More Variance Reduction (and maybe) Introduction to Animation

Computer Graphics
CMU 15-462/15-662

Review: Path Space Formulation of Light Transport

- We had been using the recursive rendering equation:

$$L_{\mathrm{o}}(\mathbf{x}, \omega_{\mathrm{o}}) = L_e(\mathbf{x}, \omega_{\mathrm{o}}) + \int_{\Omega} f_r(\mathbf{x}, \omega_{\mathrm{i}}, \omega_{\mathrm{o}})\, L_{\mathrm{i}}(\mathbf{x}, \omega_{\mathrm{i}})\, (\omega_{\mathrm{i}} \cdot \mathbf{n})\, \mathrm{d}\,\omega_{\mathrm{i}}$$

- Importance sampling can be done by making intelligent "local" choices at each step (material/lights)
- Alternatively, we can use a "path integral" formulation:

how much "light" is carried by this path?

$$I = \int_{\Omega} f(\bar{x}) d\mu(\bar{x})$$

how much of path space does this path "cover"

one particular path

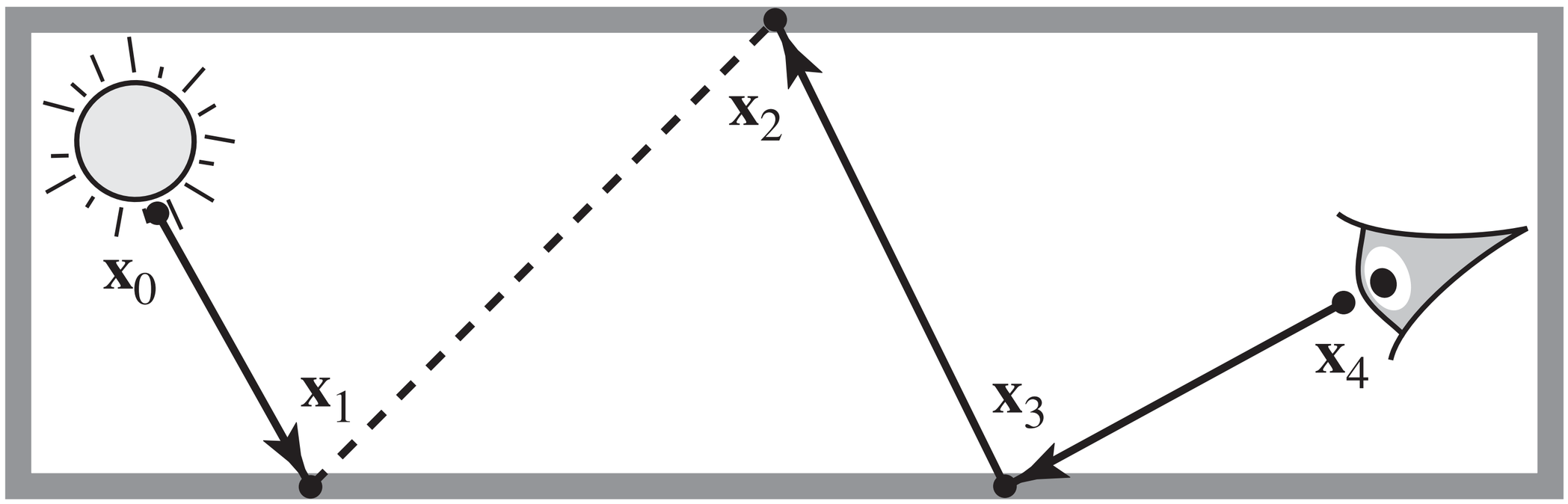
all possible paths

- Opens the door to intelligent "global" importance sampling. (But still hard!)

How do we choose paths—and path lengths?

