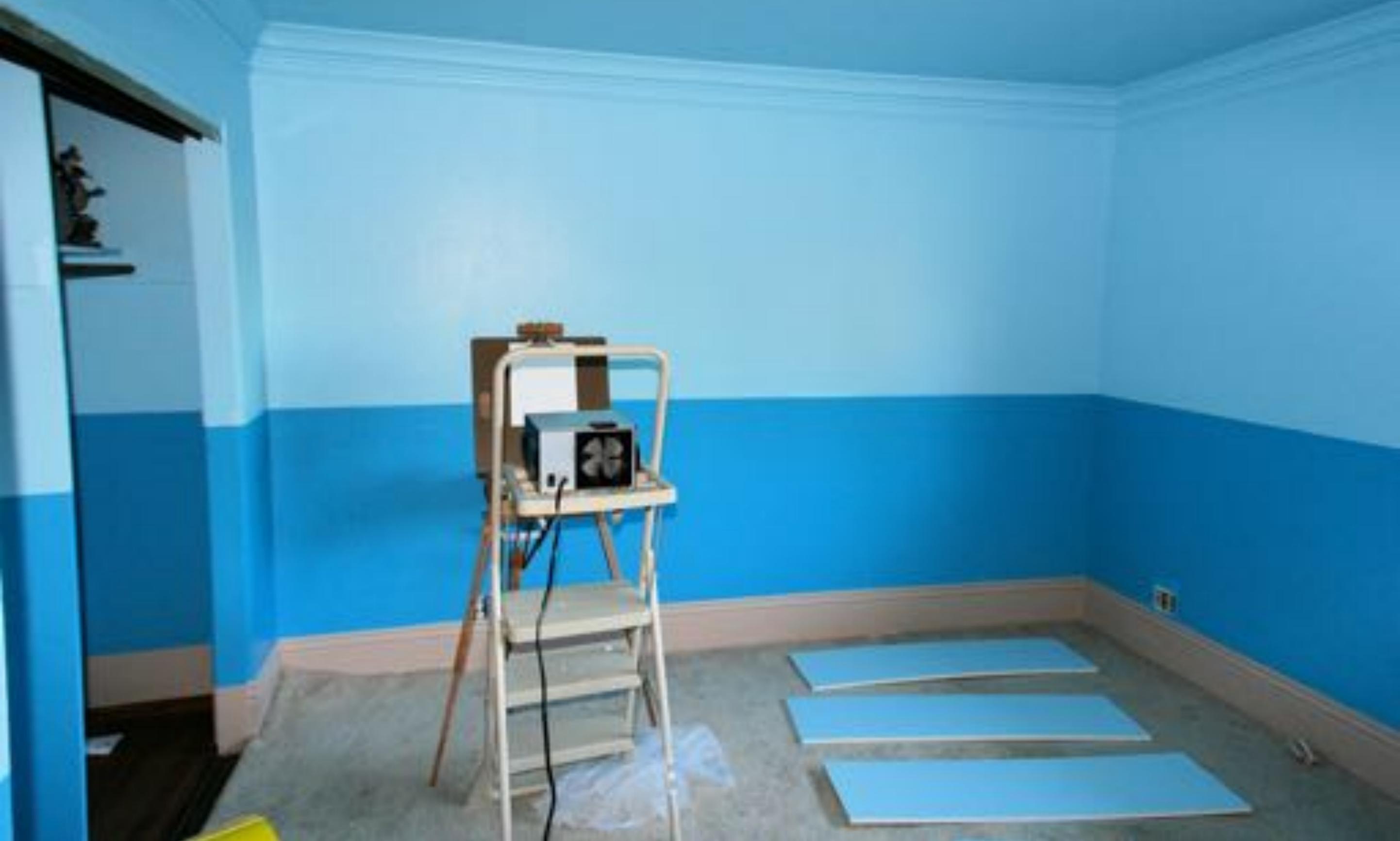


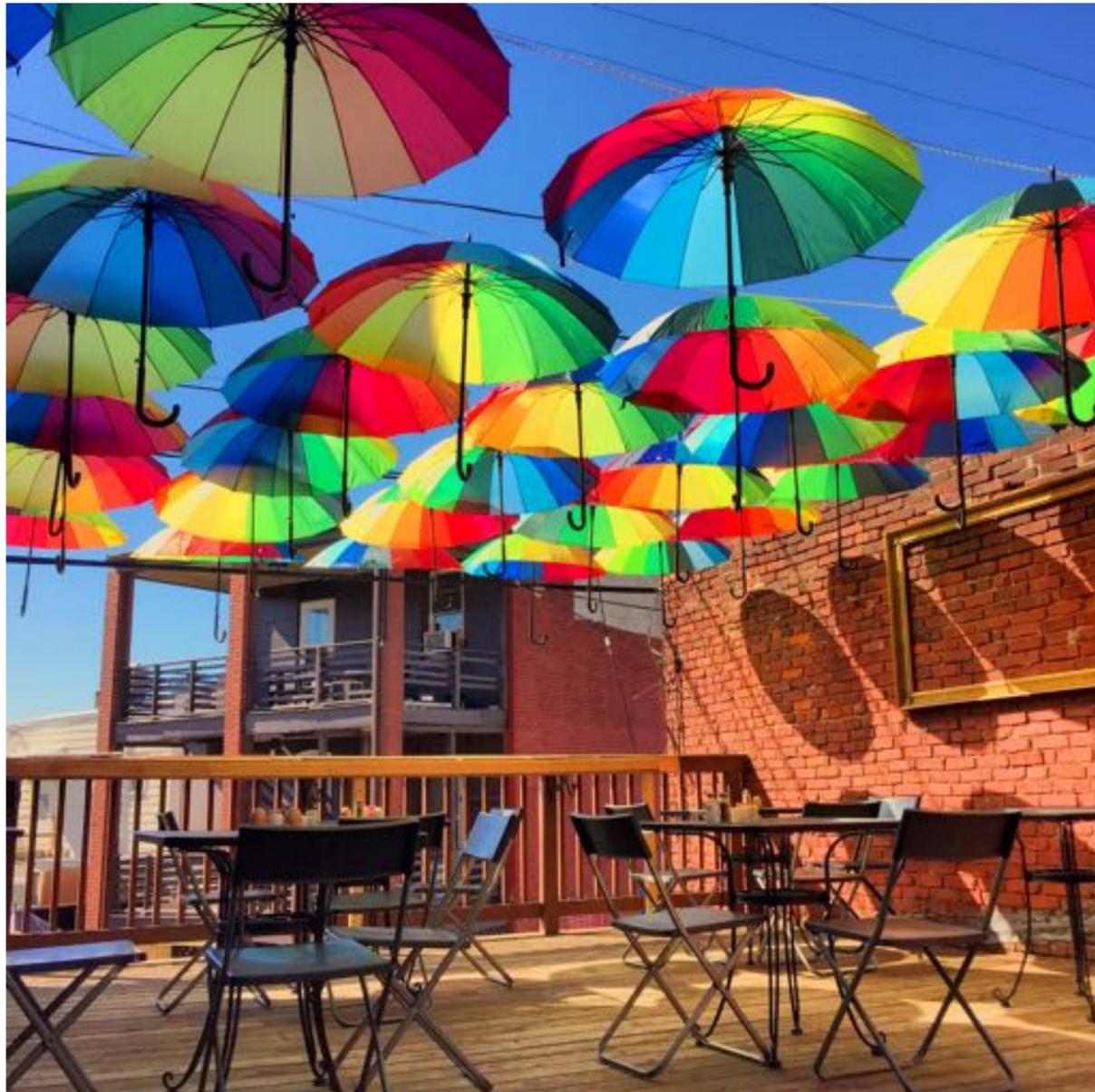
Color

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Why do we need to be able to talk precisely about color?



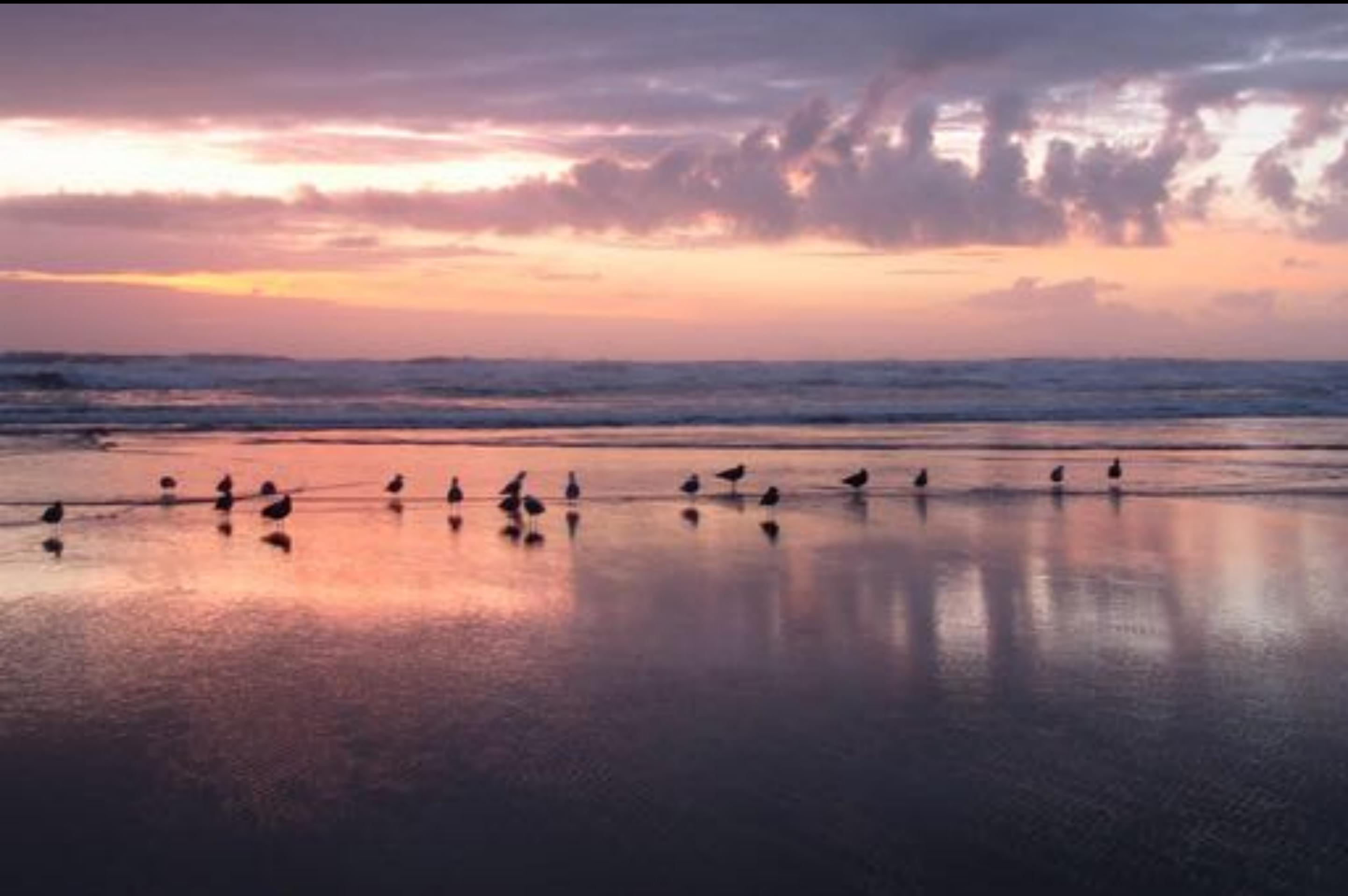




on screen



printed

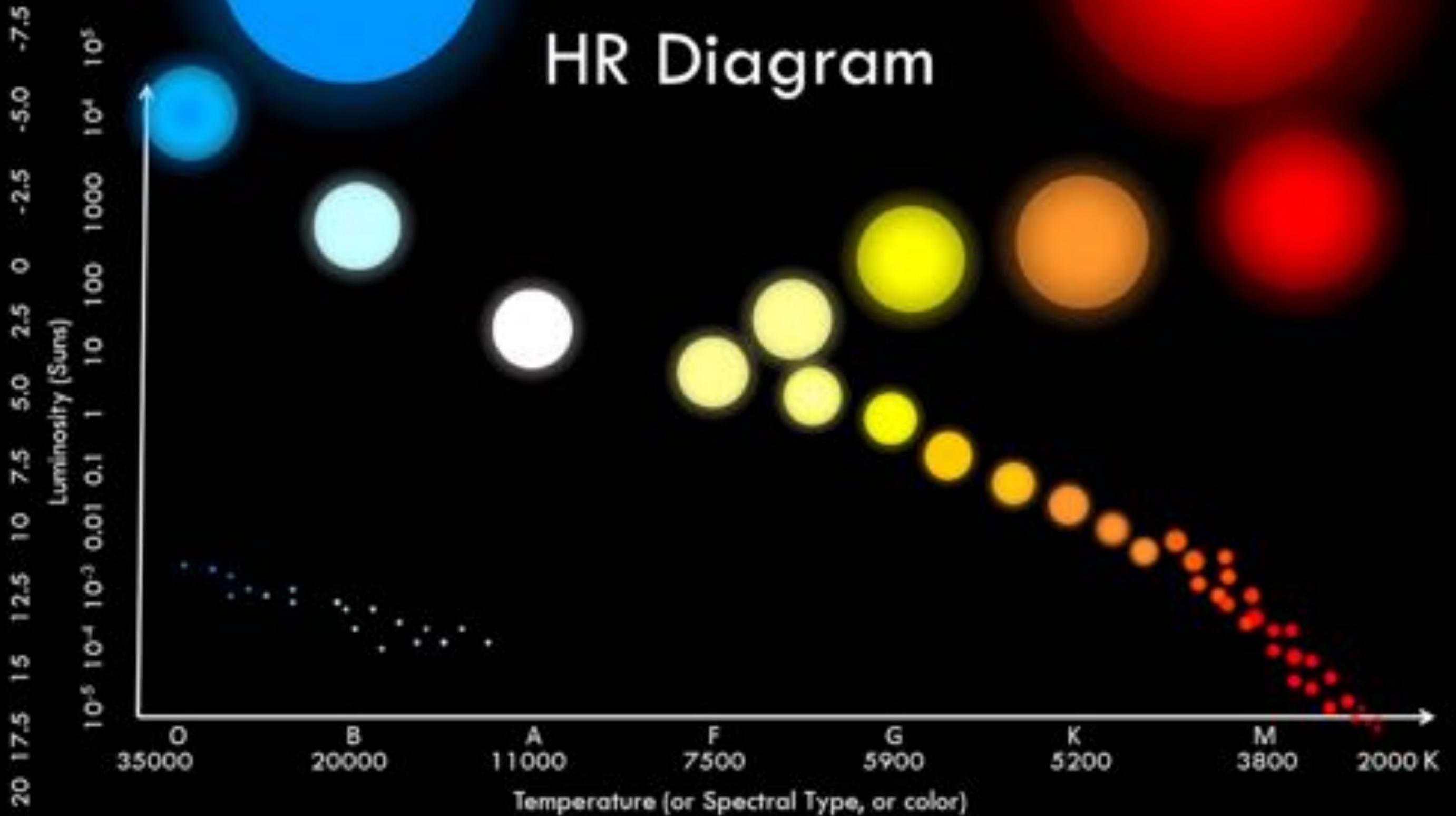


Cannon Beach, Oregon



Starry Night, Van Gogh

HR Diagram

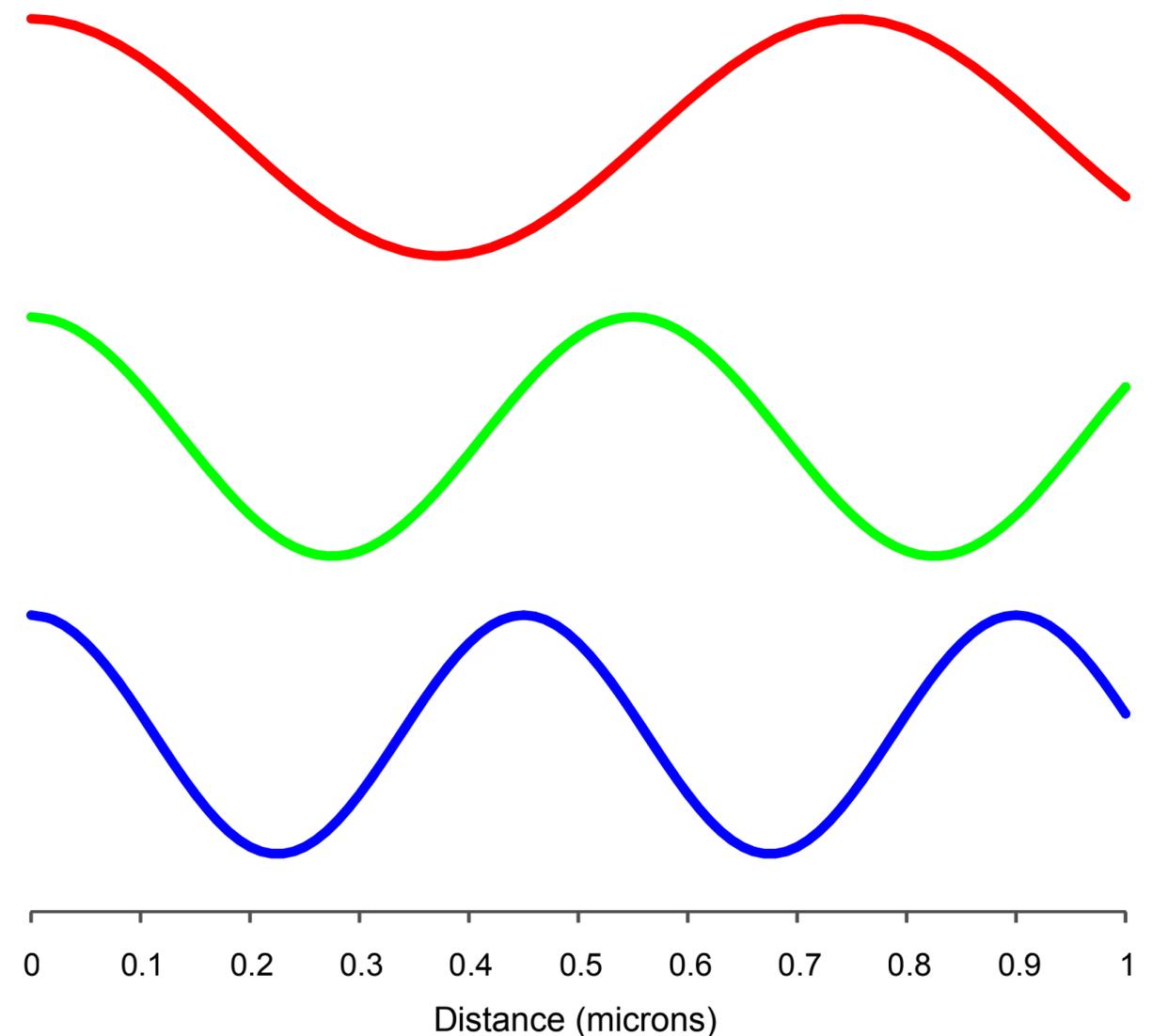


Hertzsprung-Russell diagram

What is color?

Light is EM Radiation; Color is Frequency

- Light is oscillating electric & magnetic field
- KEY IDEA: frequency determines color of light
- Q: What is the difference between frequency and wavelength?

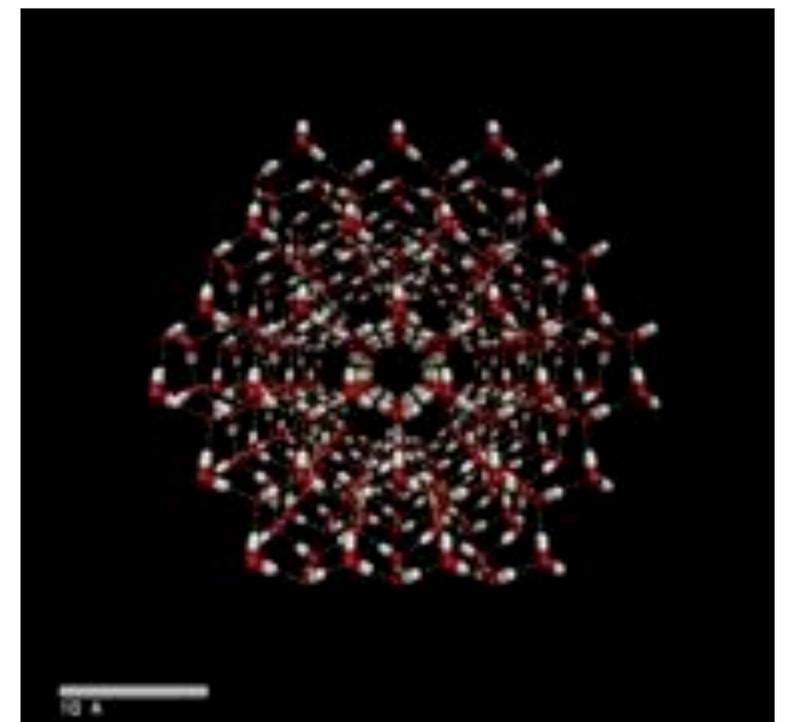
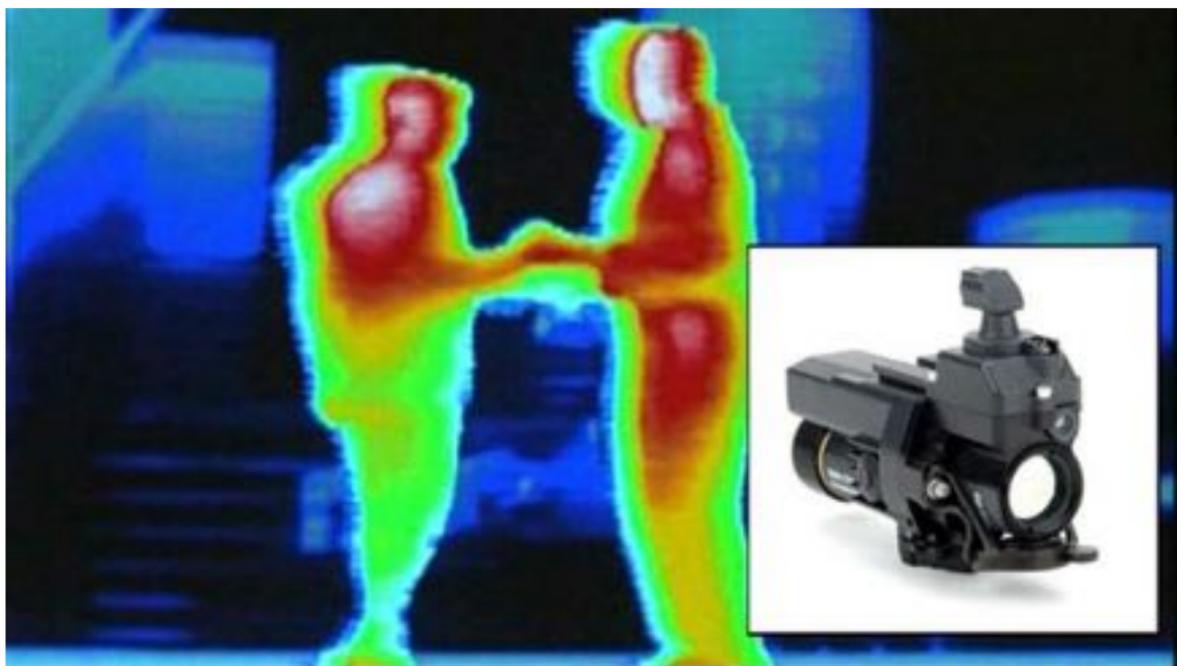
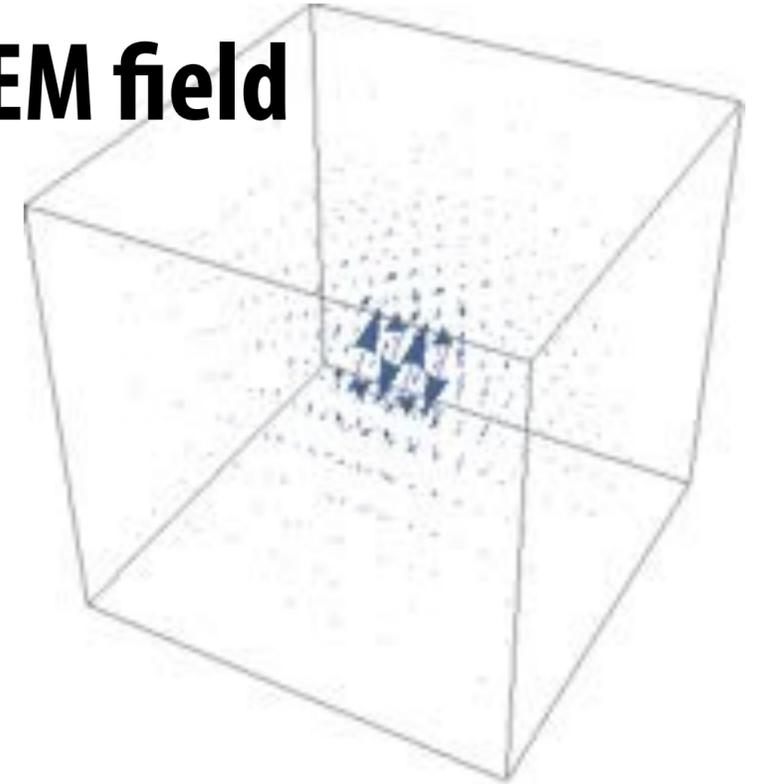




Q: Why does your stove turn red when it heats up?

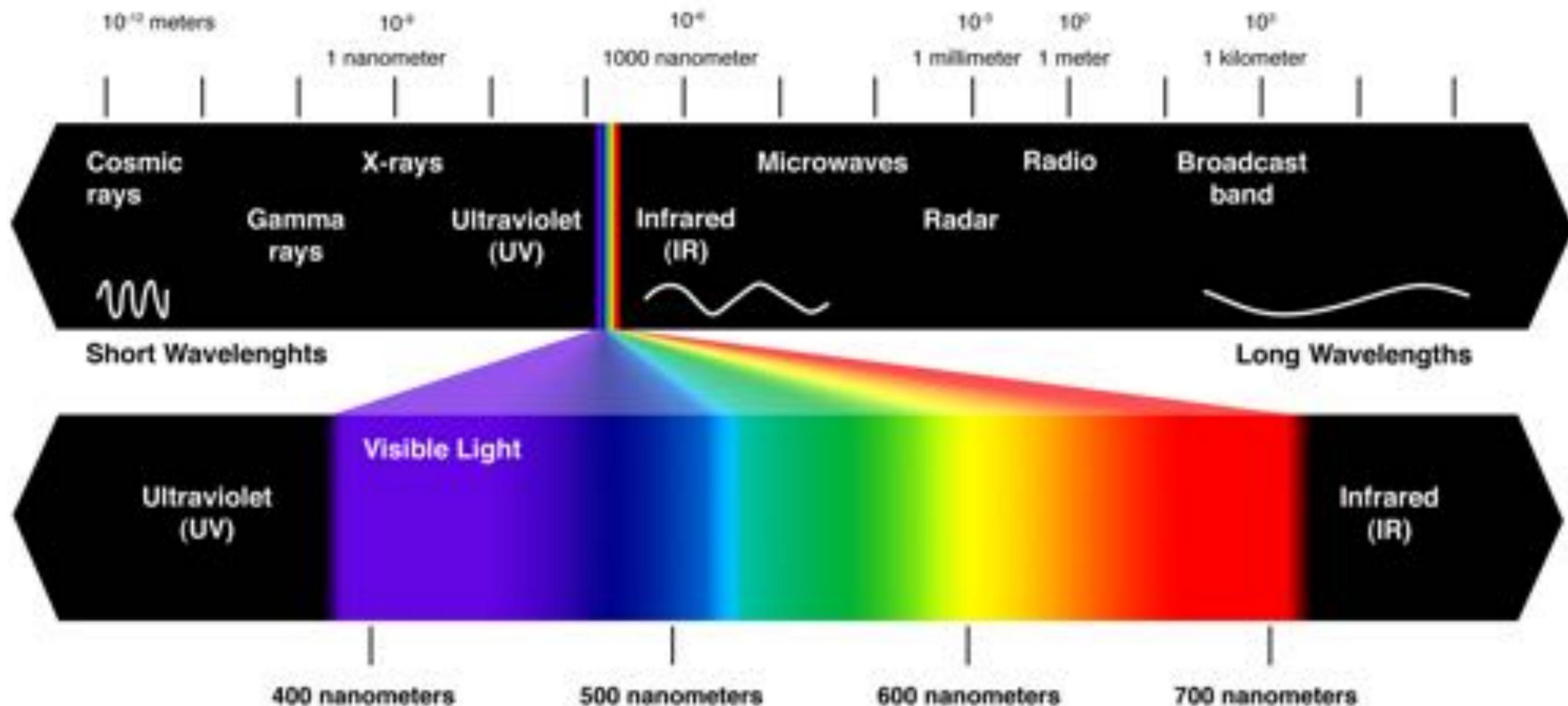
Heat generates light

- One of many ways light is produced:
- Maxwell: motion of charged particles creates EM field
- Thermodynamics: ...particles jiggle around!
- Hence, anything moving generates light
- In other words:
 - every object around you is producing color!
 - frequency determined by temperature



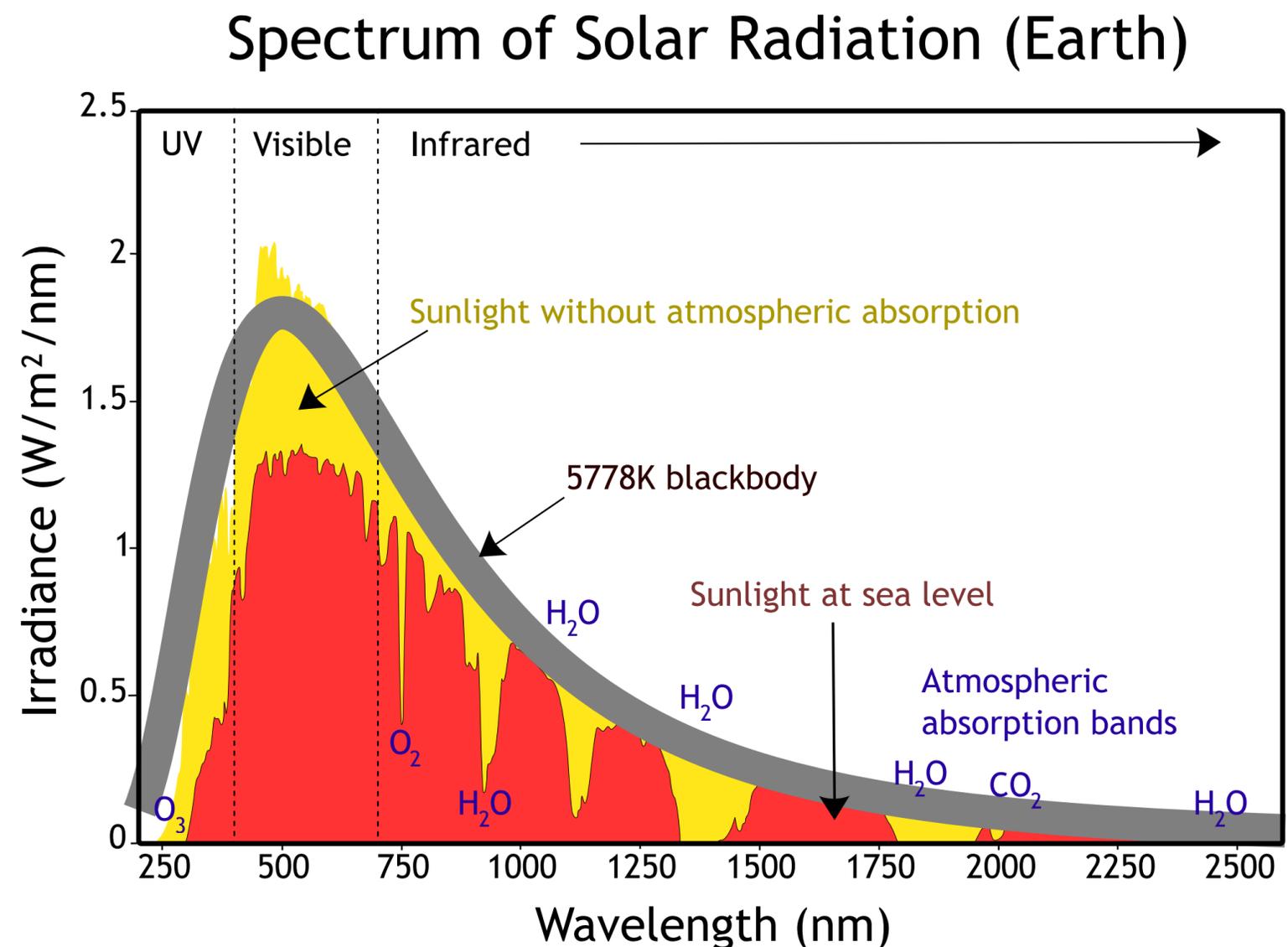
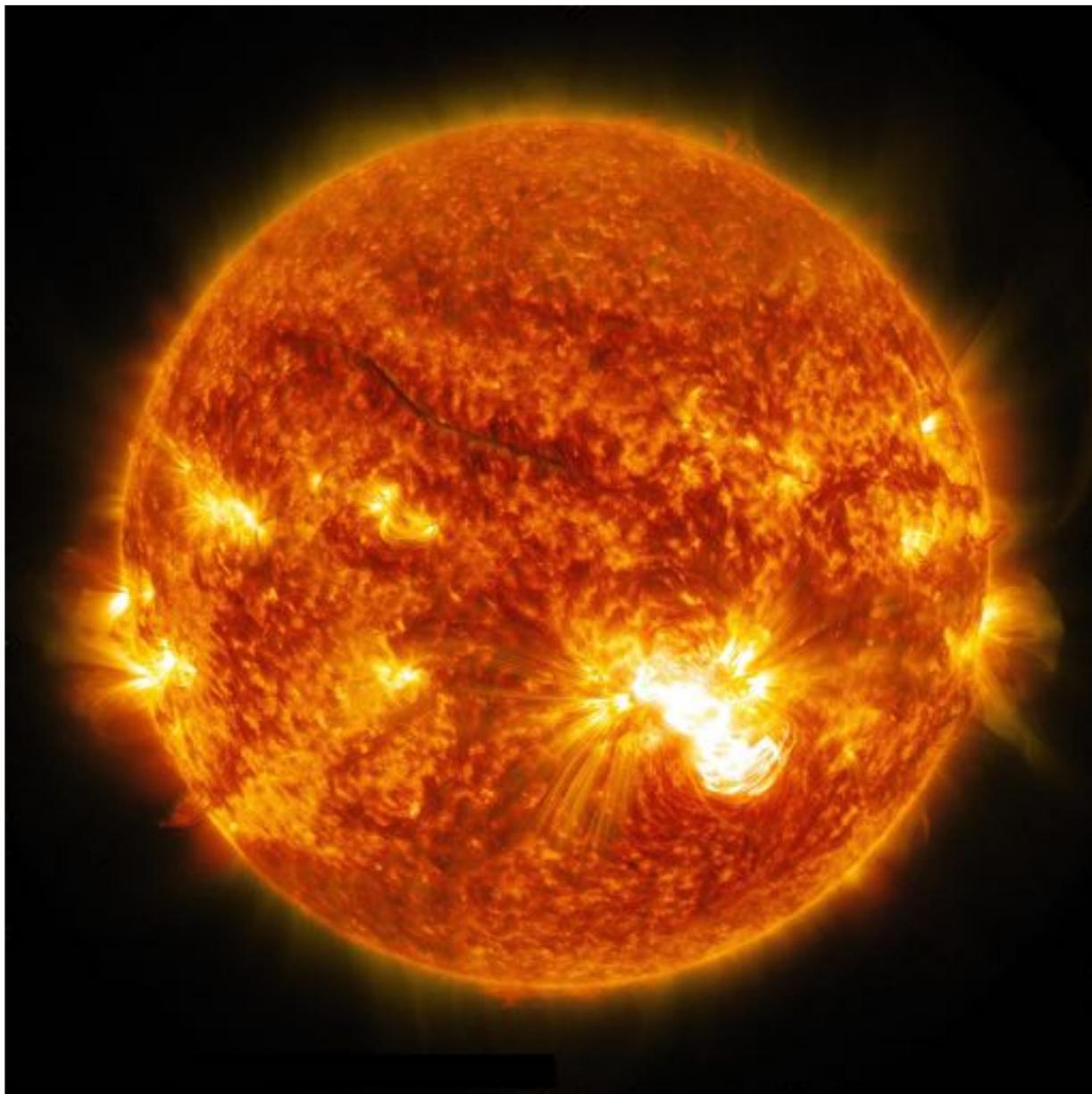
Most light is not visible!

