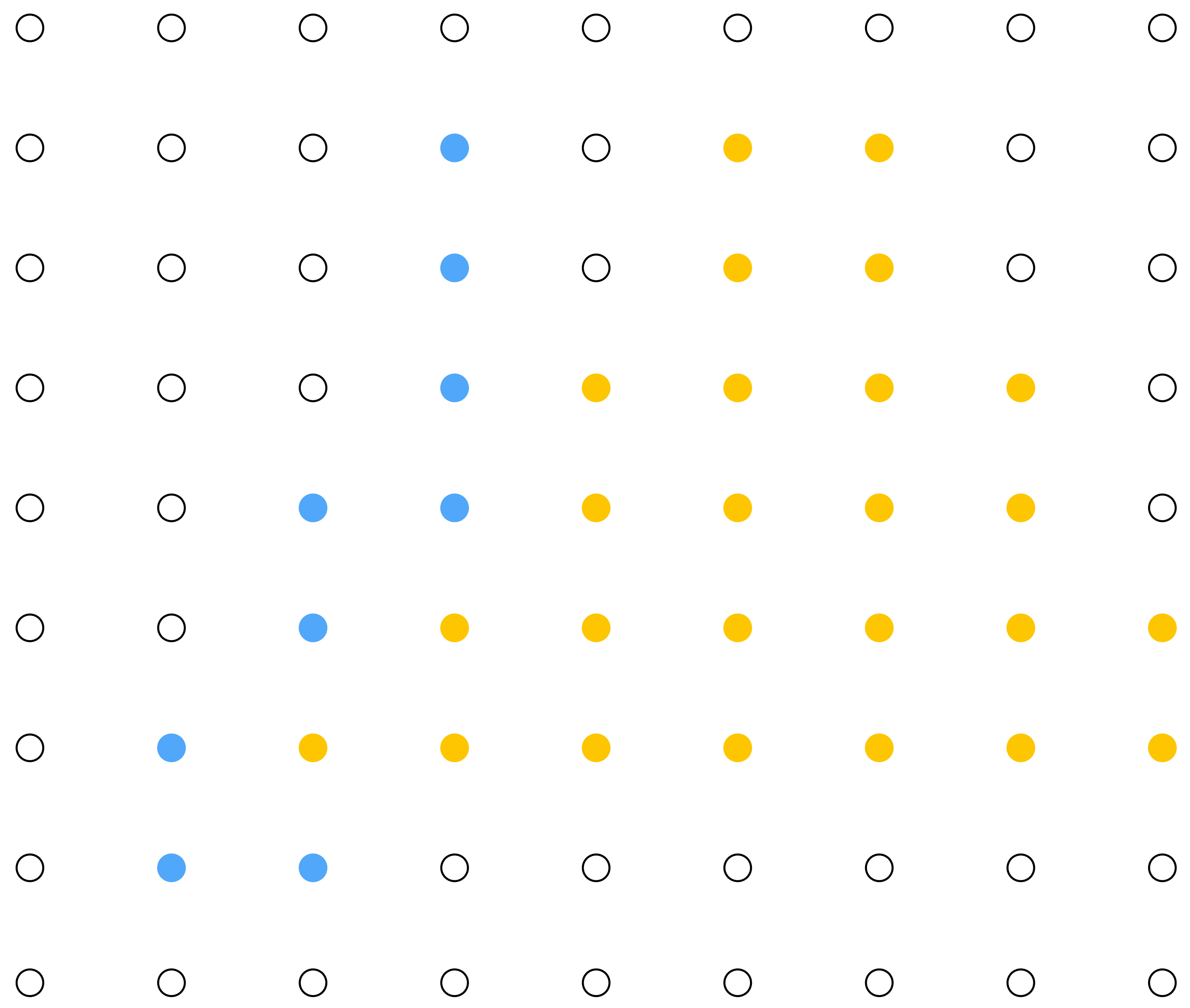
Color buffer contents



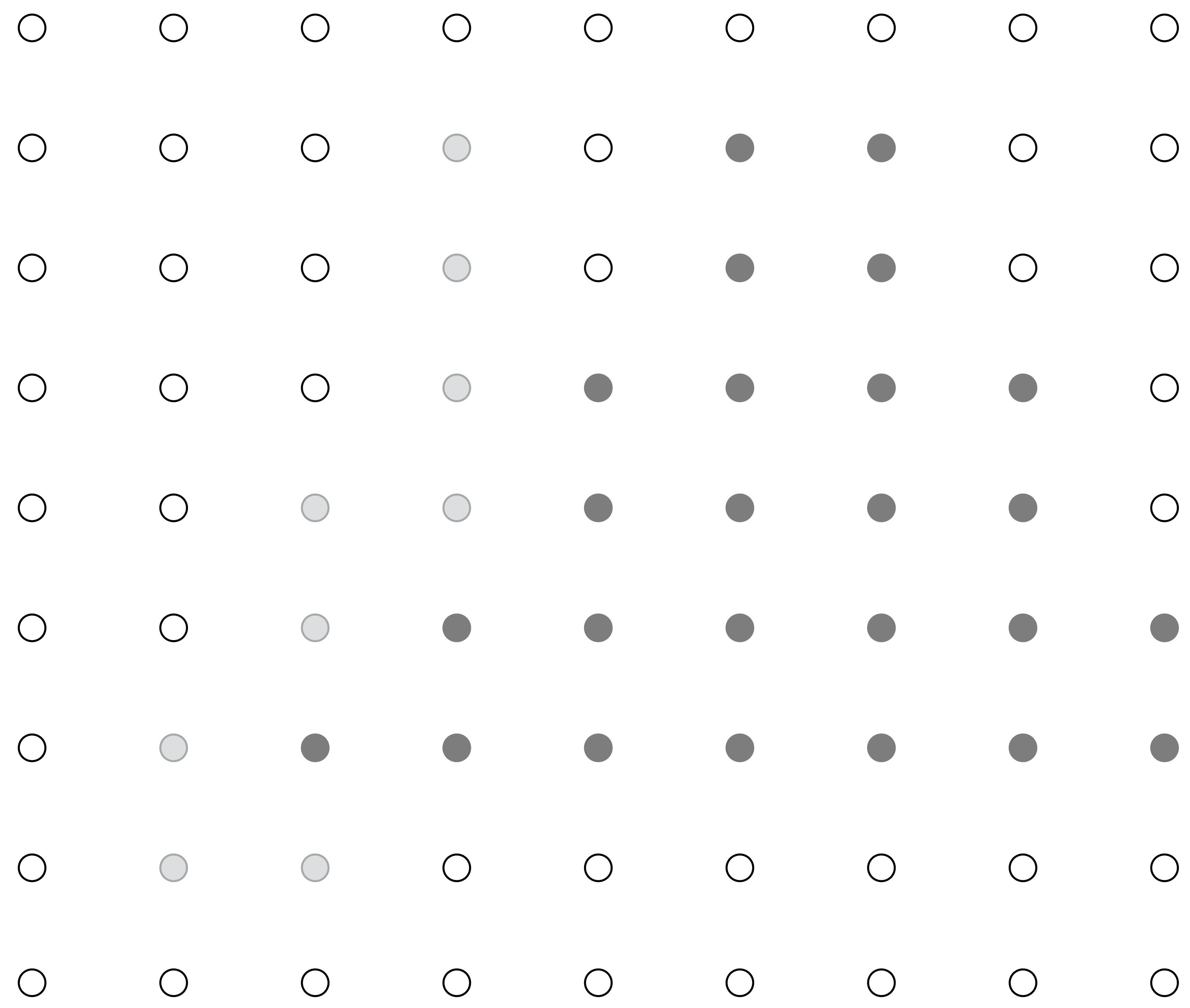
Depth buffer contents

Occlusion using the depth-buffer (Z-buffer)

After processing blue triangle:



Color buffer contents

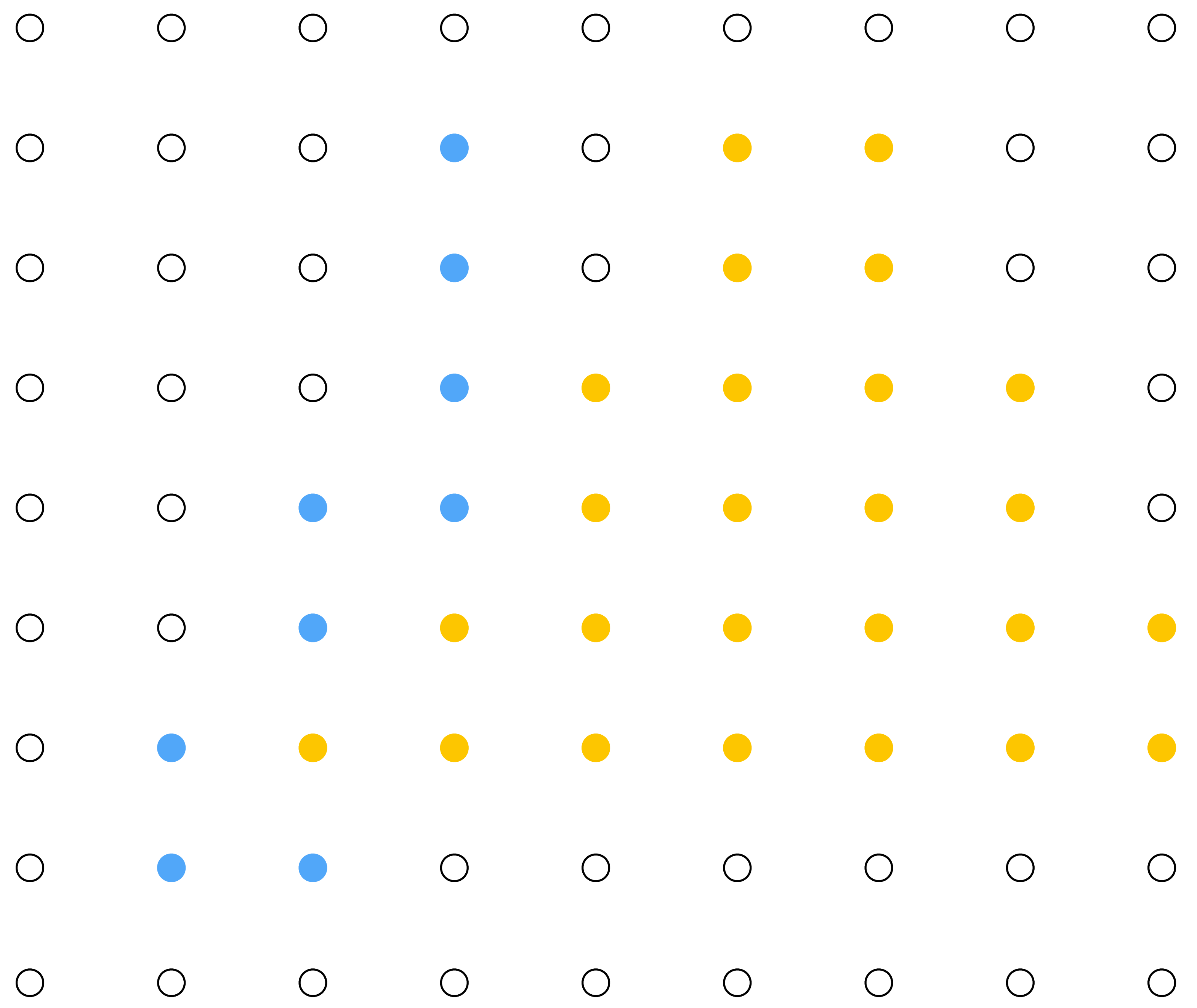


Depth buffer contents

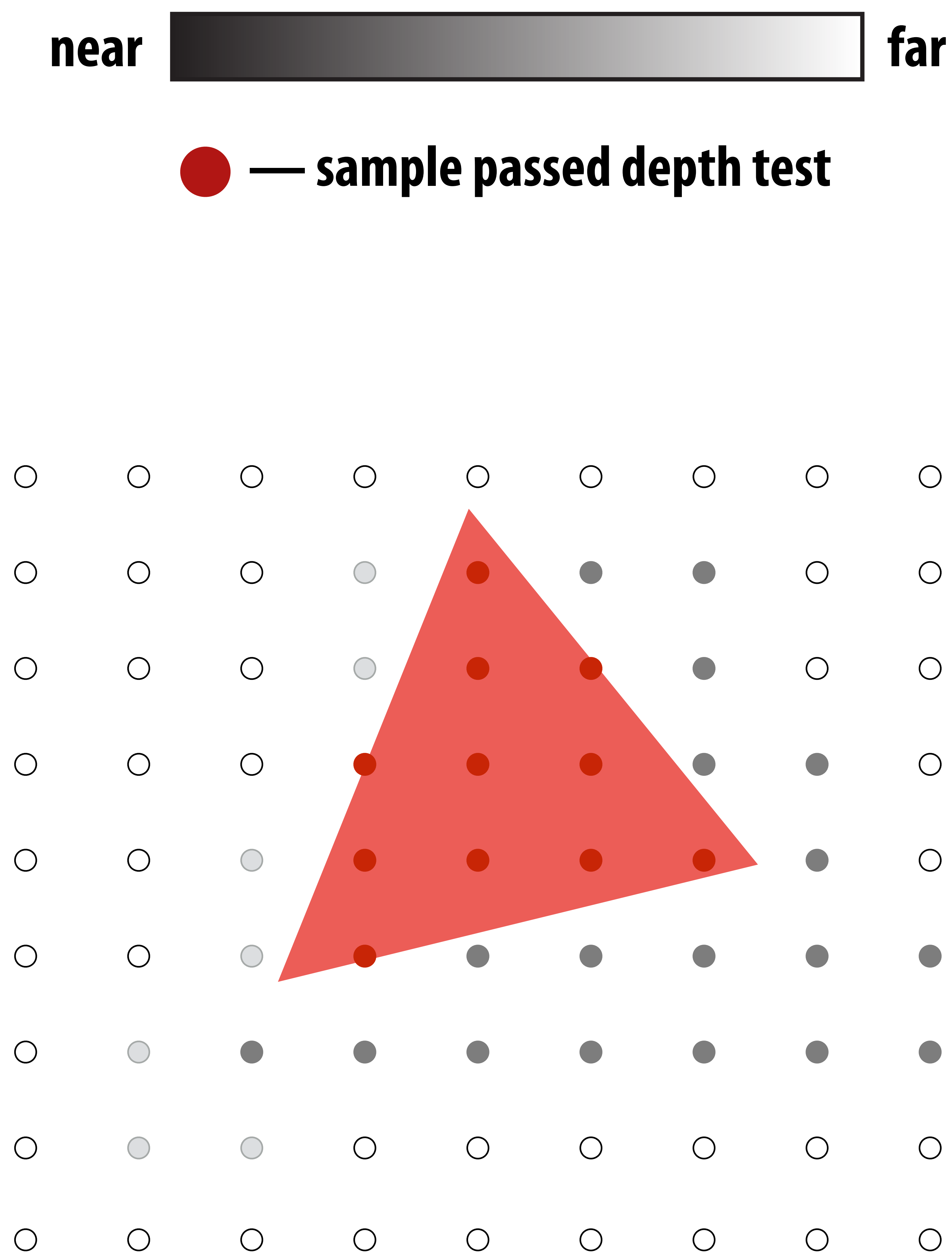
Occlusion using the depth-buffer (Z-buffer)

Processing red triangle:

depth = 0.25



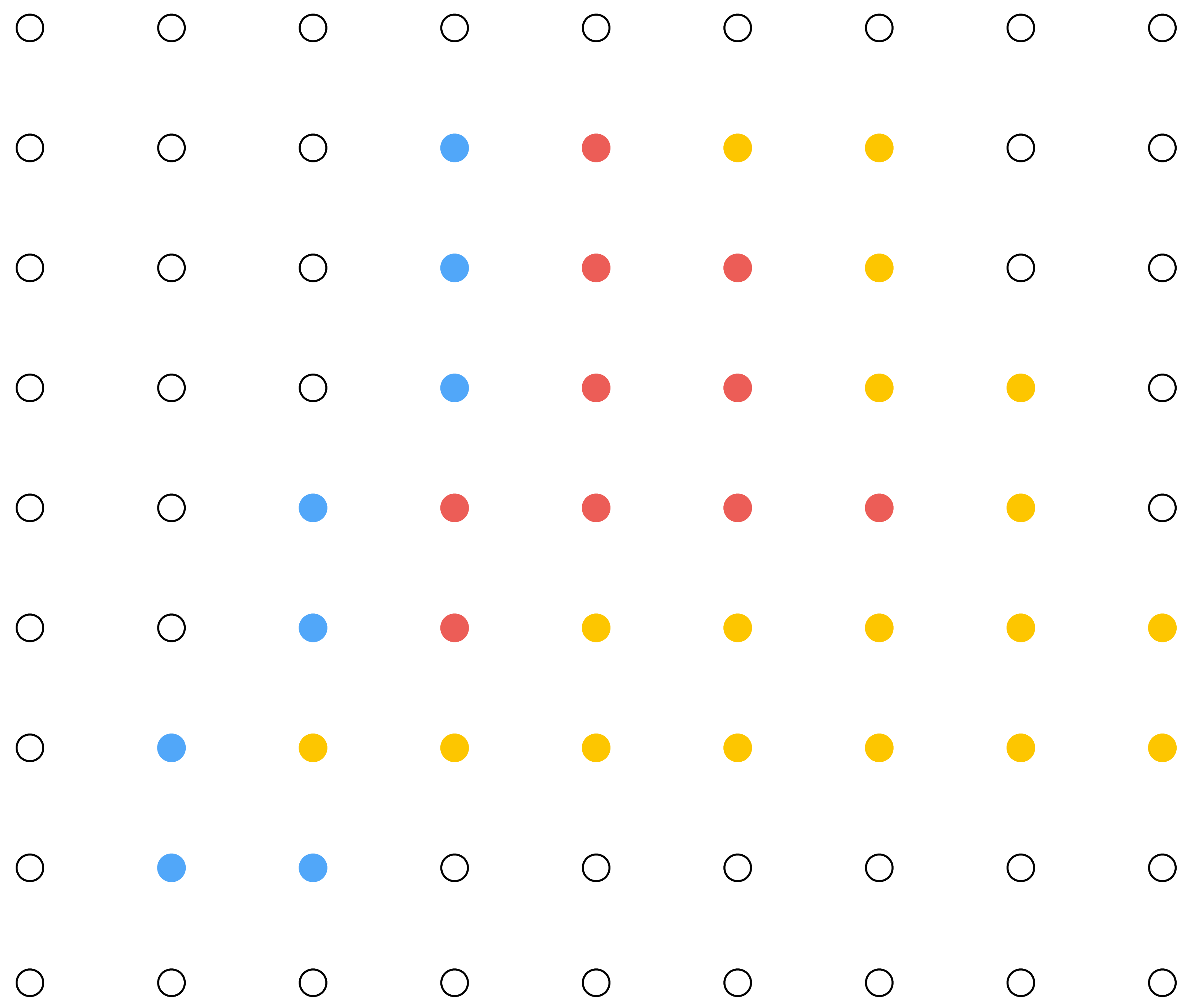
Color buffer contents



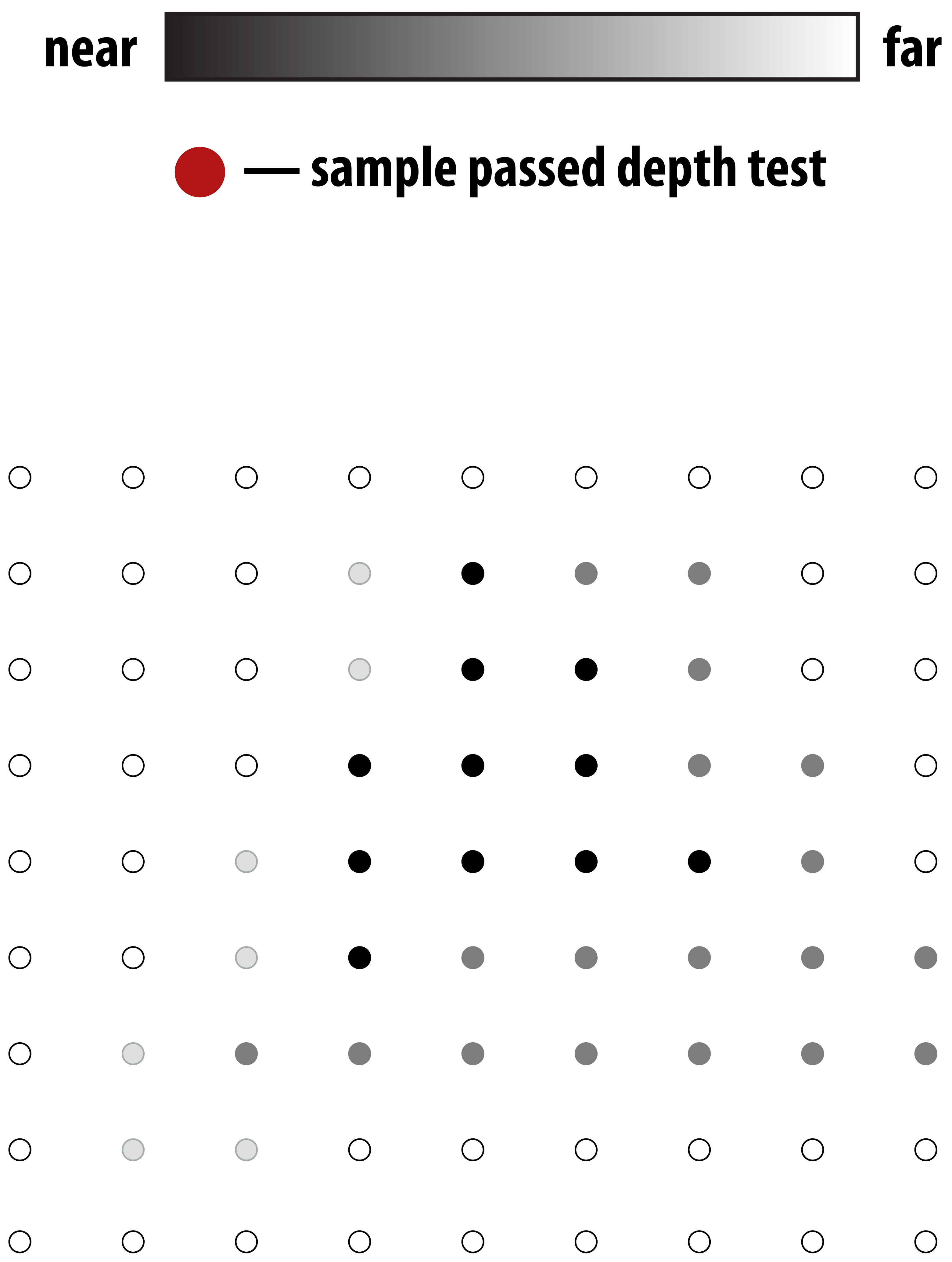
Depth buffer contents

Occlusion using the depth-buffer (Z-buffer)

After processing red triangle:



Color buffer contents



Depth buffer contents

Occlusion using the depth buffer

```
bool pass_depth_test(d1, d2)
{
   return d1 < d2;
}
```

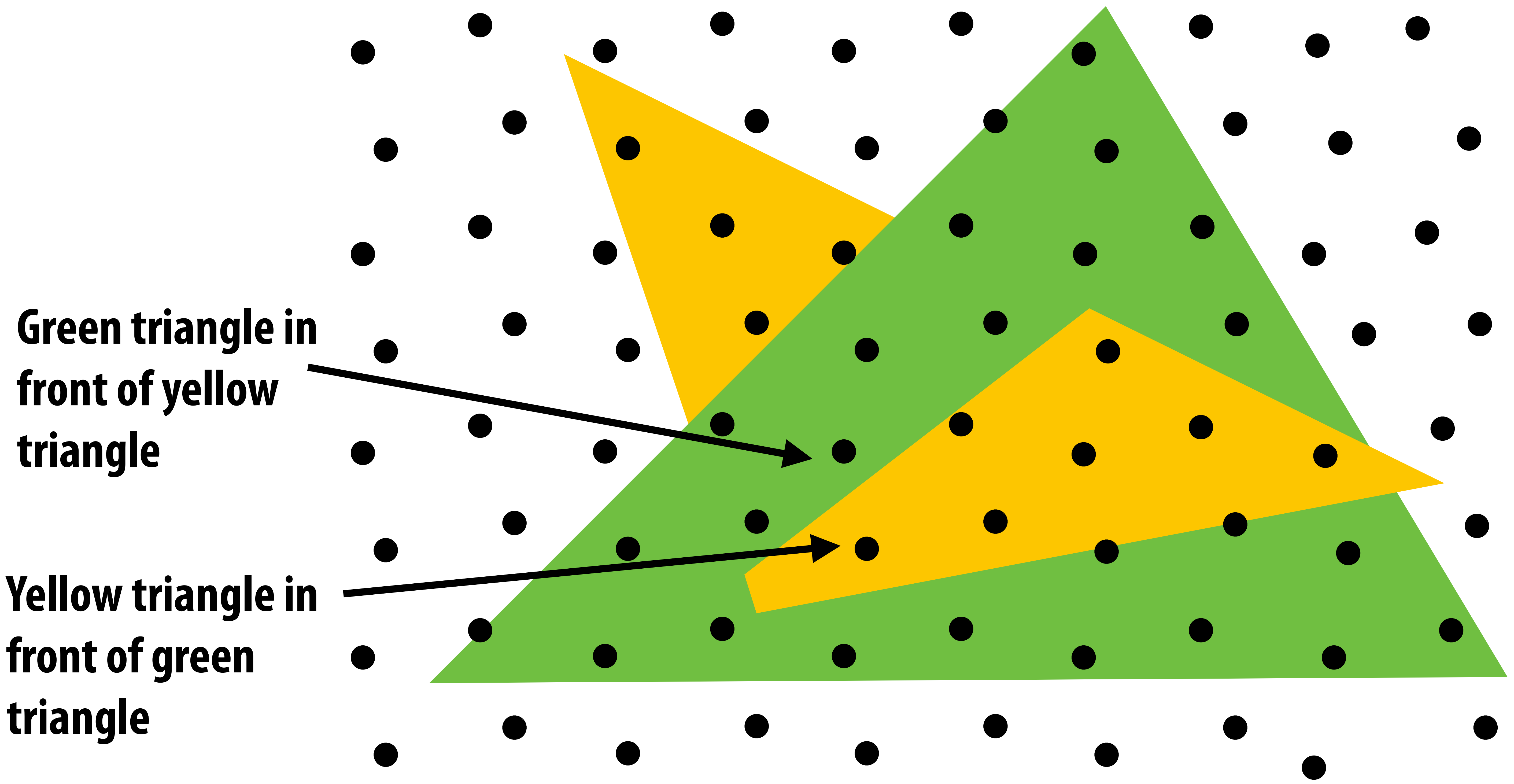
```
draw_sample(x, y, d, c) //new depth d & color c at (x,y)
{
  if( pass_depth_test( d, zbuffer[x][y] ))
  {
    // triangle is closest object seen so far at this
    // sample point. Update depth and color buffers.
    zbuffer[x][y] = d;  // update zbuffer
    color[x][y] = c;   // update color buffer
  }
  // otherwise, we've seen something closer already;
  // don't update color or depth
}
```

Depth + Intersection

Q: Does depth-buffer algorithm handle interpenetrating surfaces?

A: Of course!

Occlusion test is based on depth of triangles at a given sample point.
Relative depth of triangles may be different at different sample points.



Intersection

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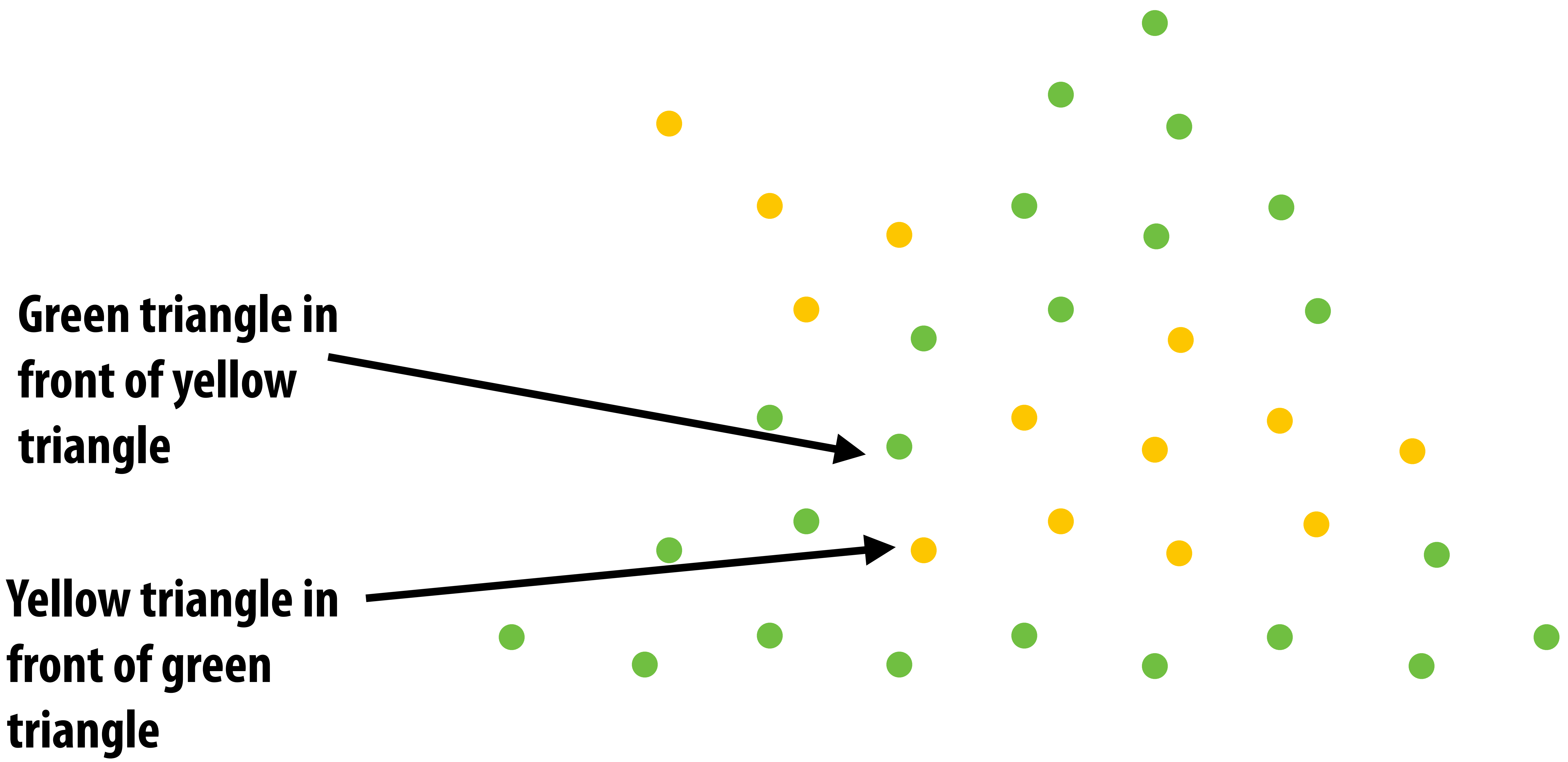


Summary: occlusion using a depth buffer

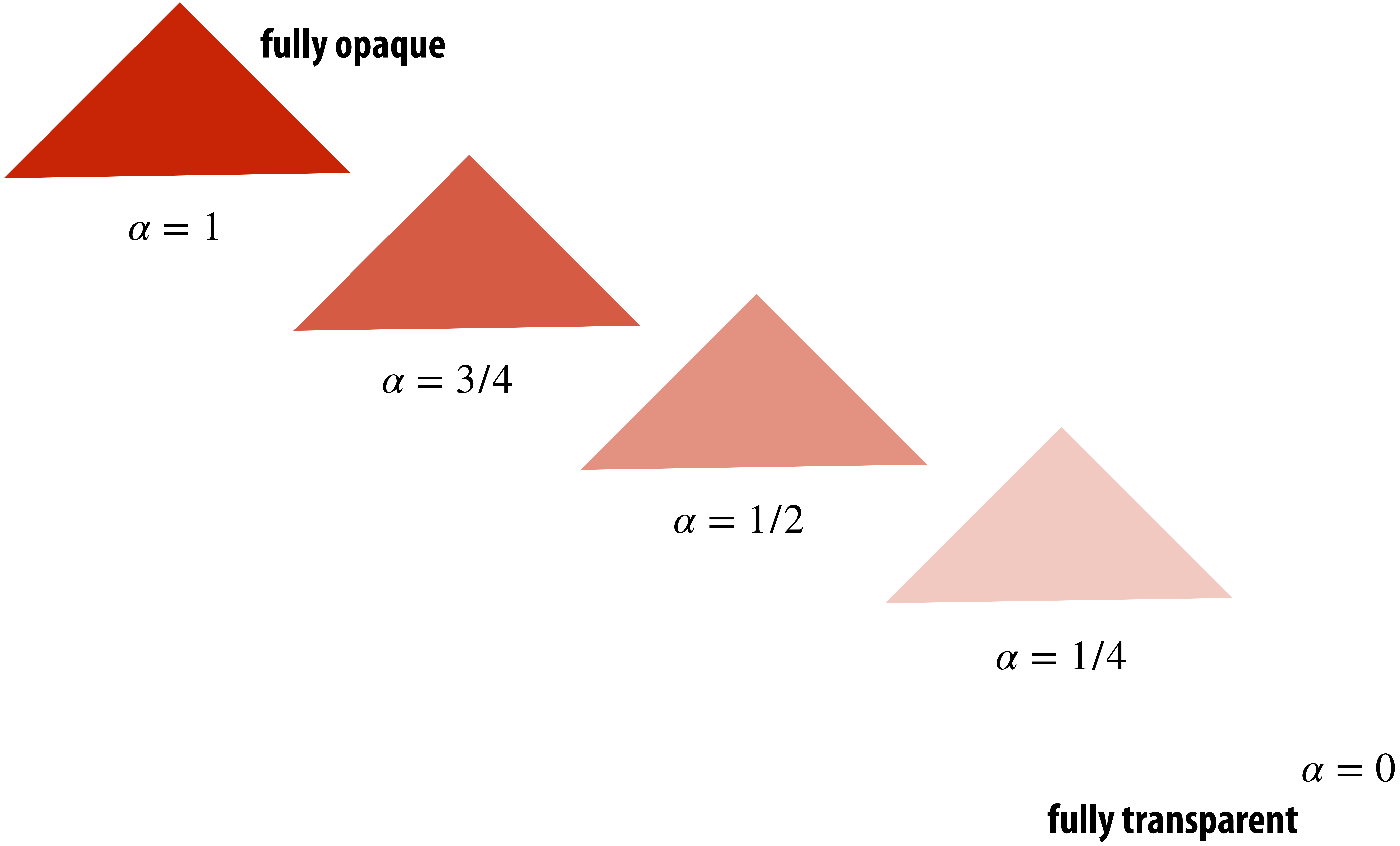
- Store one depth value per sample—this is not always going to be one per pixel!
- Constant additional space per sample
  - Hence, constant space for depth buffer
  - Doesn't depend on number of overlapping primitives!
- Constant time occlusion test per covered sample
  - Read-modify write of depth buffer if "pass" depth test
  - Just a read if "fail"
- Not specific to triangles: only requires that surface depth can be evaluated at a screen sample point

But what about semi-transparent surfaces?