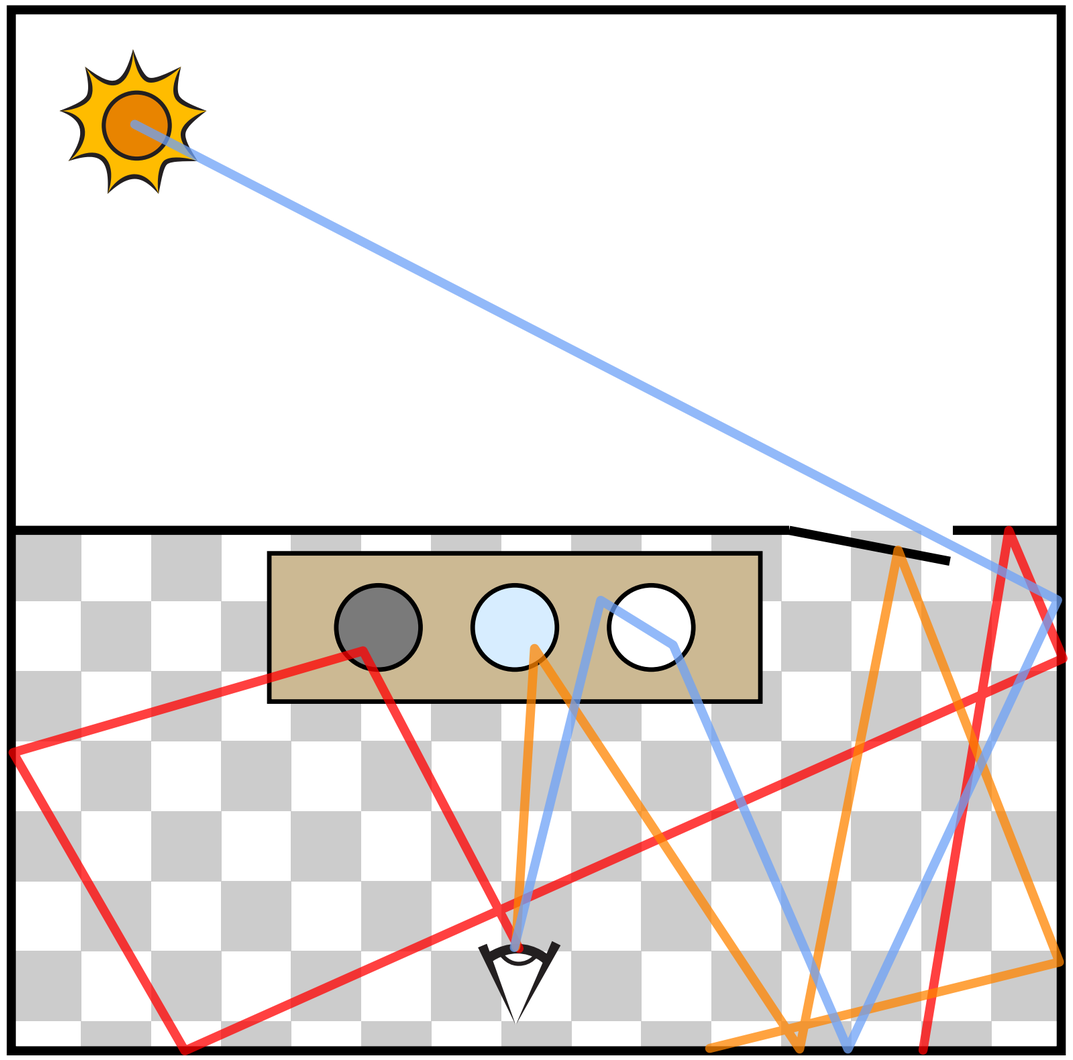
Bidirectional Path Tracing

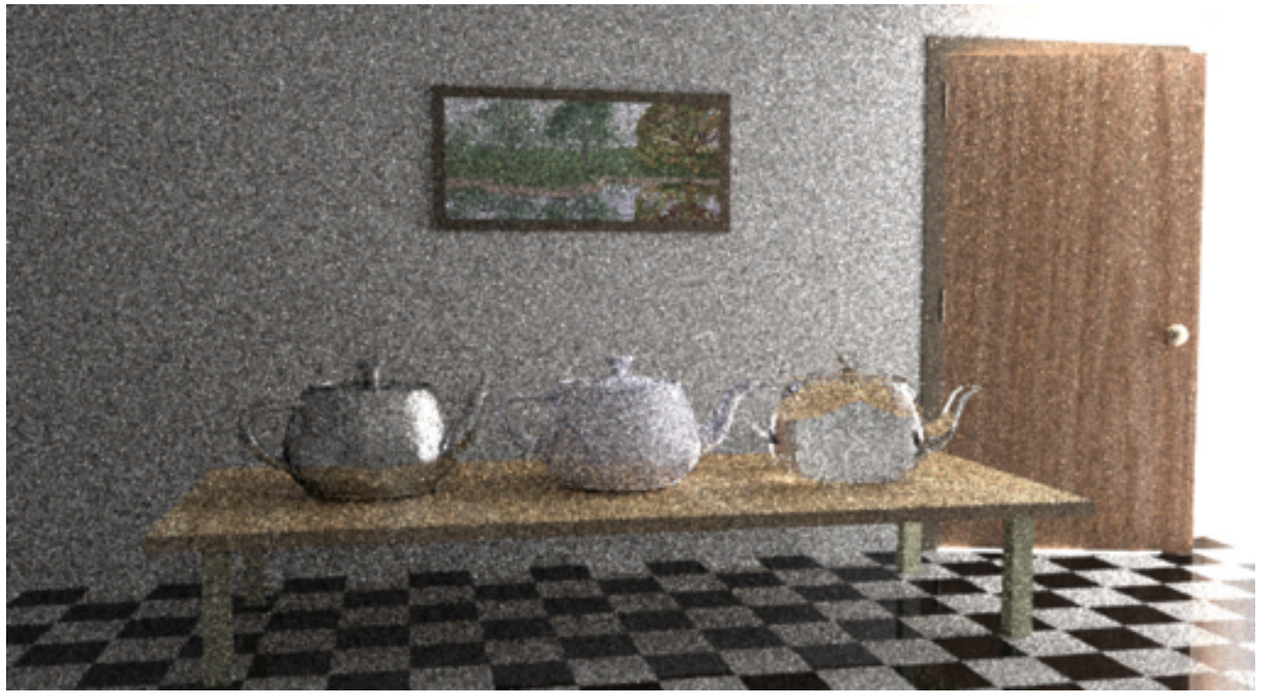
- Forward path tracing: no control over path length (hits light after n bounces, or gets terminated by Russian Roulette)
- Idea: connect paths from light, eye ("bidirectional")



- Importance sampling? Need to carefully weight contributions of path according to sampling strategy.
- (Details in Veach & Guibas, "Bidirectional Estimators for Light Transport")

Good paths can be hard to find!