- Frequencies visible by human eyes are called “visible spectrum”
- These frequencies what we normally think of as “color”



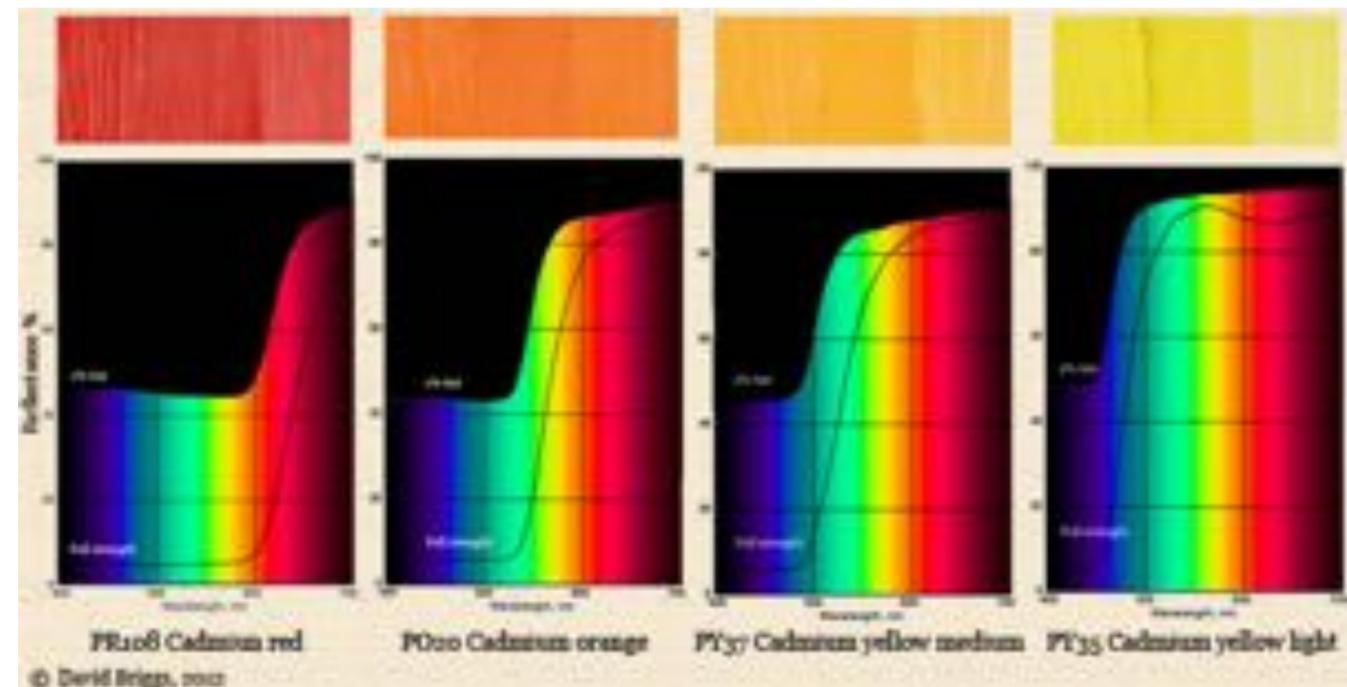
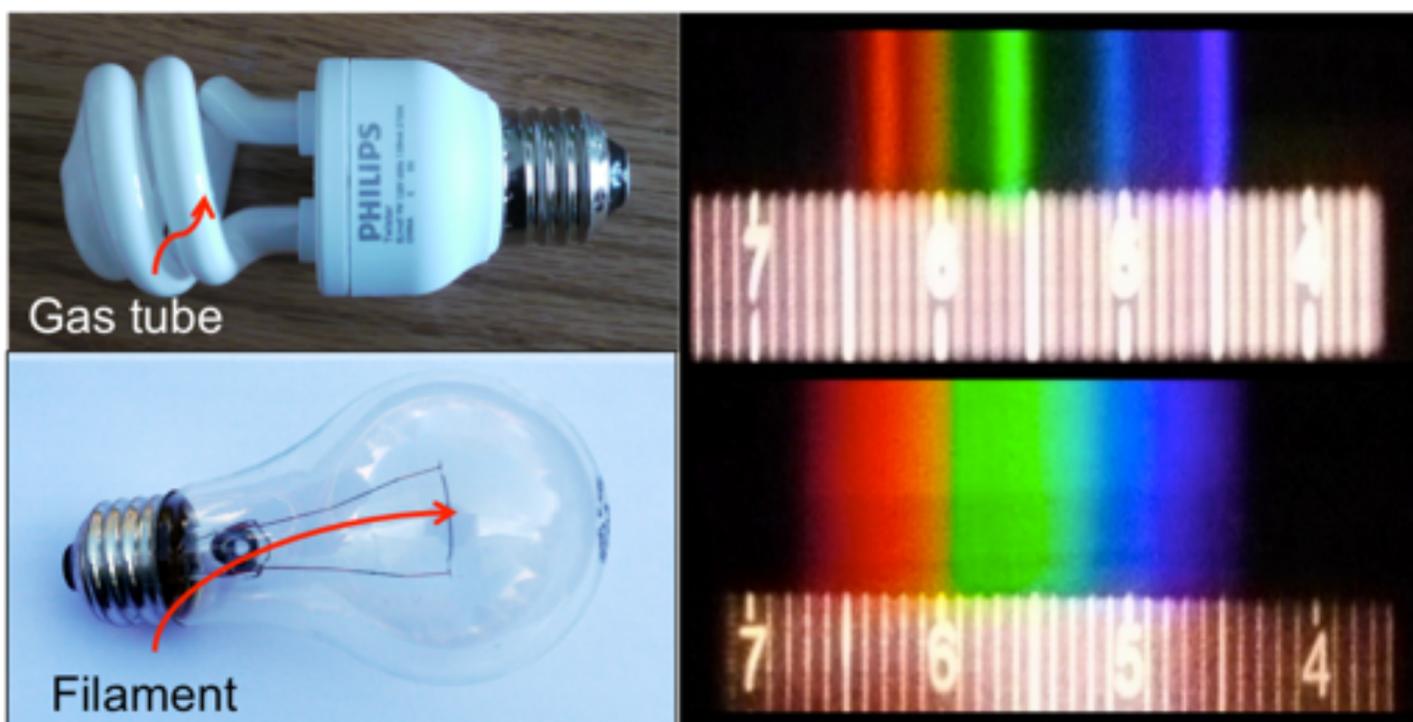
Natural light is a mixture of frequencies

- “White” light is really a mixture of all (visible) frequencies
- E.g., the light from our sun



Additive vs. Subtractive Models of Light

- **Spectrum we just saw for the sun “emission spectrum”**
 - How much light is produced (by heat, fusion, etc.)
 - Useful for, e.g., characterizing color of a lightbulb
- **Another useful description: “absorption spectrum”**
 - How much light is absorbed (e.g., turned into heat)
 - Useful for, e.g., characterizing color of paint, ink, etc.



Emission Spectrum

Describes light intensity as a function of frequency

Below: spectrum of various common light sources:

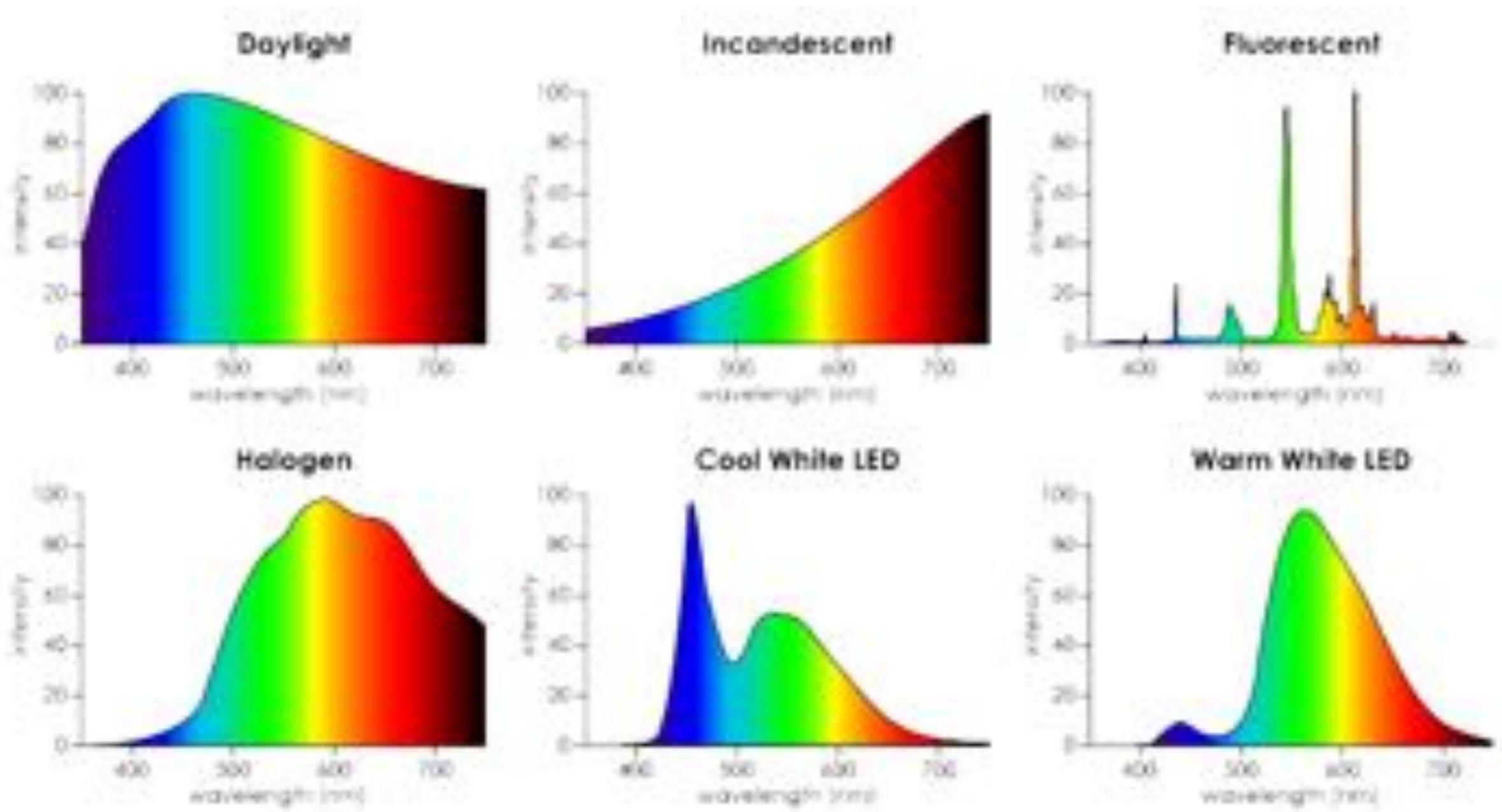


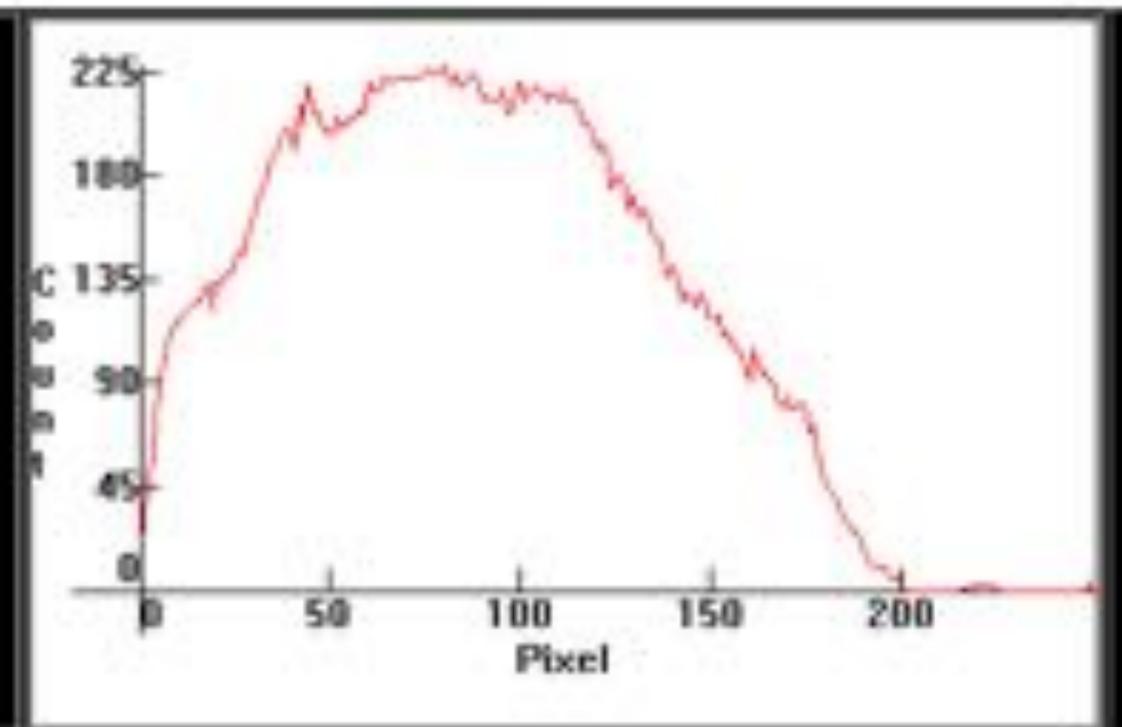
Figure credit:

Emission Spectrum—Example

- Why so many different kinds of lightbulbs on the market?
- “Quality” of light:

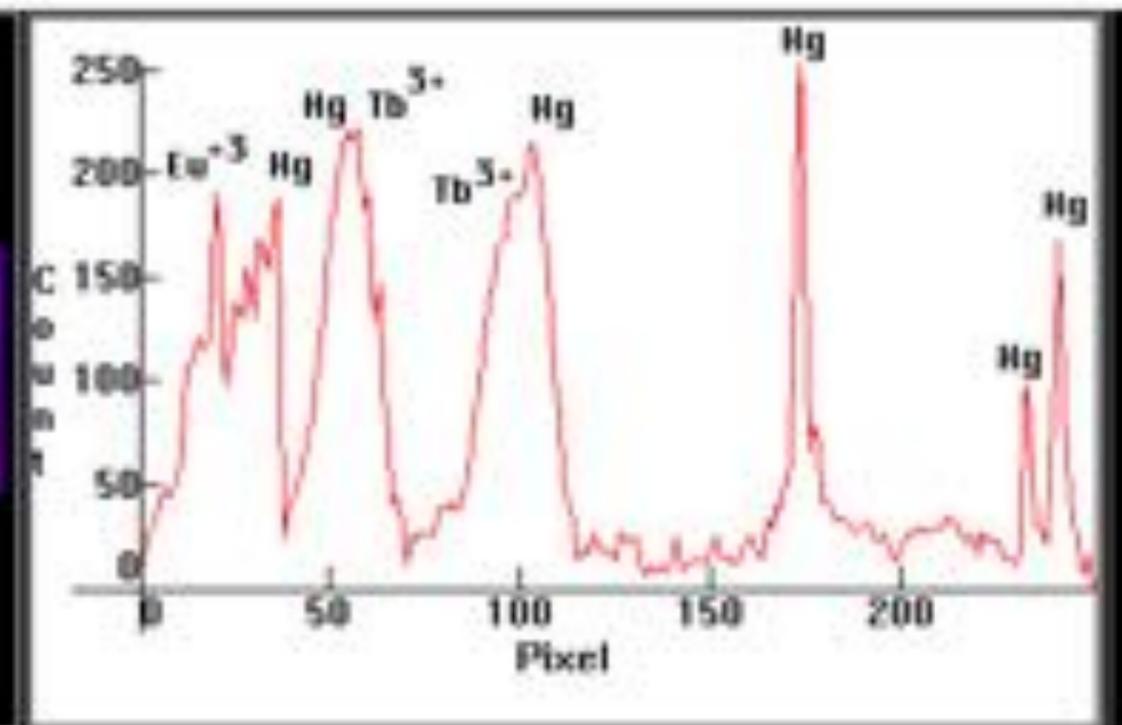
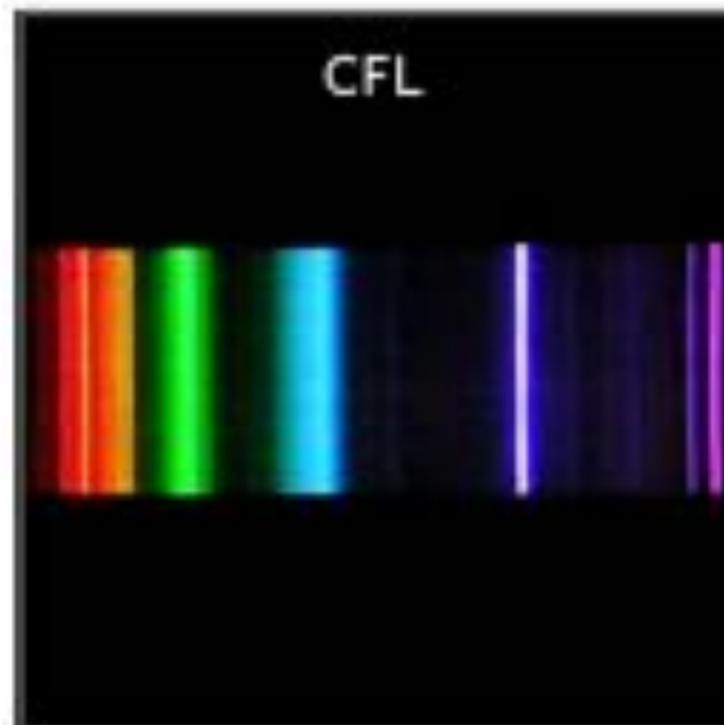
Incandescent:

- +more sun-like
- power-hungry



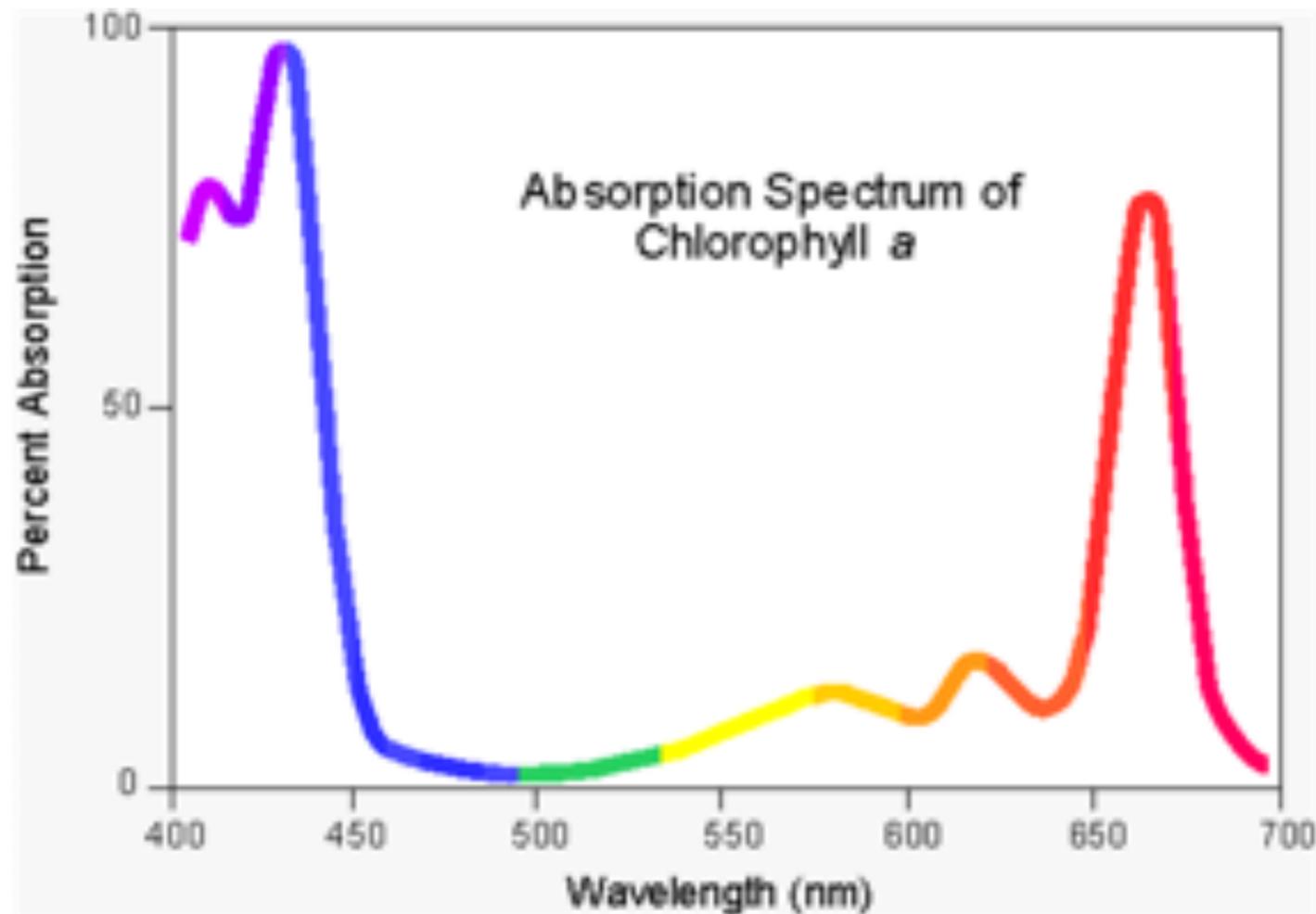
CFL:

- “choppy” spectrum
- +power efficient



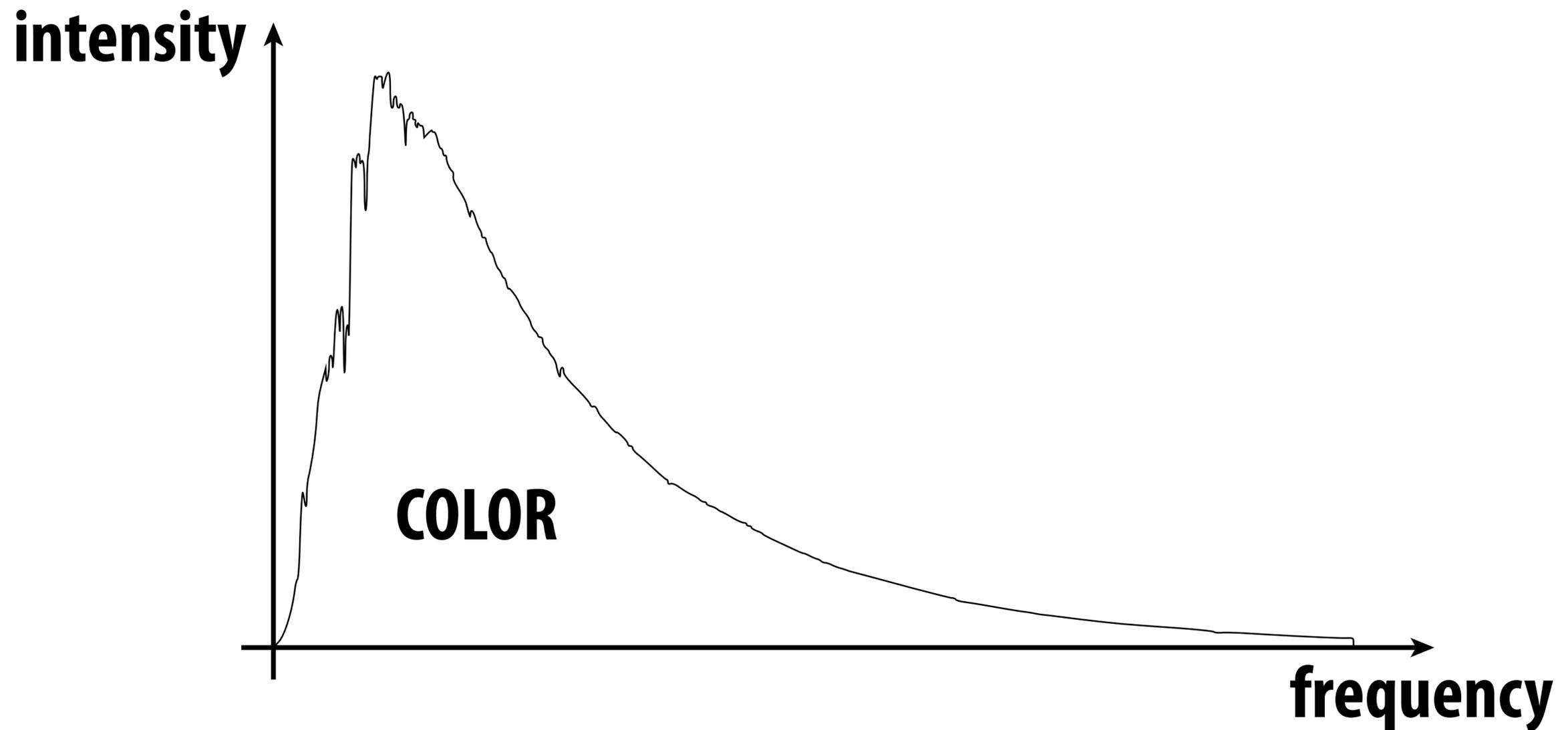
Absorption Spectrum

- Emission spectrum is intensity as a function of frequency
- Absorption spectrum is fraction absorbed as function of frequency



Q: What color is an object with this absorption spectrum?

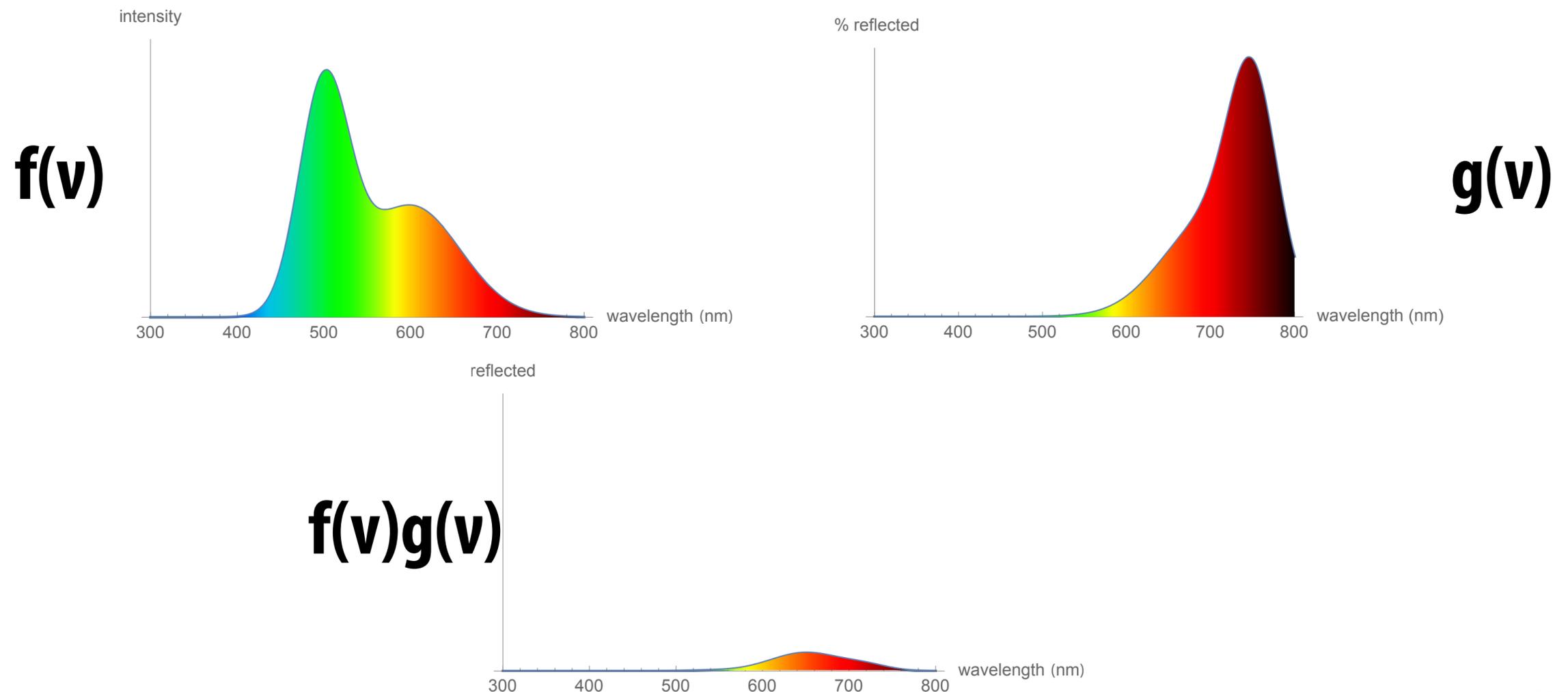
This is the fundamental description of color: intensity or absorption as a function of frequency



Everything else is merely a convenient approximation!

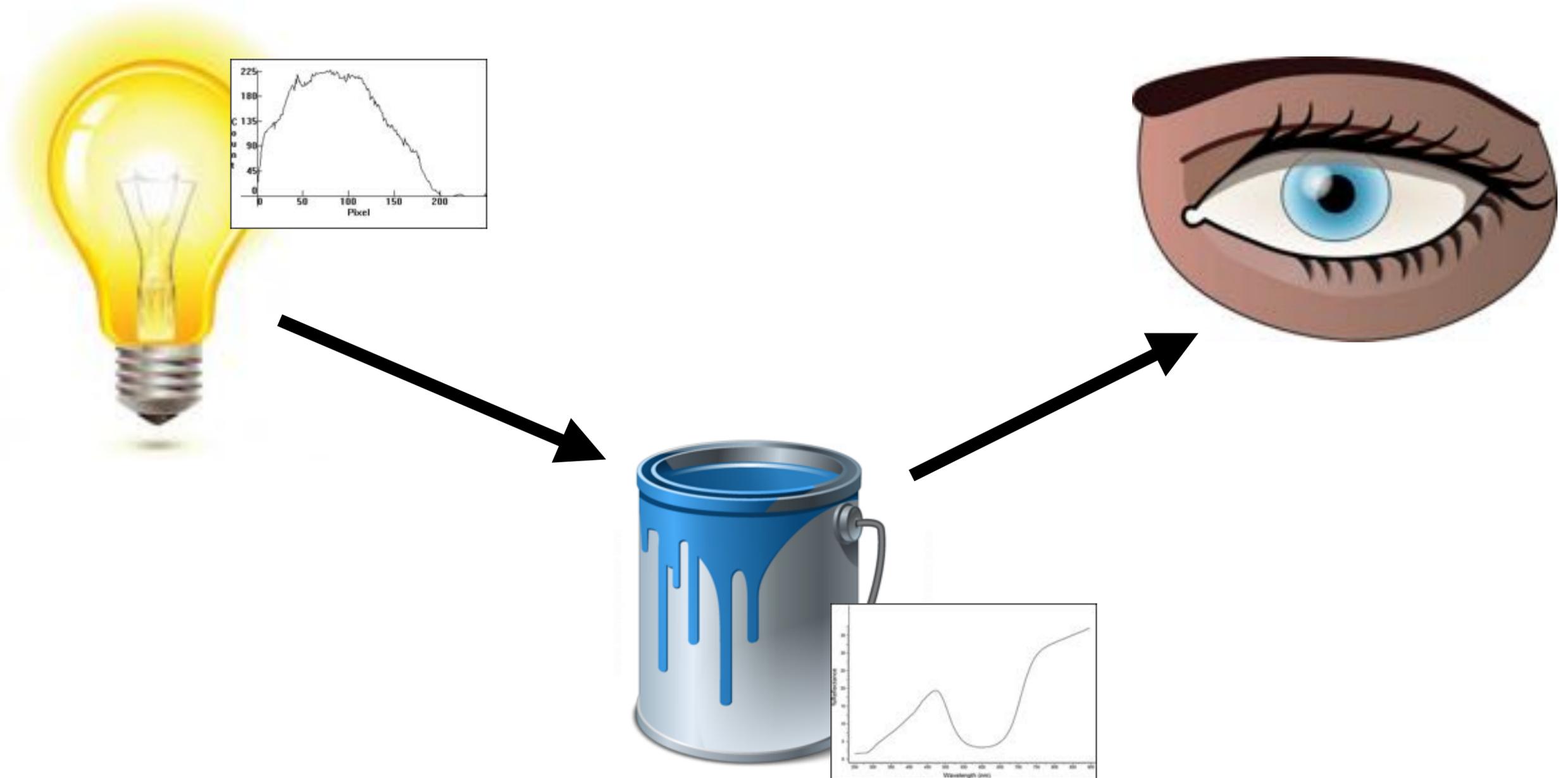
Interaction of emission and reflection

- Toy model for what happens when light gets reflected
 - ν —frequency (Greek “nu”)
 - Light source has emission spectrum $f(\nu)$
 - Surface has reflection spectrum $g(\nu)$
 - Resulting intensity is the product $f(\nu)g(\nu)$



Color reproduction is hard!