Compositing

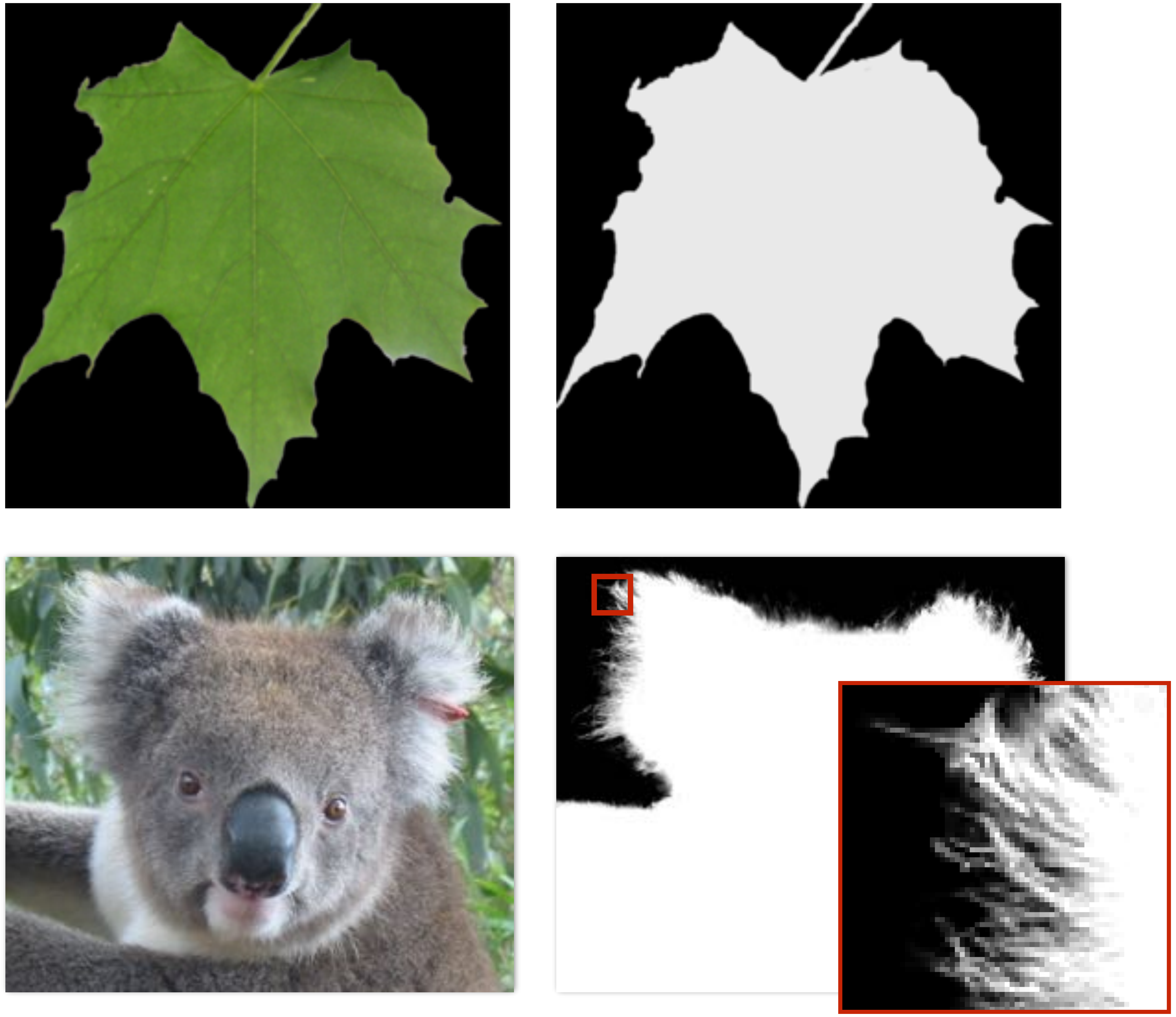
Representing opacity as alpha

An "alpha" value $0 \leq \alpha \leq 1$ describes the <u>opacity</u> of an object



Alpha channel of an image

color channels α channel

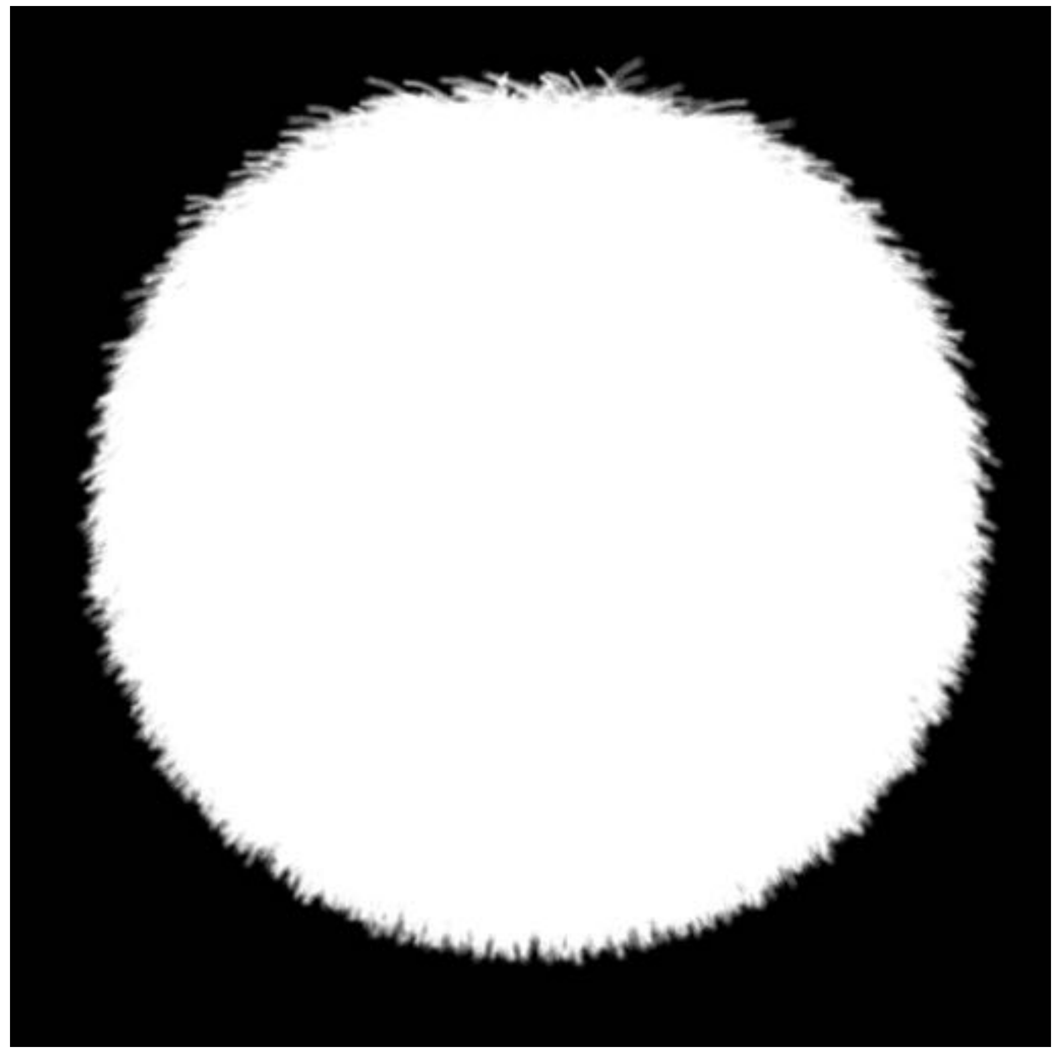


Key idea: can use α channel to composite one image on top of another.

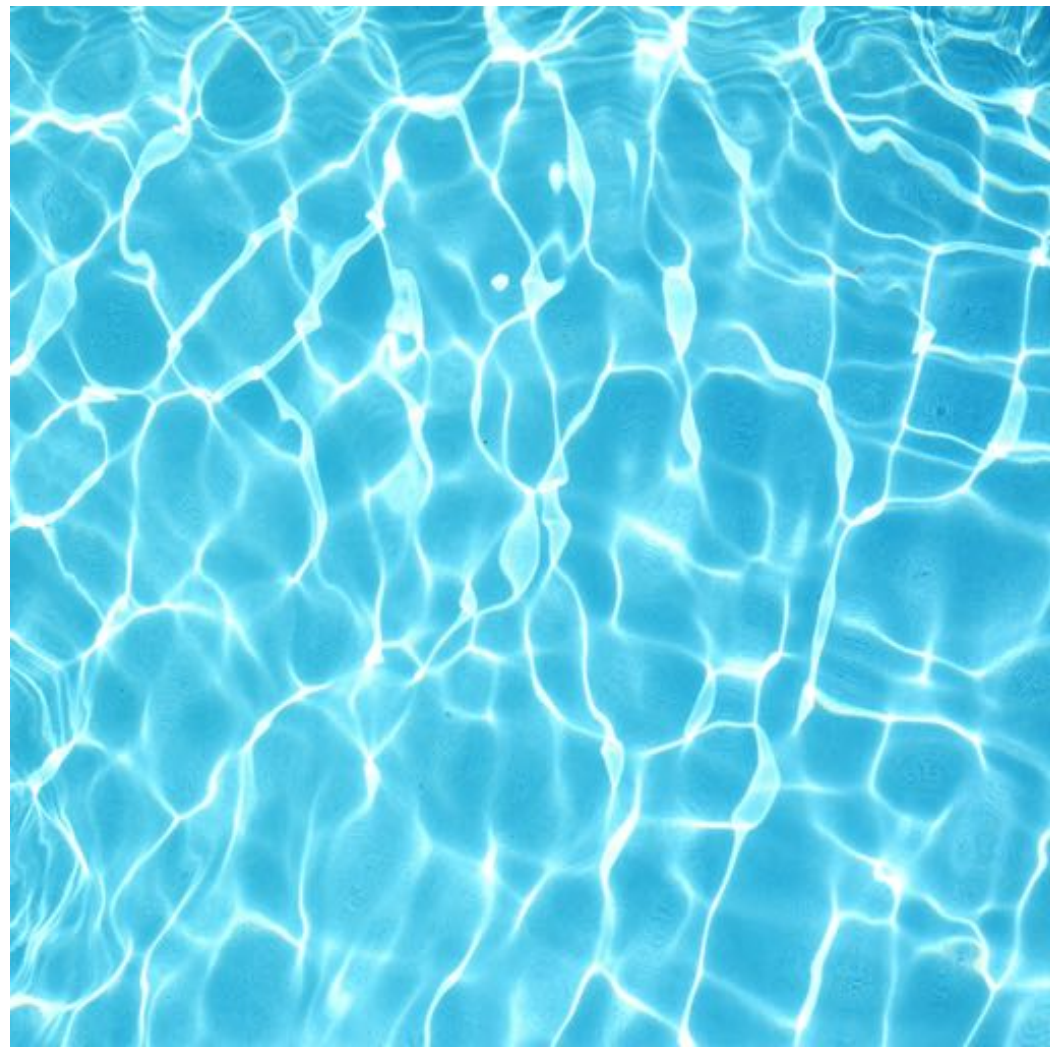
Fringing

Poor treatment of color/alpha can yield dark “fringing”:

foreground color



foreground alpha

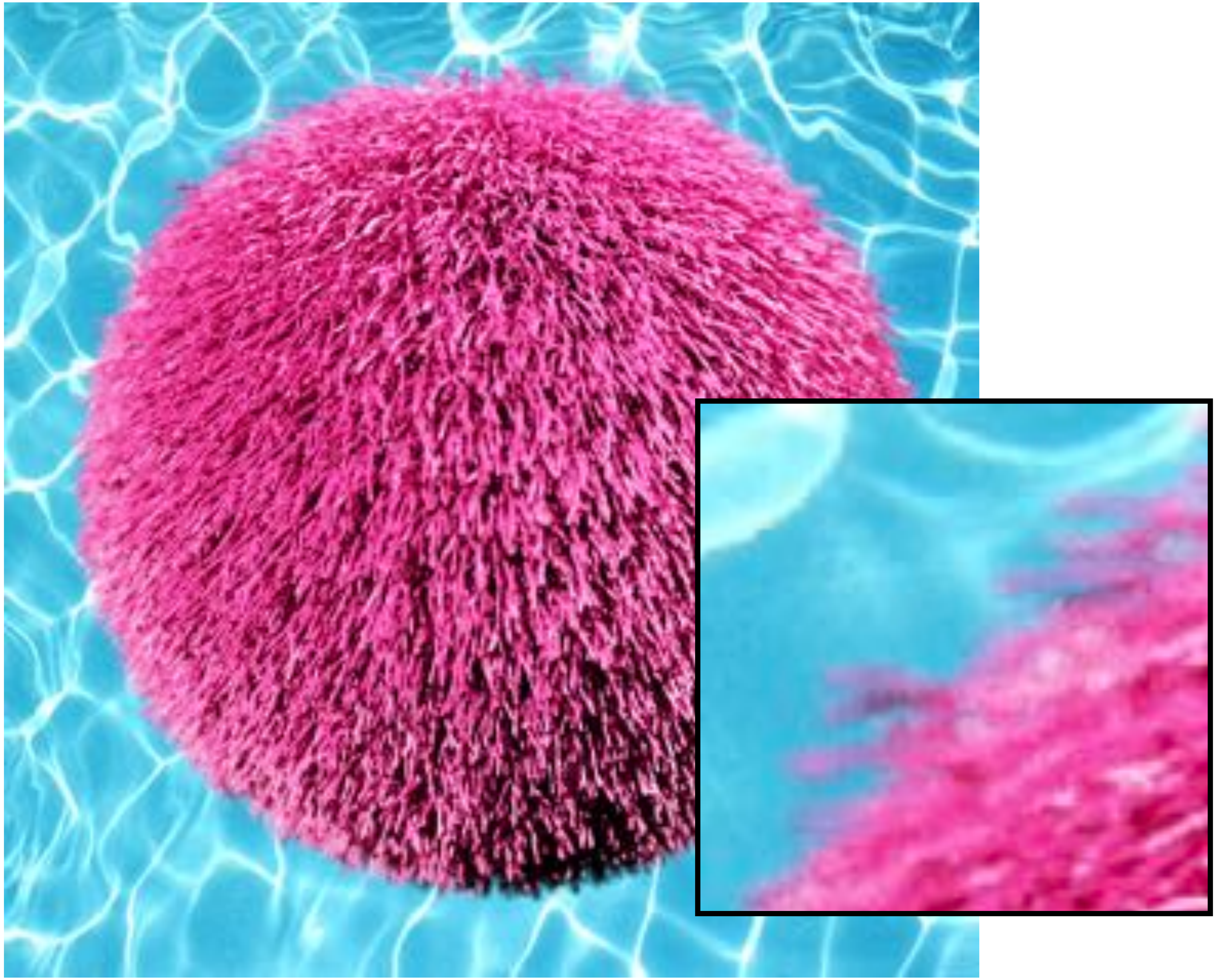


background color

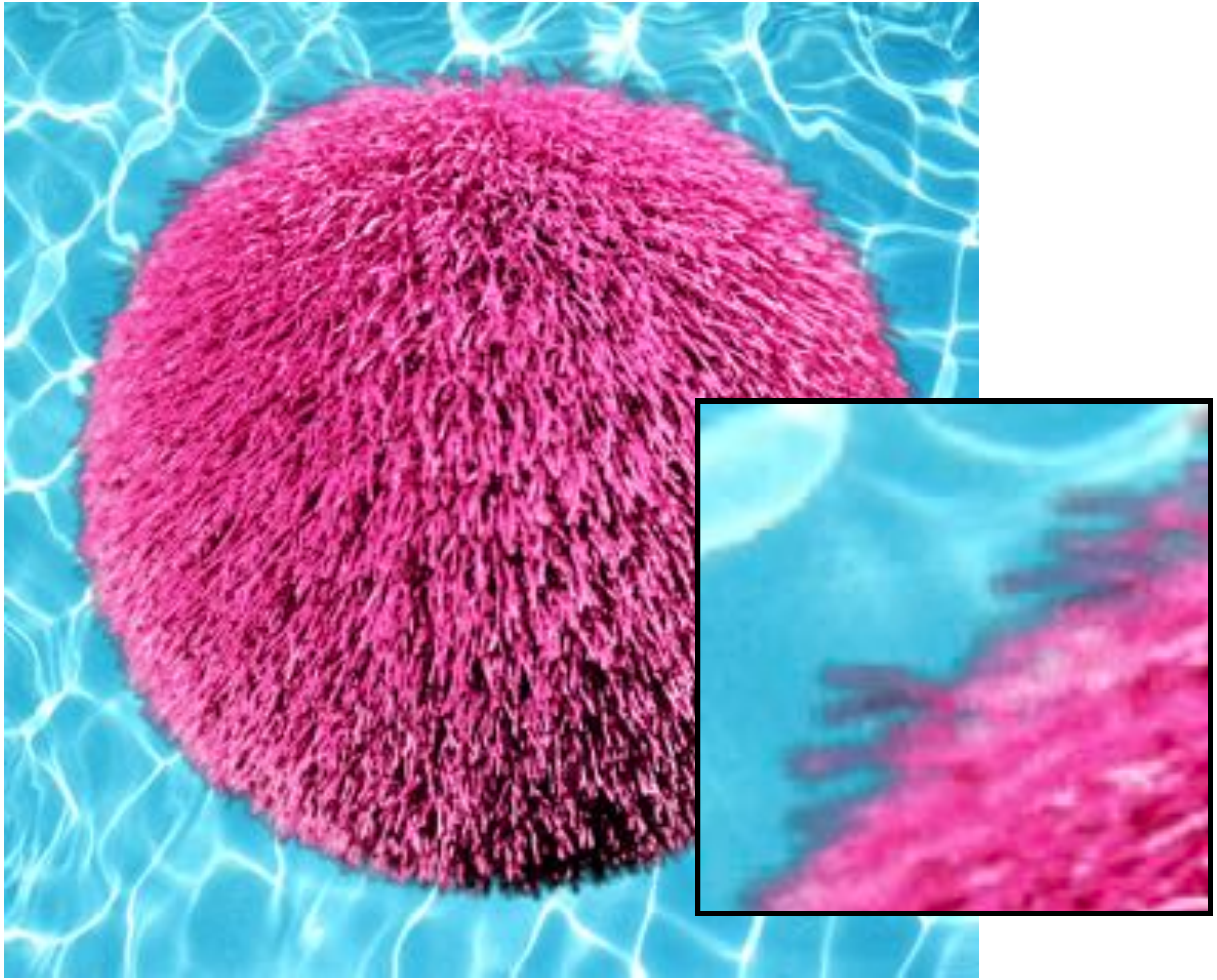
fringing

no fringing

No fringing



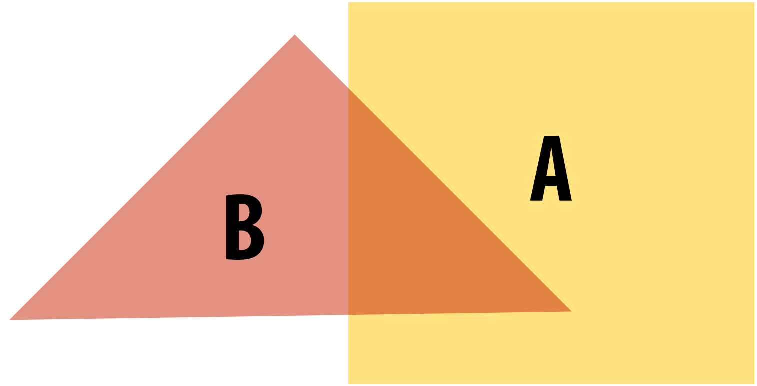
Fringing (...why does this happen?)