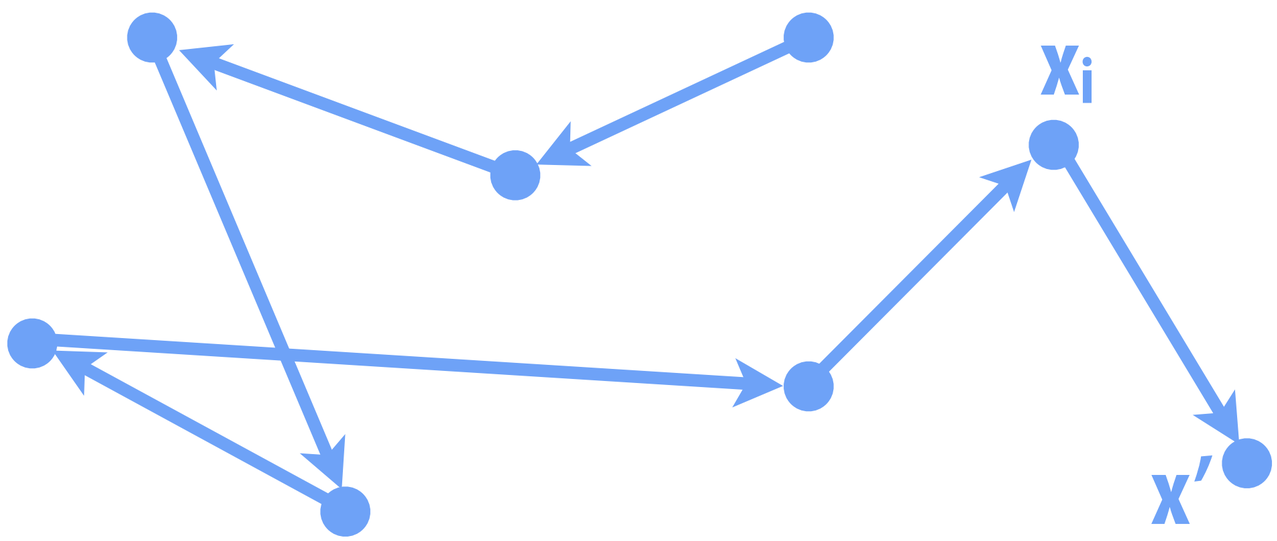
bidirectional path tracing

Metropolis light transport (MLT)

Idea:
Once we find a good path, perturb it to find nearby "good" paths.

Metropolis-Hastings Algorithm (MH)

- Standard Monte Carlo: sum up independent samples
- MH: take random walk of dependent samples ("mutations")
- Basic idea: prefer to take steps that increase sample value



$\alpha := f(x') / f(x_i)$ "transition probability"

if random # in [0,1] < α:

$x_{i+1} = x'$

else:

$x_{i+1} = x_i$

- If careful, sample distribution will be proportional to integrand
  - make sure mutations are "ergodic" (reach whole space)
  - need to take a long walk, so initial point doesn't matter ("mixing")

Metropolis-Hastings: Sampling an Image

- Want to take samples proportional to image density f
- Start at random point; take steps in (normal) random direction
- Occasionally jump to random point (ergodicity)
- Transition probability is "relative darkness" $f(x')/f(x_i)$