- Color clearly starts to get complicated as we start combining emission and absorption/reflection (real-world challenge!)



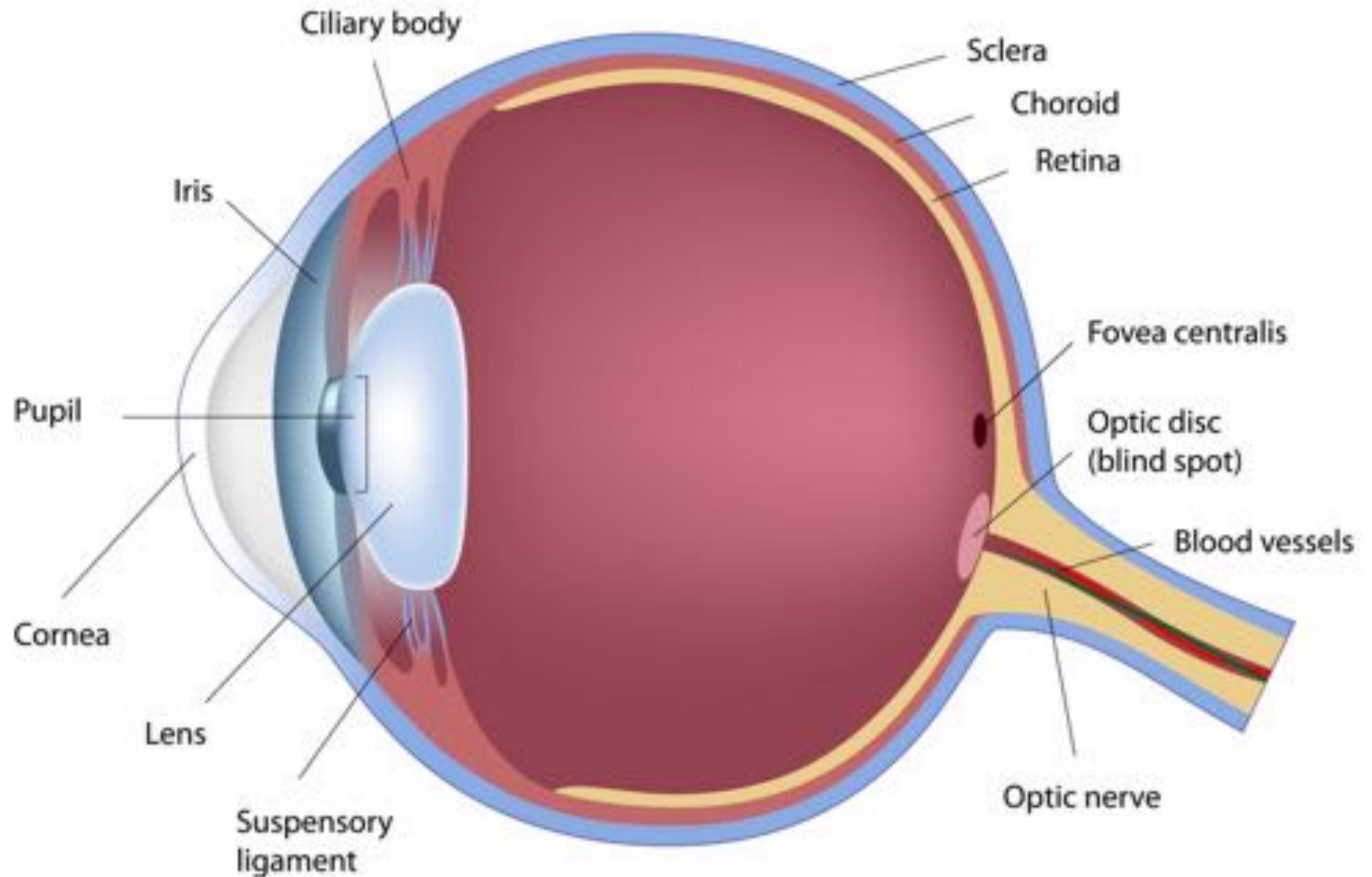
(What color ink should we use to get the desired appearance?)

...And what about perception?

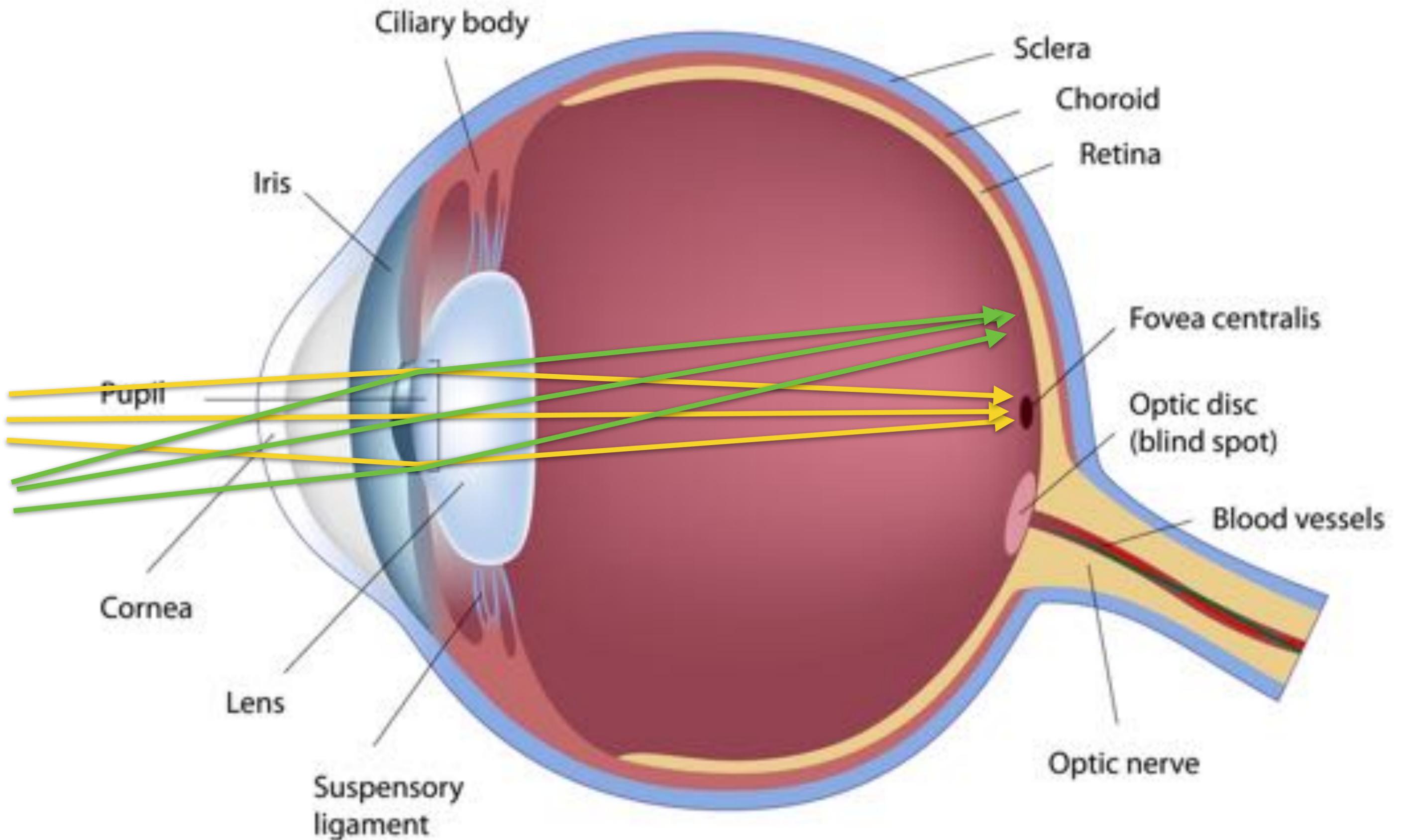
Q: What color is this dress?



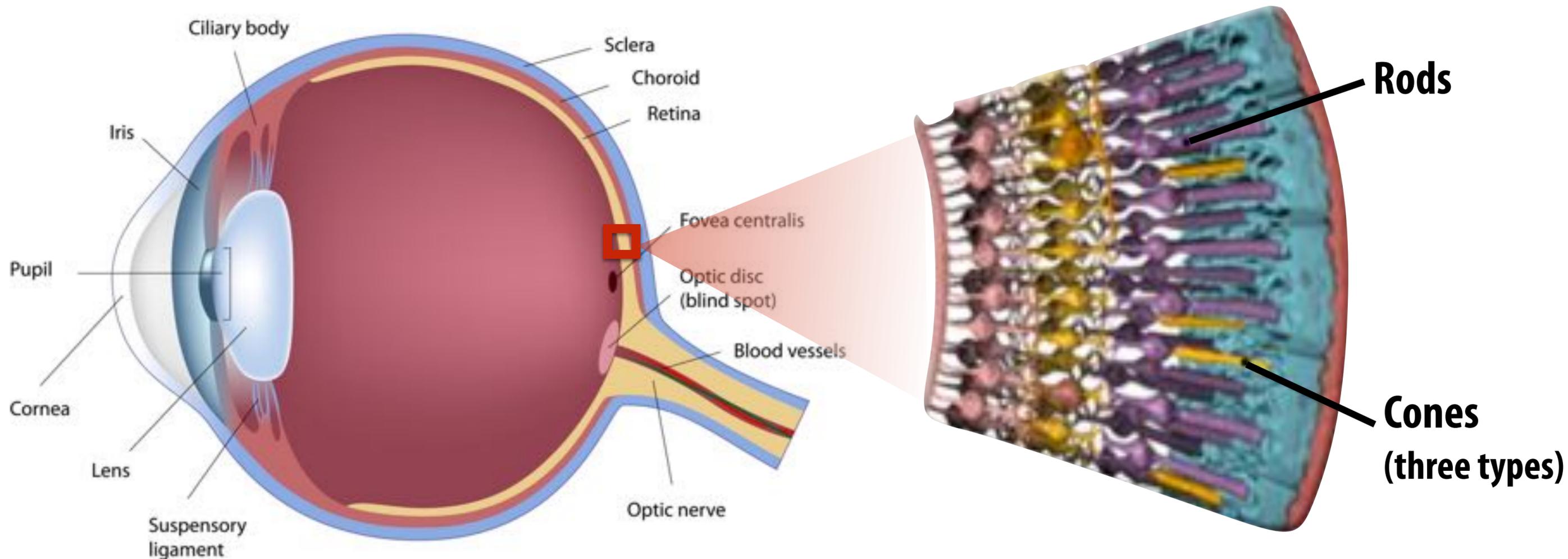
The eye



The eye (optics)

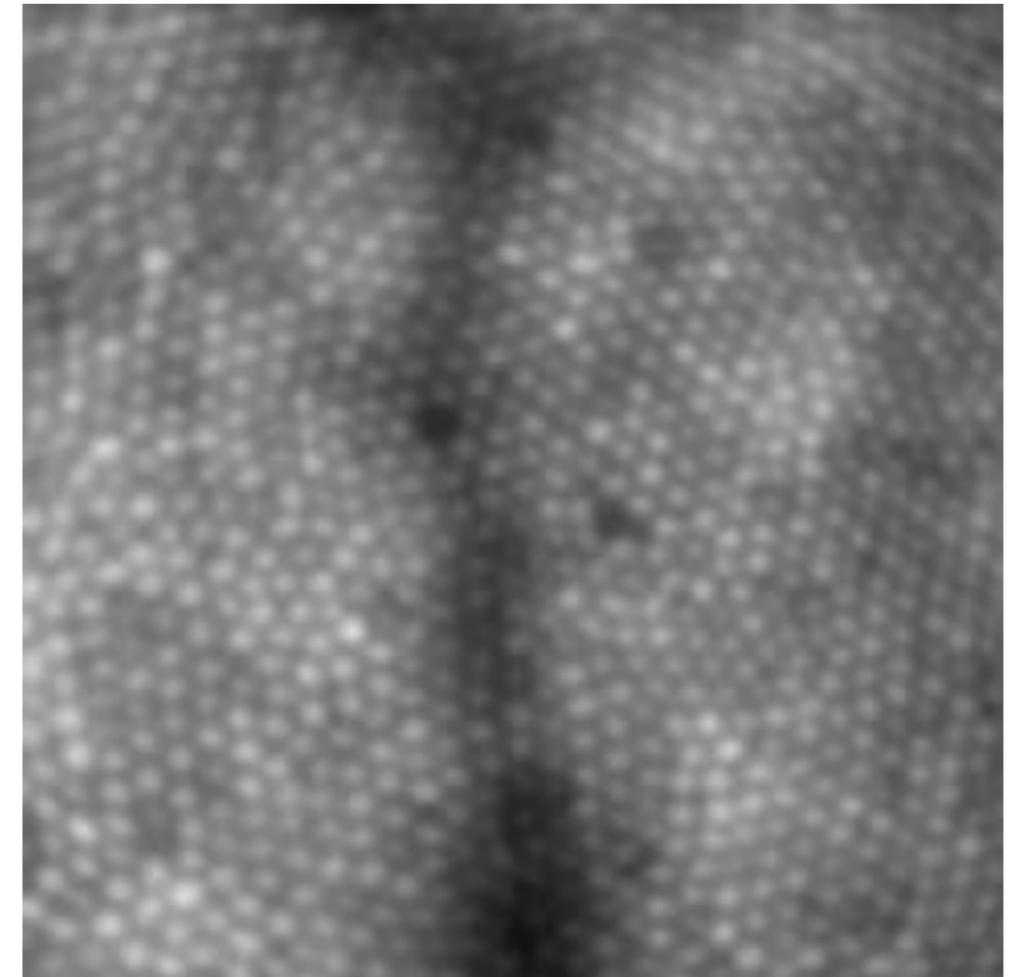
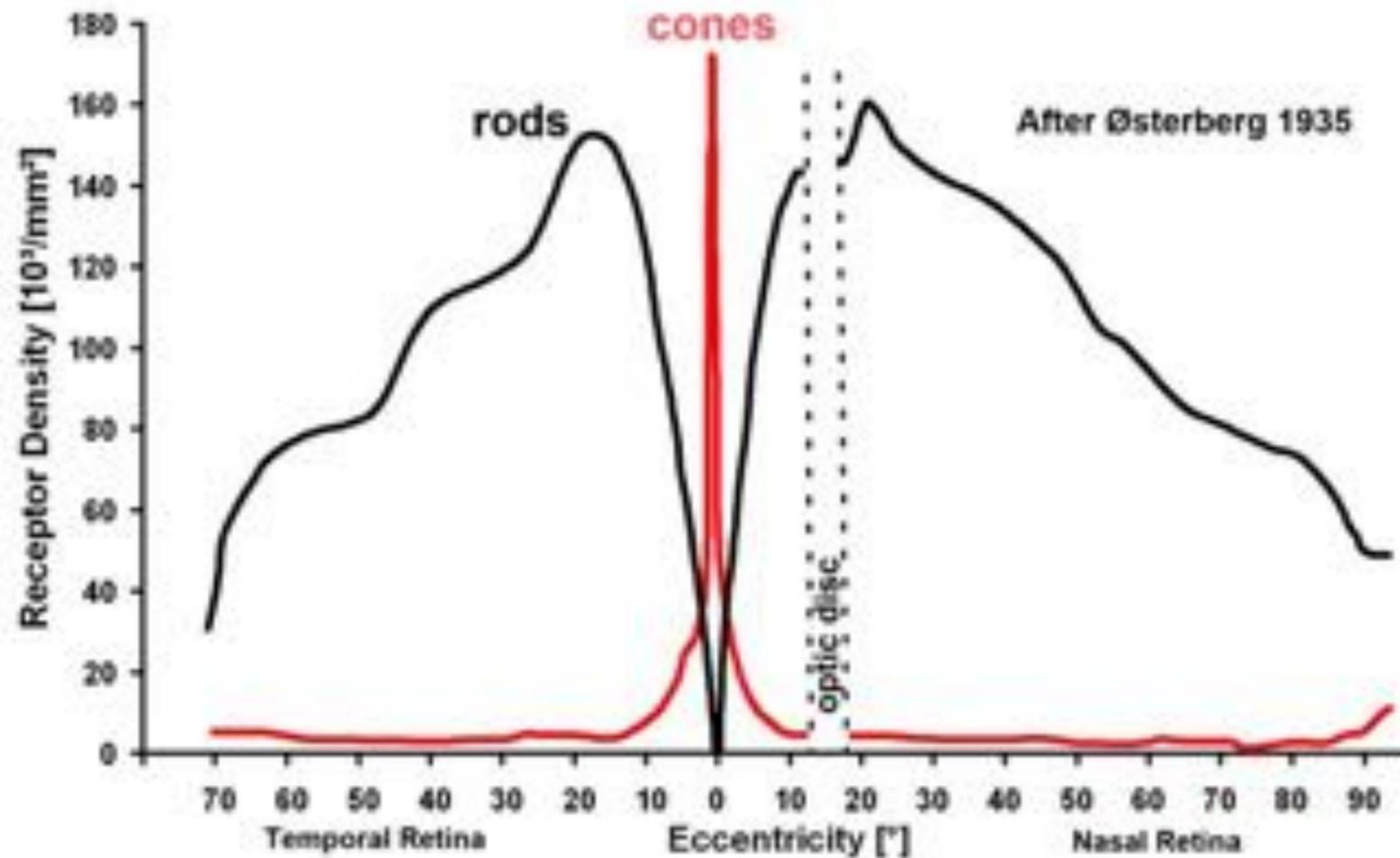


The eye's photoreceptor cells: rods & cones



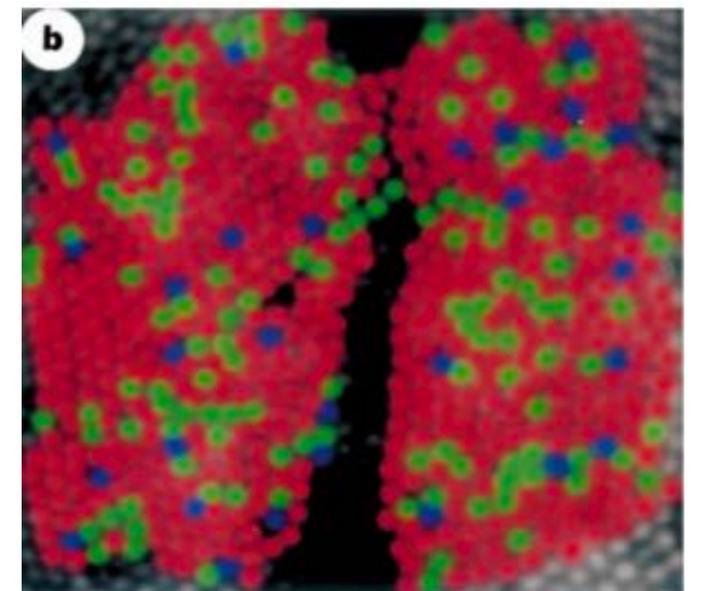
- **Rods are primary receptors under dark viewing conditions (scotopic conditions)**
 - Approx. 120 million rods in human eye
- **Cones are primary receptors under high-light viewing conditions (photopic conditions, e.g., daylight)**
 - Approx. 6-7 million cones in the human eye
 - Each of the three types of cone feature a different spectral response. This will be critical to color vision (much more on this in the coming slides)

Density of rods and cones in the retina



[Roorda 1999]

- Highest density of cones is in fovea
(best color vision at center of where human is looking)
- Note “blind spot” due to optic nerve



ACTIVITY: Rods vs. Cones

- **Grab someone and try it at home!**
 - **Have them hold up colored markers in peripheral vision**
 - **All you have to do is say what color it is (easy!)**



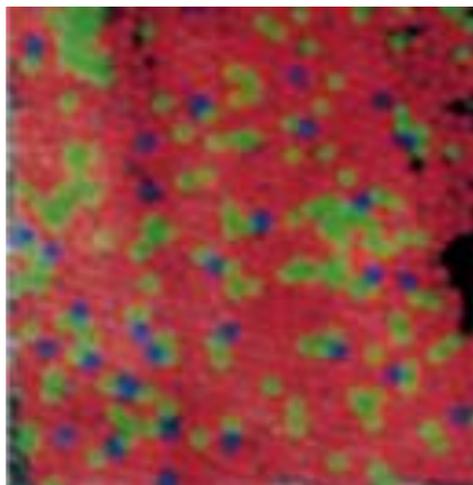
Spectral response of cones

Three types of cones: S, M, and L cones (corresponding to peak response at short, medium, and long wavelengths)

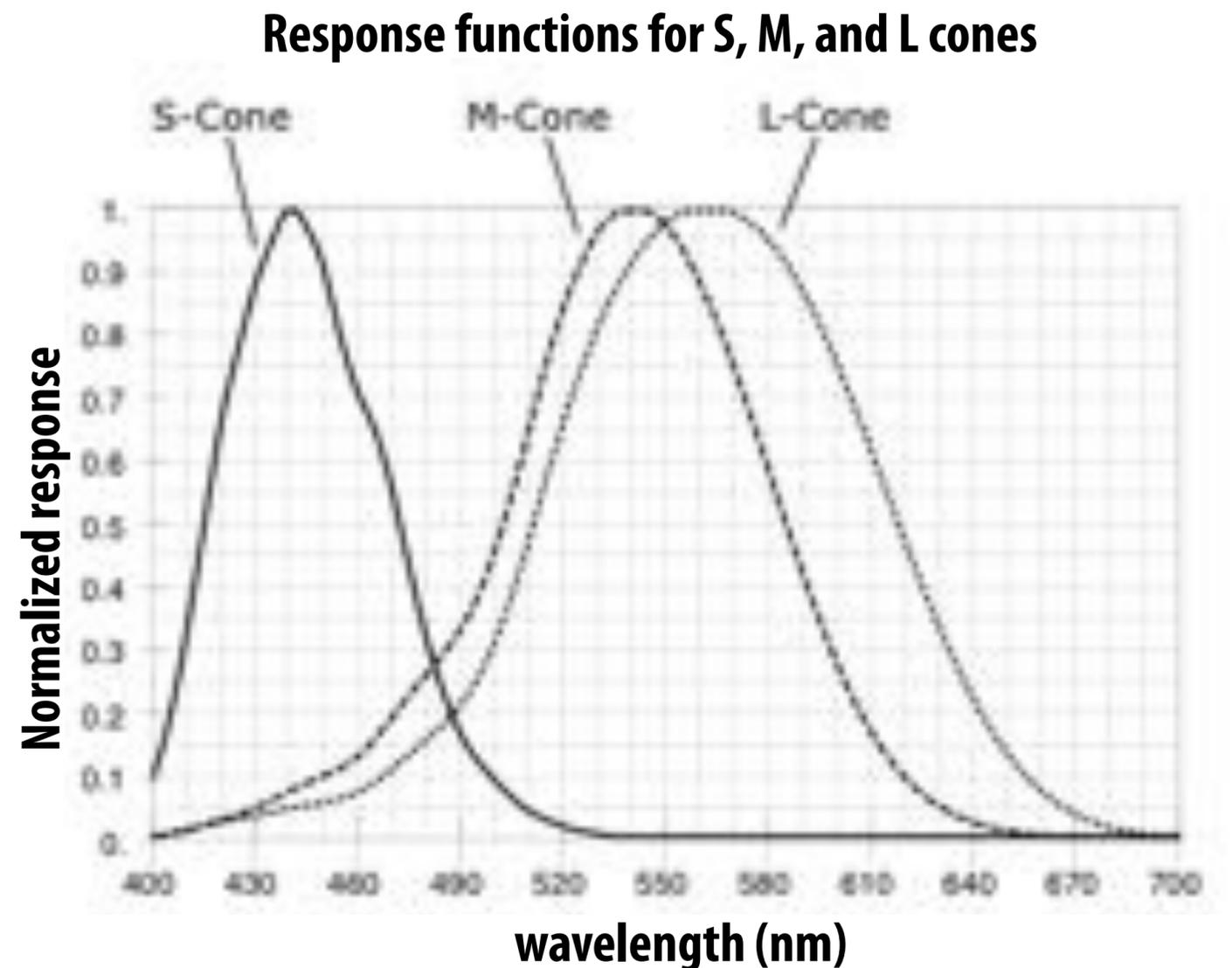
$$S = \int_{\lambda} \Phi(\lambda) S(\lambda) d\lambda$$

$$M = \int_{\lambda} \Phi(\lambda) M(\lambda) d\lambda$$

$$L = \int_{\lambda} \Phi(\lambda) L(\lambda) d\lambda$$

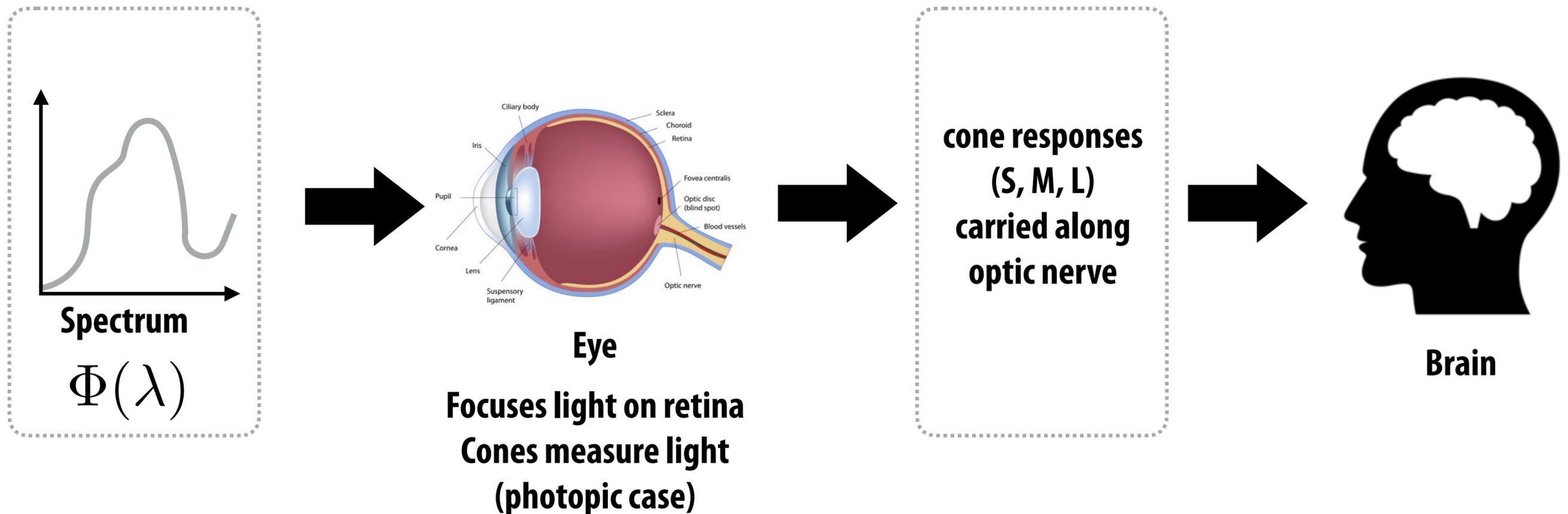


Uneven distribution of cone types in eye
~64% of cones are L cones, ~ 32% M cones



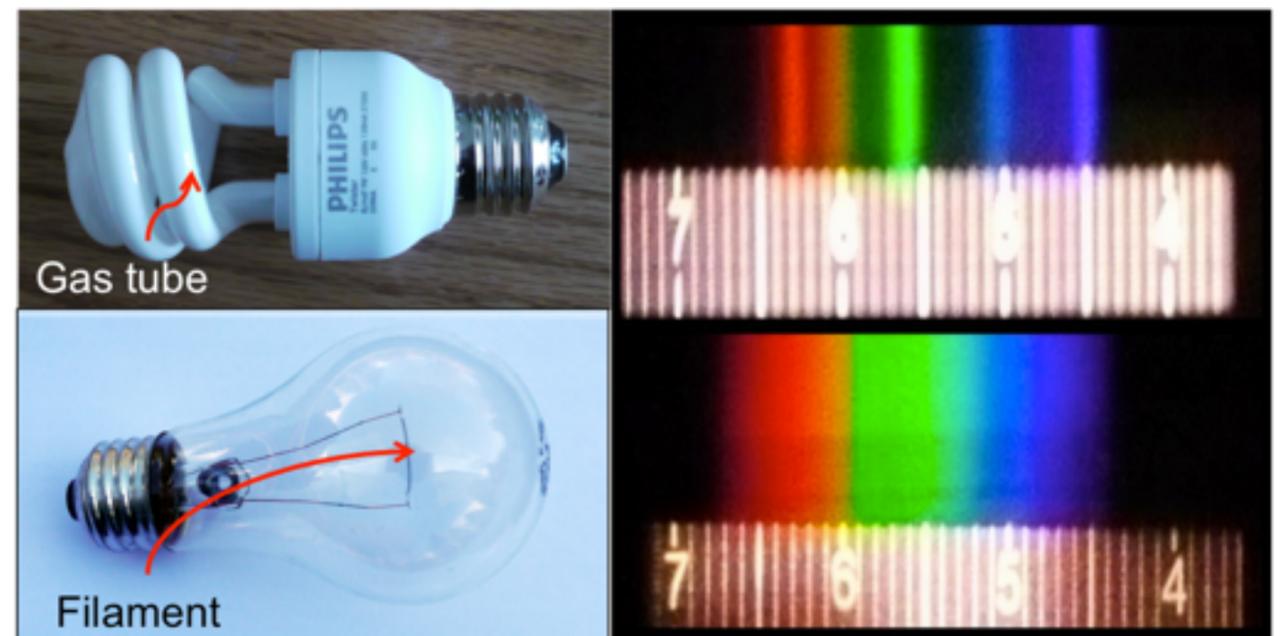
The human visual system

- Human eye does not directly measure the spectrum of incoming light
 - i.e., the brain does not receive “a spectrum” from the eye
- The eye measures three response values = (S, M, L). The result of integrating the incoming spectrum against response functions of S, M, L-cones



Metamers

- **Metamers = two different spectra that integrate to the same (S,M,L) response!**
- **The fact that metameters exist is critical to color reproduction: we don't have to reproduce the exact same spectrum that was present in a real world scene in order to reproduce the perceived color on a monitor (or piece of paper, or paint on a wall)**
- **...On the other hand, combination of light & paint could still cause trouble—different objects appearing “wrong” under different lighting conditions.**

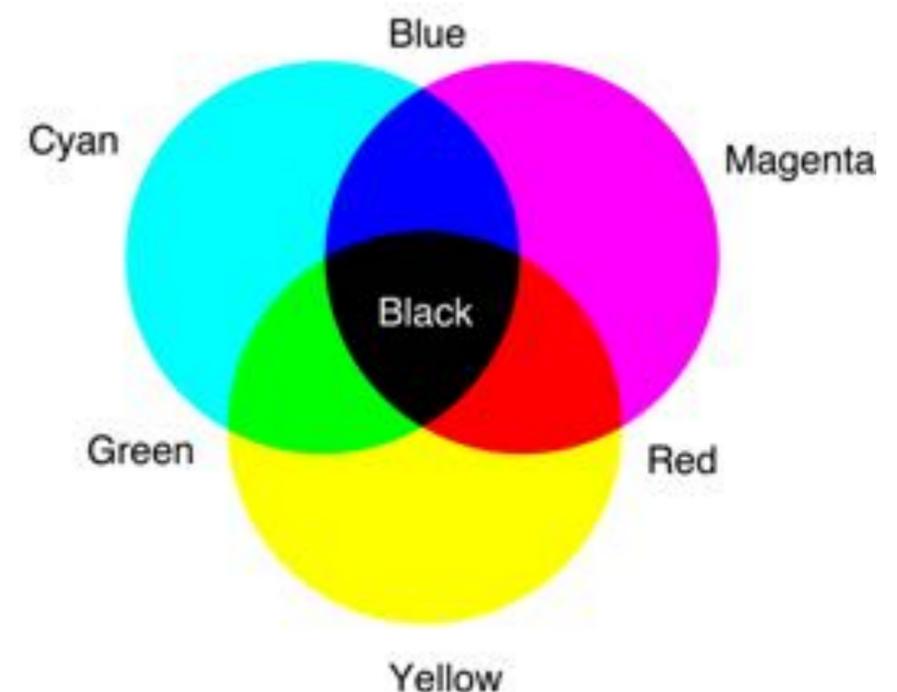
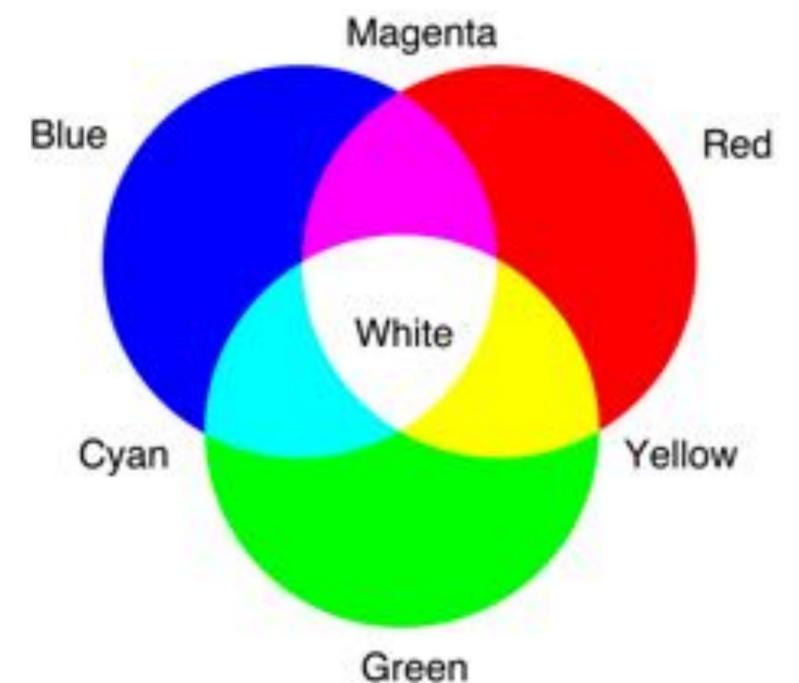


Ok, so color can get pretty complicated!