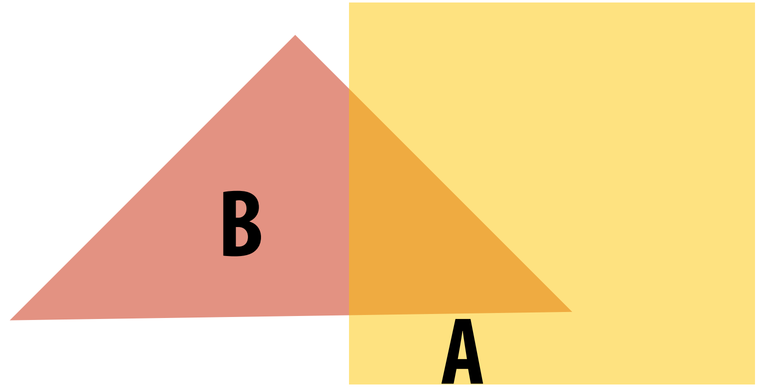
Over operator:

Composites image B with opacity α_B over image A with opacity α_A

Informally, captures behavior of "tinted glass"



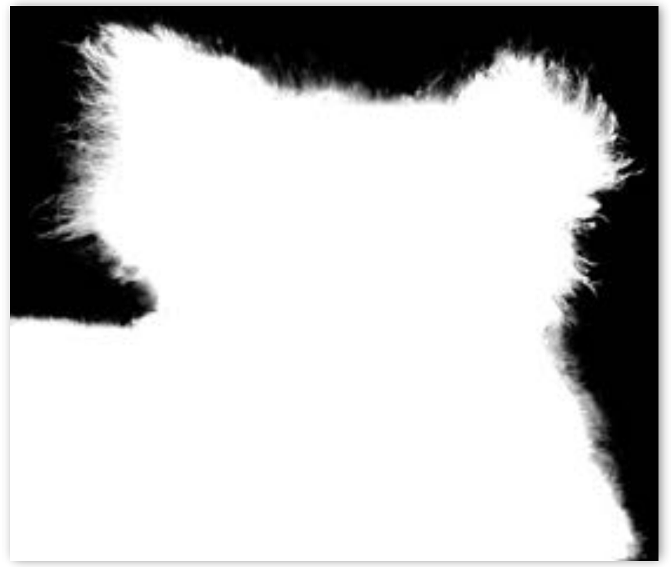
B over A



A over B

Notice: "over" is not commutative

A over $B \neq B$ over A



Koala over NYC

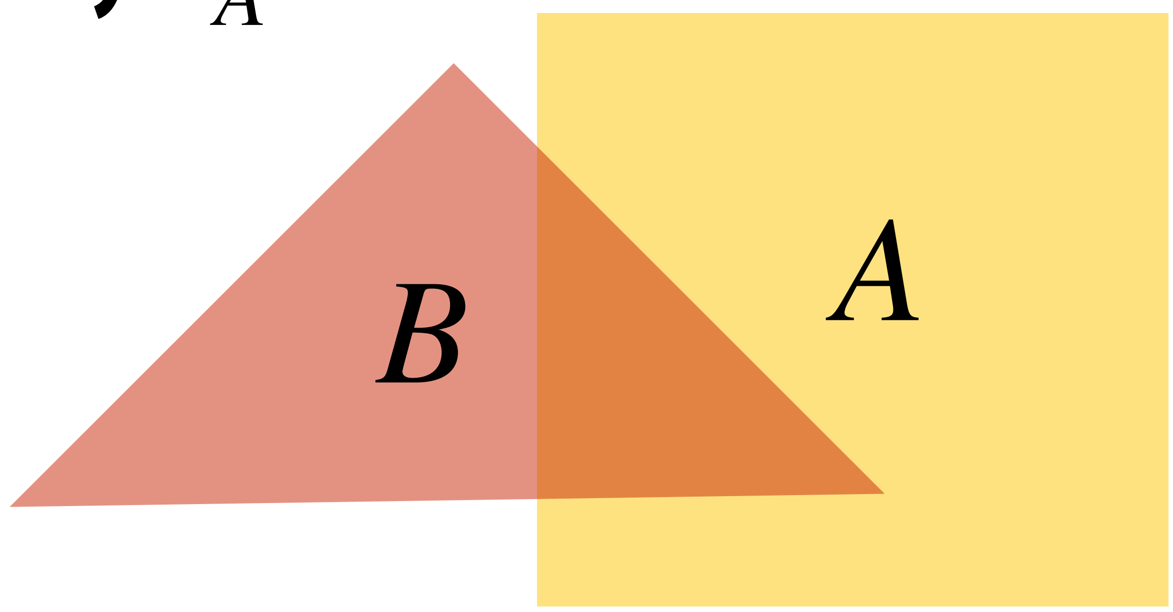
Porter & Duff "Compositing Digital Images" (1984)

Over operator: non-premultiplied alpha

Composite image B with opacity α_B over image A with opacity α_A

A first attempt:

$$A = (A_r, A_g, A_b)$$
$$B = (B_r, B_g, B_b)$$



B over A

Composite color:

what B lets through

$$C = \alpha_B B + (1 - \alpha_B)\alpha_A A$$

appearance of semi-transparent B

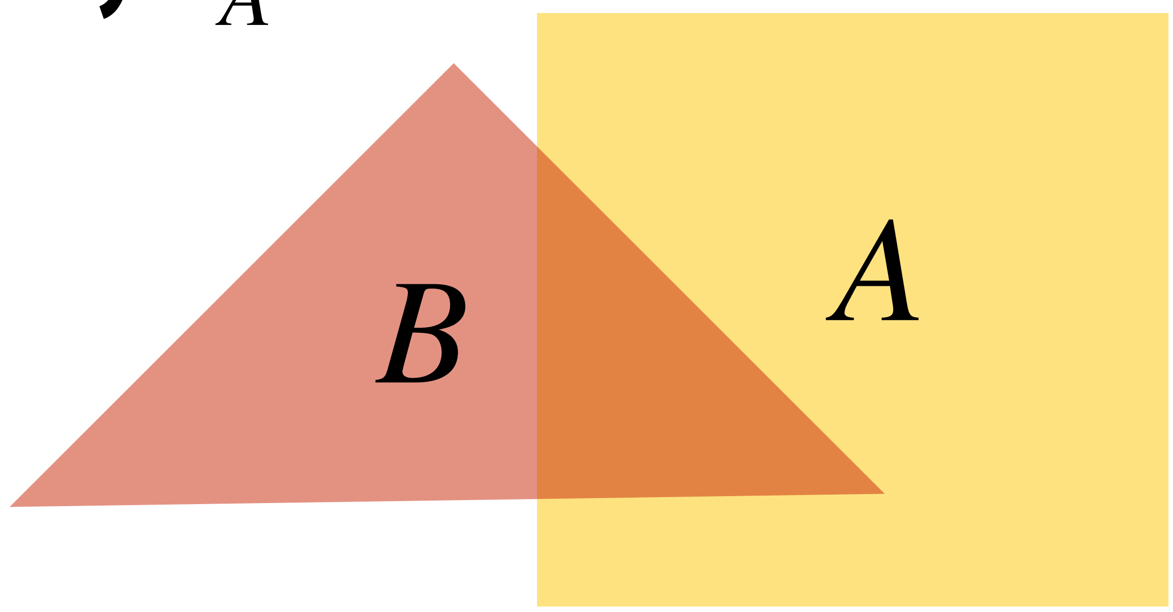
appearance of semi-transparent A

Composite alpha:

$$\alpha_C = \alpha_B + (1 - \alpha_B)\alpha_A$$

Over operator: premultiplied alpha

Composite image B with opacity α_B over image A with opacity α_A



B **over** A

Premultiplied alpha—multiply color by α, <u>then</u> composite:

$$A' = (\alpha_A A_r,\ \alpha_A A_g,\ \alpha_A A_b, \alpha_A)$$

$$B' = (\alpha_B B_r,\ \alpha_B B_g,\ \alpha_B B_b, \alpha_B)$$

$$C' = B' + (1 - \alpha_B)A'$$

Notice premultiplied alpha composites alpha just like how it composites rgb.
(Non-premultiplied alpha composites alpha differently than rgb.)

"Un-premultiply" to get final color:

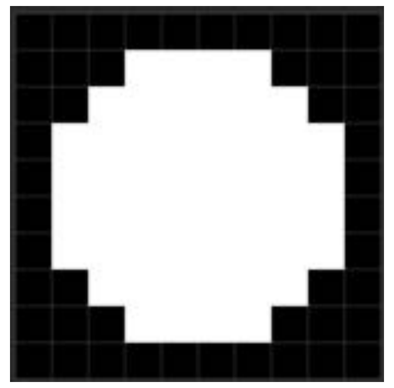
$$(C_r,\ C_g,\ C_b,\ \alpha_C) \Longrightarrow (C_r/\alpha_C, C_g/\alpha_C, C_b/\alpha_C)$$

Q: Does this division remind you of anything?

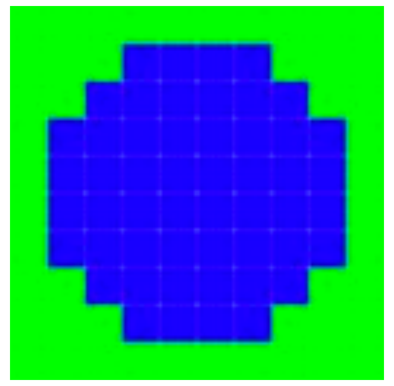
Compositing with & without premultiplied α

Suppose we upsample an image w/ an α channel, then composite it onto a background:

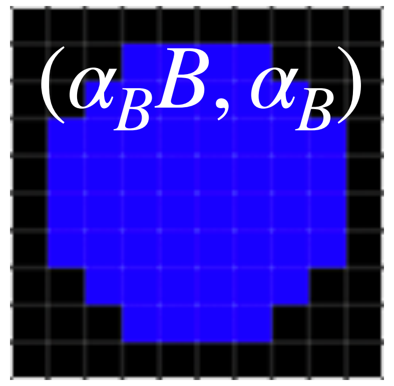
image B



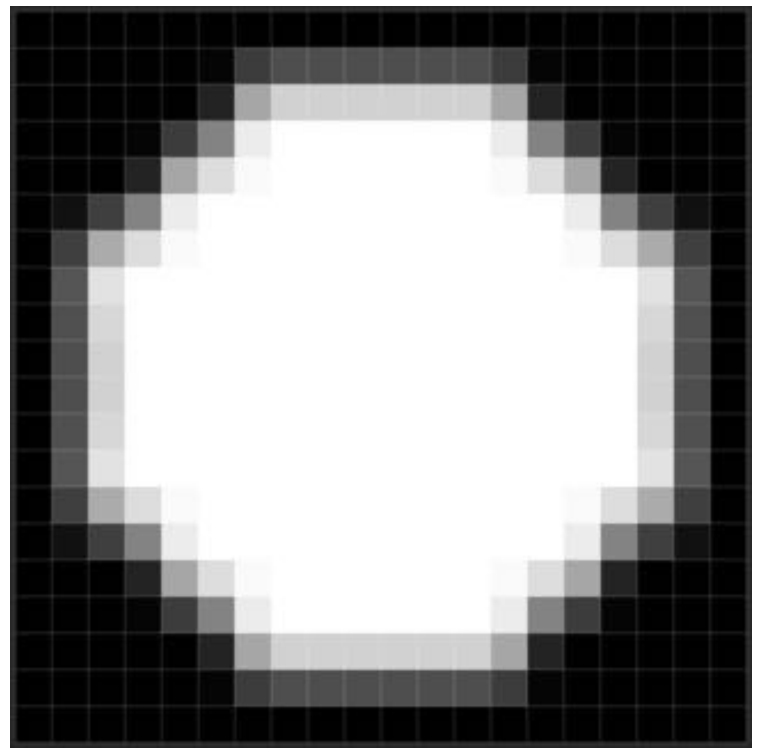
original alpha



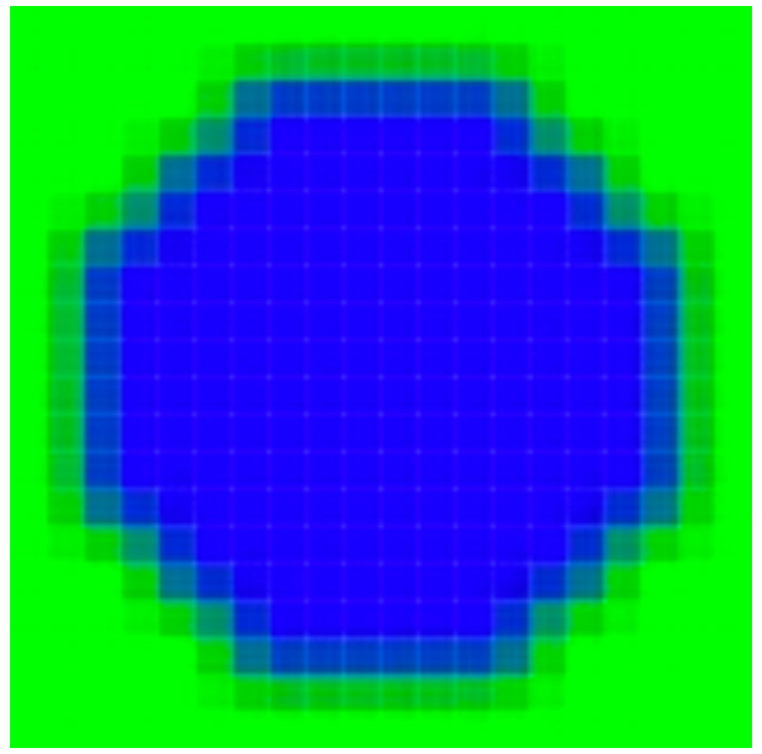
original color



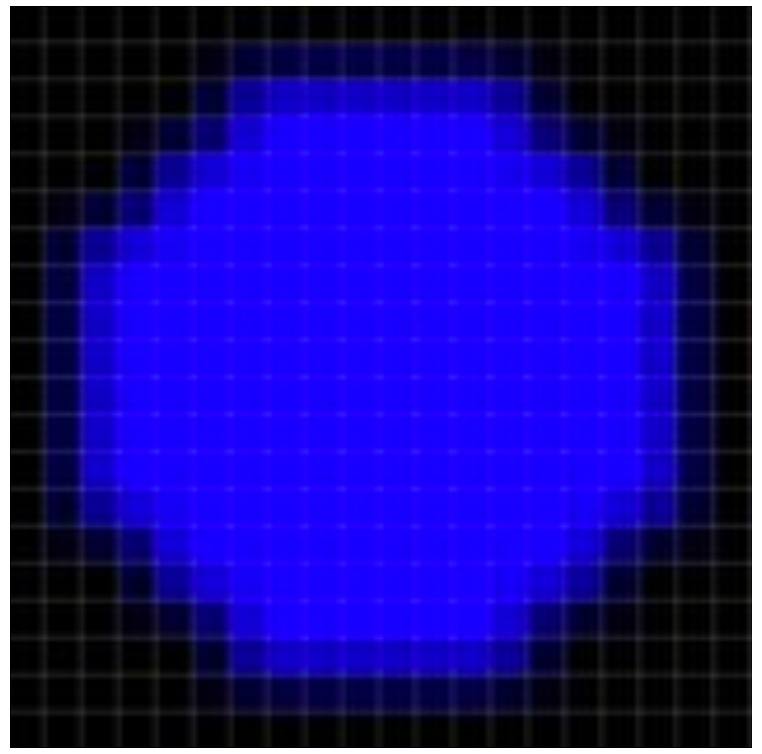
premultiplied color



upsampled alpha

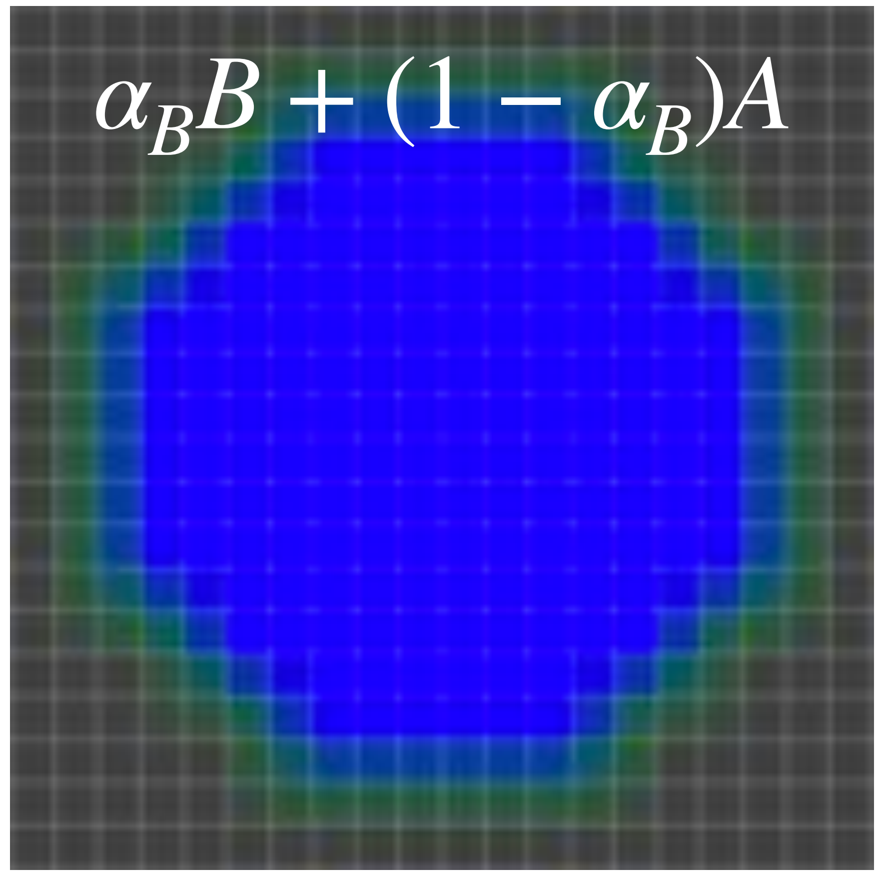


upsampled color

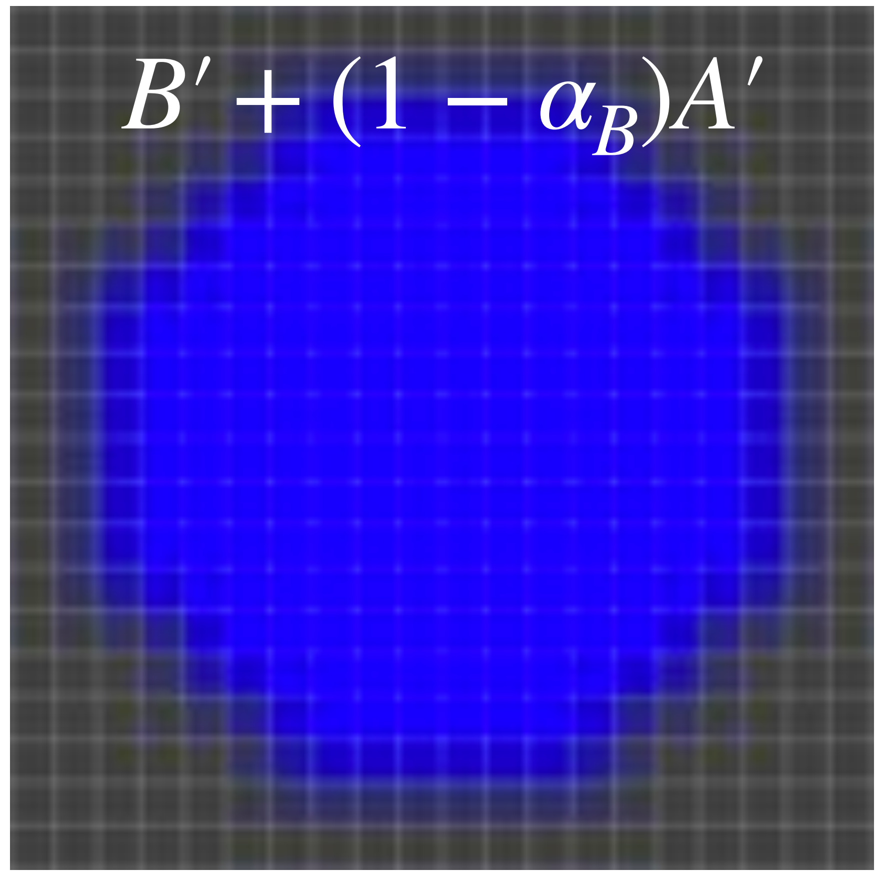


upsampled premultiplied color

new background A
$(\alpha_A = 1)$



B over A
non-premultiplied



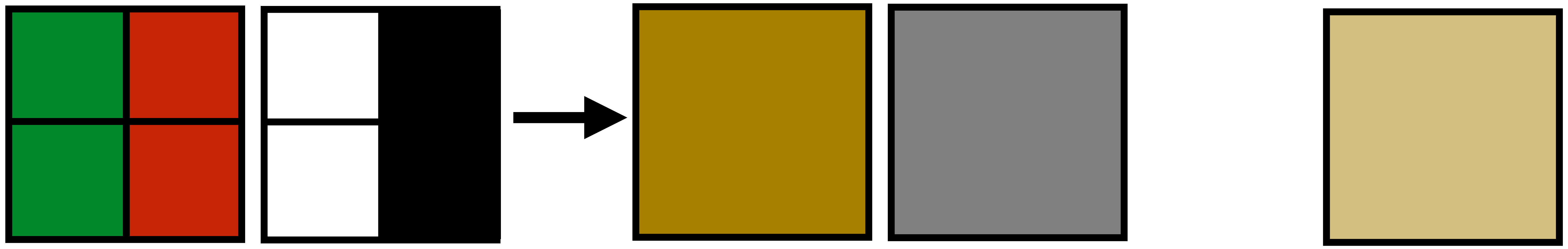
B over A
premultiplied

Q: Why do we get the "green fringe" when we don't premultiply?

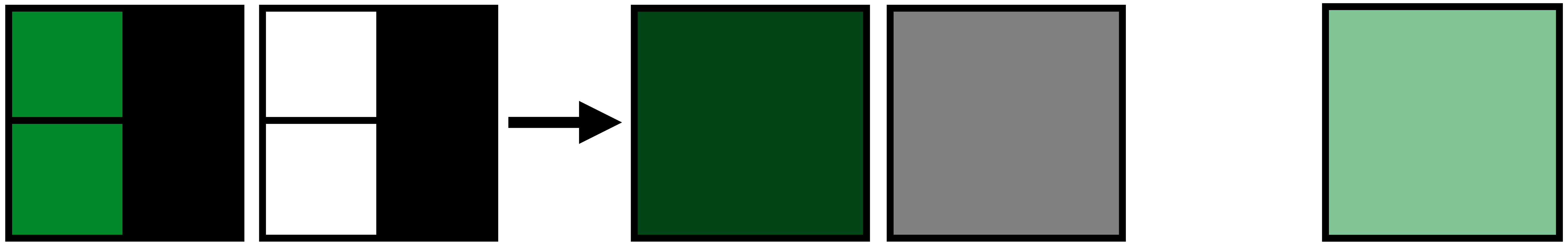
Similar problem with non-premultiplied α

Consider pre-filtering (downsampling) a texture with an alpha matte

desired downsampled result



input color | input α | filtered color | filtered α | composited over white

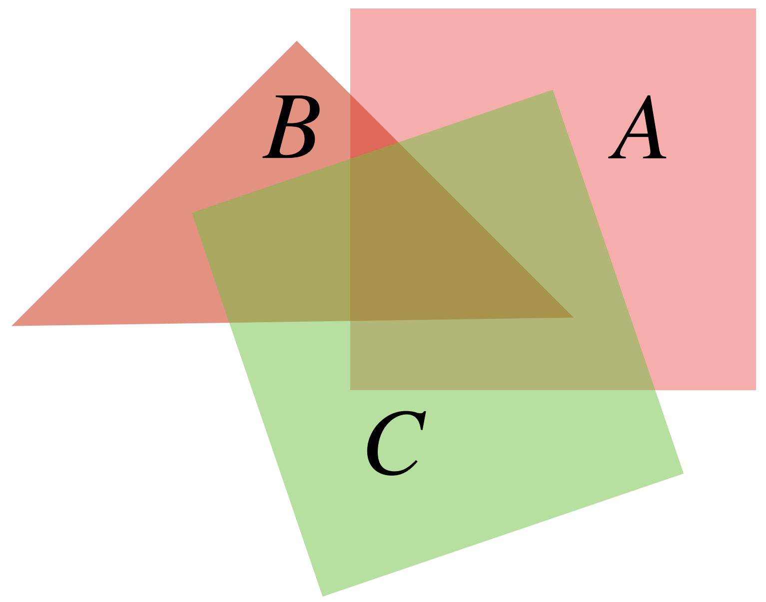


premultiplied color | premultiplied α | filtered color | filtered α | composited over white

More problems: applying "over" repeatedly

Composite image C with opacity α_C over B with opacity α_B over image A with opacity α_A

Premultiplied alpha is closed under composition; non-premultiplied alpha is not!



C over B over A

Example: composite 50% bright red over 50% bright red (where "bright red" = $(1,0,0)$, and $\alpha = 0.5$)

non-premultiplied

color

$$.5(1,0,0) + (1\text{-}.5).5(1,0,0)$$
$$\Downarrow$$
$$(0.75,0,0)$$

too dark!

alpha

$$.5 + (1\text{-}.5).5 = .75$$

premultiplied

color

$$(.5,0,0,.5)+(1\text{-}.5)(.5,0,0,.5)$$
$$\Downarrow$$
$$(.75,0,0.75)$$
$$\Downarrow \text{ divide by } \alpha$$

bright red $(1,0,0)$

alpha

$$\alpha = 0.75$$

Summary: advantages of premultiplied alpha

- Compositing operation treats all channels the same (color and α)
- Fewer arithmetic operations for "over" operation than with non-premultiplied representation
- Closed under composition (repeated "over" operations)
- Better representation for filtering (upsampling/downsampling) images with alpha channel
- Fits naturally into rasterization pipeline (homogeneous coordinates)

Strategy for drawing semi-transparent primitives

Assuming all primitives are semi-transparent, and color values are encoded with premultiplied alpha, here's a strategy for rasterizing an image:

```
over(c1, c2)
{
   return c1.rgba + (1-c1.a) * c2.rgba;
}
```

```
update_color_buffer( x, y, sample_color, sample_depth )
{
   if (pass_depth_test(sample_depth, zbuffer[x][y])
   {
       // (how) should we update depth buffer here??
       color[x][y] = over(sample_color, color[x][y]);
  }
}
```

Q: What is the assumption made by this implementation?

Triangles must be rendered in back to front order!

Putting it all together

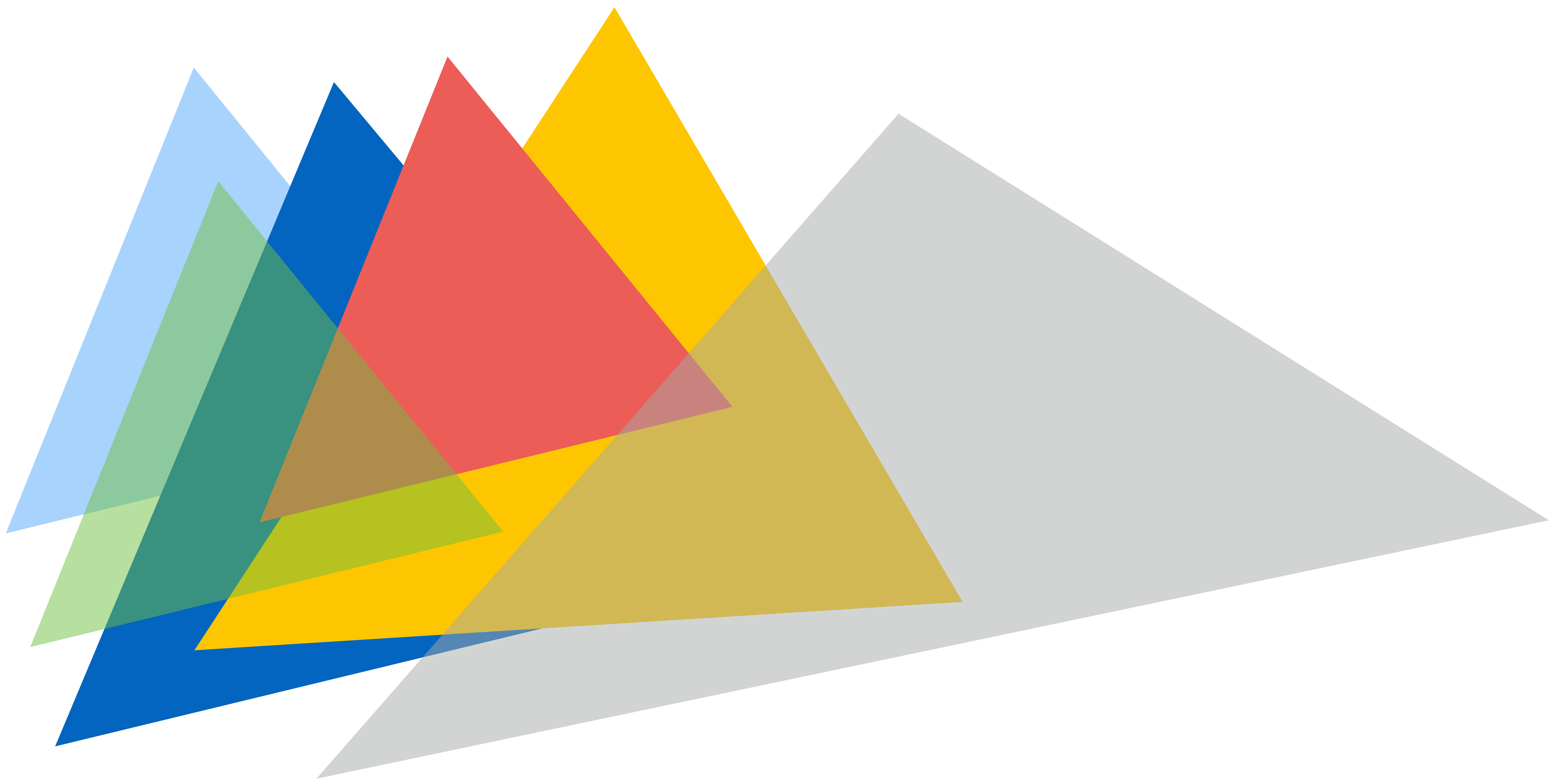
What if we have a mixture of opaque and transparent triangles?

<u>Step 1:</u> render opaque primitives (in any order) using depth-buffered occlusion

If pass depth test, triangle overwrites value in color buffer at sample

<u>Step 2:</u> disable depth buffer update, render semi-transparent surfaces in back-to-front order.

If pass depth test, triangle is composited OVER contents of color buffer at sample



End-to-end rasterization pipeline

Goal: turn inputs into an image!

Inputs:

```
positions = {
    v0x, v0y, v0z,
    v1x, v1y, v1x,
    v2x, v2y, v2z,
    v3x, v3y, v3x,
    v4x, v4y, v4z,
    v5x, v5y, v5x
};
```

Object-to-camera-space transform $T \in \mathbb{R}^{4\times4}$

Perspective projection transform $P \in \mathbb{R}^{4\times4}$

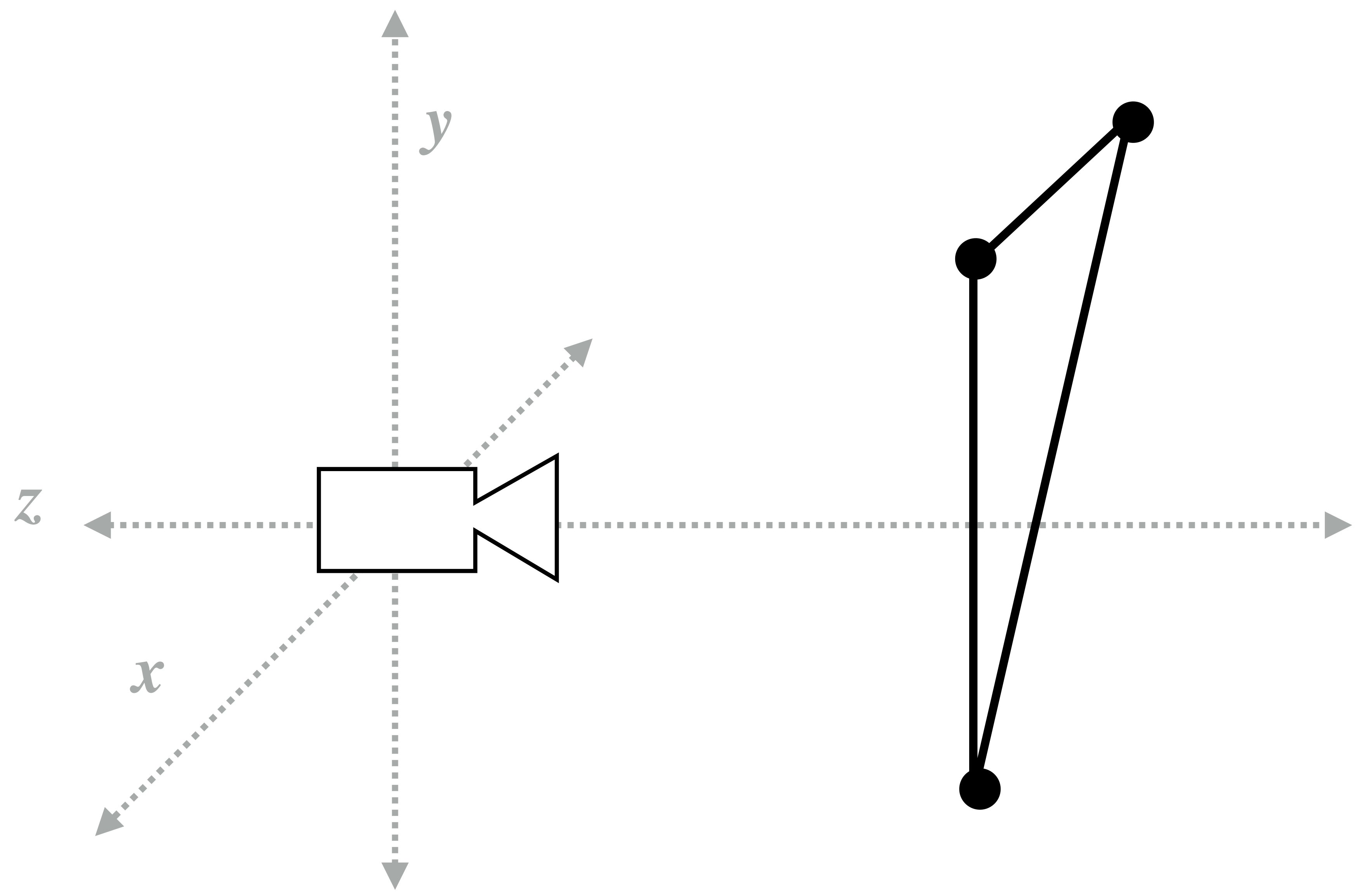
Size of output image (W, H)

At this point we have almost all the tools we need to make an image…

Let's review!