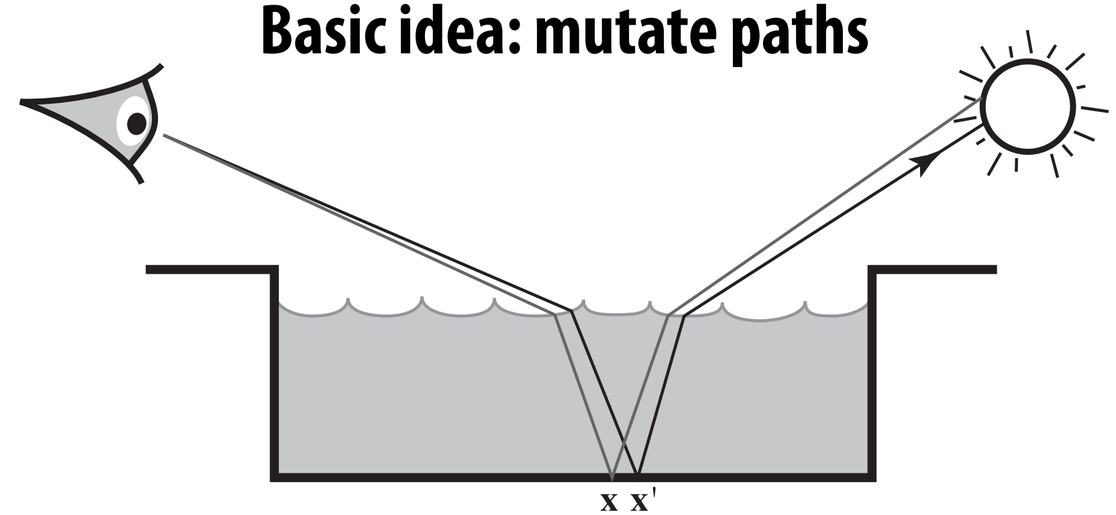
short walk

long walk

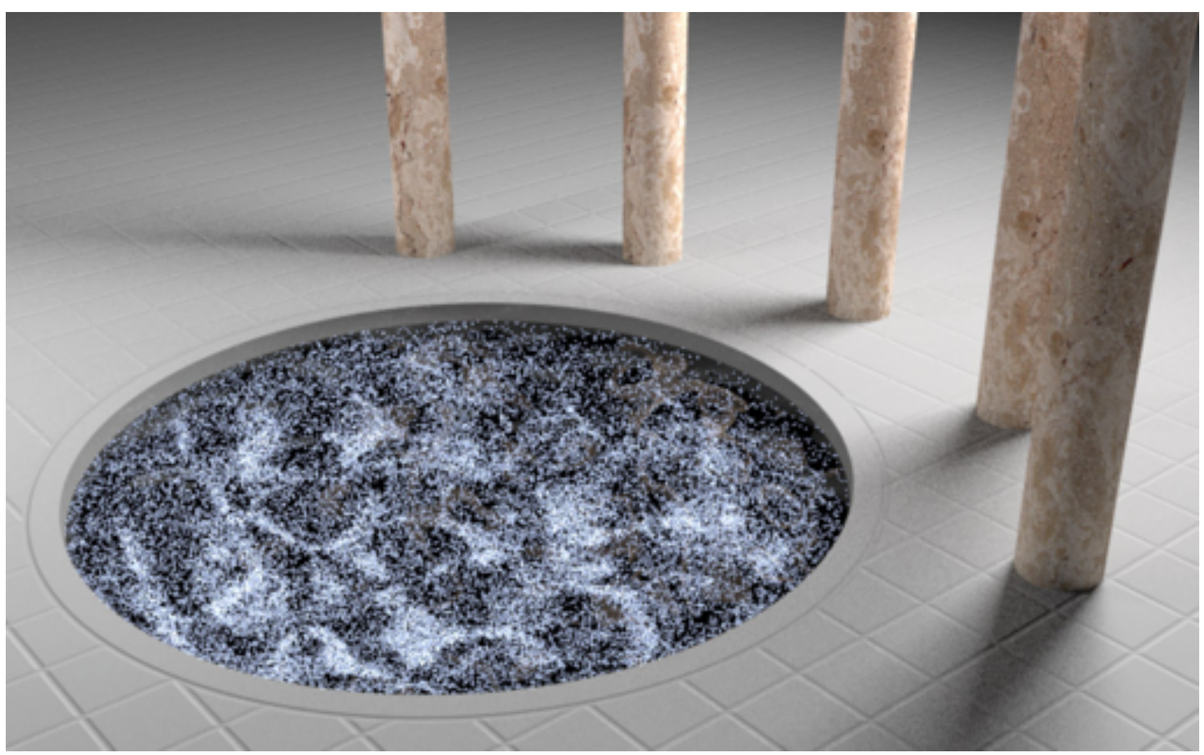
(original image)