How do we encode it in a simple(r) way?

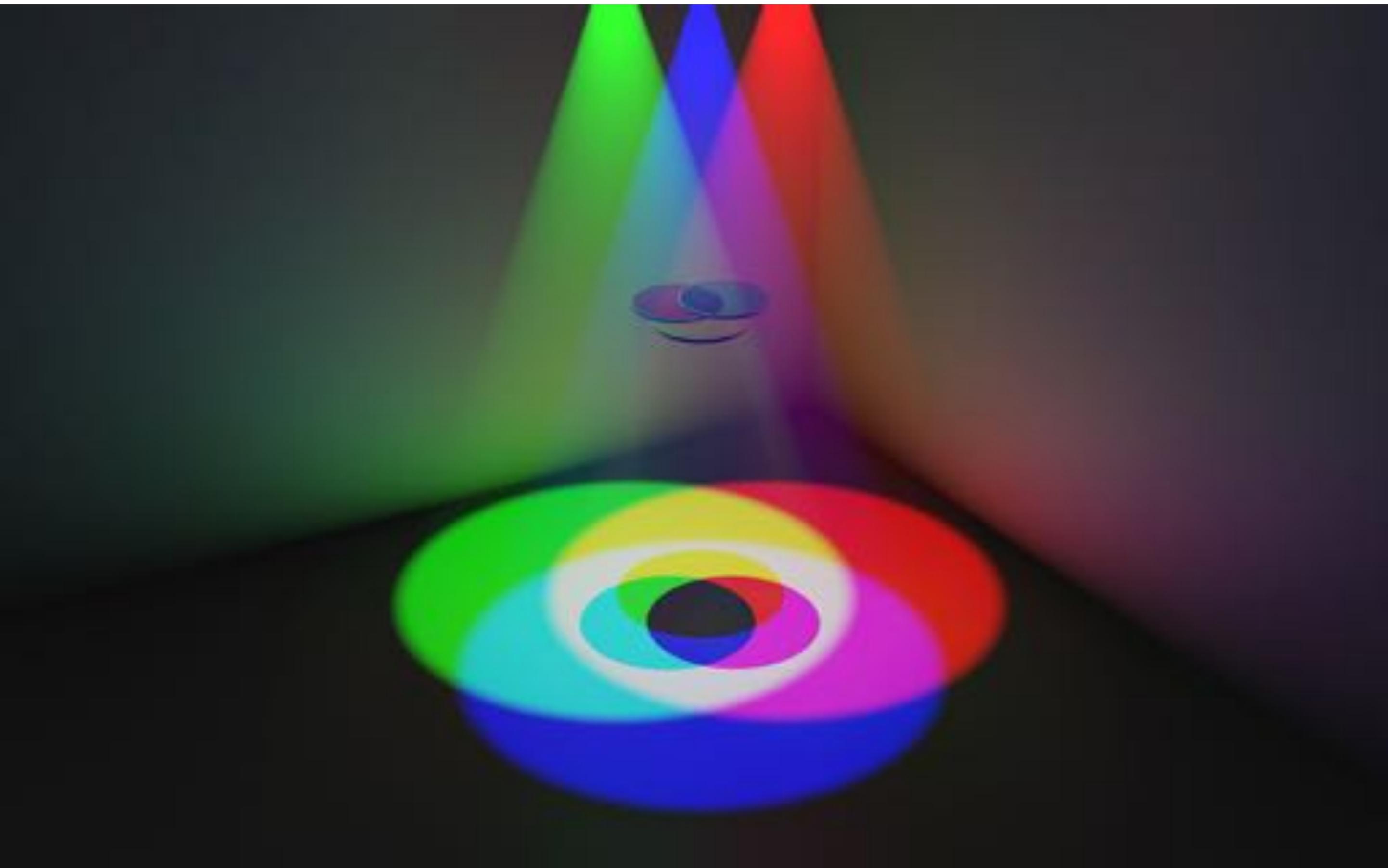
Additive vs. Subtractive Color Models

- **Just like we had emission & absorption spectra, we have additive and subtractive* color models**
- **Additive**
 - **Used for, e.g., combining colored lights**
 - **Prototypical example: RGB**
- **Subtractive**
 - **Used for, e.g., combining paint colors**
 - **Prototypical example: CMYK**



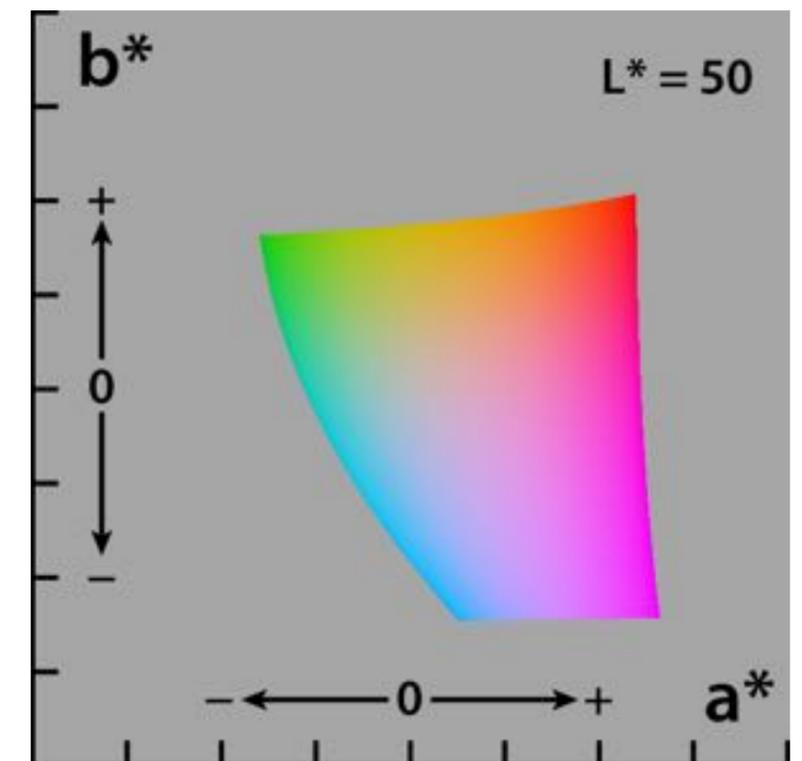
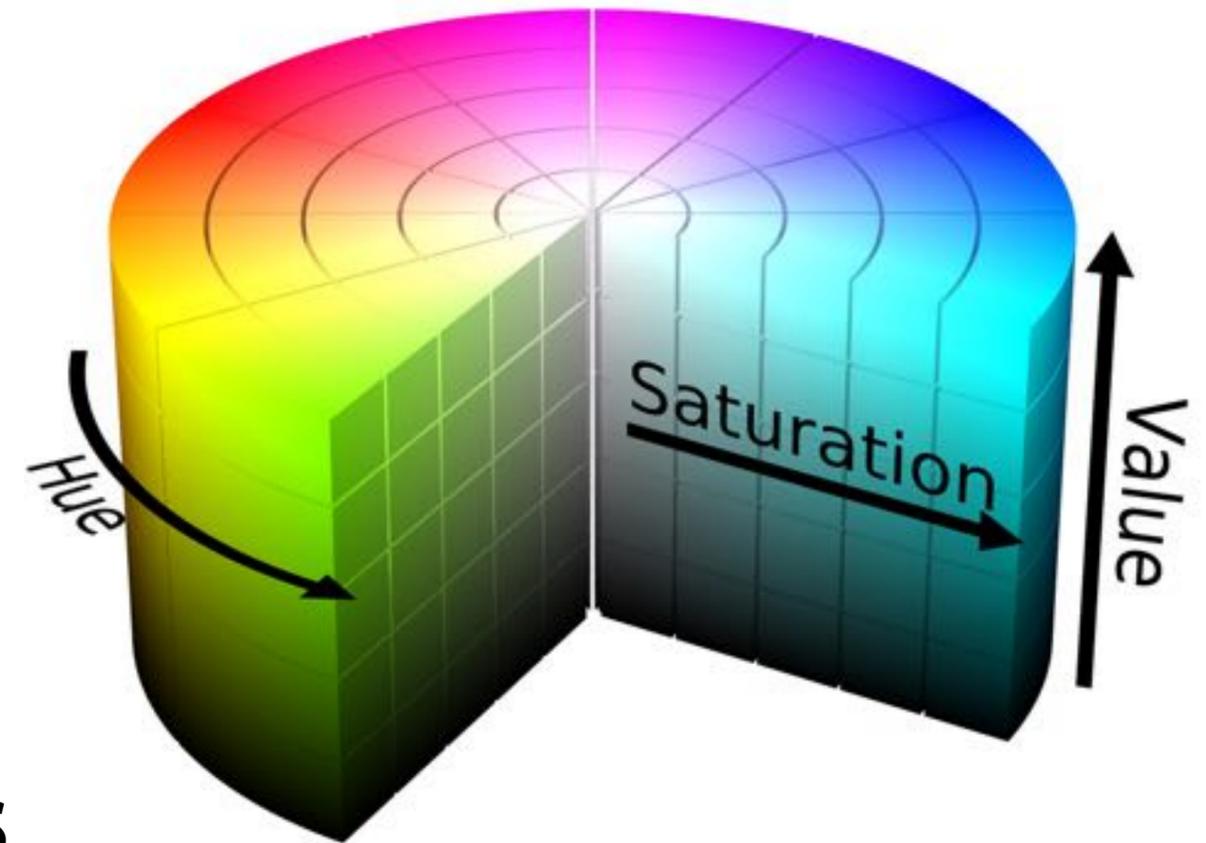
*A better name than subtractive might be multiplicative, since we multiply to get the final color!

Let's shed some light on this picture...



Other Common Color Models

- **HSV**
 - hue, saturation, value
 - more intuitive than RGB/CMYK
- **SML—physiological model**
 - corresponds to stimulus of cones
 - not practical for most color work
- **XYZ—preceptually-driven model**
 - Y captures luminance (intensity)
 - X,Z capture chromaticity (color)
 - related to, but different from, SML
- **Lab—“perceptually uniform” modification of XYZ**



Example: Y'CbCr color model

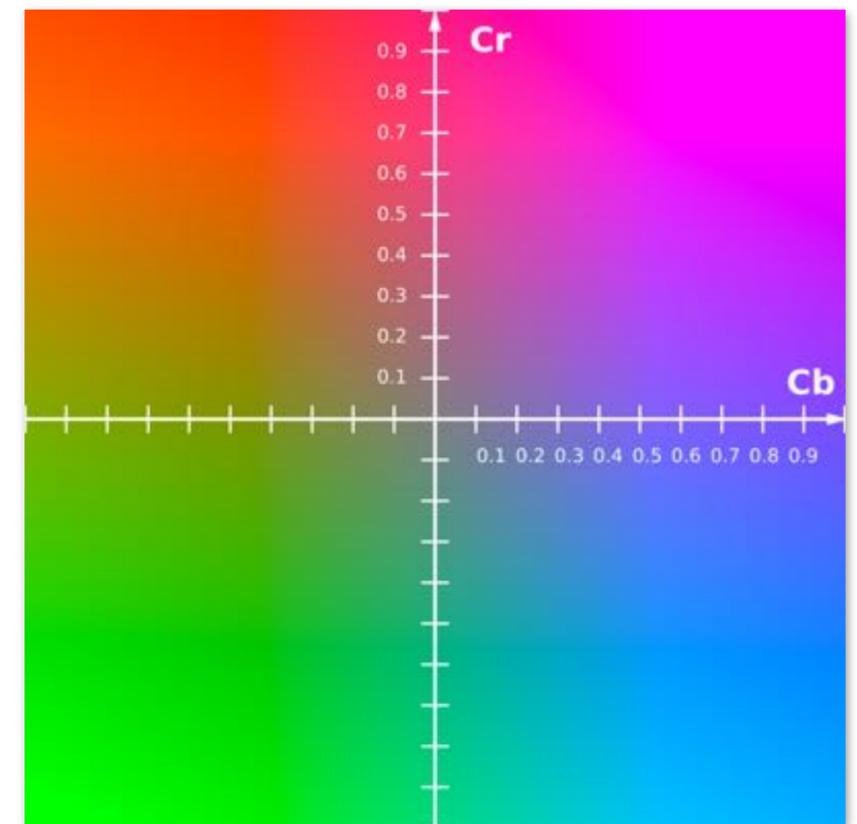
- Common for modern digital video
- Y' = luma: perceived luminance (same as L^* in CIELAB)
- Cb = blue-yellow deviation from gray
- Cr = red-cyan deviation from gray



Y'



Cr





Original picture



**Contents of CbCr color channels downsampled by a factor of 20 in each dimension
(400x reduction in number of samples)**



Full resolution sampling of luma (Y')



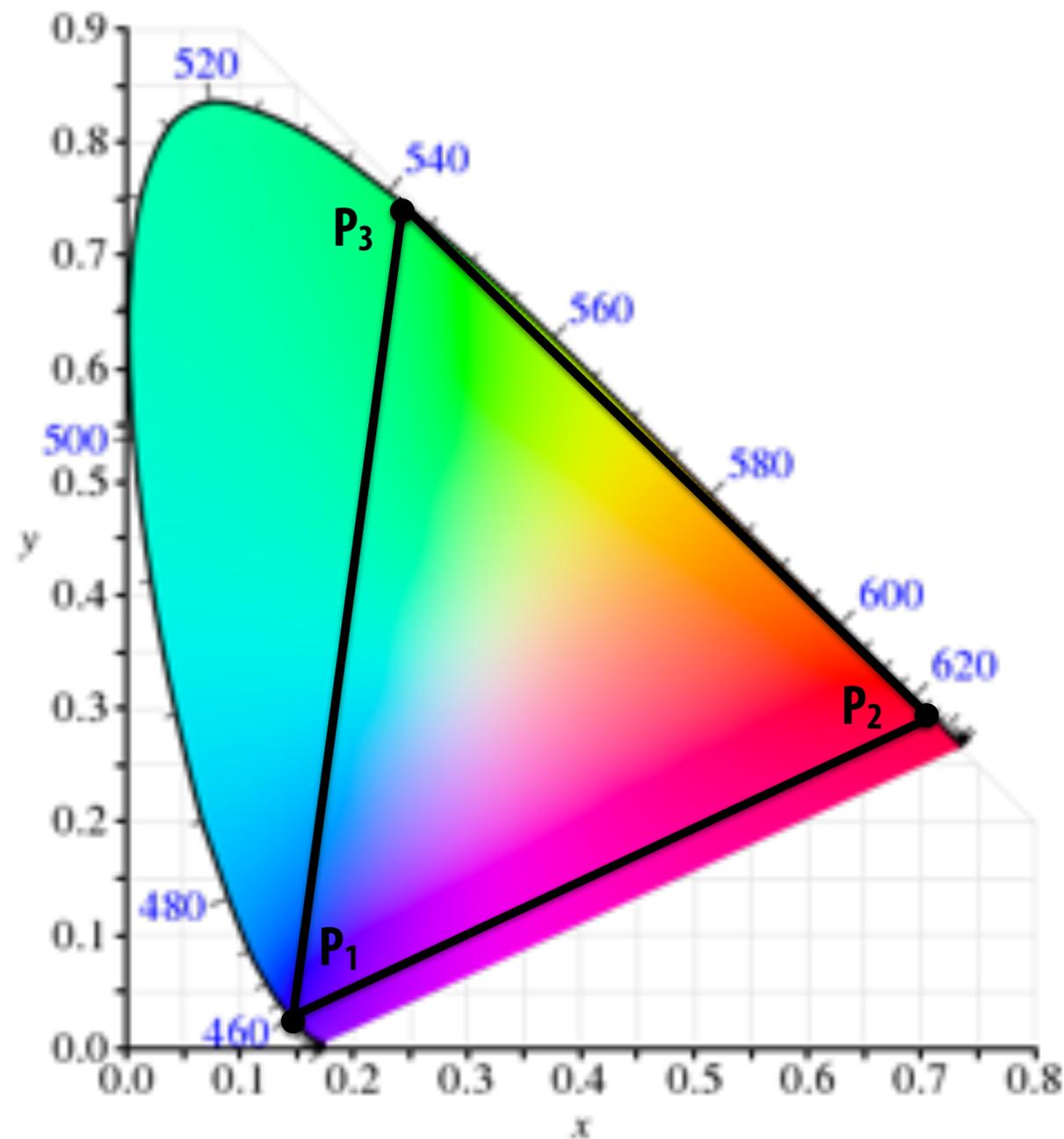
**Reconstructed result
(looks pretty good)**



Original picture

Chromaticity Diagrams

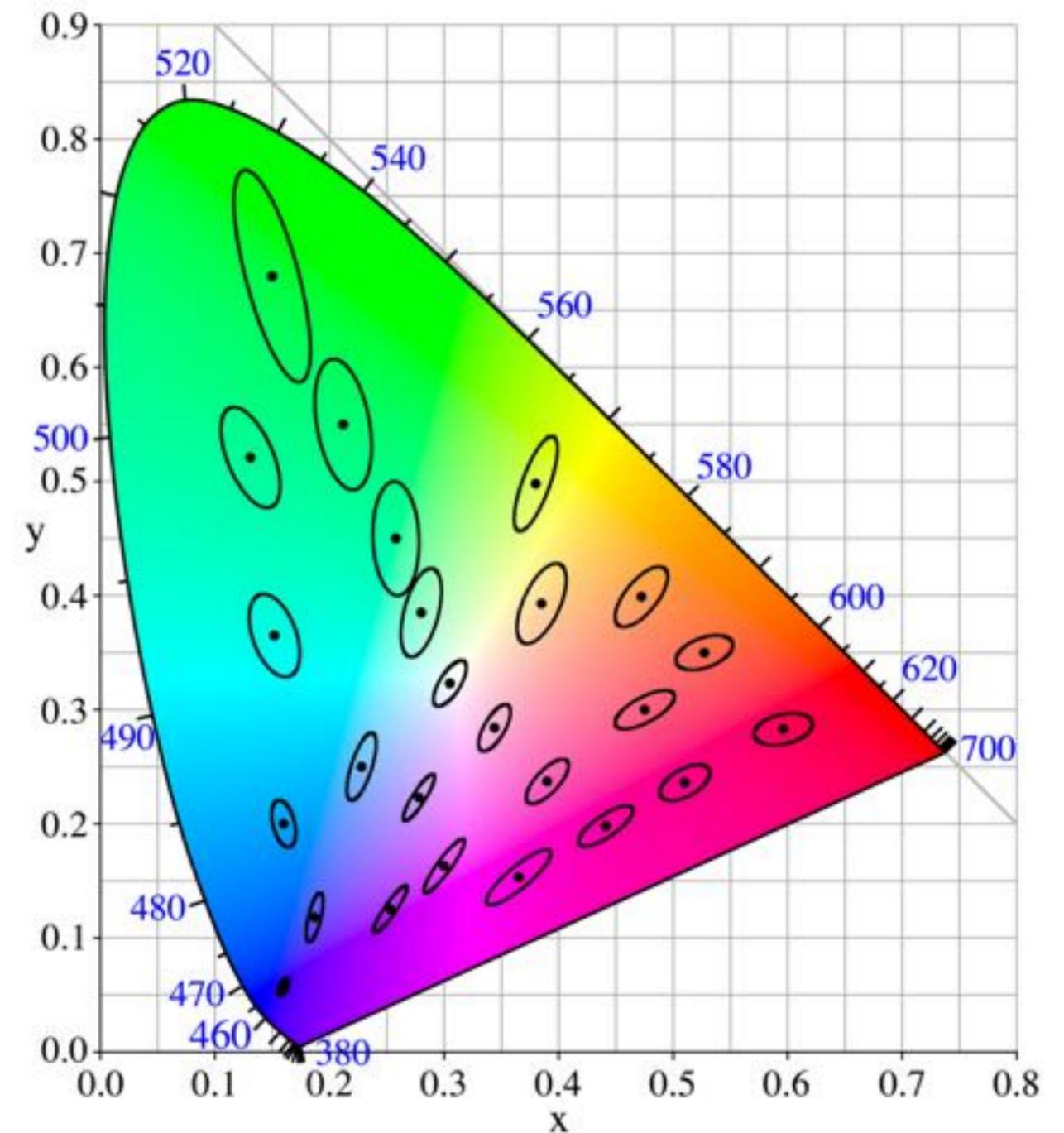
- Chromaticity is the intensity-independent component of a color
- Chromaticity diagram used to visualize extent of a color space



A display with primaries with chromacities P_1 , P_2 , P_3 can create colors that are combinations of these primaries (colors that fall within the triangle)

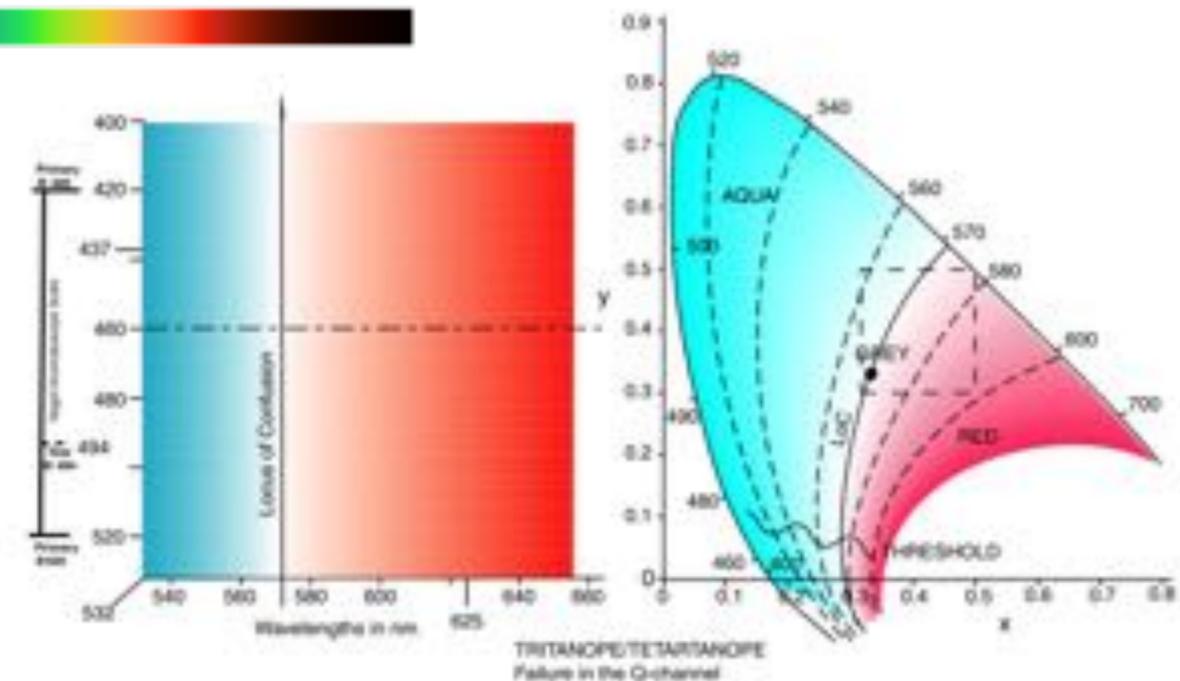
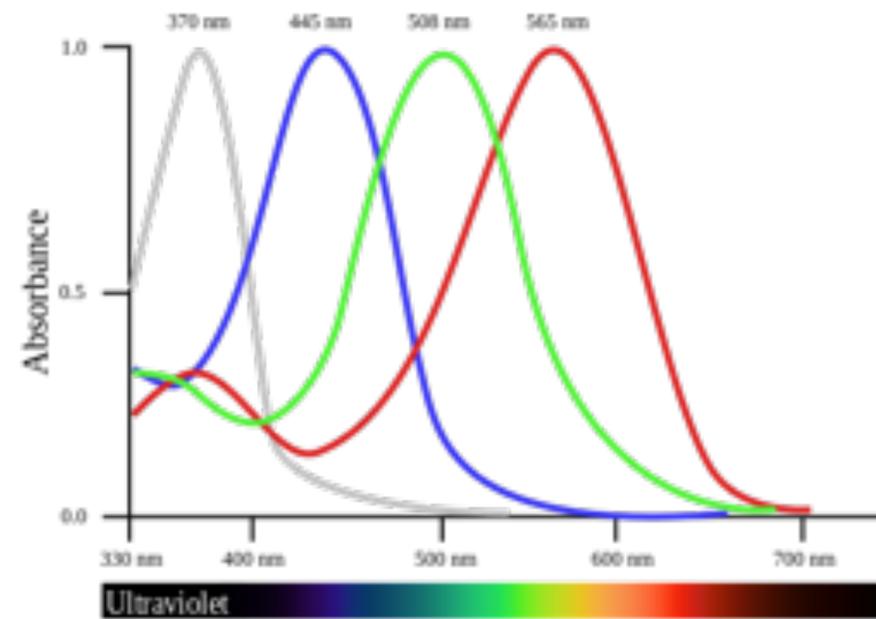
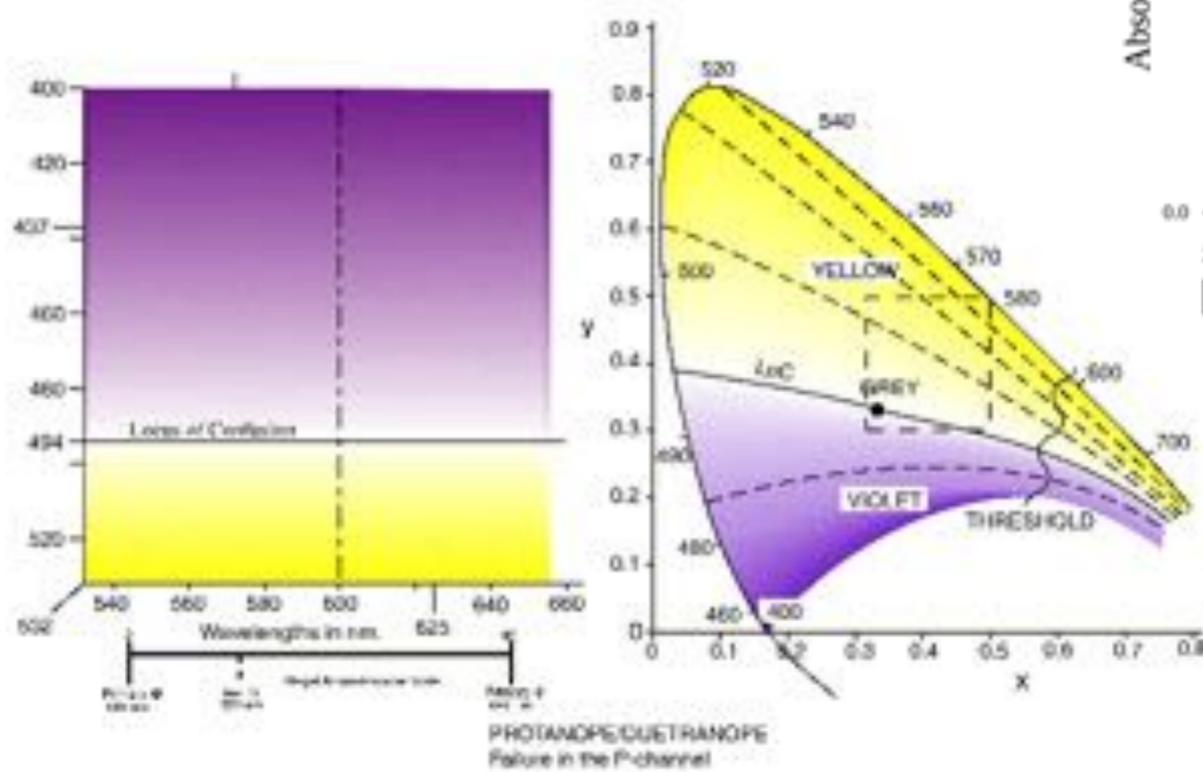
Color Acuity (MacAdam Ellipse)

- In addition to range of colors visible, one might be interested in how sensitive people are to changes in color
- Each ellipse corresponds to a region of “just noticeable differences” of color (chromaticity)
- So, if you want to make two colors distinct, at bare minimum should avoid overlapping ellipses...