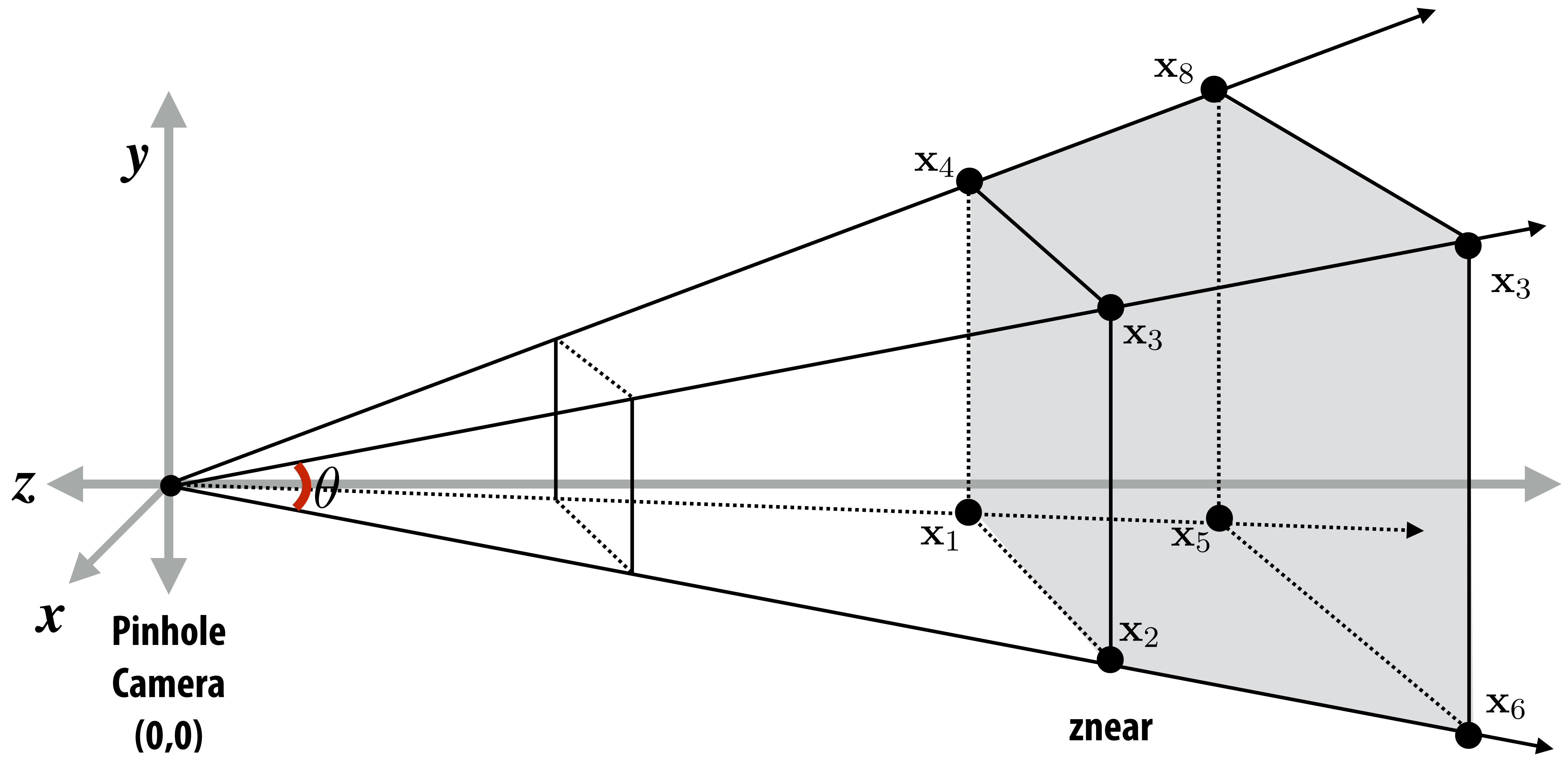
Step 1:

Transform triangle vertices into camera space

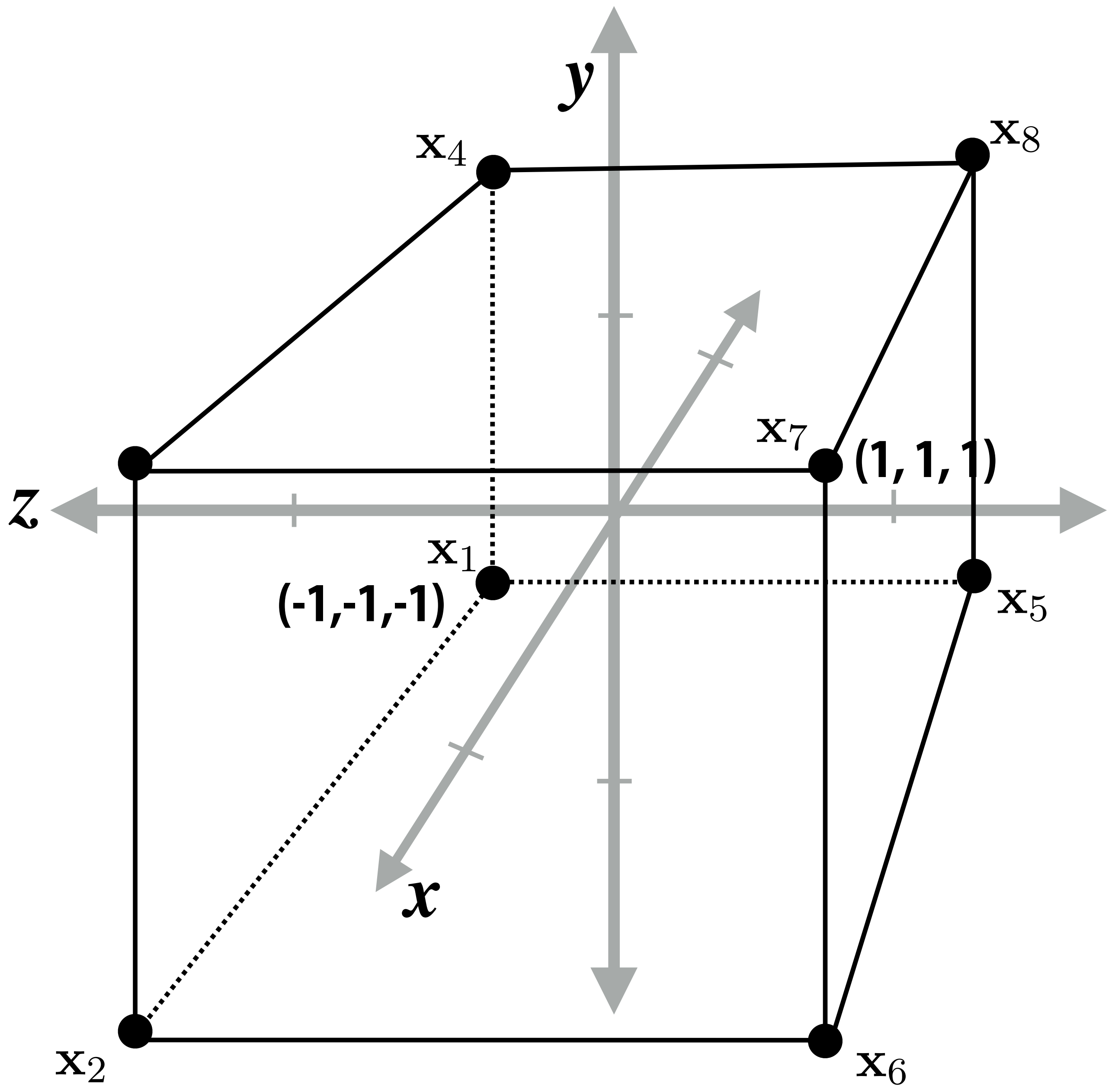


Step 2:

Apply perspective projection transform to transform triangle vertices into normalized coordinate space



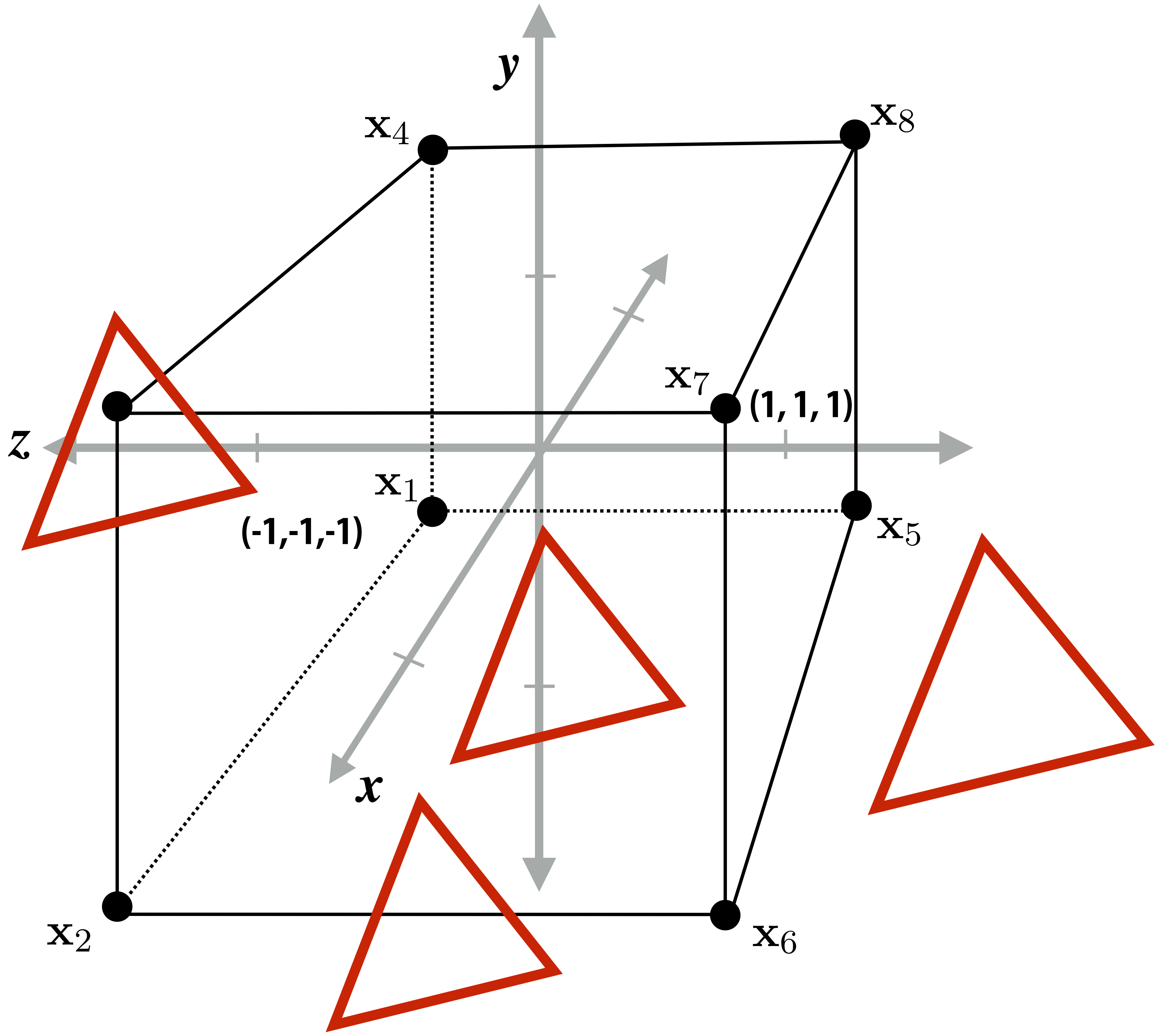
Camera-space positions: 3D



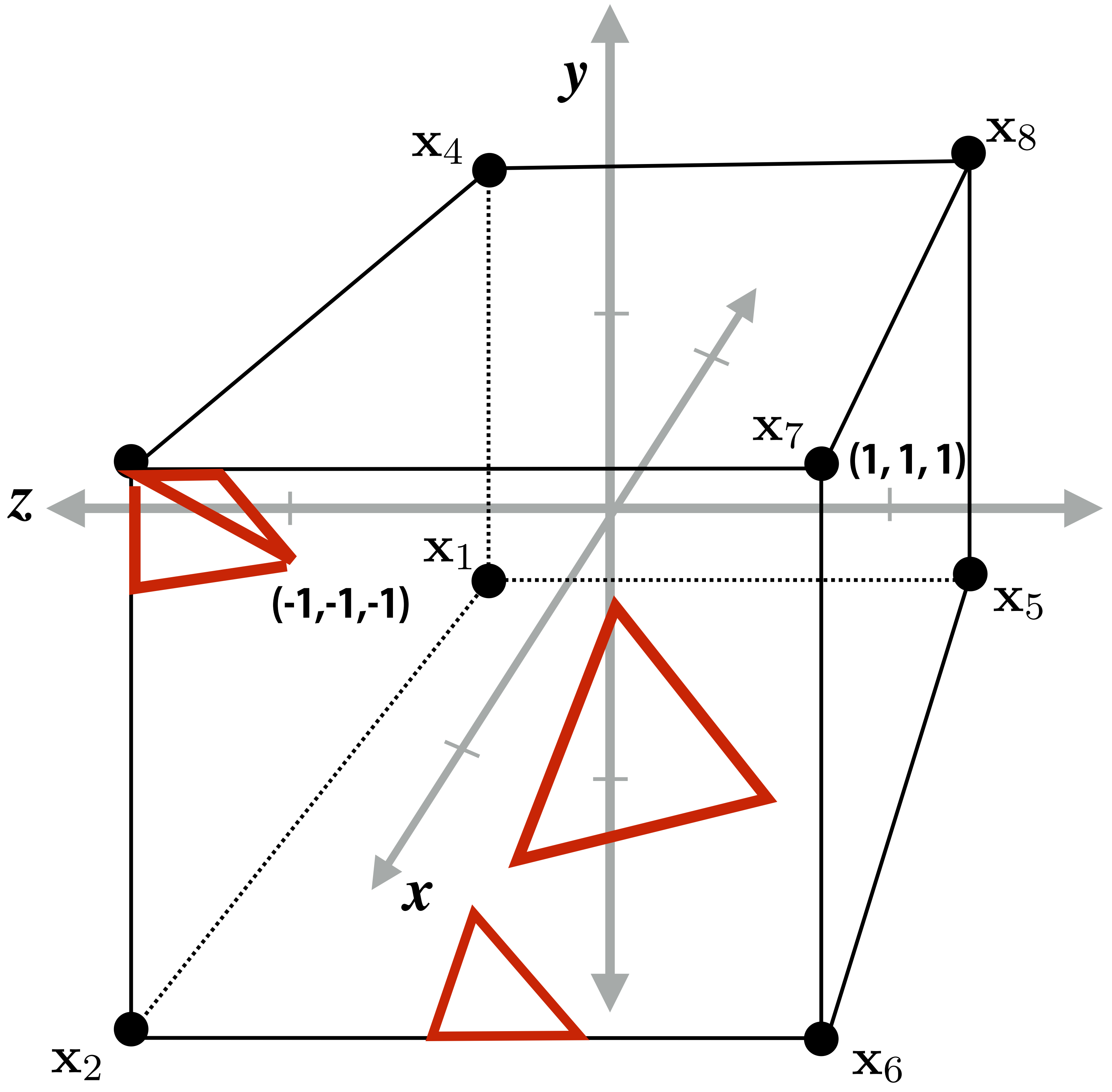
Normalized space positions

Step 3: clipping

- Discard triangles that lie complete outside the unit cube (culling)
  - They are off screen, don't bother processing them further
- Clip triangles that extend beyond the unit cube to the cube
  - (possibly generating new triangles)