Metropolis Light Transport

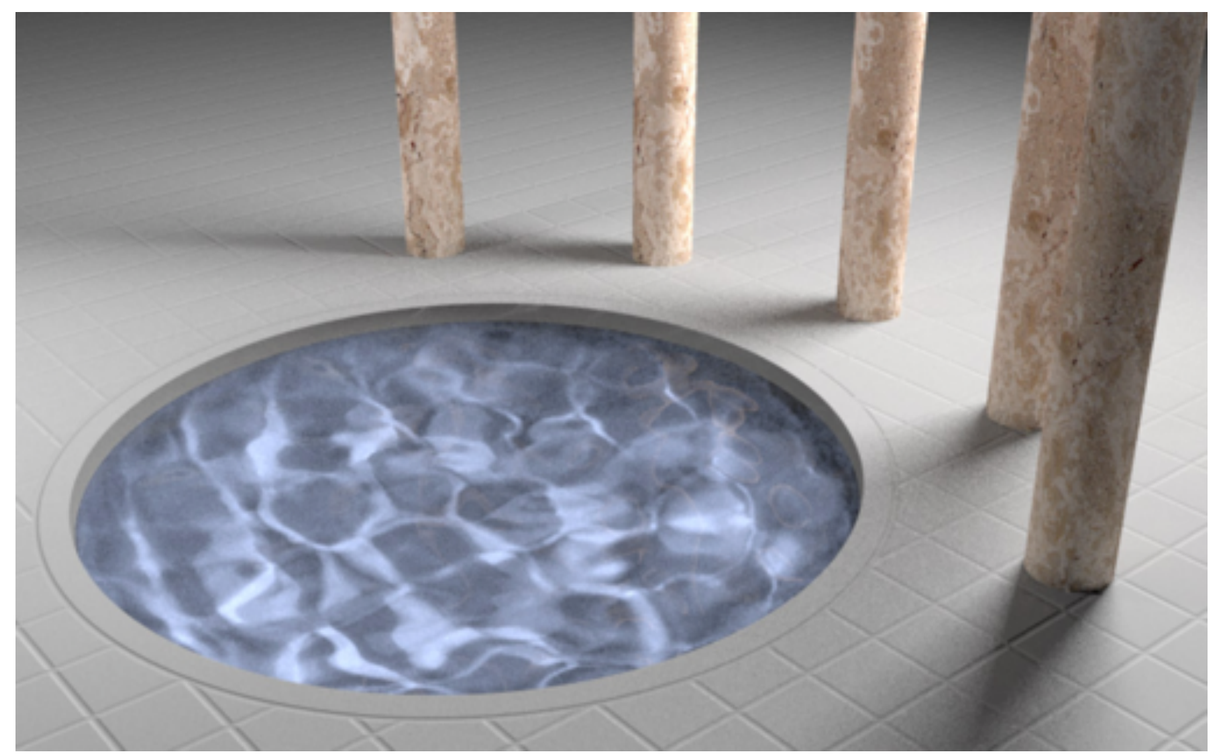
Basic idea: mutate paths



(For details see Veach, "Robust Monte Carlo Methods for Light Transport Simulation")



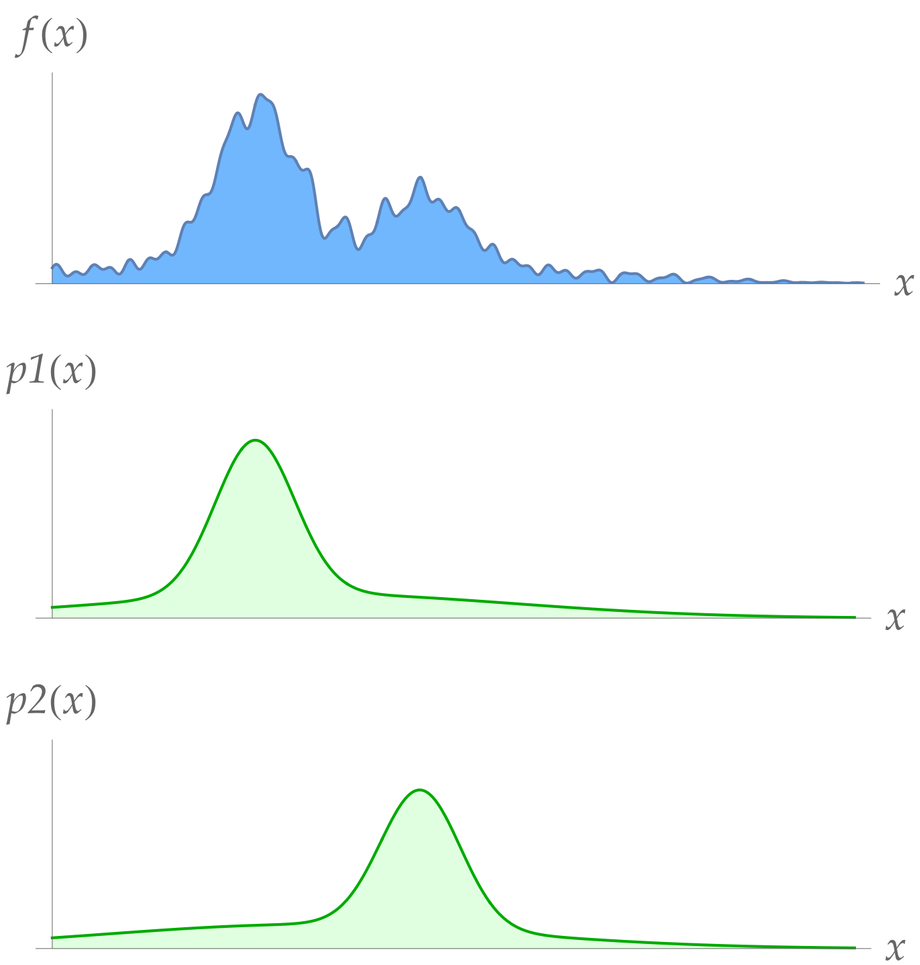
path tracing



Metropolis light transport (same time)