Nonstandard Color Vision

- Morphological differences in eye can cause people (& animals) to see different ranges of color (e.g., more/fewer cone types)
- Alternative chromaticity diagrams help visualize color gamut, useful for designing, e.g., widely-accessible interfaces



Radiometry

Computer Graphics
CMU 15-462/15-662

So far, we have been discussing color

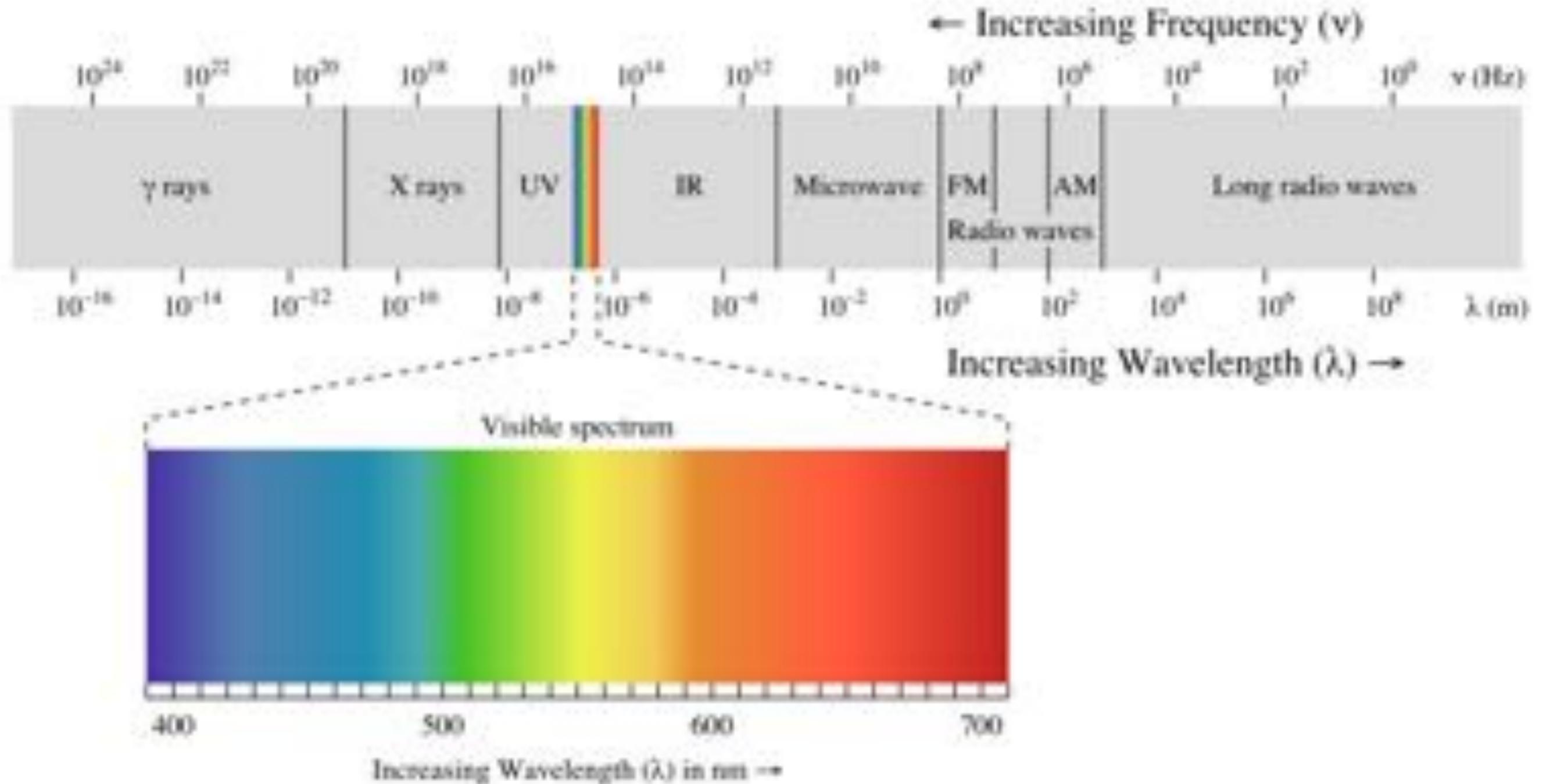


Image credit: Licensed under CC BY-SA 3.0 via Commons

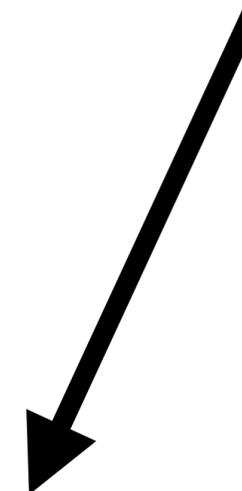
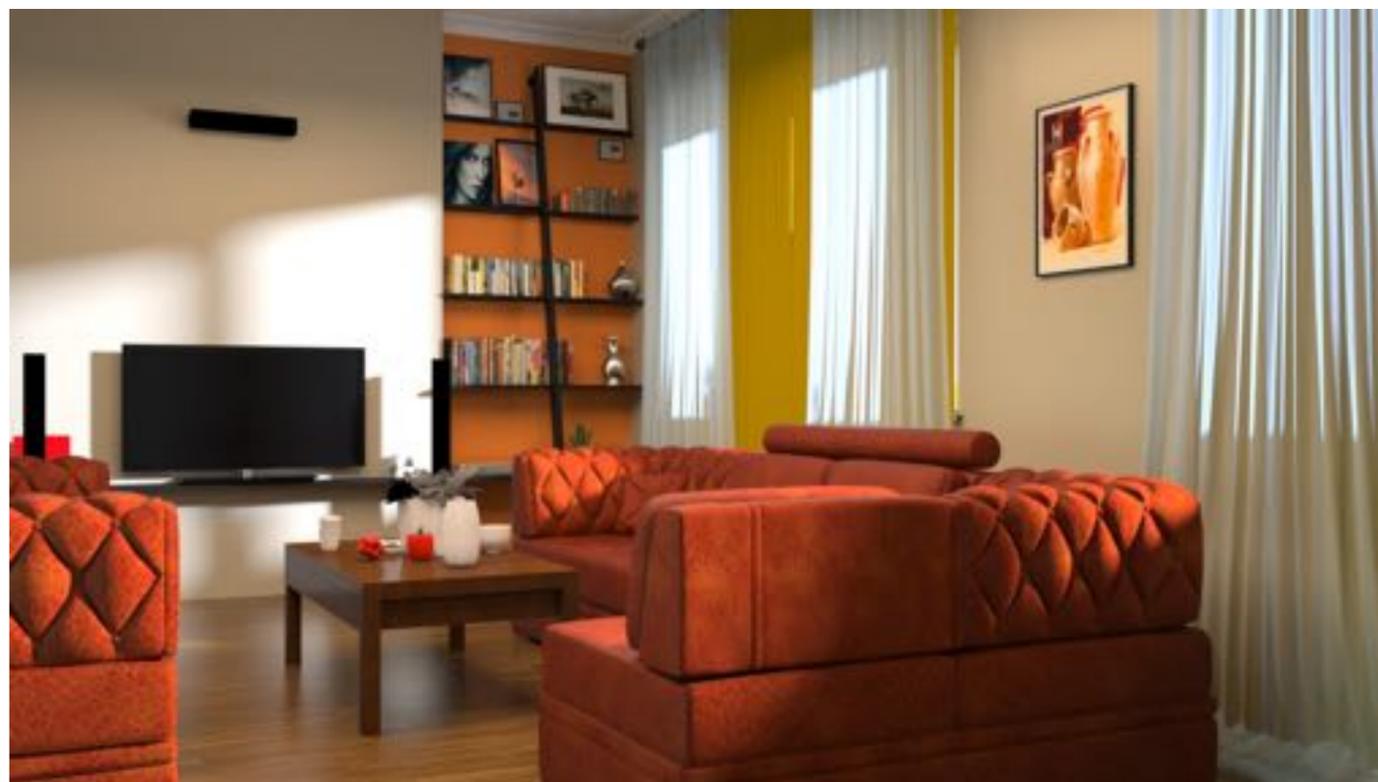
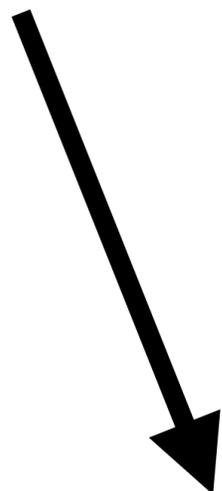
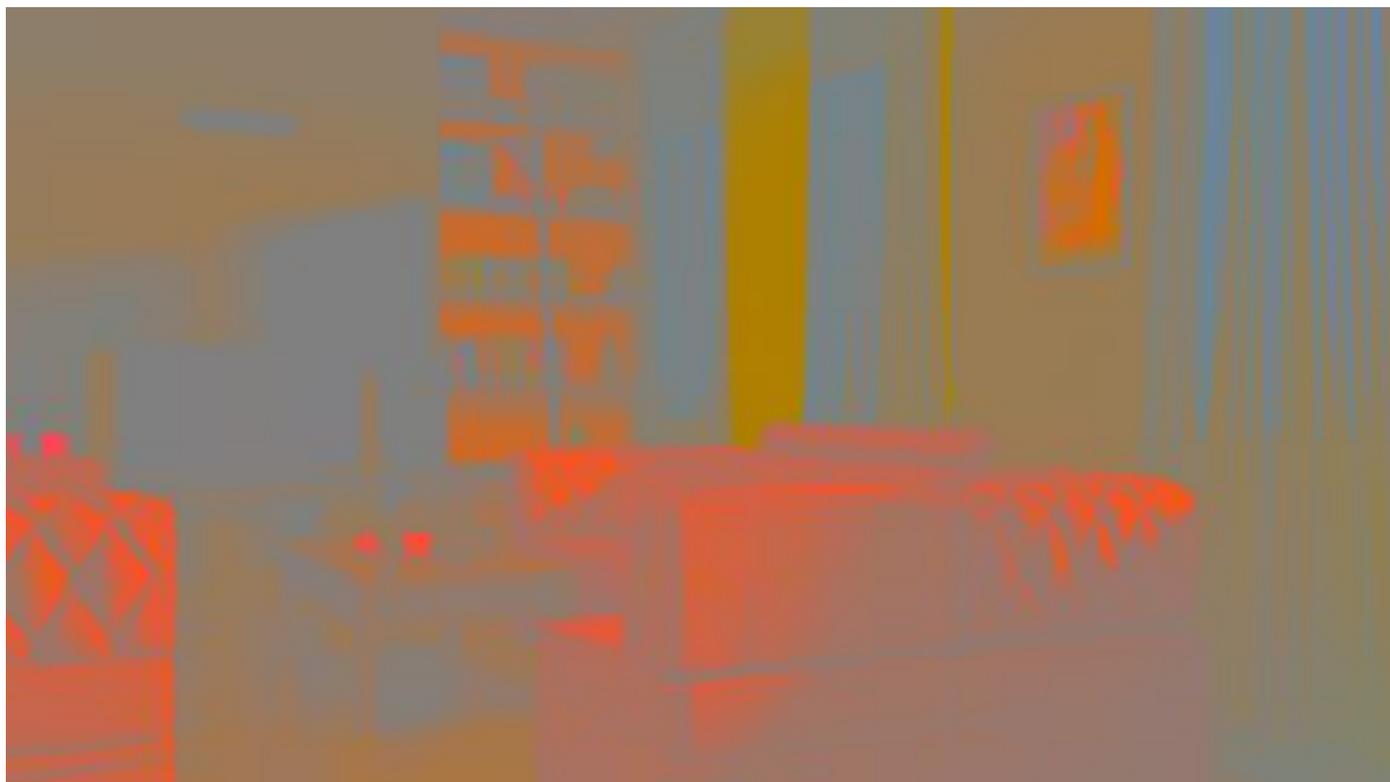
https://commons.wikimedia.org/wiki/File:EM_spectrum.svg#/media/File:EM_spectrum.svg

Rendering is more than just color!

- Also need to know how much light hits each pixel:

color

intensity



image

How do we quantify measurements of light?

Radiometry

- **System of units and measures for measuring EM radiation (light)**
- **Geometric optics model of light**
 - **Photons travel in straight lines**
 - **Represented by rays**
 - **Wavelength \ll size of objects**
 - **No diffraction, interference, ...**
 - **LOTS of terminology!**
 - **Focus first on concepts**
 - **Terminology comes second**



What do we want to measure (and why?)



- **Many physical processes convert energy into photons**
 - **E.g., incandescent lightbulb turns heat into light (blackbody radiation)**
 - **Nuclear fusion in stars (sun!) generates photons**
 - **Etc.**
- **Each photon carries a small amount of energy**
- **Want some way of recording “how much energy”**
- **Energy of photons hitting an object ~ “brightness”**
 - **Film, eyes, CCD sensor, sunburn, solar panels, ...**
 - **Need this information to make accurate (and beautiful!) images**
- **Simplifying assumption: “steady state” process**
 - **How long does it take for lighting to reach steady state?**

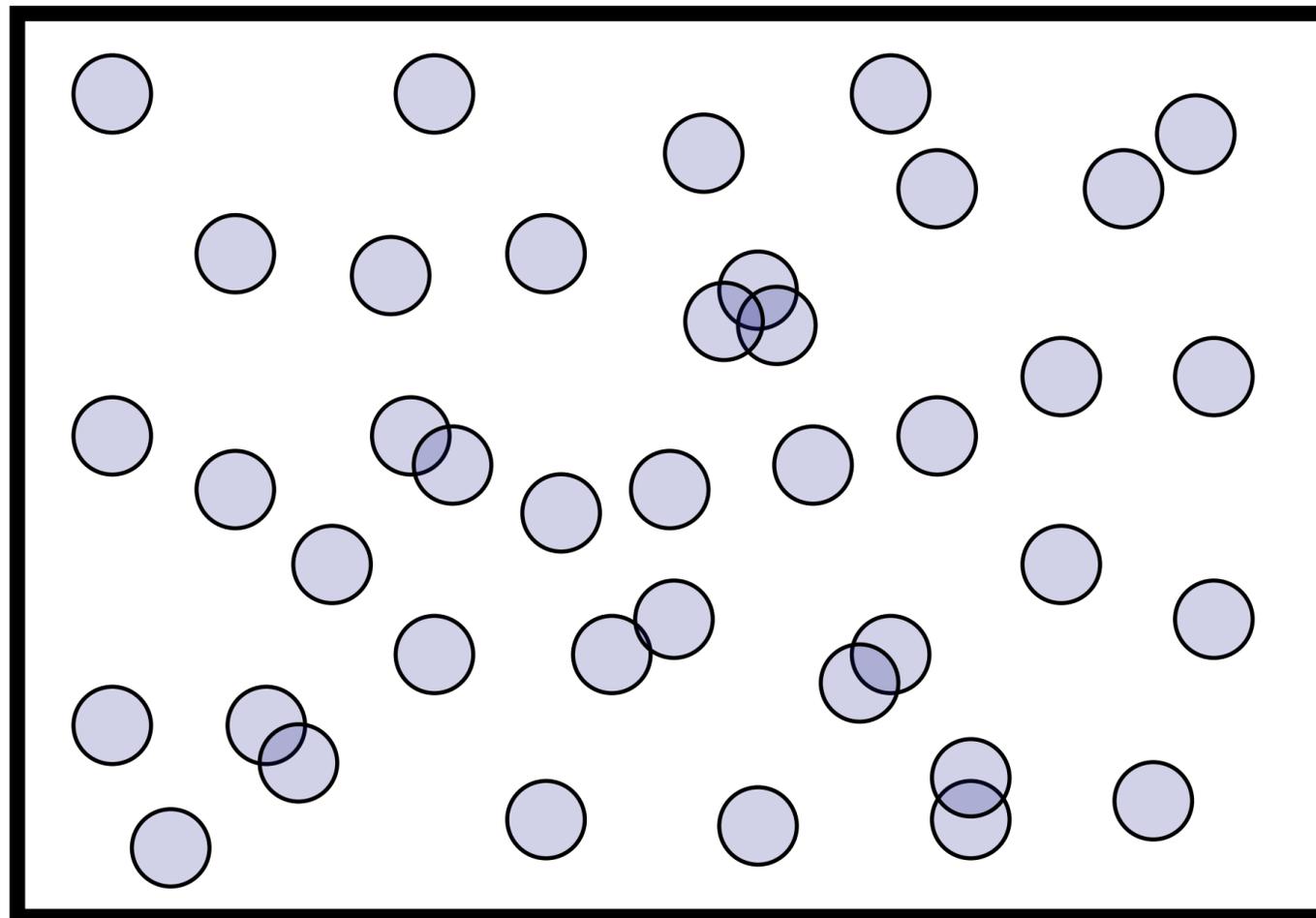
Imagine every photon is a little rubber ball hitting the scene:



How can we record this process? What information should we store?

Radiant energy is “total # of hits”

- One idea: just store the total number of “hits” that occur anywhere in the scene, over the complete duration of the scene
- This quantity captures the total energy of all the photons hitting the scene*

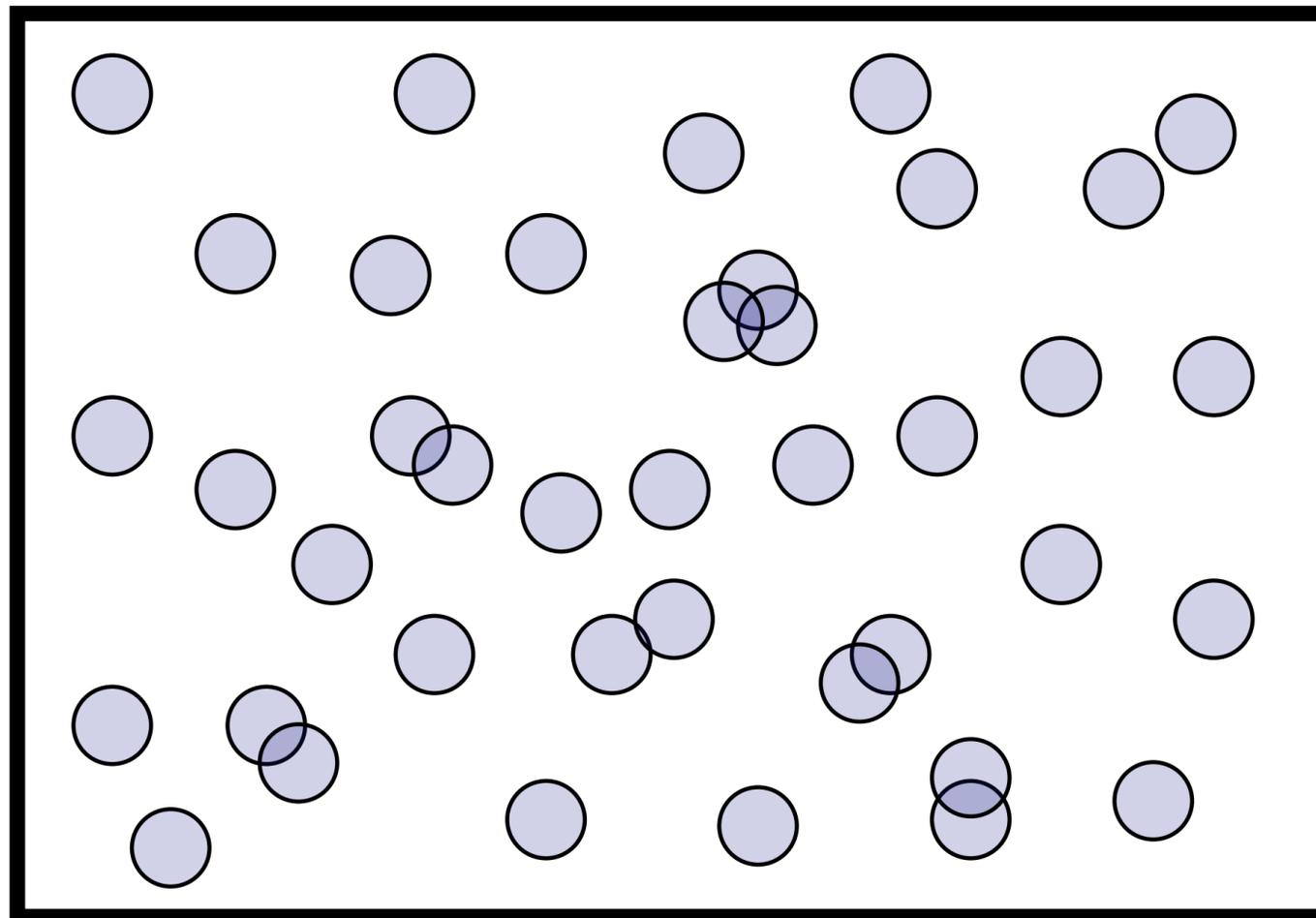


“Radiant energy”: 40

*Eventually we will care about constants & units. But these will not help our conceptual understanding...

Radiant flux is “hits per second”

- For illumination phenomena at the level of human perception, usually safe to assume equilibrium is reached immediately.
- So, rather than record total energy over some (arbitrary) duration, may make more sense to record total hits per second

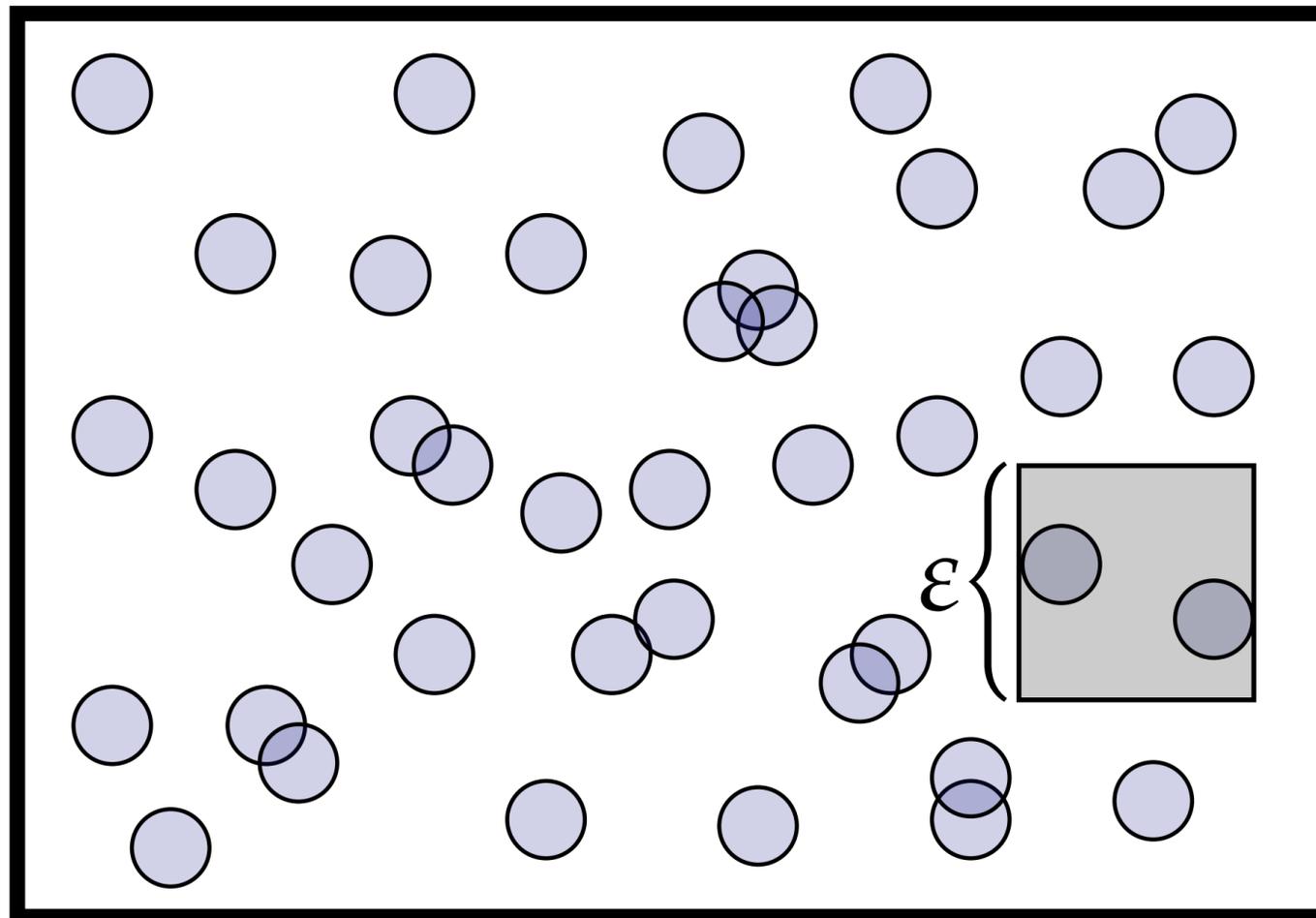


(Our video takes .05s to display each “hit”!)

Estimate of “radiant flux”: $40 \text{ hits}/2s = 20 \text{ hits/s}$

Irradiance is “#hits per second, per unit area”

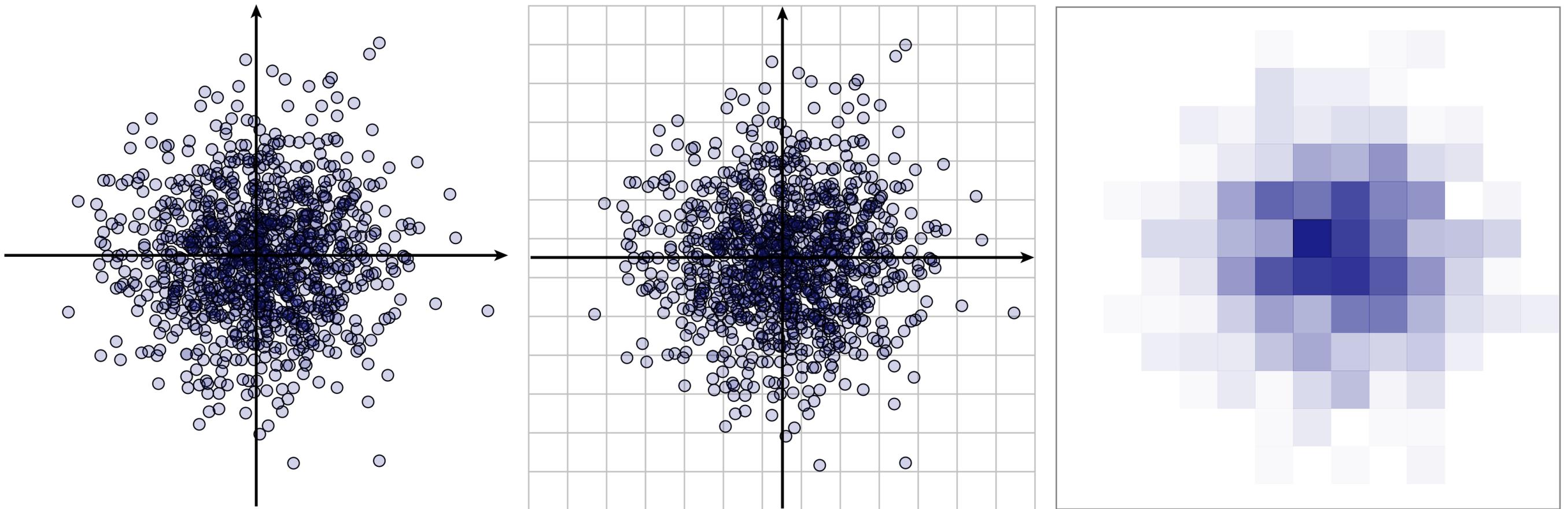
- Typically we want to get more specific than just the total
- To make images, also need to know where hits occurred
- So, compute hits per second in some “really small” area, divided by area:



Estimate of “radiant energy density”: $2/\epsilon^2$

Image generation as irradiance estimation

- From this point of view, our goal in image generation is to estimate the irradiance at each point of an image (or really: the total radiant flux per pixel...):



Recap so far...

Radiant Energy
(total number of hits)

Radiant Energy Density
(hits per unit area)

Radiant Flux
(total hits per second)

Radiant Flux Density
a.k.a. *Irradiance*
(hits per second per unit area)

Ok, but how about some units...

Measuring illumination: radiant energy

- How can we be more precise about the amount of energy?
- Said we were just going to count “the number of hits,” but do all hits contribute same amount of energy?
- Energy of a single photon:

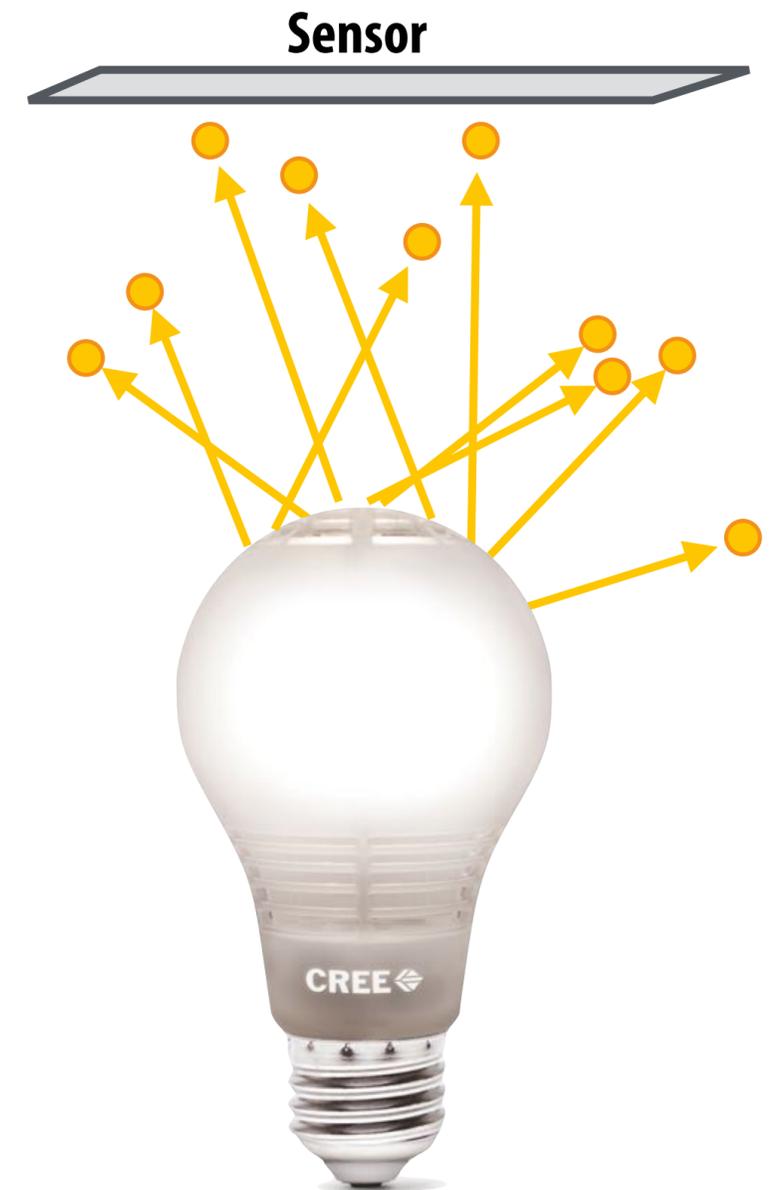
$$Q = \frac{hc}{\lambda}$$

Planck's constant h speed of light c wavelength (color!) λ

$$h \approx 6.626 \times 10^{-34} \text{ J}\cdot\text{s} \quad (\text{Joules times seconds})$$

$$c \approx 3.00 \times 10^8 \text{ m/s} \quad (\text{meters per second})$$

$$\lambda \approx 390\text{--}700 \times 10^{-3} \text{ m (visible)}$$



Q: What are units for a photon?

$$\frac{(\text{J} \times \text{s})(\text{m/s})}{\text{m}} = \text{J}$$

Aside: Units are a powerful debugging tool!

Measuring illumination: radiant flux (power)

- **Flux: energy per unit time (Watts) received by the sensor (or emitted by the light)**

$$\Phi = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \left[\frac{\text{J}}{\text{s}} \right]$$

← "Watts"

- **Can also go the other direction: time integral of flux is total radiant energy**

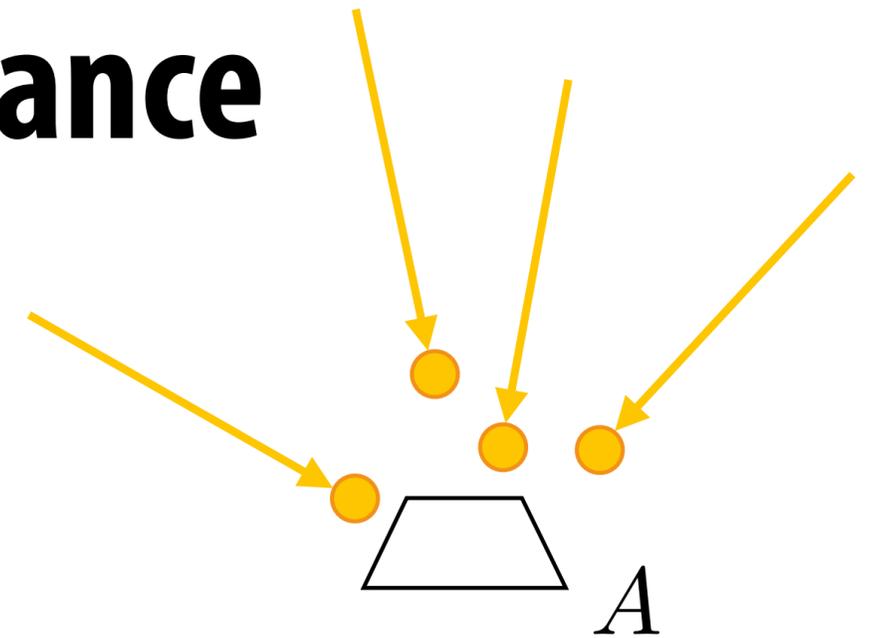
$$Q = \int_{t_0}^{t_1} \Phi(t) dt$$

(Units?)



Measuring illumination: irradiance

- Radiant flux: time density of energy
- Irradiance: area density of radiant flux



Given a sensor of with area A , we can consider the average flux over the entire sensor area:

$$\frac{\Phi}{A}$$

Irradiance (E) is given by taking the limit of area at a single point on the sensor:

$$E(p) = \lim_{\Delta \rightarrow 0} \frac{\Delta \Phi(p)}{\Delta A} = \frac{d\Phi(p)}{dA} \left[\frac{\text{W}}{\text{m}^2} \right]$$

Recap, with units

Radiant Energy

(total number of hits)

Joules (J)

Radiant Energy Density

(hits per unit area)

Joules per square meter (J/m^2)

Radiant Flux

(total hits per second)

Joules per second (J/s) = Watts (W)

Radiant Flux Density

a.k.a. *Irradiance*

(hits per second per unit area)

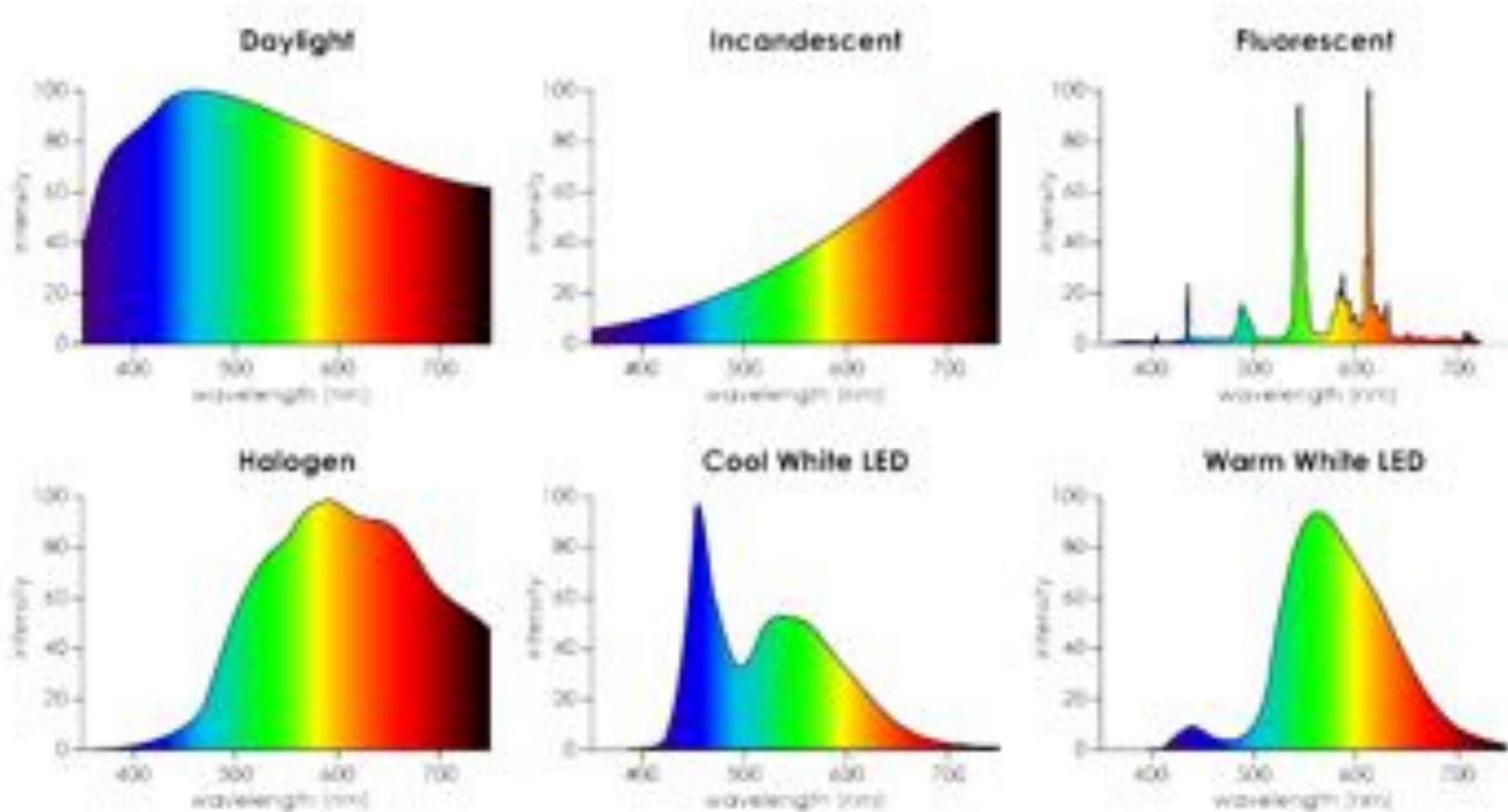
Watts per square meter (W/m^2)

What about color?

**How might we quantify, say, the
“amount of **green**?”**

Spectral power distribution

- Describes irradiance per unit wavelength (units?)



Energy per unit time per unit area per unit wavelength...