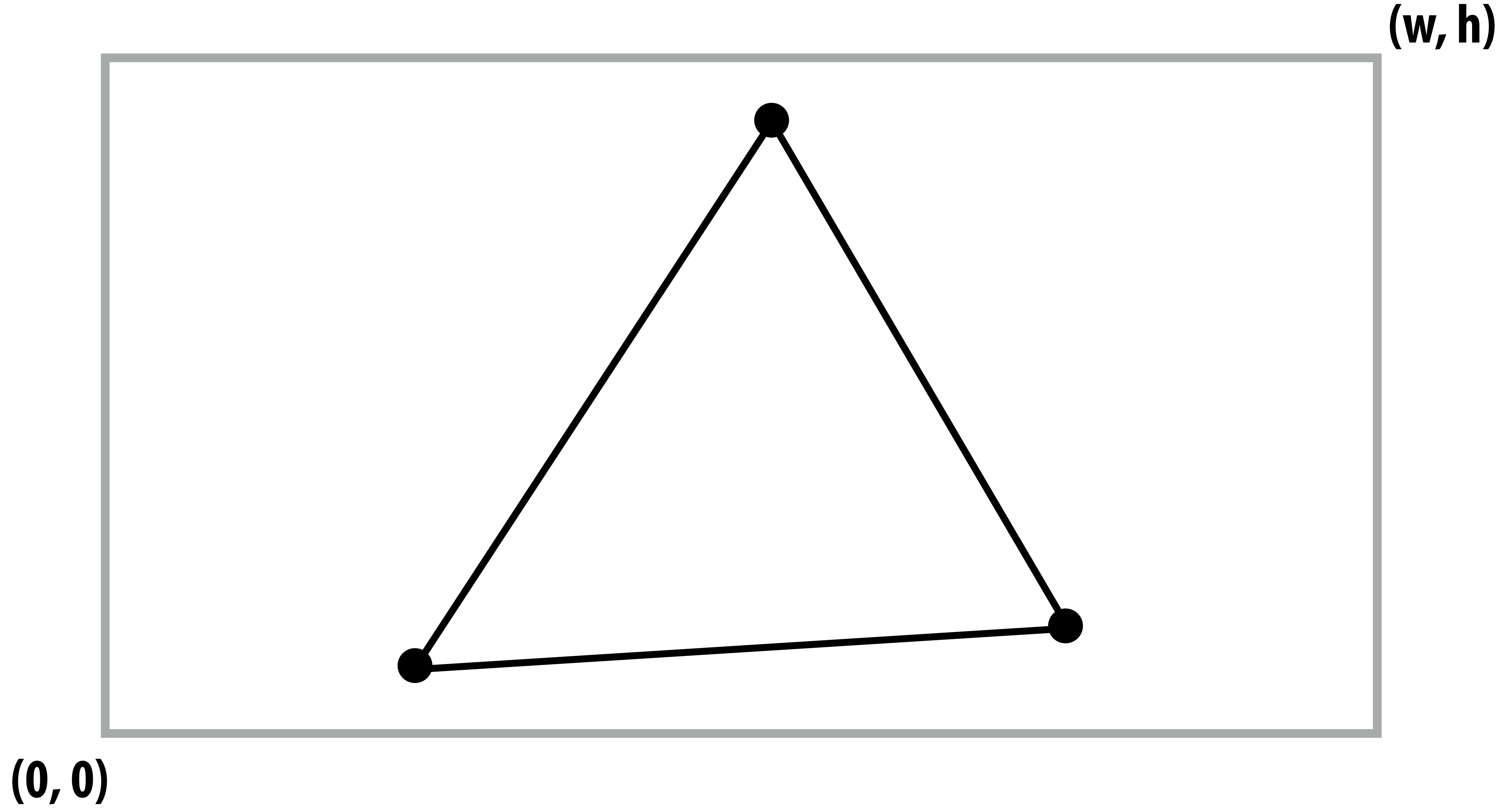
Triangles before clipping



Triangles after clipping

Step 4: transform to screen coordinates

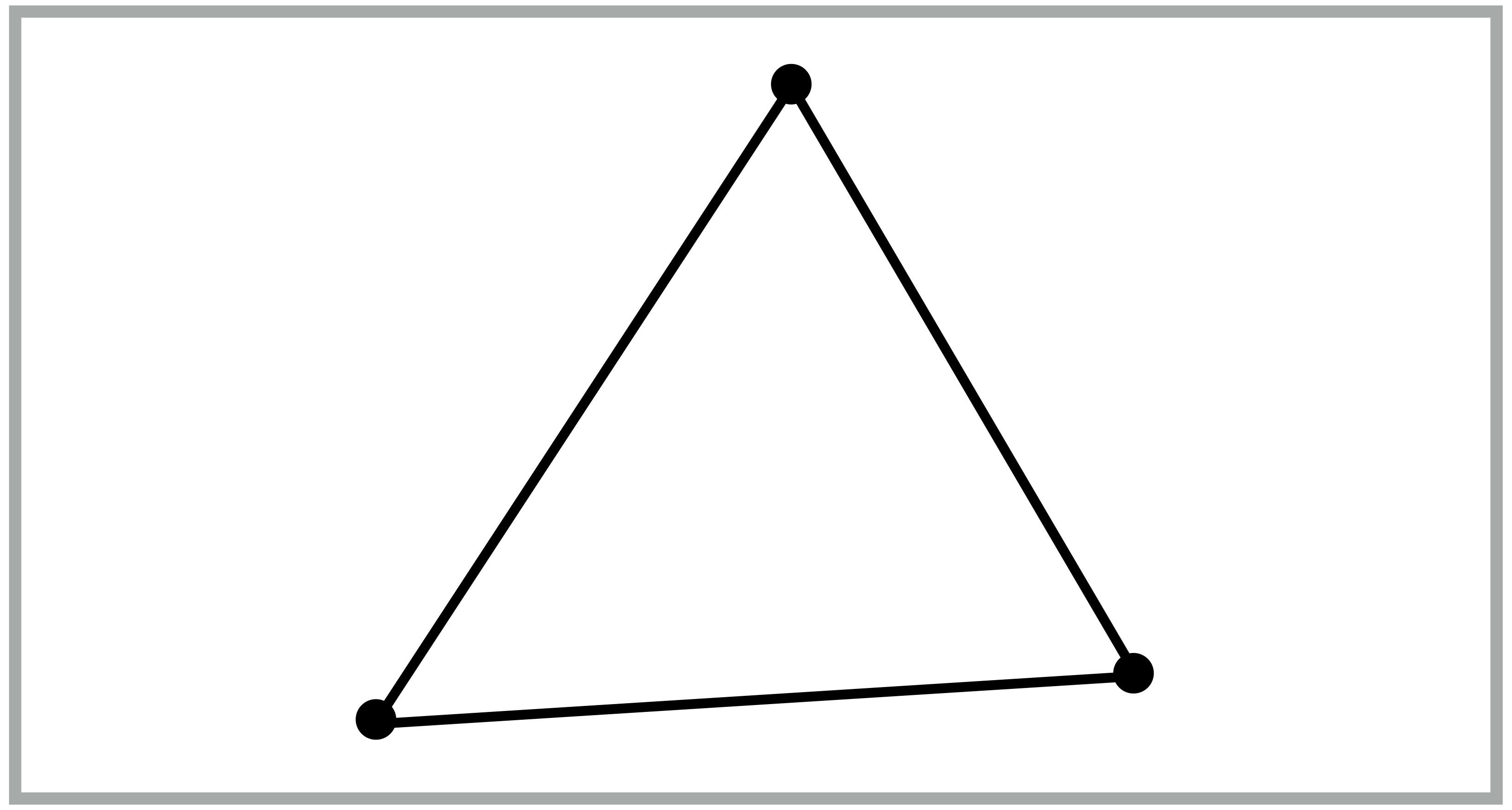
Perform homogeneous divide, transform vertex xy positions from normalized coordinates into screen coordinates (based on screen w,h)



Step 5: setup triangle (triangle preprocessing)

Before rasterizing triangle, can compute a bunch of data that will be used by all fragments, e.g.,

- **triangle edge equations**
- **triangle attribute equations**
- **etc.**



$$\mathbf{E}_{01}(x, y) \qquad \mathbf{U}(x, y)$$

$$\mathbf{E}_{12}(x, y) \qquad \mathbf{V}(x, y)$$

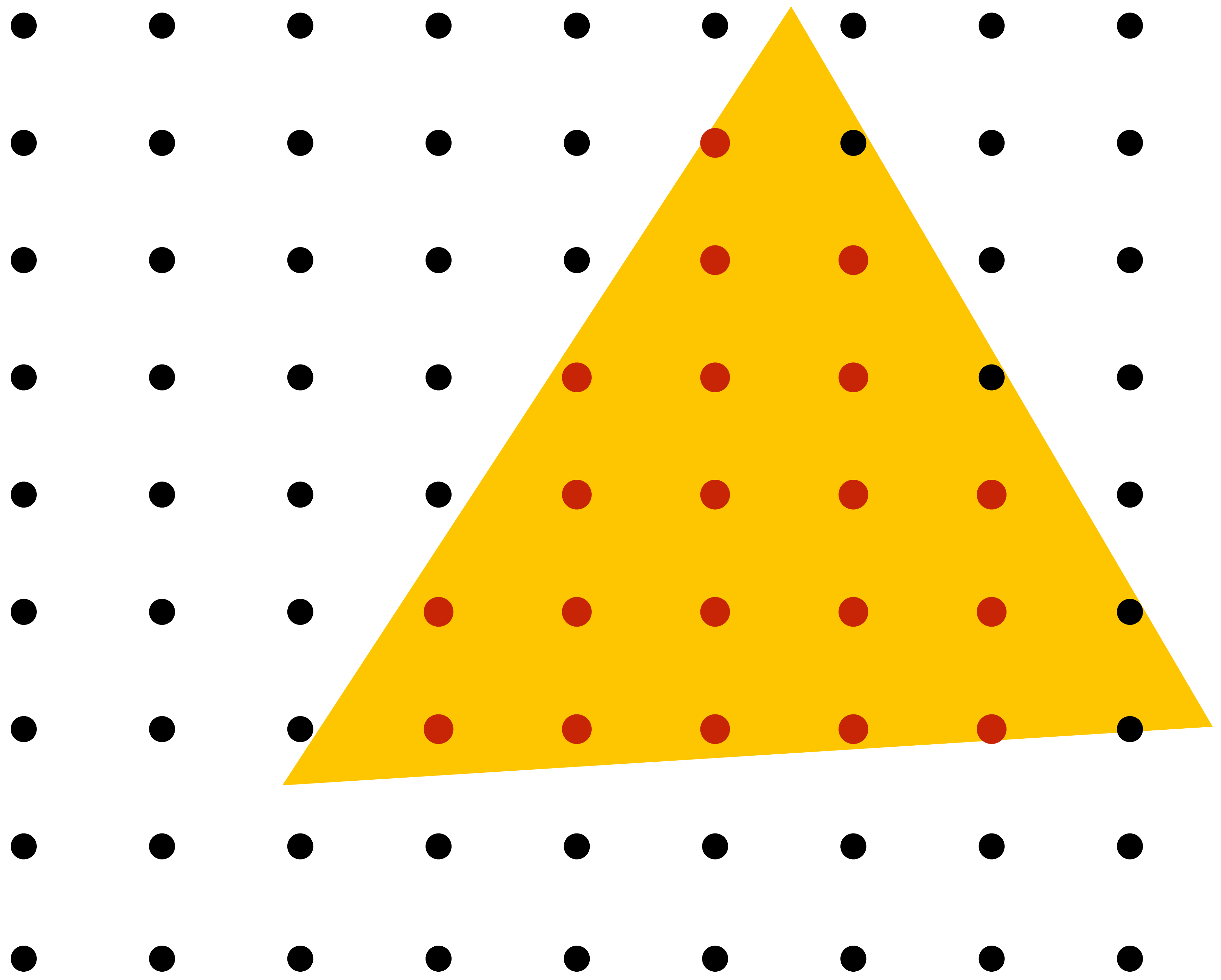
$$\mathbf{E}_{20}(x, y)$$

$$\frac{1}{\mathbf{w}}(x, y)$$

$$\mathbf{Z}(x, y)$$

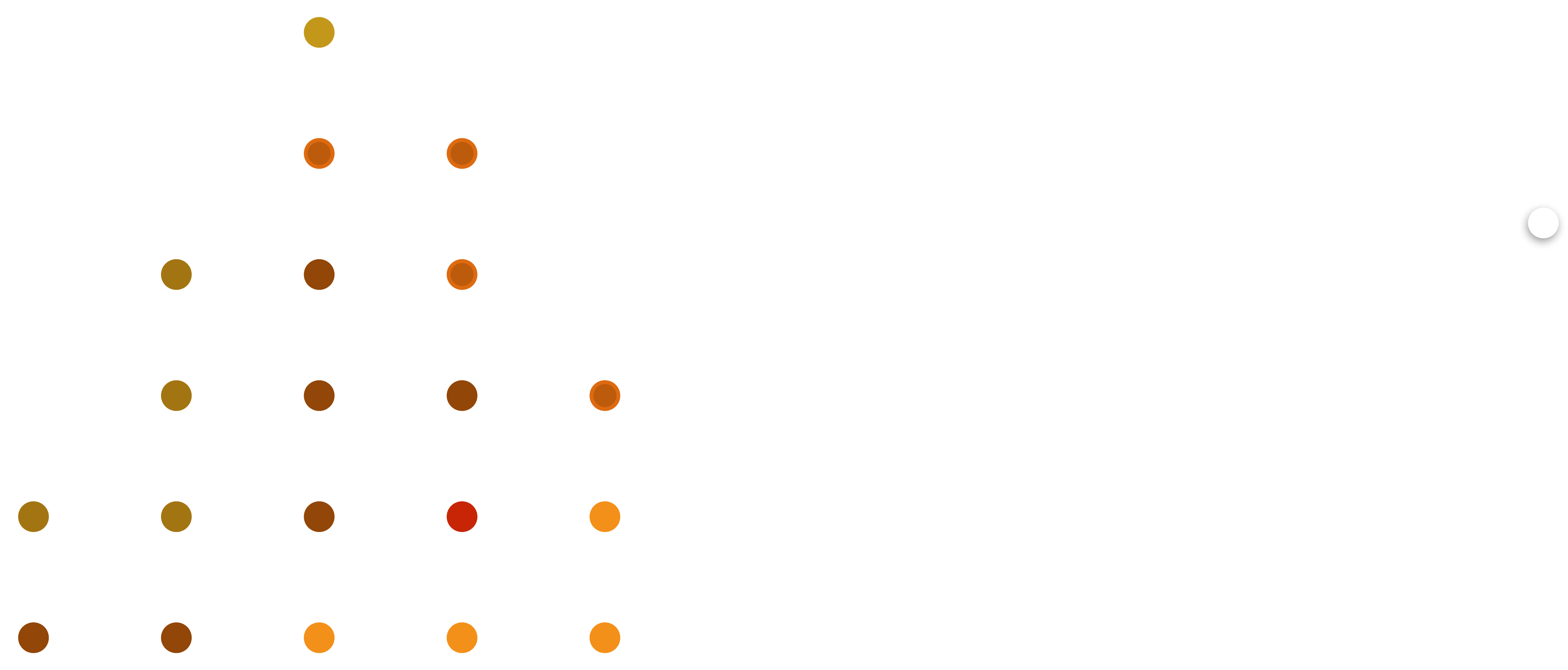
Step 6: sample coverage

Evaluate attributes z, u, v at all covered samples



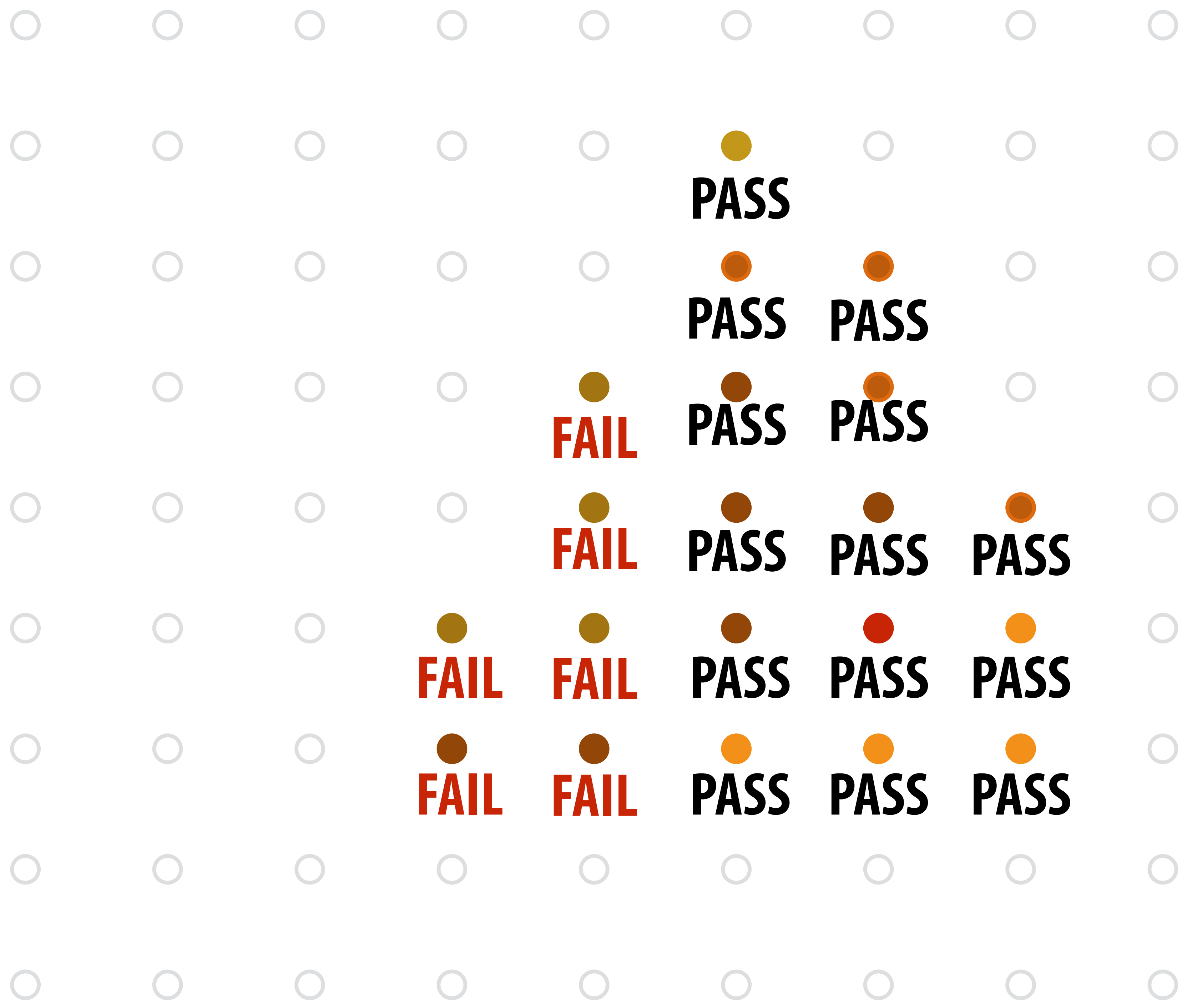
Step 6: compute triangle color at sample point

e.g., interpolate from vertices using barycentric coordinates

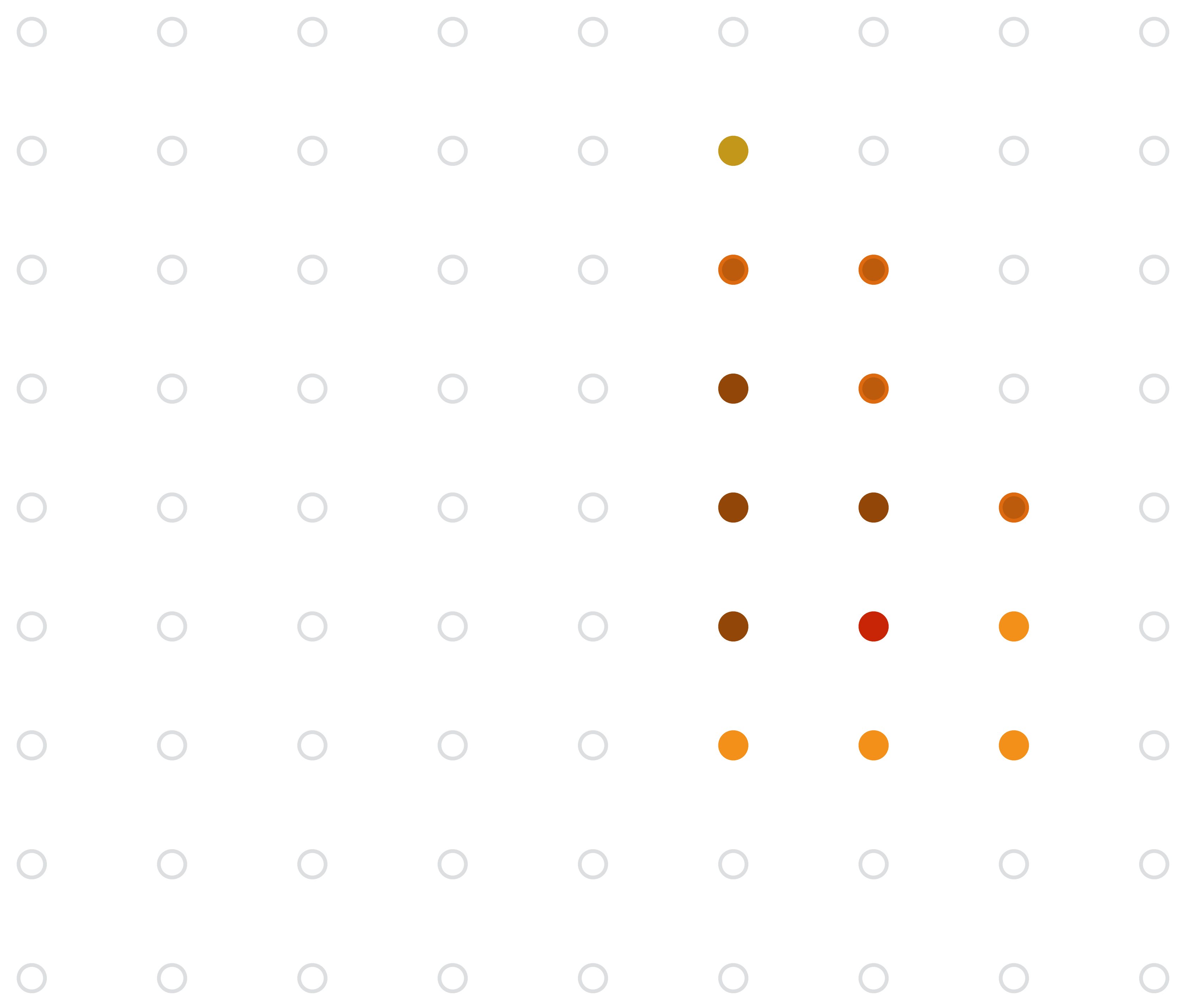


Step 7: perform depth test (if enabled)