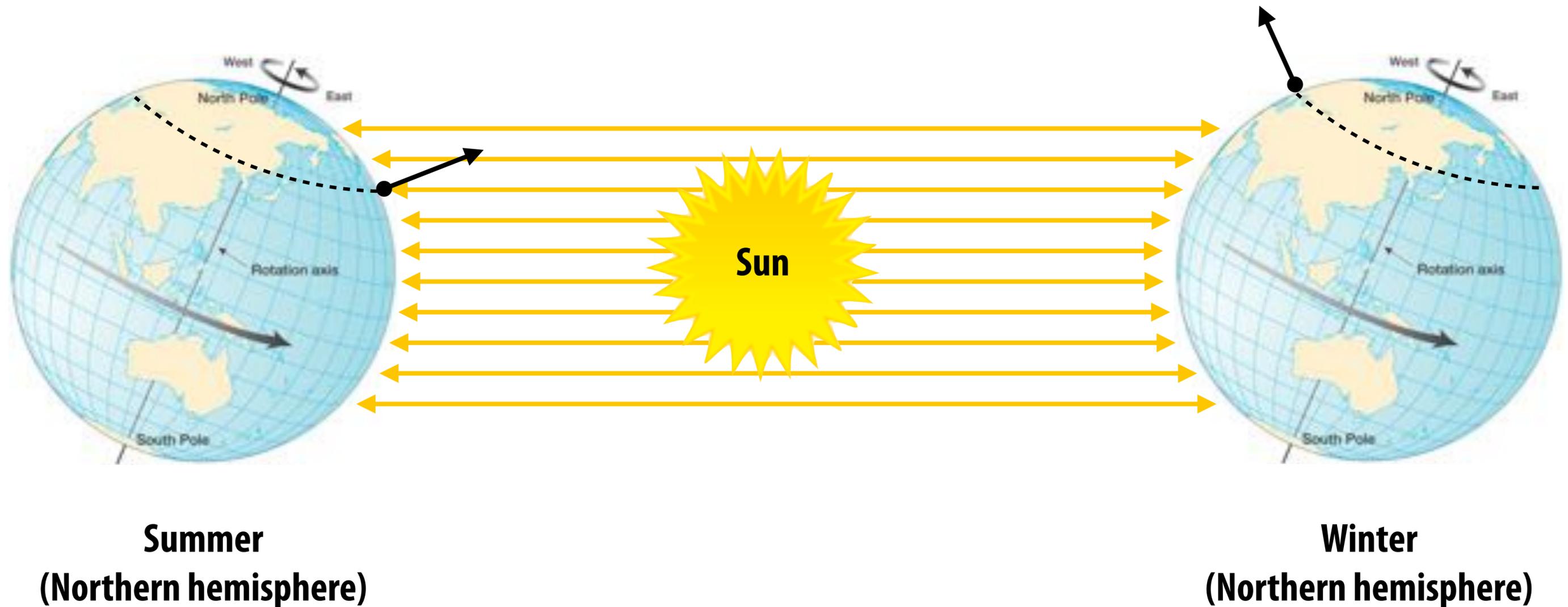
Figure credit:

**Given what we now know
about radiant energy...**



**Why do some parts of a
surface look lighter or darker?**

Why do we have seasons?



Earth's axis of rotation: $\sim 23.5^\circ$ off axis

Beam power in terms of irradiance

Consider beam with flux Φ incident on surface with area A

irradiance
(energy per time,
per area)

radiant flux
(energy per time)

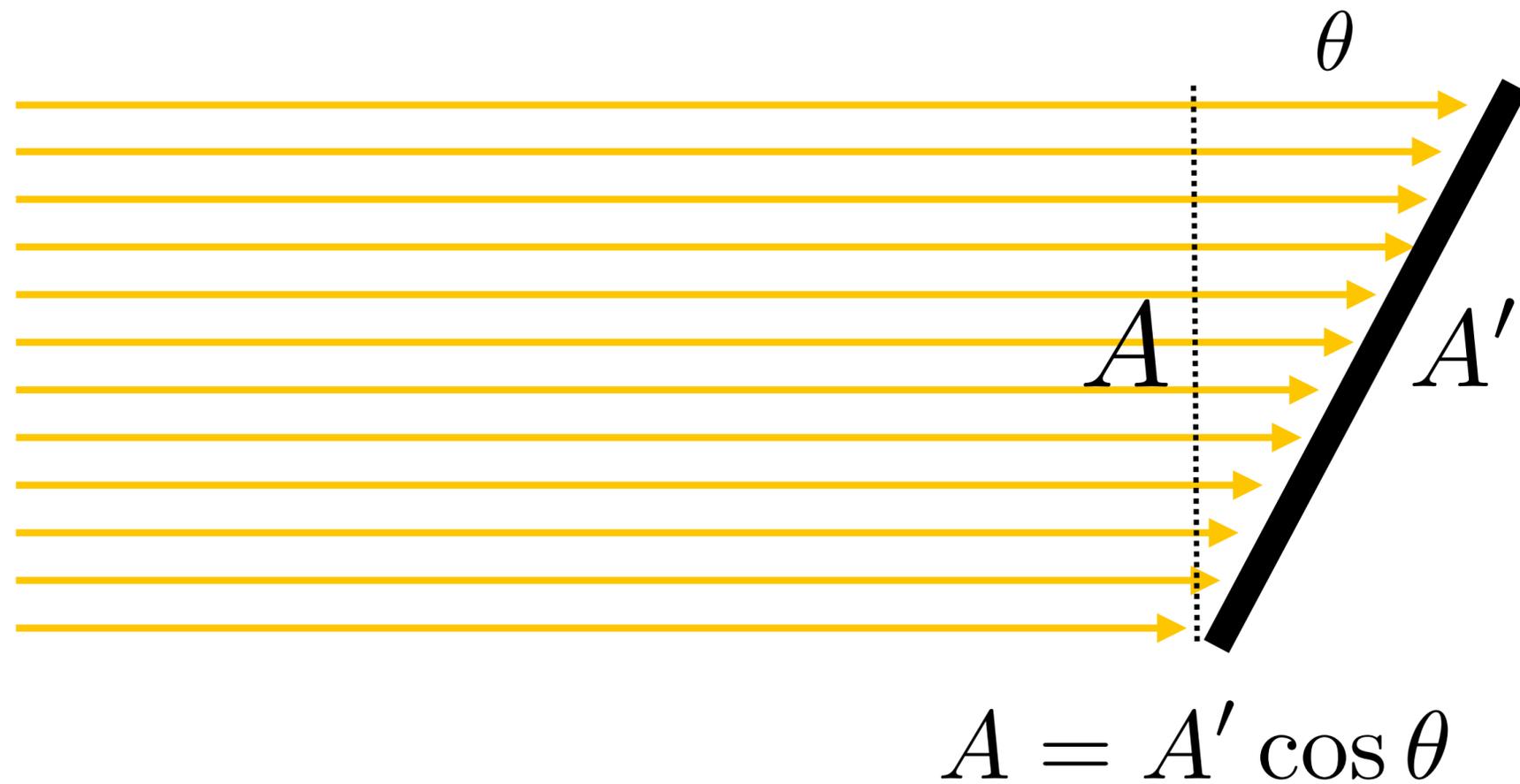
$$E = \frac{\Phi}{A}$$

$$\Phi = EA$$



Projected area

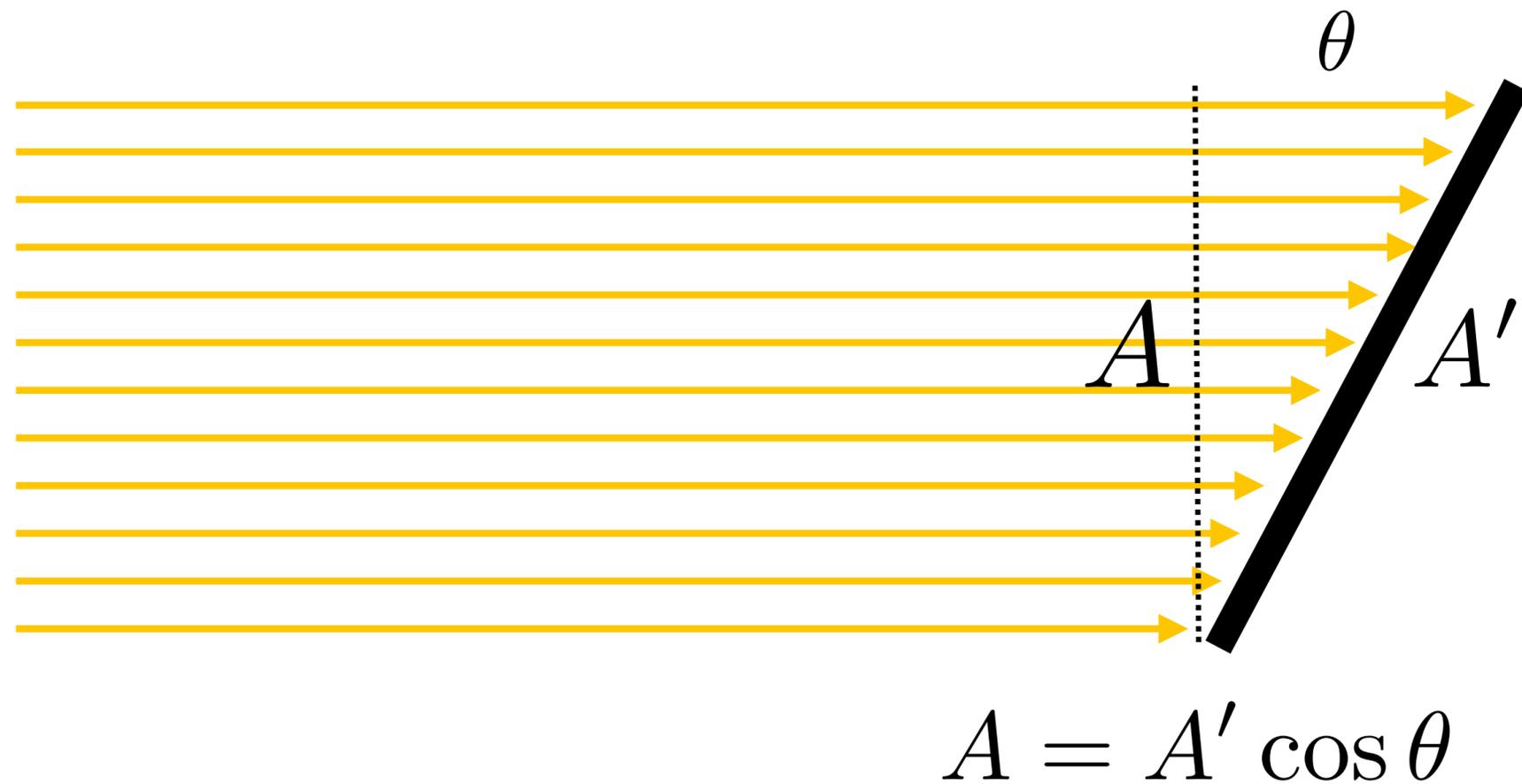
Consider beam with flux Φ incident on angled surface with area A'



A = projected area of surface relative to direction of beam

Lambert's Law

Irradiance at surface is proportional to cosine of angle between light direction and surface normal.

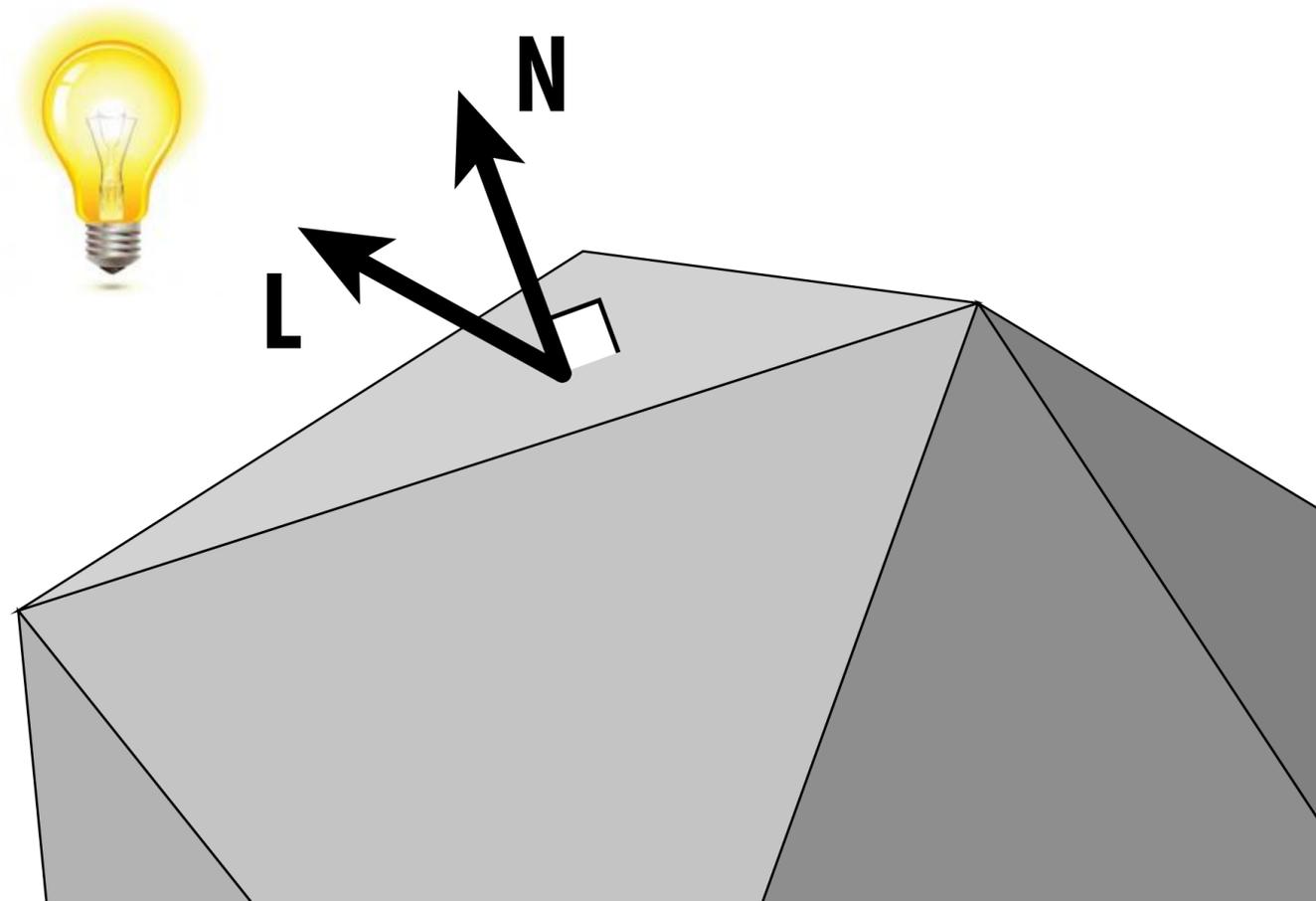


$$E = \frac{\Phi}{A'} = \frac{\Phi \cos \theta}{A}$$

“N-dot-L” lighting

- Most basic way to shade a surface: take dot product of unit surface normal (N) and unit direction to light (L)

```
double surfaceColor( Vec3 N, Vec3 L )  
{  
    return dot( N, L );  
}
```

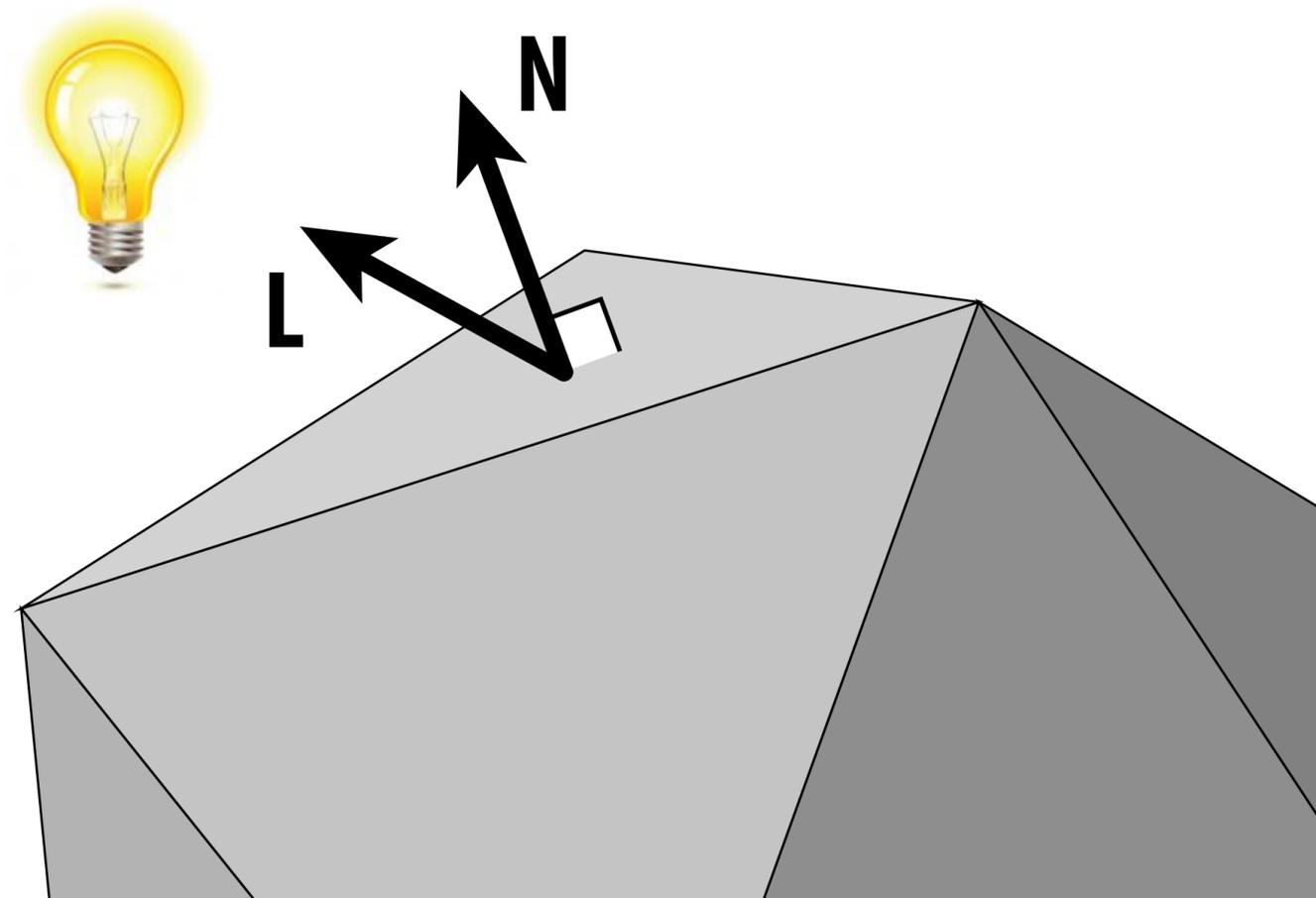


(Q: What's wrong with this code?)

“N-dot-L” lighting

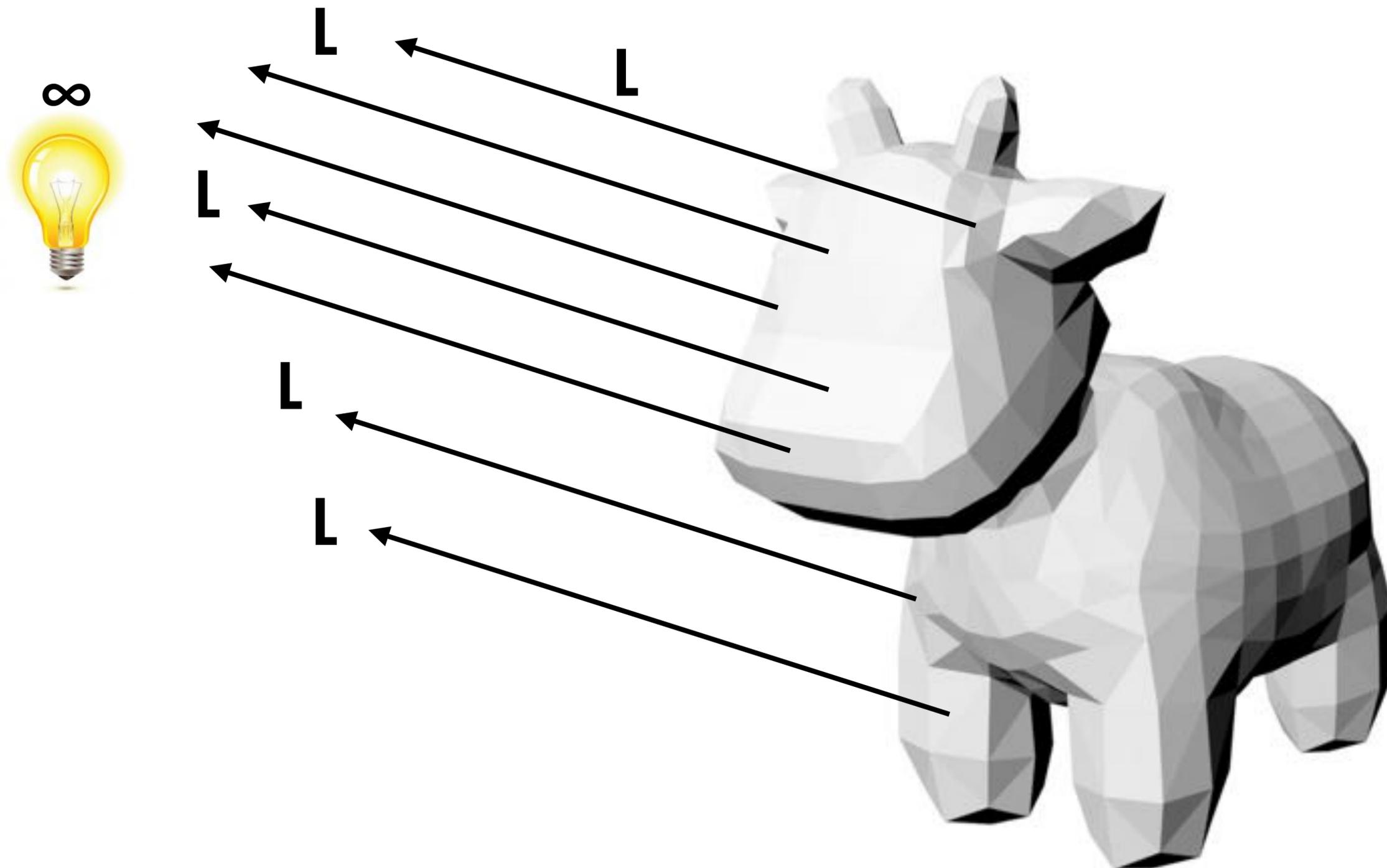
- Most basic way to shade a surface: take dot product of unit surface normal (N) and unit direction to light (L)

```
double surfaceColor( Vec3 N, Vec3 L )  
{  
    return max( 0., dot( N, L ) );  
}
```



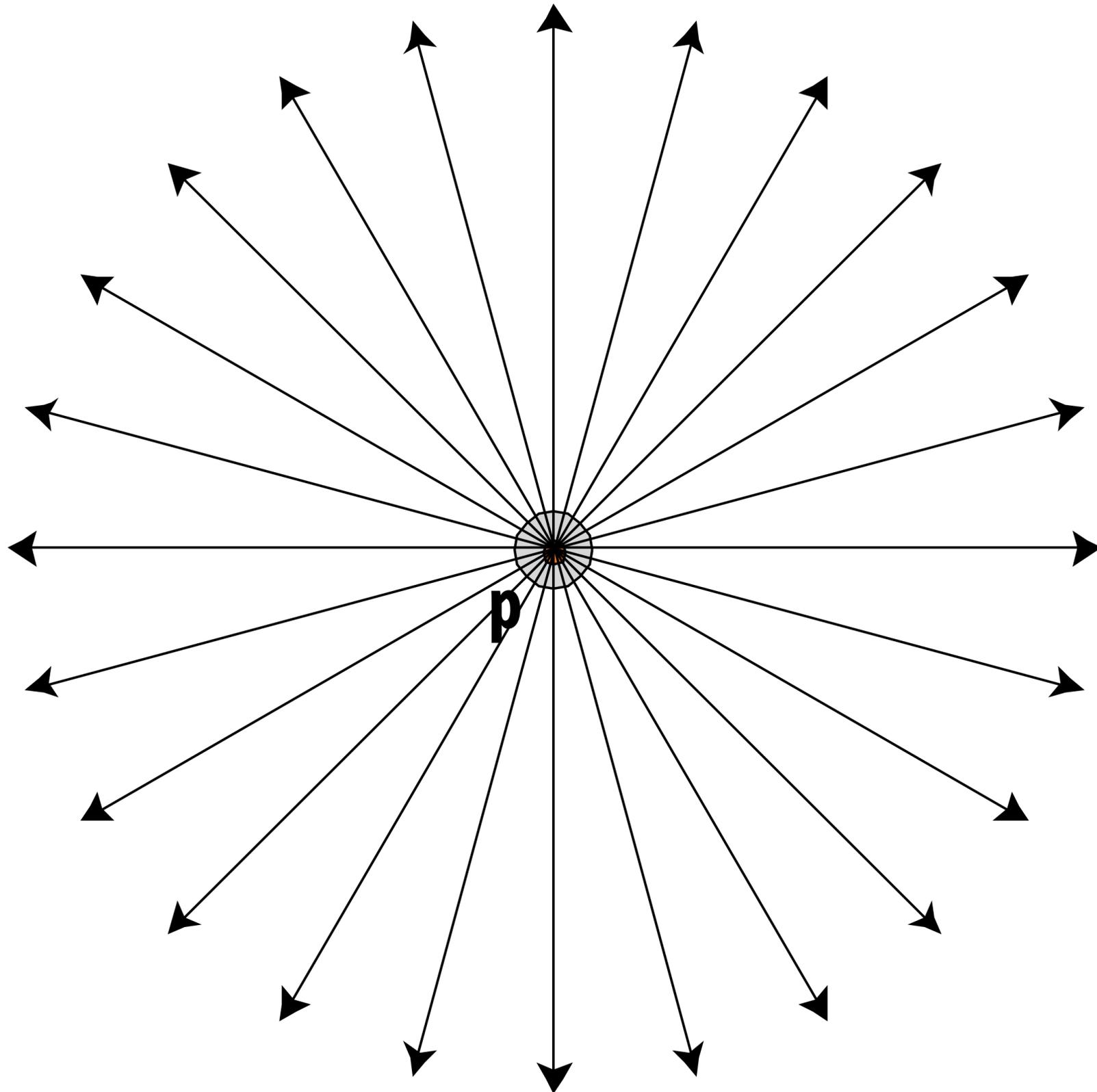
Example: “directional” lighting

- Common abstraction: infinitely bright light source “at infinity”
- All light directions (L) are therefore identical



Isotropic point source

Slightly more realistic model...



Suppose our light is such that:

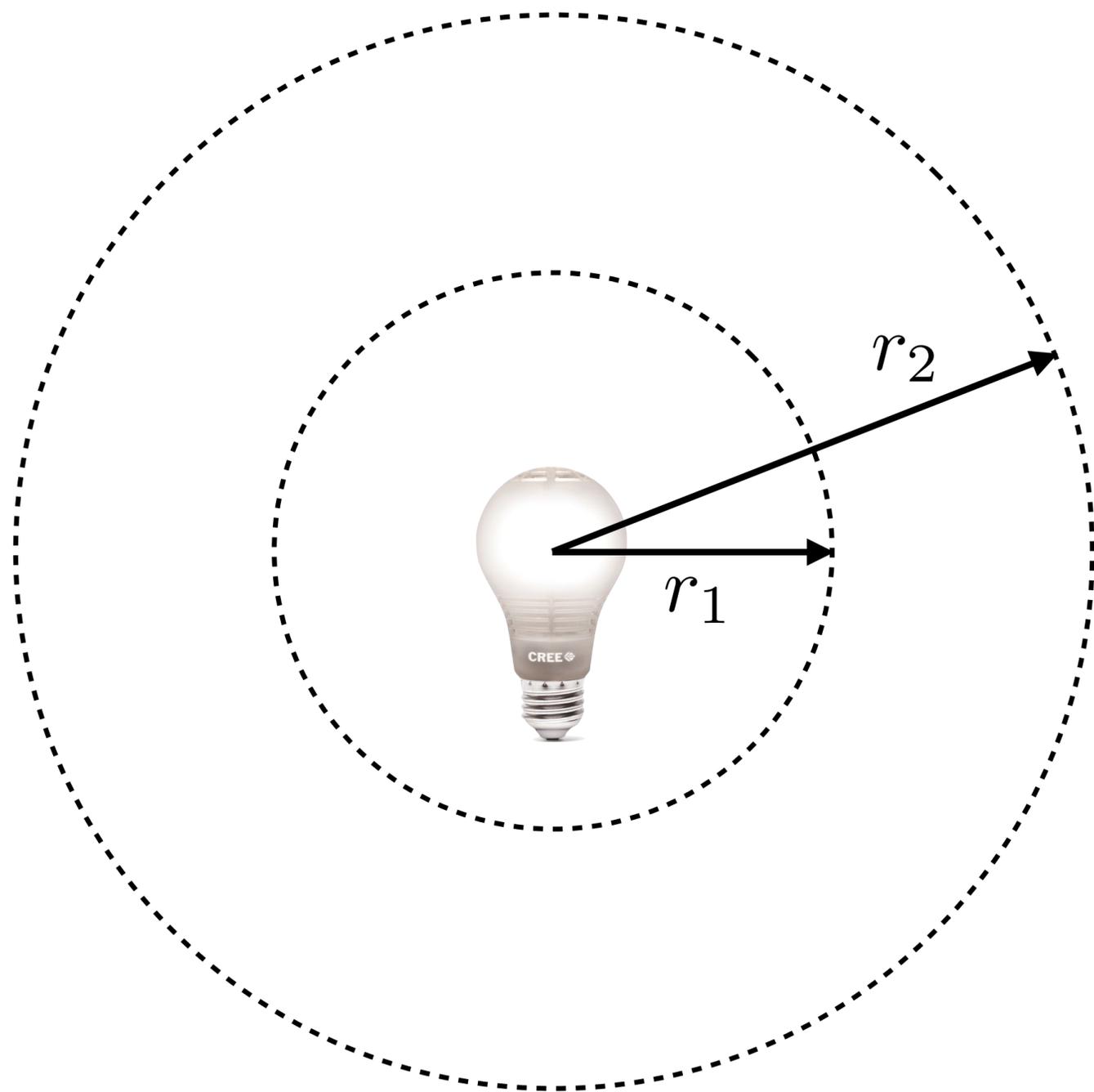
total radiant flux

"intensity"

$$\Phi = \int_{S^2} I \, d\omega$$
$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Irradiance falloff with distance



Since same amount of energy is distributed over larger and larger spheres, has to get darker quadratically with distance.

Assume light is emitting flux Φ in a uniform angular distribution

Compare irradiance at surface of two spheres:

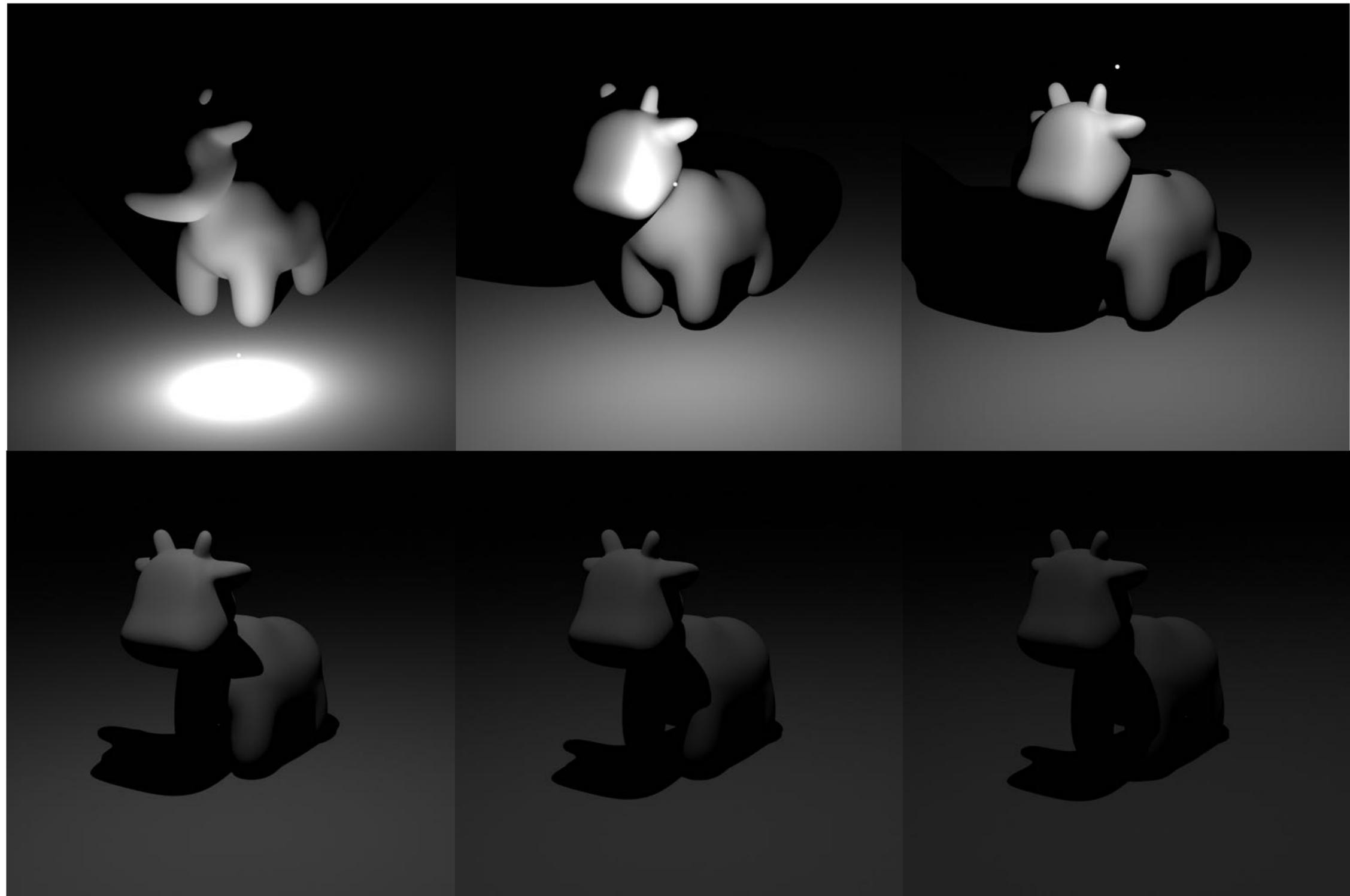
$$E_1 = \frac{\Phi}{4\pi r_1^2} \rightarrow \Phi = 4\pi r_1^2 E_1$$

$$E_2 = \frac{\Phi}{4\pi r_2^2} \rightarrow \Phi = 4\pi r_2^2 E_2$$

$$\frac{E_2}{E_1} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

What does quadratic falloff look like?

Single point light, move in 1m increments:

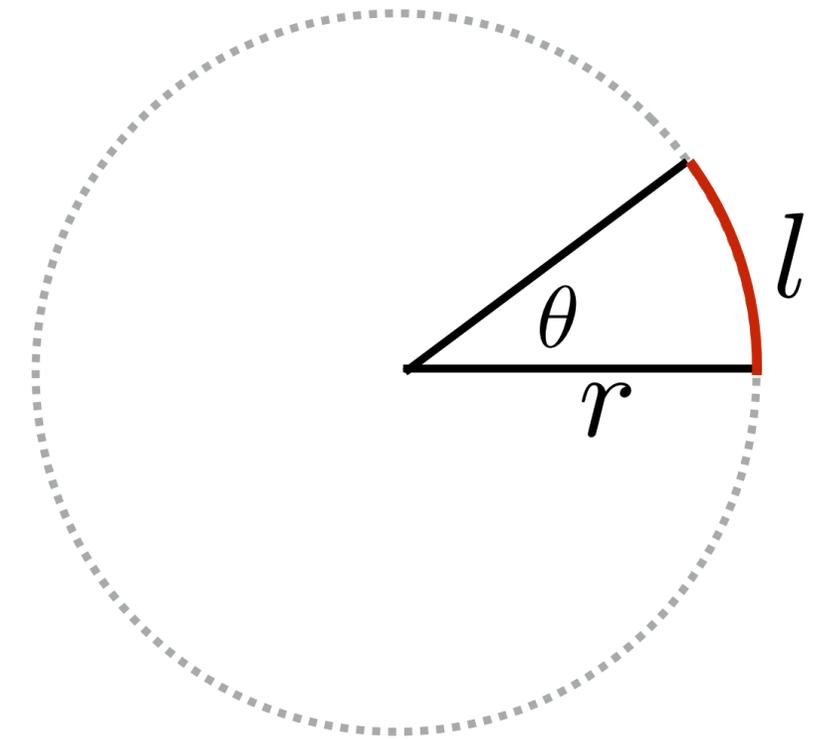


...things get dark fast!

Angles and solid angles

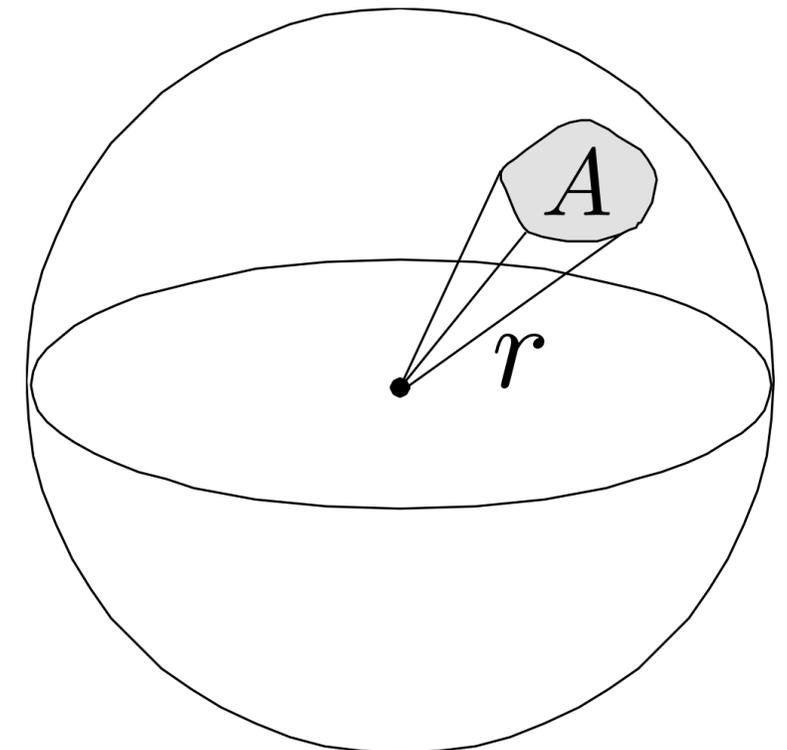
- **Angle: ratio of subtended arc length on circle to radius**

- $\theta = \frac{l}{r}$
- **Circle has 2π radians**



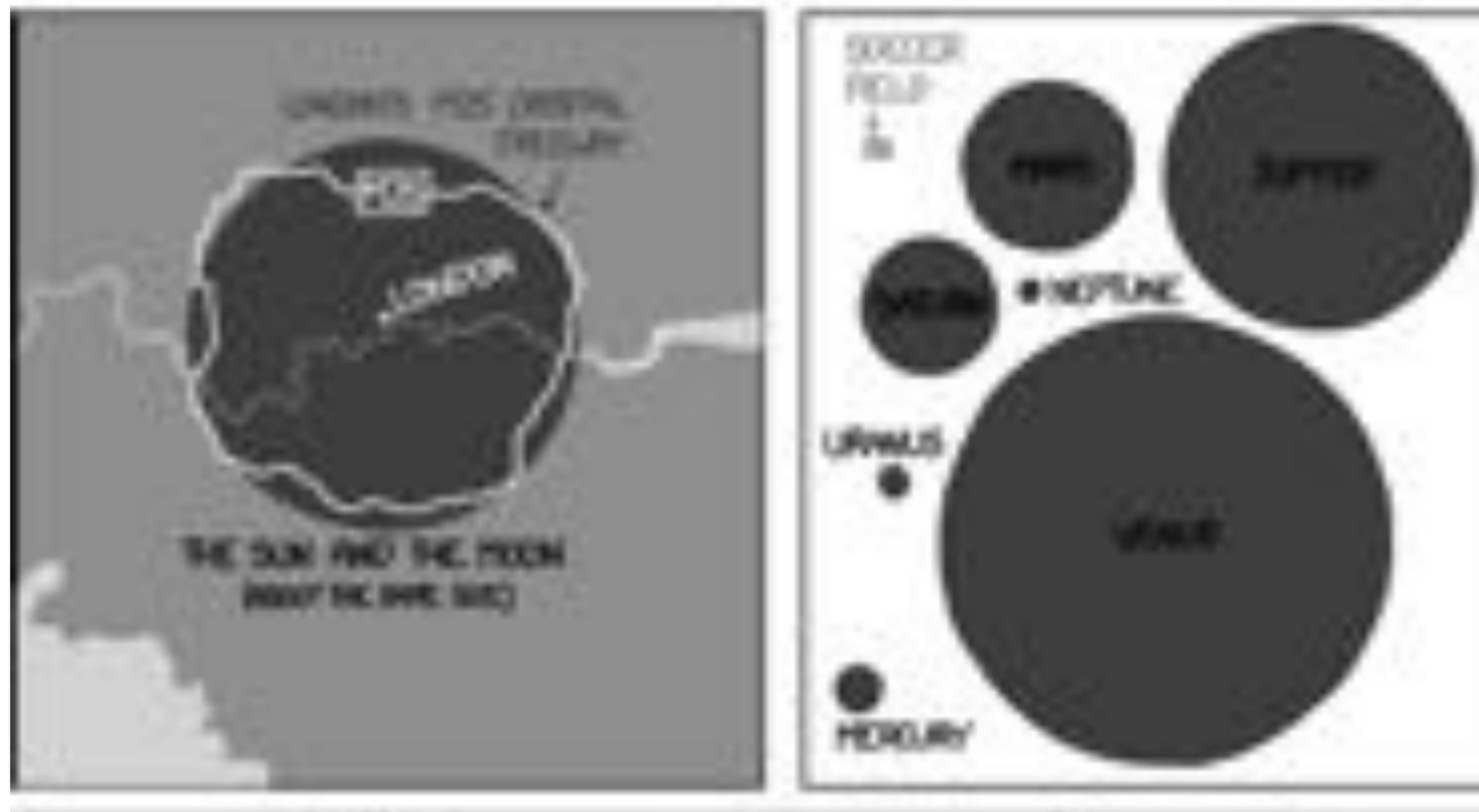
- **Solid angle: ratio of subtended area on sphere to radius squared**

- $\Omega = \frac{A}{r^2}$
- **Sphere has 4π steradians**



Solid angles in practice

THE SIZE OF THE PART OF EARTH'S SURFACE DIRECTLY UNDER VARIOUS SPACE OBJECTS



- Sun and moon both subtend $\sim 60\mu$ sr as seen from earth

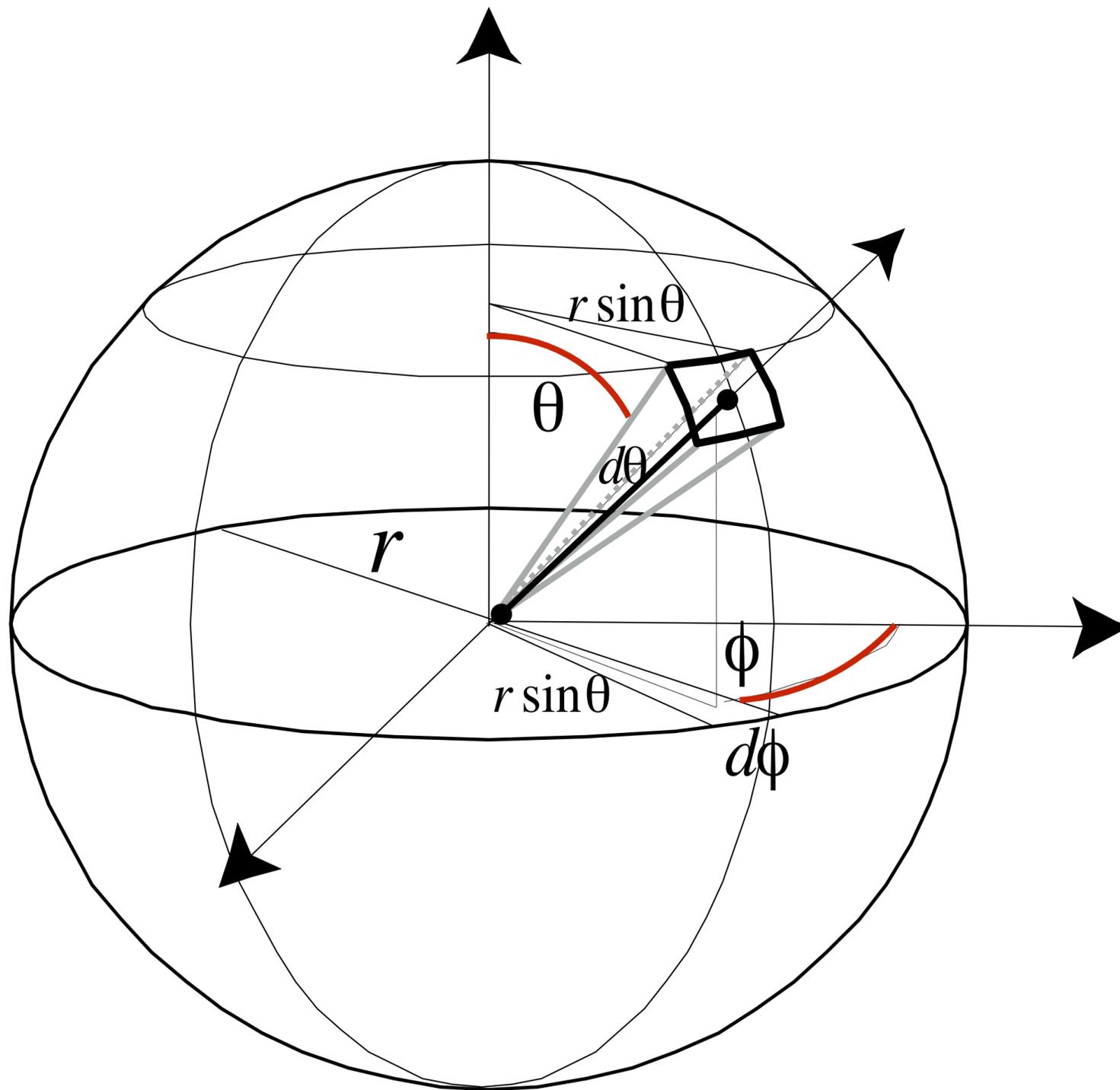
- Surface area of earth: $\sim 510\text{M km}^2$

- Projected area:

$$510\text{Mkm}^2 \frac{60\mu\text{sr}}{4\pi\text{sr}} = 510 \frac{15}{\pi} \approx 2400\text{km}^2$$

<http://xkcd.com/1276/>

Differential solid angle



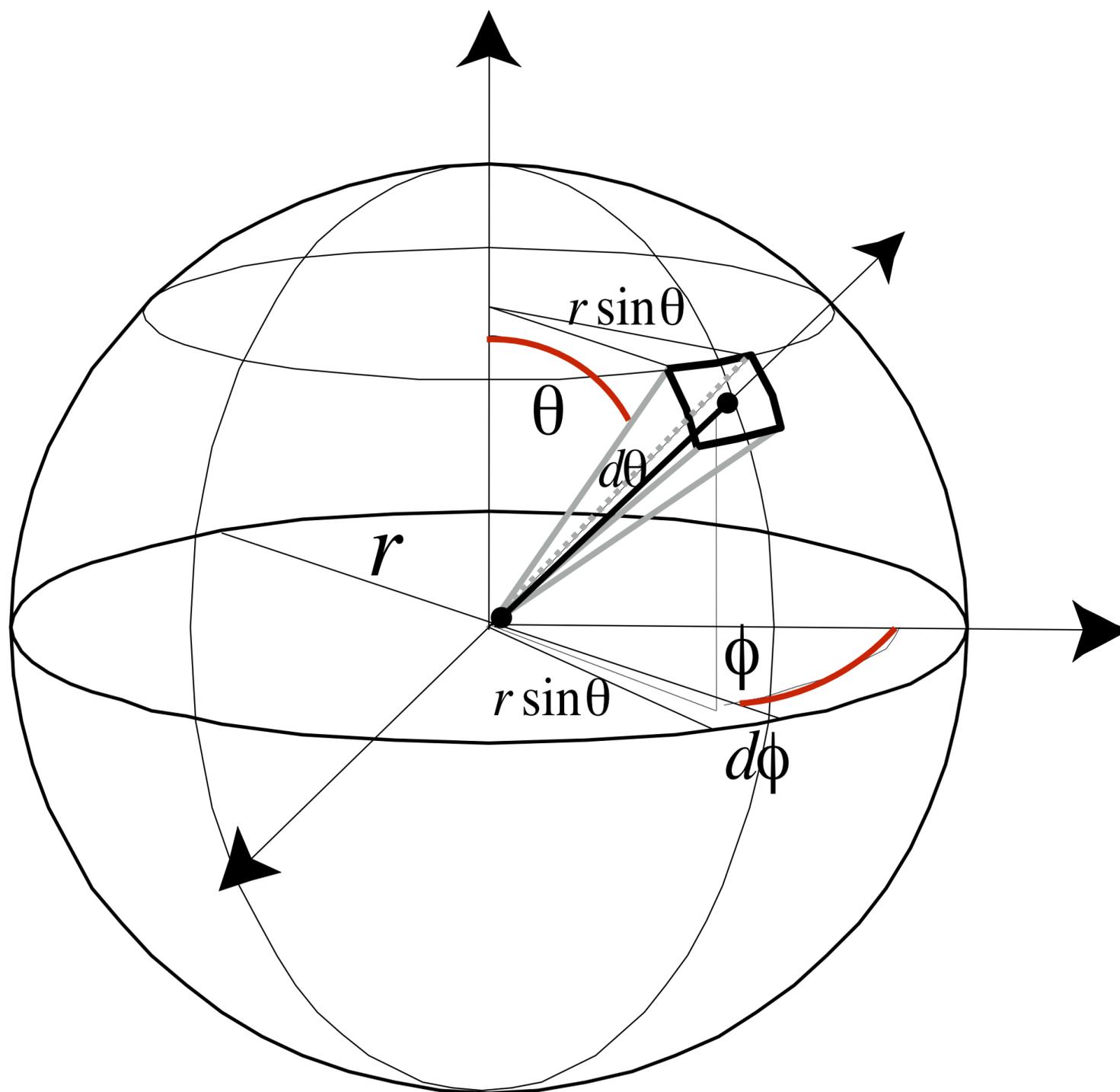
Consider a tiny area swept out by a tiny angle in each direction...

$$\begin{aligned} dA &= (r d\theta)(r \sin \theta d\phi) \\ &= r^2 \sin \theta d\theta d\phi \end{aligned}$$

$$d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi$$

Differential solid angle is just that same tiny area projected onto the unit sphere

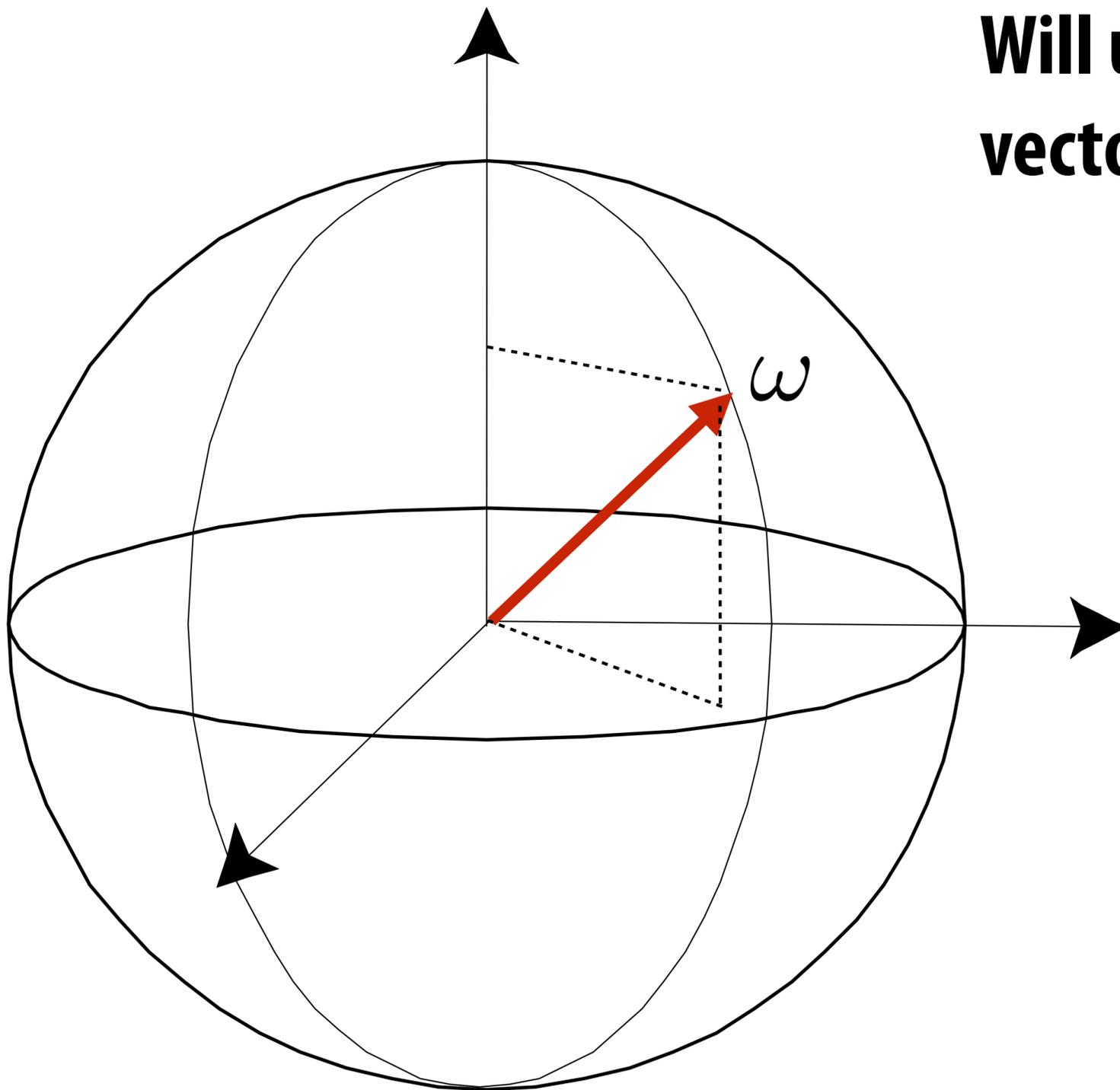
Differential solid angle



$$\begin{aligned}\Omega &= \int_{S^2} d\omega \\ &= \int_0^{2\pi} \int_0^\pi \sin \theta \, d\theta \, d\phi \\ &= 4\pi\end{aligned}$$

ω as a direction vector

Will use ω to denote a direction vector (unit length)

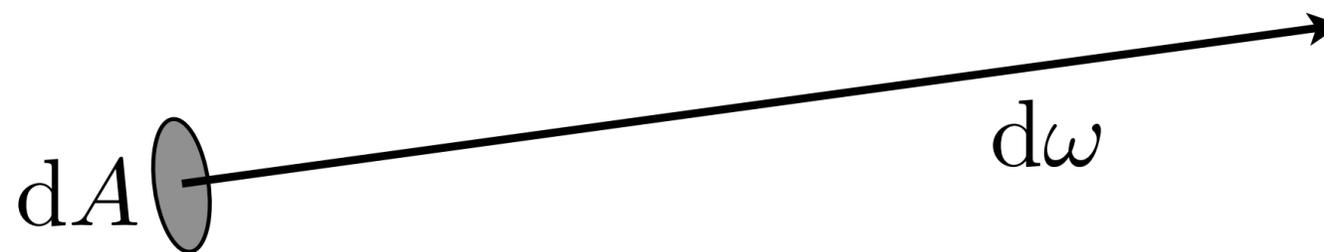


Radiance

- Radiance is the solid angle density of irradiance

$$L(p, \omega) = \lim_{\Delta \rightarrow 0} \frac{\Delta E_\omega(p)}{\Delta \omega} = \frac{dE_\omega(p)}{d\omega} \left[\frac{\text{W}}{\text{m}^2 \text{ sr}} \right]$$

where E_ω denotes that the differential surface area is oriented to face in the direction ω



In other words, radiance is energy along a ray defined by origin point p and direction ω

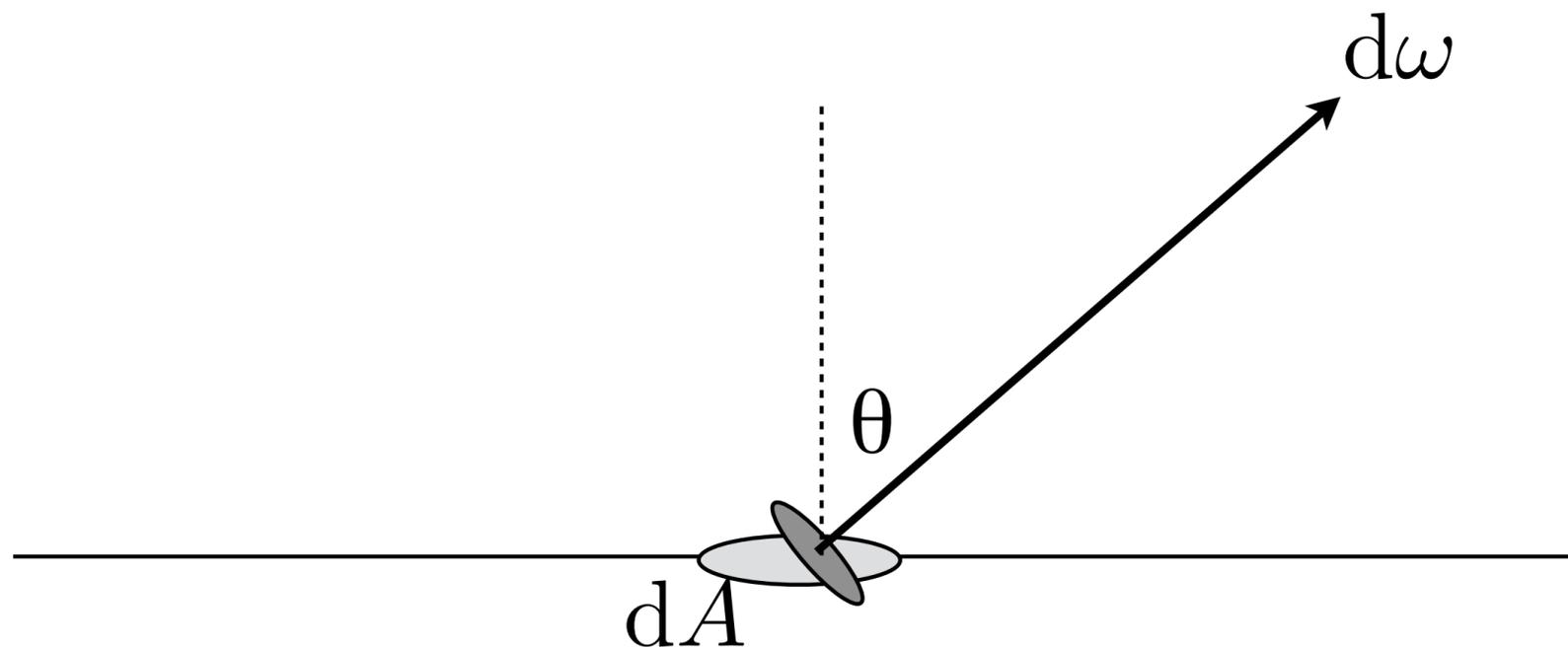
Energy per unit time per unit area per unit solid angle...!

Surface Radiance

- Equivalently,

$$L(p, \omega) = \frac{dE(p)}{d\omega \cos \theta} = \frac{d^2\Phi(p)}{dA d\omega \cos \theta}$$

- Previous slide described measuring radiance at a surface oriented in ray direction
 - **cos(theta) accounts for different surface orientation**



Spectral Radiance

- To summarize, radiance is: **radiant energy per unit time per unit area per unit solid angle**
- To really get a complete description of light we have to break this down just one more step: **radiant energy per unit time per unit area per unit solid angle per unit wavelength**
- Q: What additional information do we now get?
- A: Color!

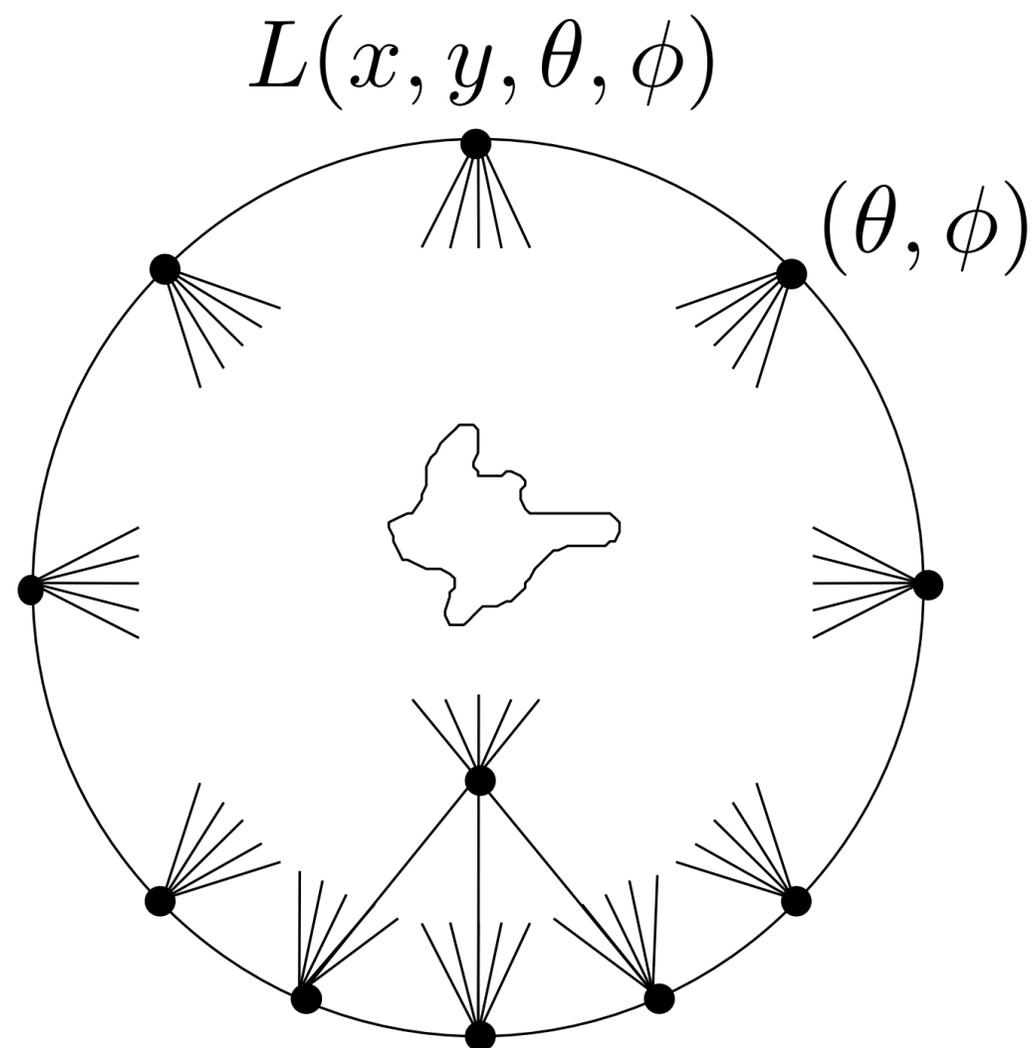


Why do we break energy down to this granularity? (spectral radiance)

Because once we have spectral radiance, we have a complete description of the light in an environment!