Also update depth value at covered samples (if necessary)



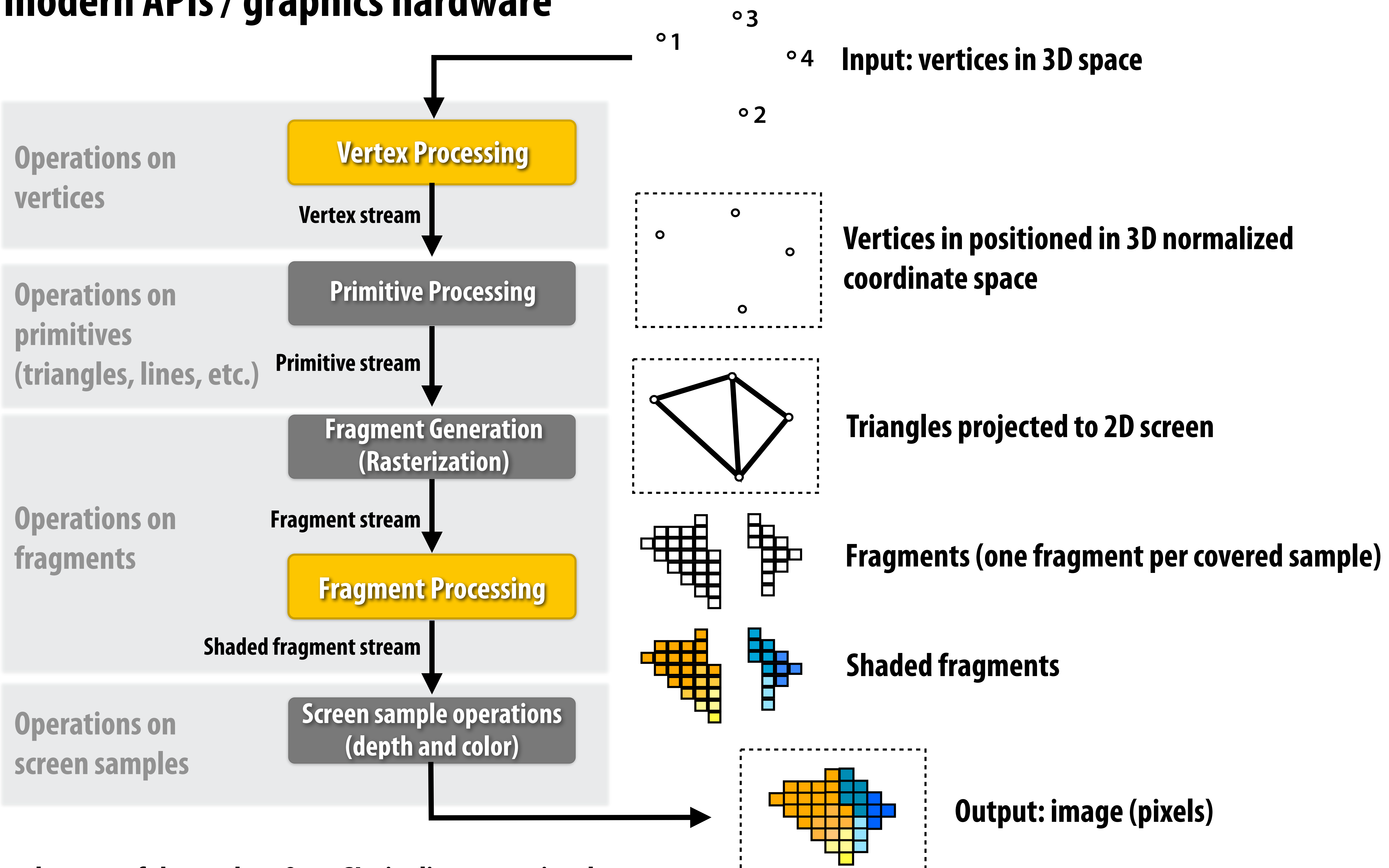
Step 8: update color buffer* (if depth test passed)



* Possibly using OVER operation for transparency

OpenGL/Direct3D graphics pipeline

Our rasterization pipeline doesn't look much different from "real" pipelines used in modern APIs / graphics hardware

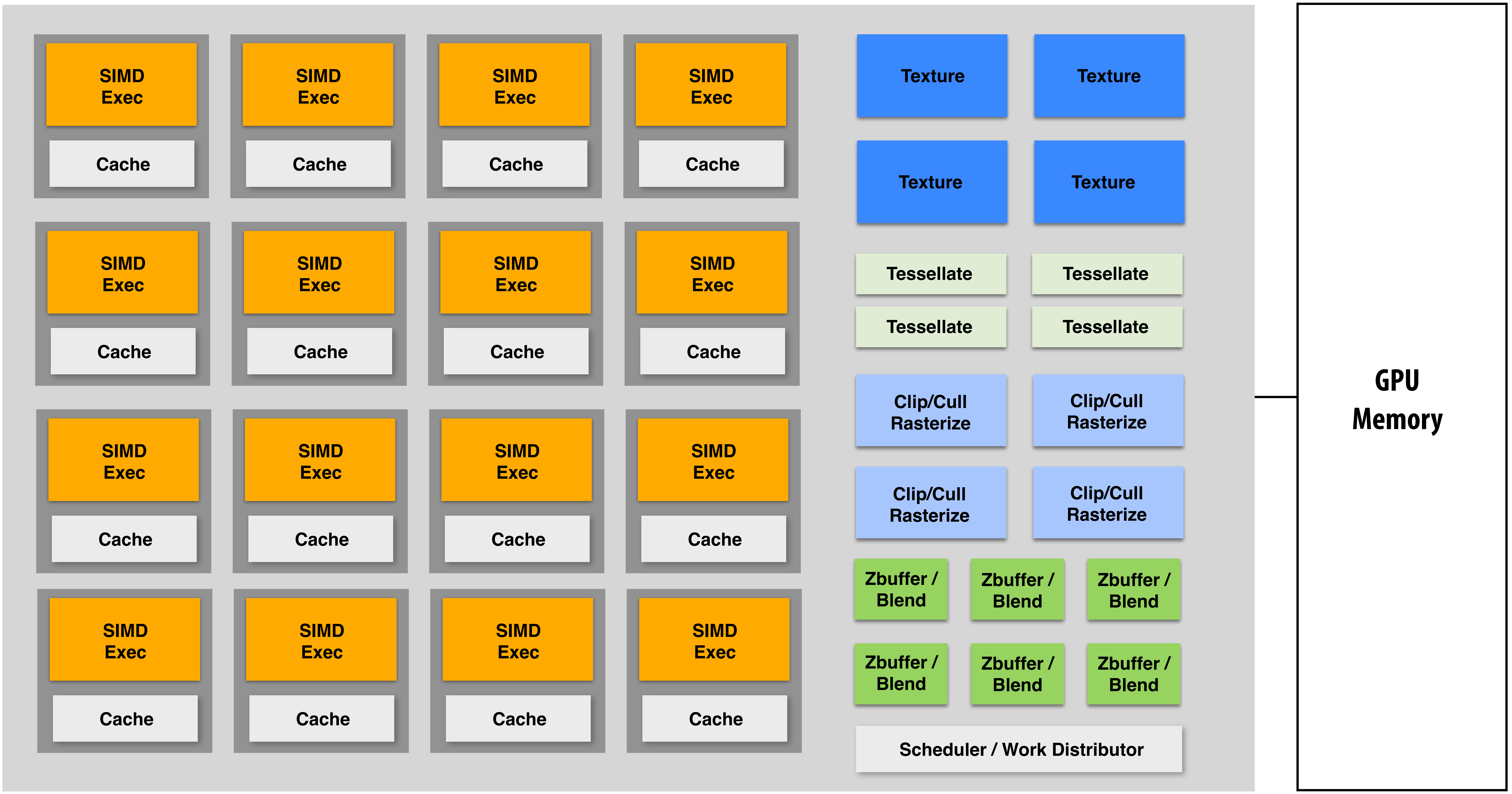


* Several stages of the modern OpenGL pipeline are omitted

GPU: heterogeneous, multi-core processor

Modern GPUs offer ~35 TFLOPs of performance for generic vertex/fragment programs ("compute")

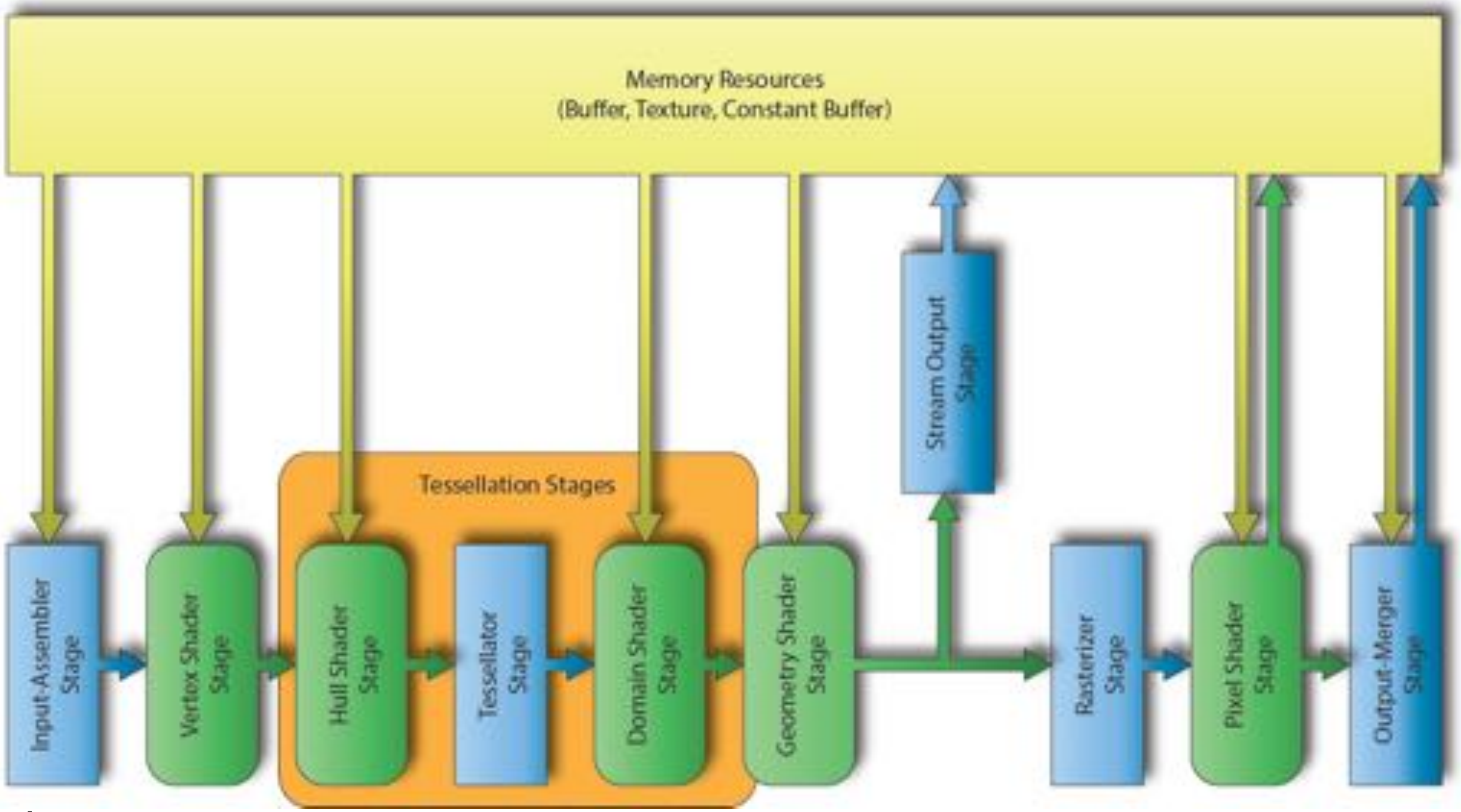
still enormous amount of fixed-function compute over here



This part (mostly) not used by CUDA/OpenCL; raw graphics horsepower still greater than compute!

Modern Rasterization Pipeline

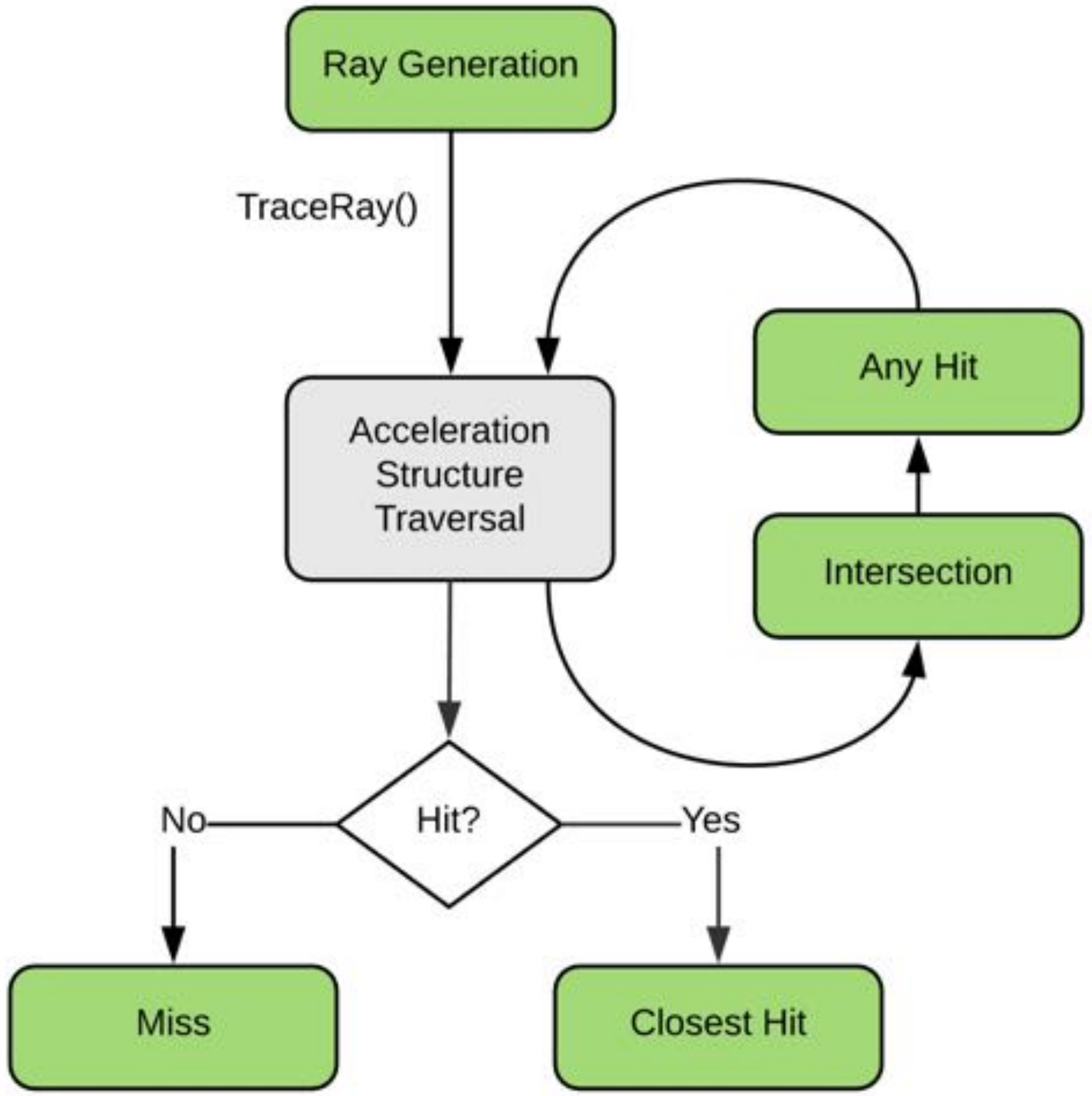
- Trend toward more generic (but still highly parallel!) computation:
  - make stages programmable
    - replace fixed function vertex, fragment processing
    - add geometry, tessellation shaders
    - generic "compute" shaders (whole other story…)
  - more flexible scheduling of stages



(DirectX 12 Pipeline)

Ray Tracing in Graphics Pipeline

- **More recently: specialized pipeline for ray tracing (NVIDIA RTX)**



https://devblogs.nvidia.com/introduction-nvidia-rtx-directx-ray-tracing/

GPU Ray Tracing Demo ("Marbles at Night")

Next time: Texture Mapping and Supersampling