Field radiance: the light field

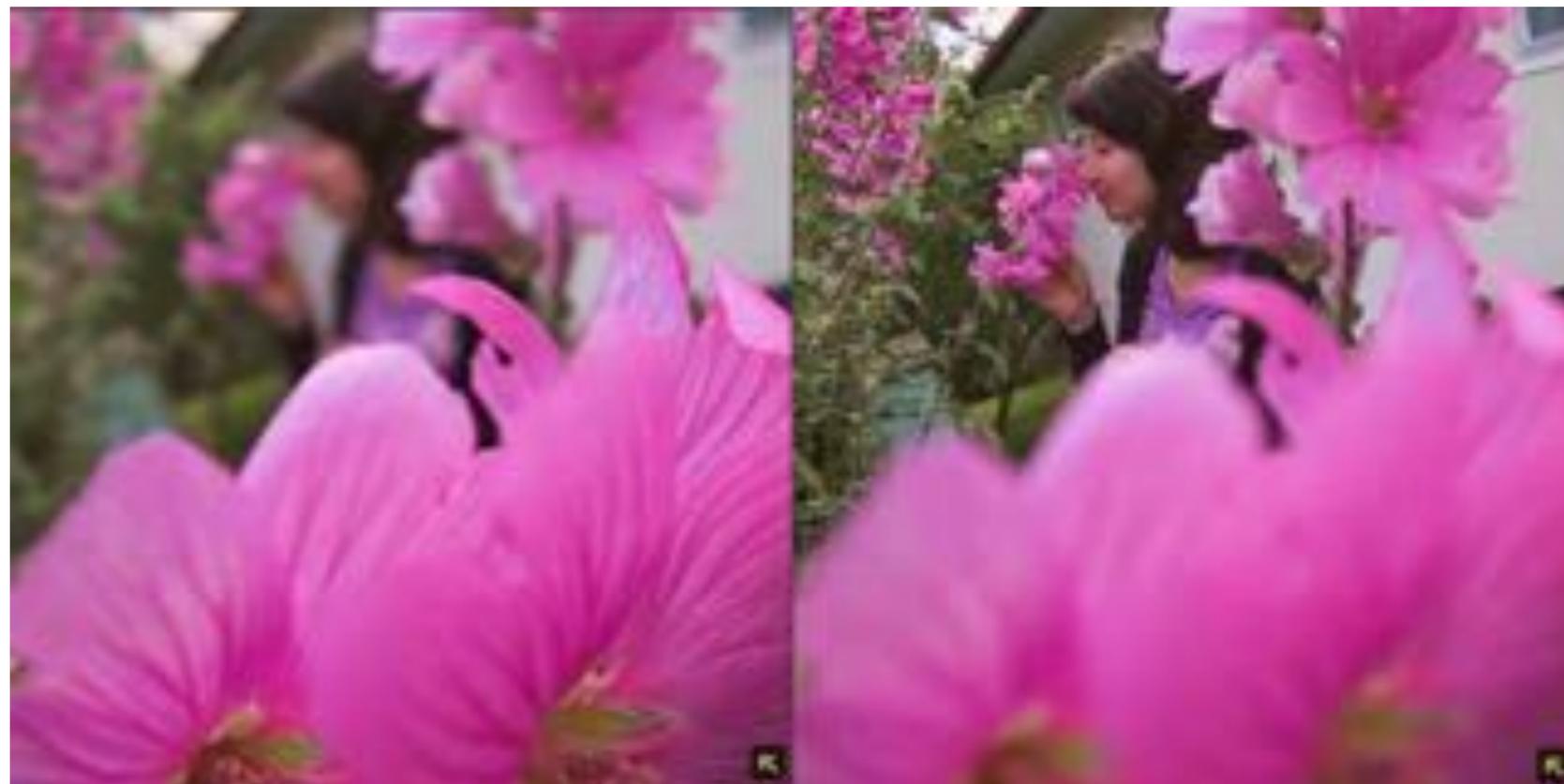
- Light field = radiance function on rays
- Radiance is constant along rays *
- Spherical gantry: captures 4D light field (all light leaving object)



* in a vacuum

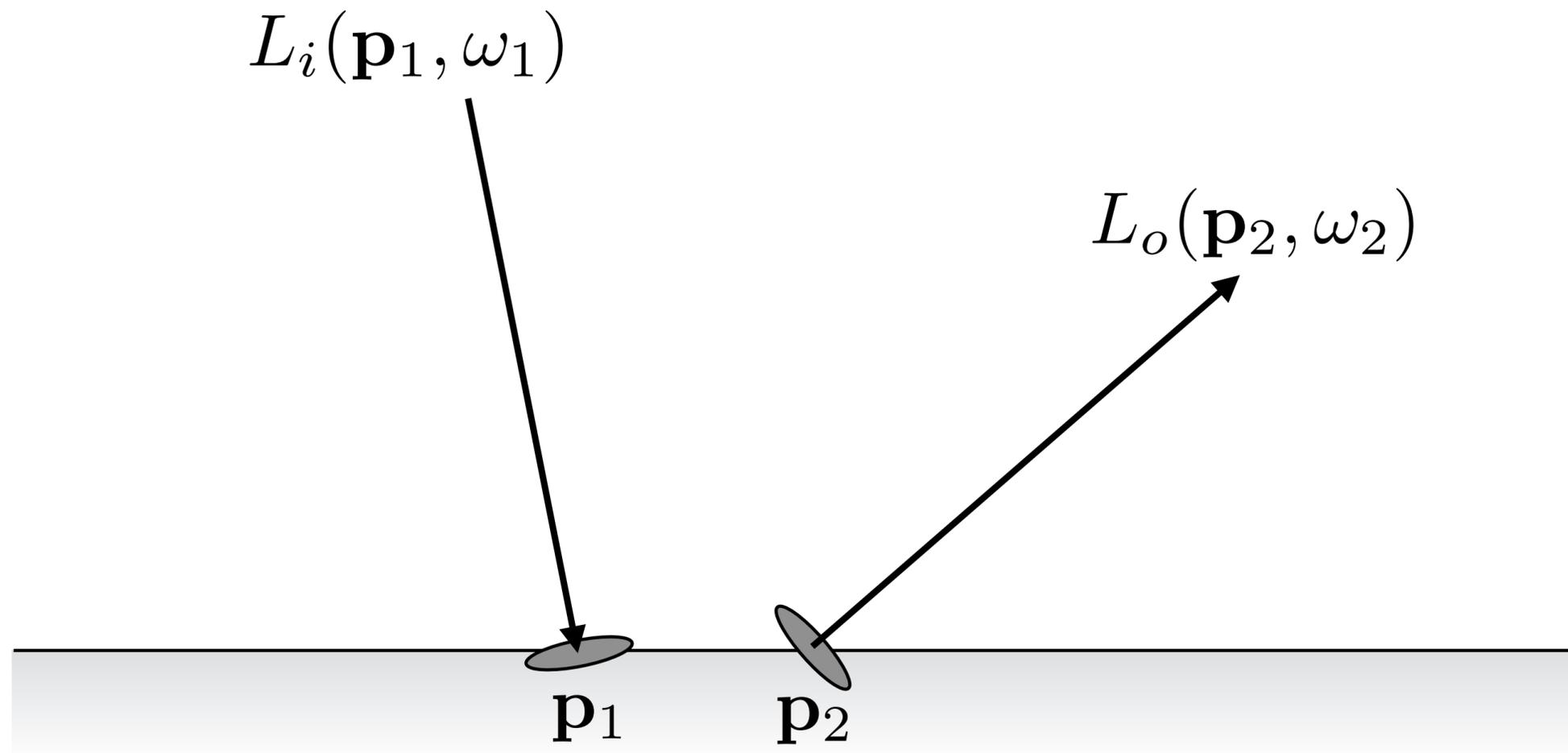
Light Field Photography

- A standard camera captures a small “slice” of the light field
- Light field cameras capture a “bigger slice,” recombine information to get new images after taking the photo



Incident vs. Exitant Radiance

- Often need to distinguish between incident radiance and exitant radiance functions at a point on a surface



In general: $L_i(\mathbf{p}, \omega) \neq L_o(\mathbf{p}, \omega)$

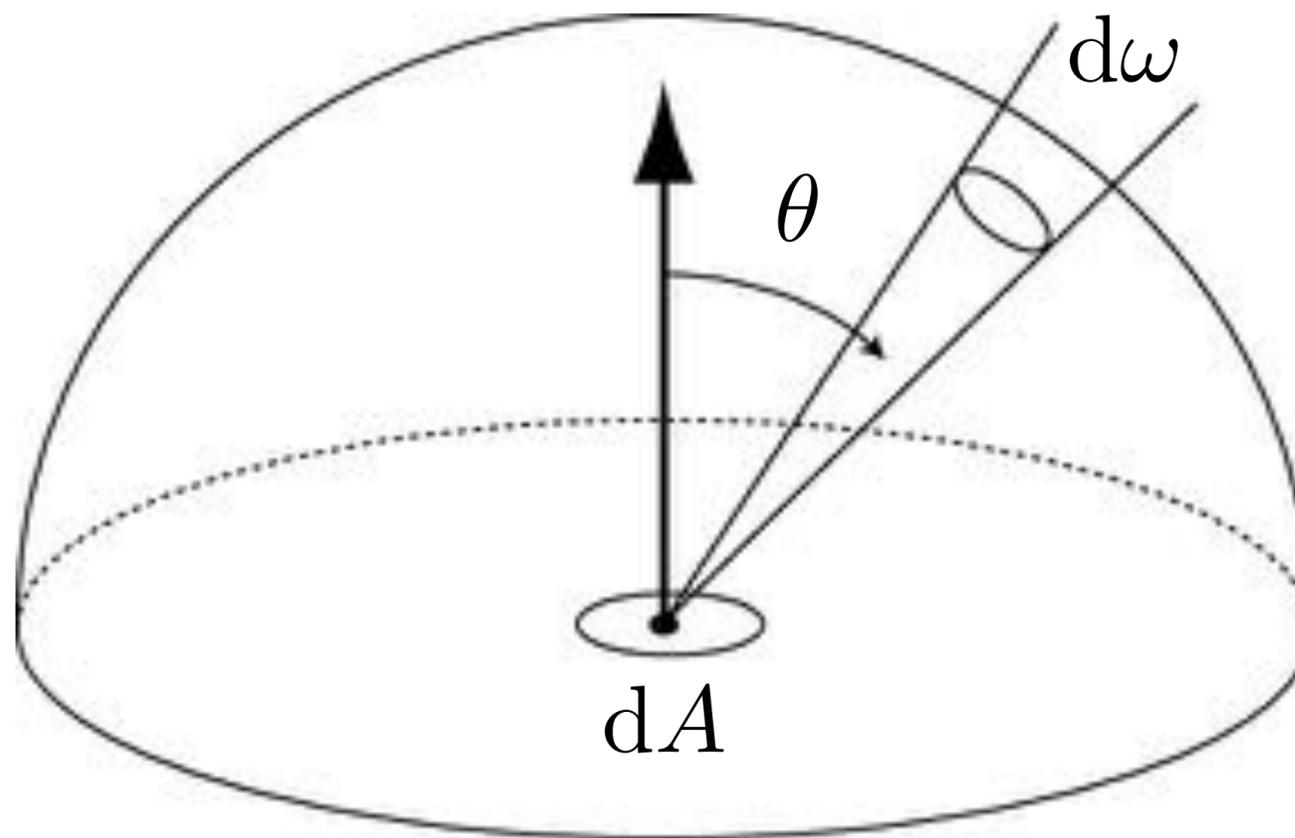
Properties of radiance

- Radiance is a fundamental field quantity that characterizes the distribution of light in an environment
 - Radiance is the quantity associated with a ray
 - **Rendering is all about computing radiance**
- Radiance is constant along a ray (in a vacuum)
- A pinhole camera measures radiance

Irradiance from the environment

Computing flux per unit area on surface, due to incoming light from all directions:

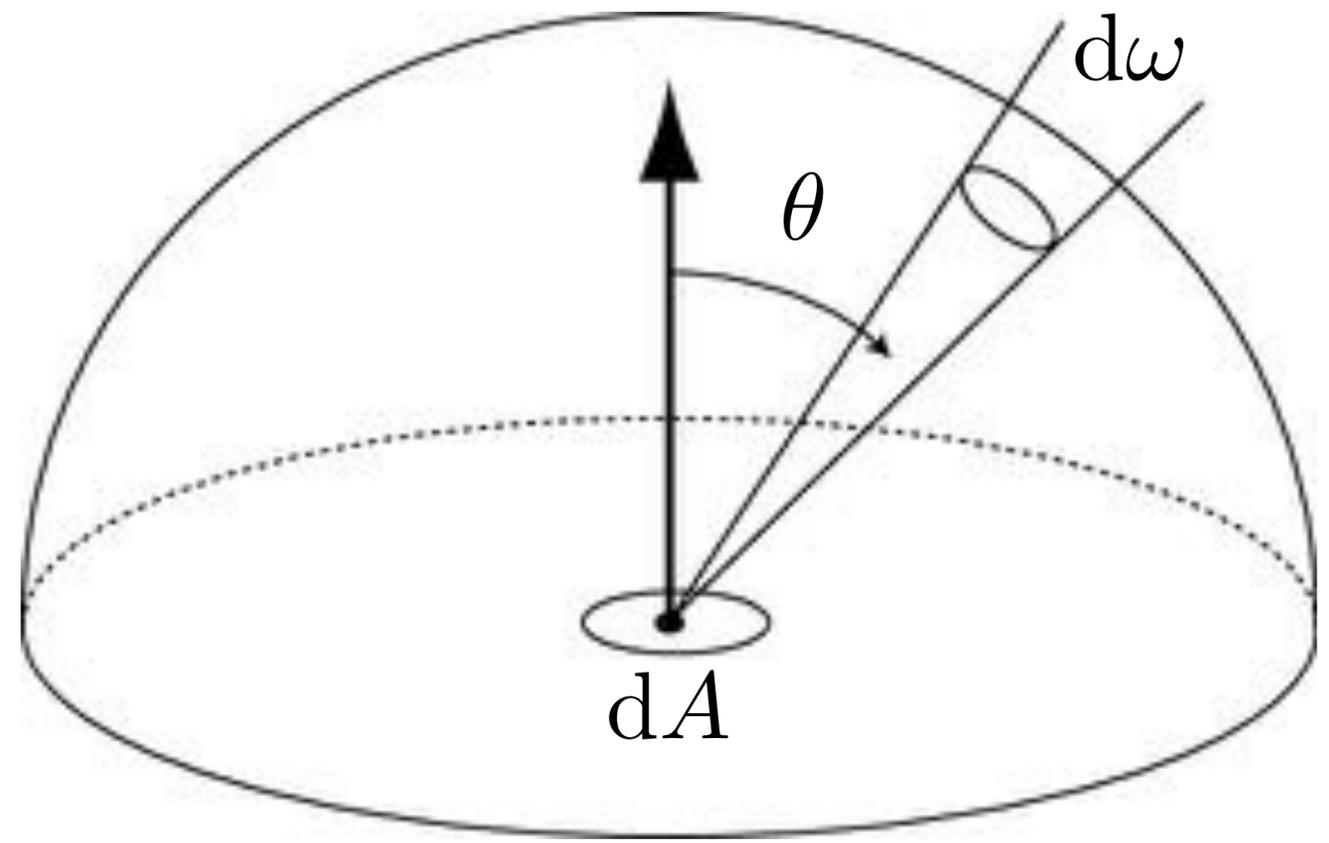
$$E(\mathbf{p}) = \int_{H^2} L_i(\mathbf{p}, \omega) \cos \theta d\omega$$



(This is what we often want to do for rendering!)

Simple case: irradiance from uniform hemispherical source

$$\begin{aligned} E(\mathbf{p}) &= \int_{H^2} L_i(\mathbf{p}, \omega) \cos \theta d\omega \\ &= L \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \cos \theta \sin \theta d\theta d\phi \\ &= L\pi \end{aligned}$$



Example of hemispherical light source



Q: Why didn't we just get the same constant $L\pi$ (white) at every point?

Irradiance from a uniform area source

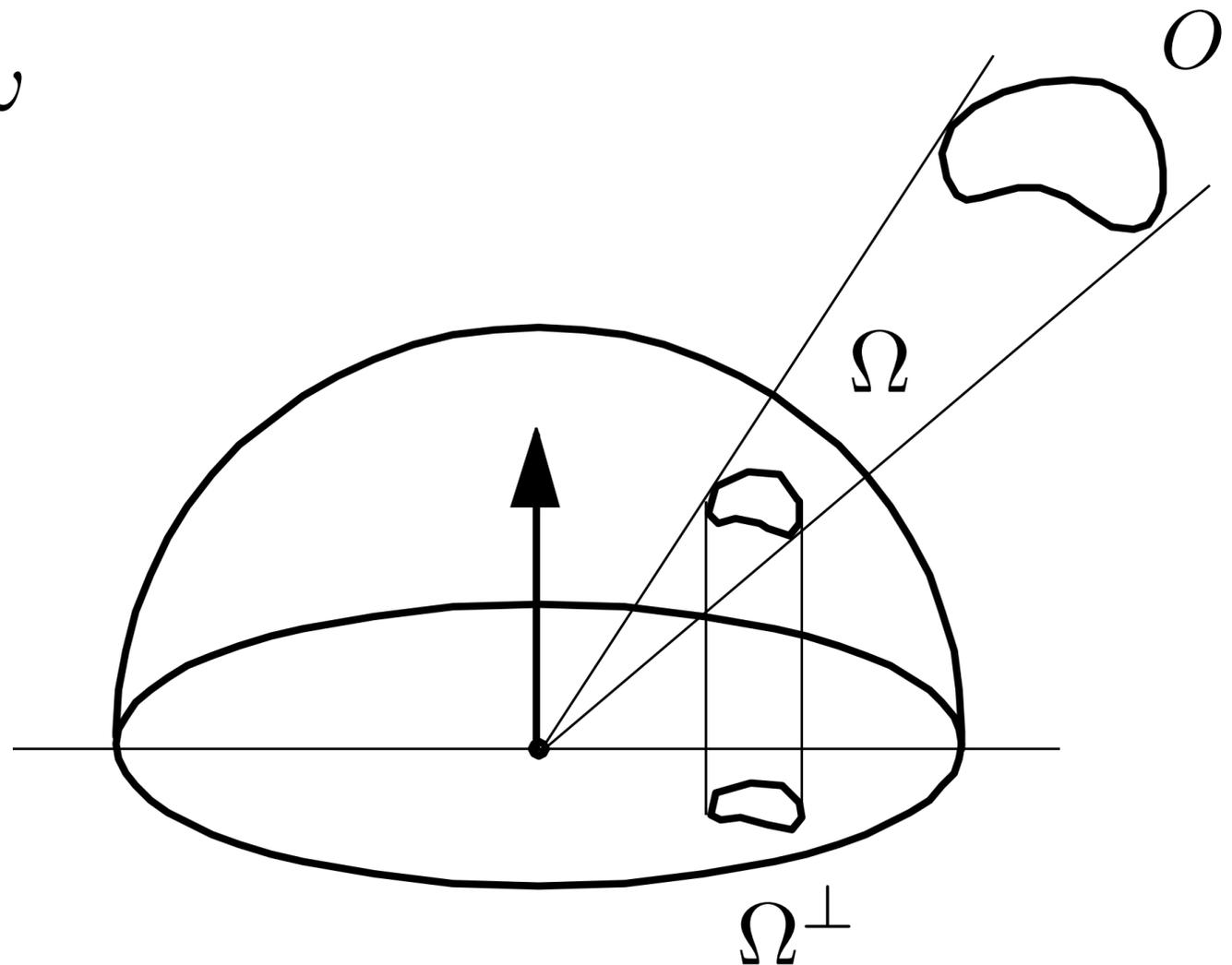
(source emits radiance L)

$$\begin{aligned} E(p) &= \int_{H^2} L(p, \omega) \cos \theta \, d\omega \\ &= L \int_{\Omega} \cos \theta \, d\omega \\ &= L\Omega^\perp \end{aligned}$$



Projected solid angle:

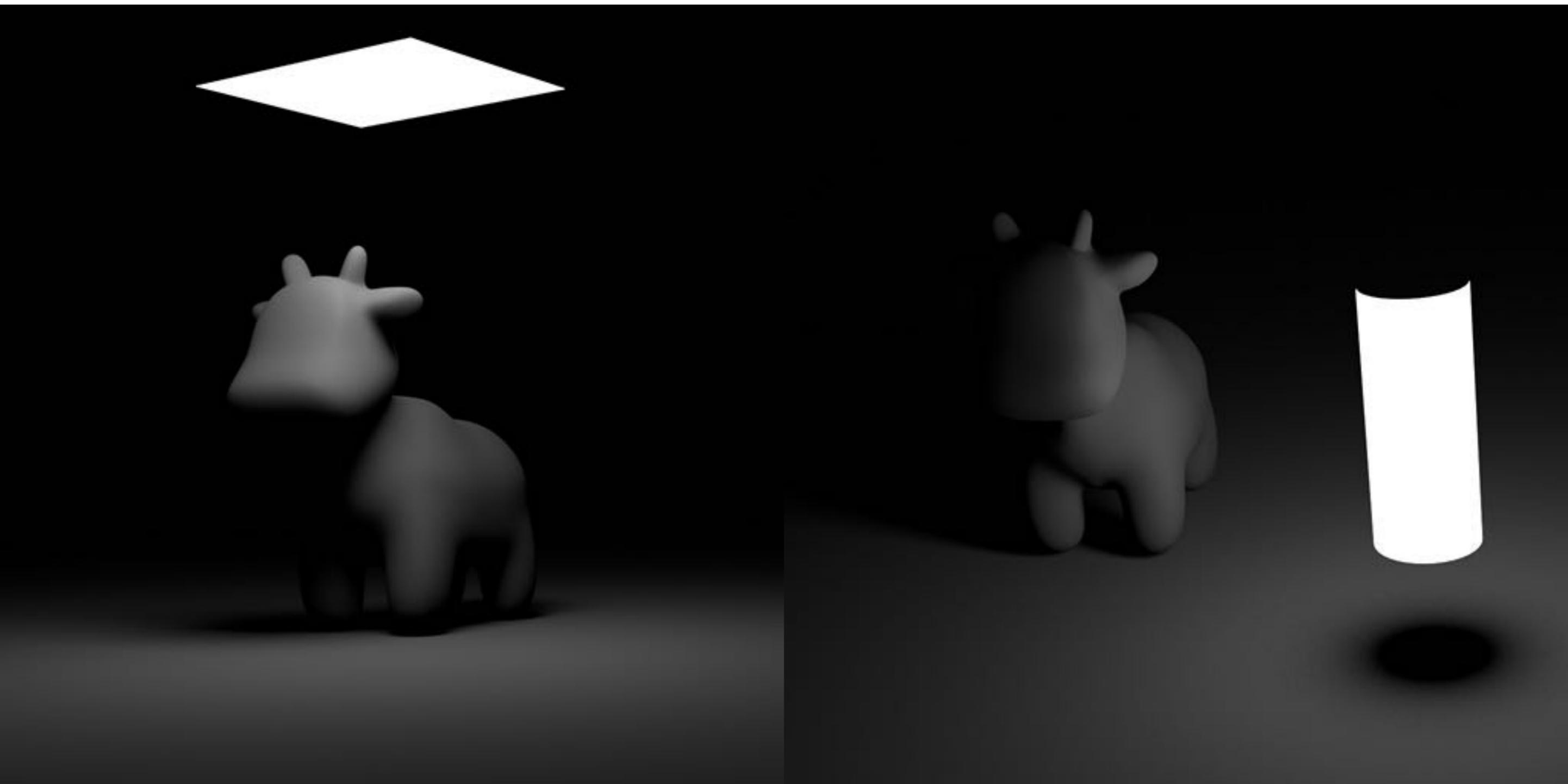
- **Cosine-weighted solid angle**
- **Area of object O projected onto unit sphere, then projected onto plane**



$$d\omega^\perp = |\cos \theta| \, d\omega$$

Examples of Area Light Sources

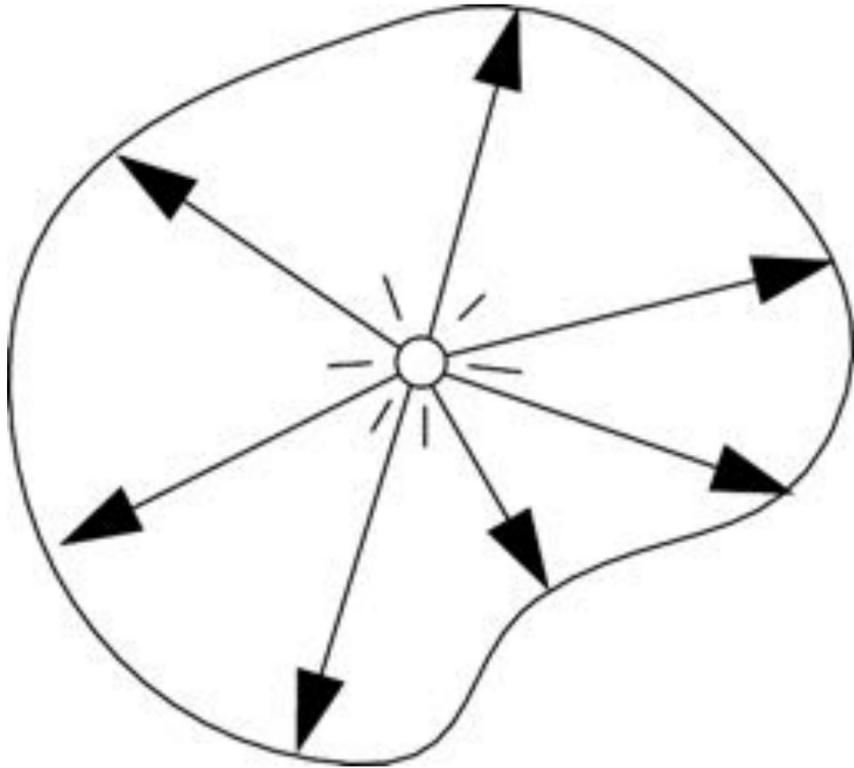
Generally “softer” appearance than point lights:



...and better model of real-world lights!

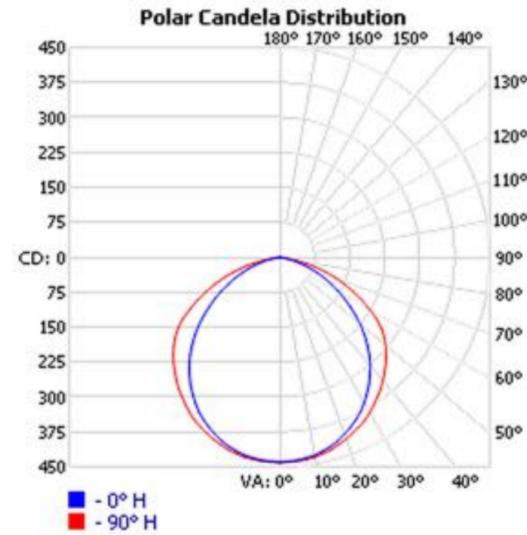
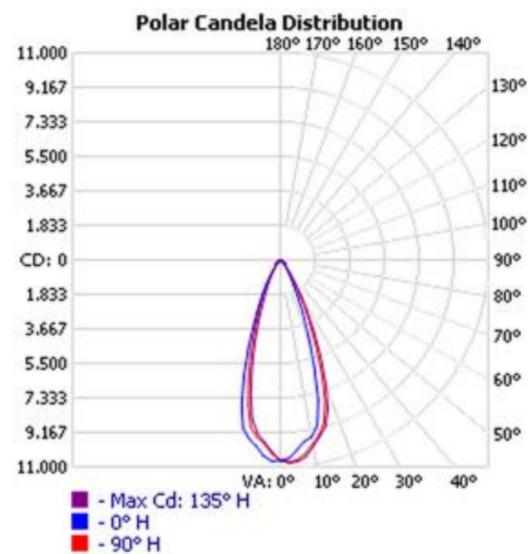
Measuring illumination: radiant intensity

- Power per solid angle emanating from a point source

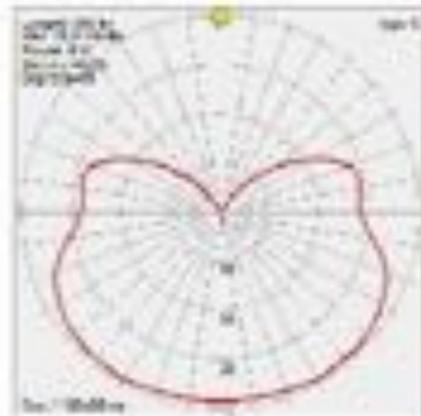
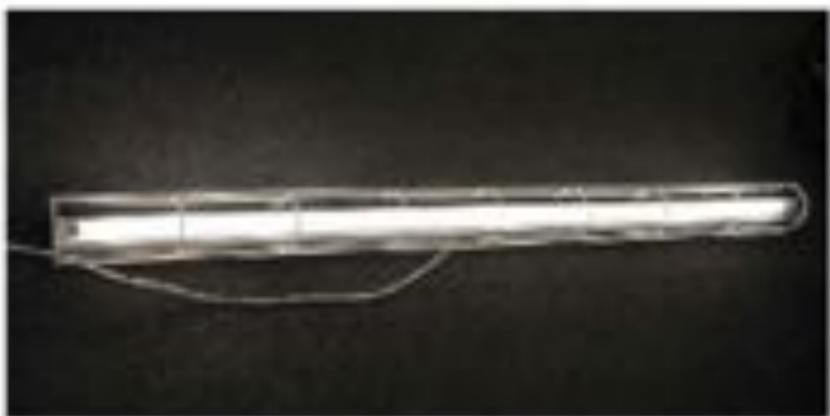


$$I(\omega) = \frac{d\Phi}{d\omega} \left[\frac{\text{W}}{\text{sr}} \right]$$

More realistic light models via “goniometry”



Goniometric diagram measures light intensity as function of angle.



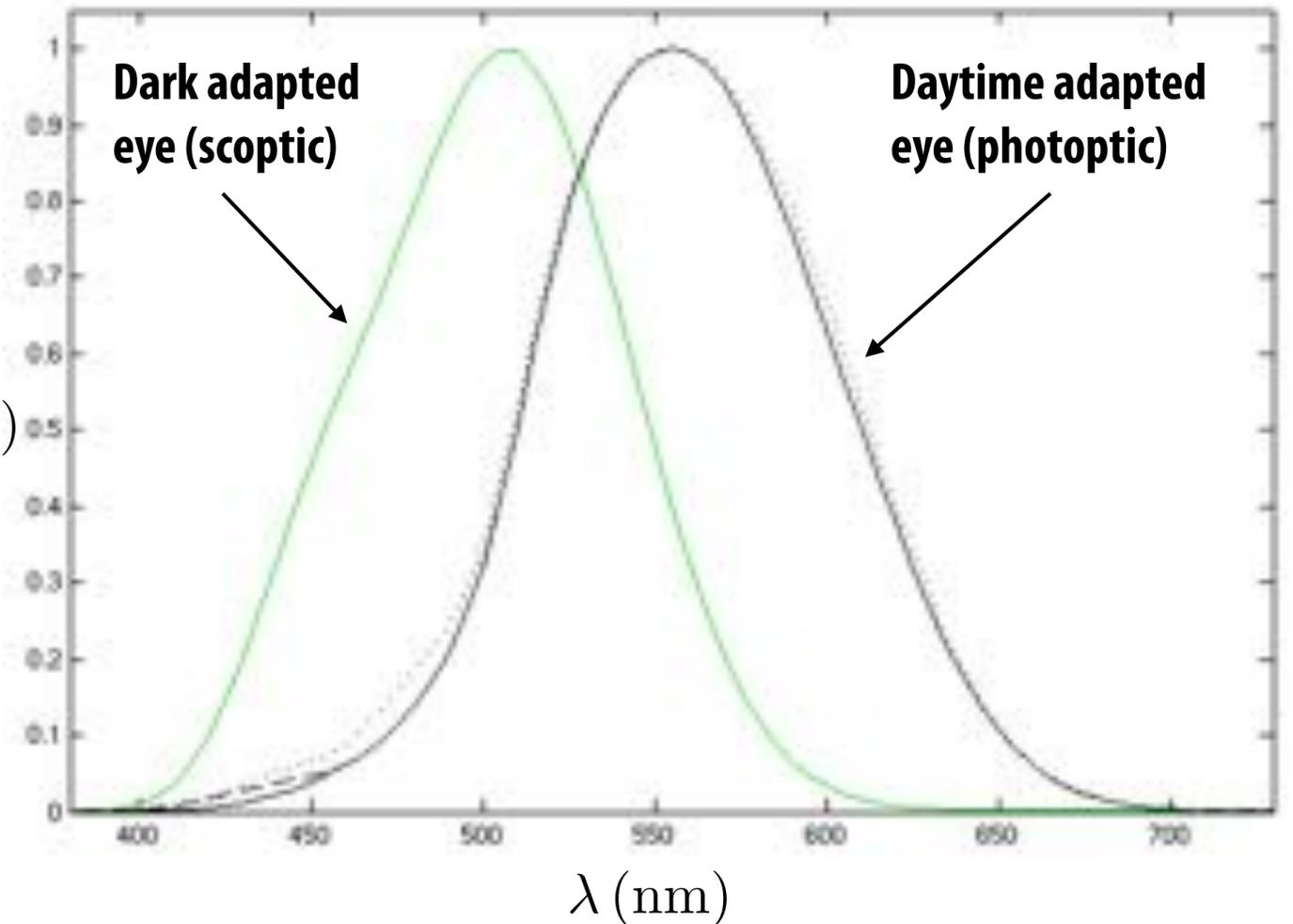
<http://www.mpi-inf.mpg.de/resources/mpimodel/v1.0/luminaires/index.html>

<http://www.visual-3d.com/tools/photometricviewer/>

Photometry: light + humans

- All radiometric quantities have equivalents in photometry
- Photometry: accounts for response of human visual system $V(\lambda)$ to electromagnetic radiation
- Luminance (Y) is photometric quantity that corresponds to radiance: integrate radiance over all wavelengths, weight by eye's luminous efficacy curve, e.g.:

$$Y(p, \omega) = \int_0^{\infty} L(p, \omega, \lambda) V(\lambda) d\lambda$$



Radiometric and photometric terms

Physics	Radiometry	Photometry
Energy	Radiant Energy	Luminous Energy
Flux (Power)	Radiant Power	Luminous Power
Flux Density	Irradiance (incoming) Radiosity (outgoing)	Illuminance (incoming) Luminosity (outgoing)
Angular Flux Density	Radiance	Luminance
Intensity	Radiant Intensity	Luminous Intensity

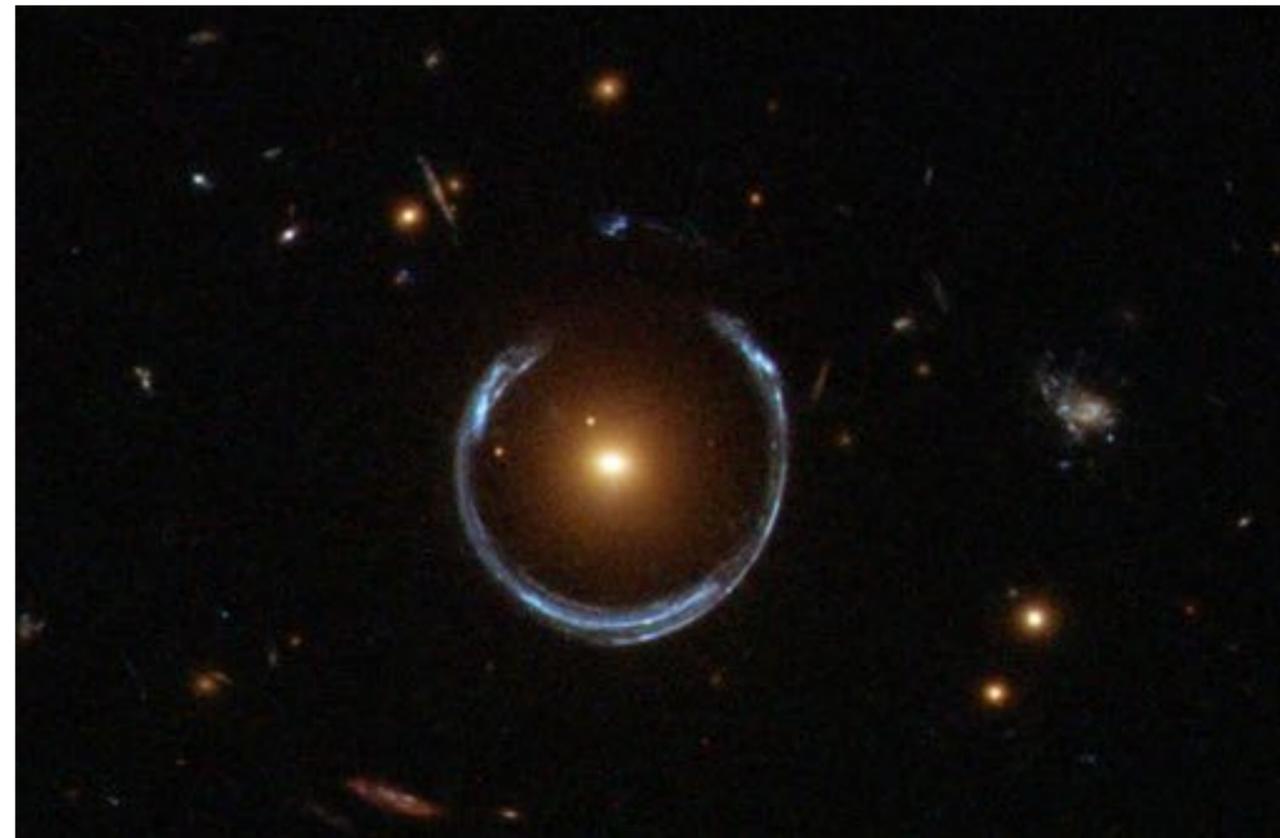
Photometric Units

Photometry	MKS	CGS	British
Luminous Energy	Talbot	Talbot	Talbot
Luminous Power	Lumen	Lumen	Lumen
Illuminance Luminosity	Lux	Phot	Footcandle
Luminance	Nit, Apostlib, Blondel	Stilb Lambert	Footlambert
Luminous Intensity	Candela	Candela	Candela

“Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?” —James Kajiya

What information are we missing?

- At the beginning, adopted “geometric optics” model of light
- Miss out on small-scale effects (e.g., diffraction/iridescence)
- Also large-scale effects (e.g., bending of light due to gravity)



Next time...

- More toward our goal of realistic rendering
- Materials, scattering, etc.