Multiple Importance Sampling (MIS)

- Many possible importance sampling strategies
- Which one should we use for a given integrand?
- MIS: combine strategies to preserve strengths of all of them



sum over strategies

sum over samples

jth sample taken with ith strategy

$$\frac{1}{N}\sum_{i=1}^{n}\sum_{j=1}^{n_i}\frac{f(x_{ij})}{\sum_k c_k p_k(x_{ij})}$$

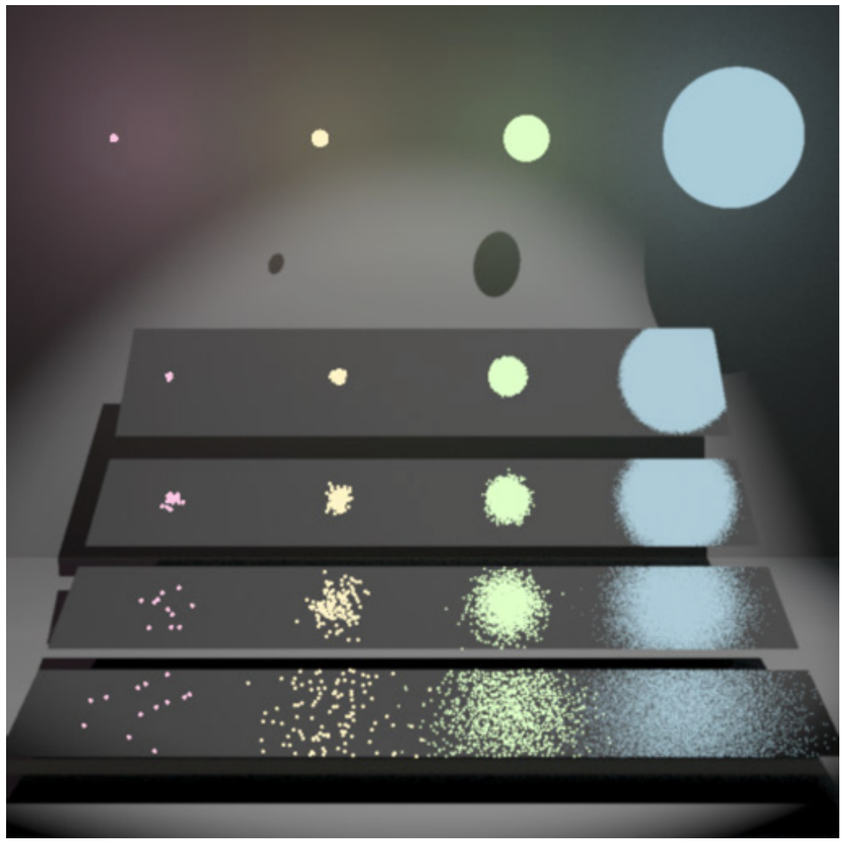
total # of samples

fraction of samples taken w/ kth strategy

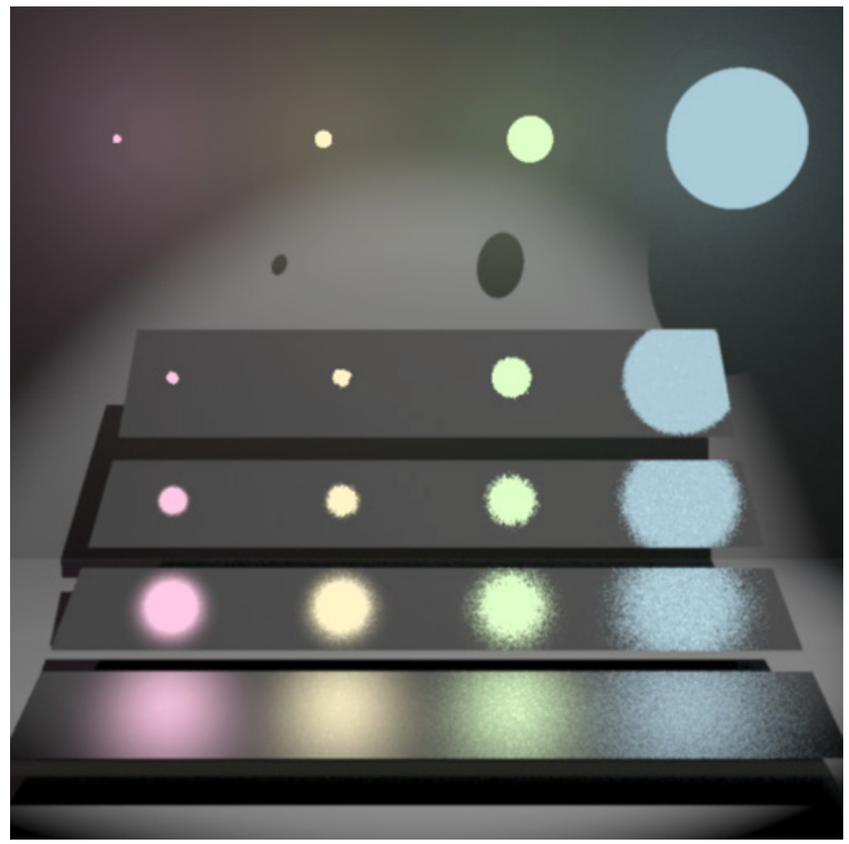
kth importance density

Still, several improvements possible (cutoff, power, max)—see Veach & Guibas.

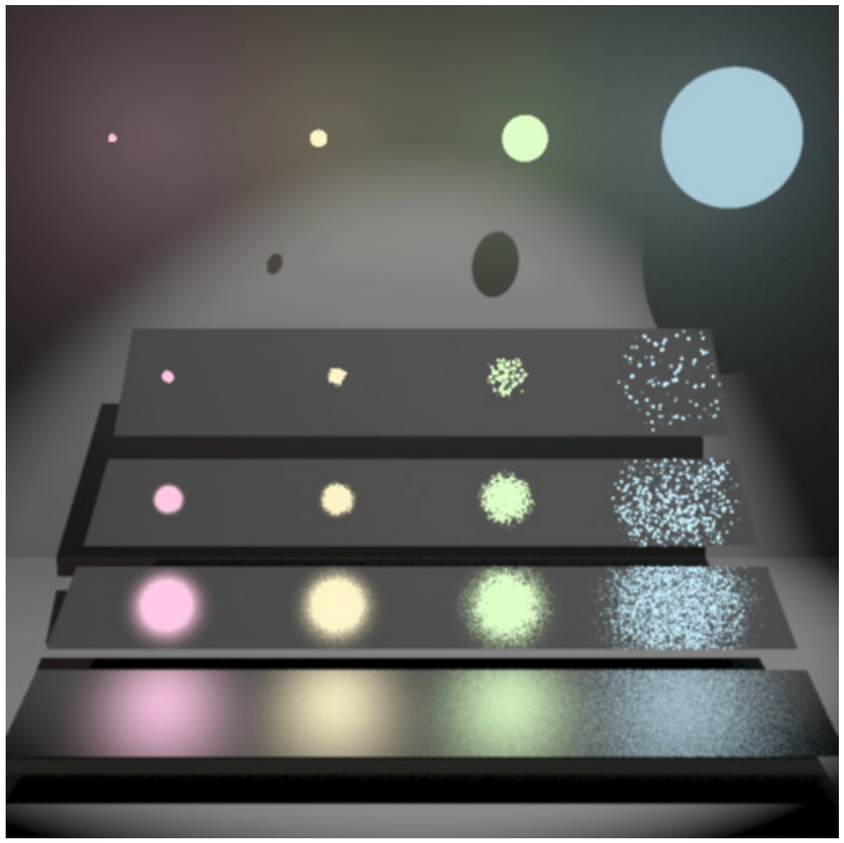
Multiple Importance Sampling: Example



sample materials



multiple importance sampling
(power heuristic)



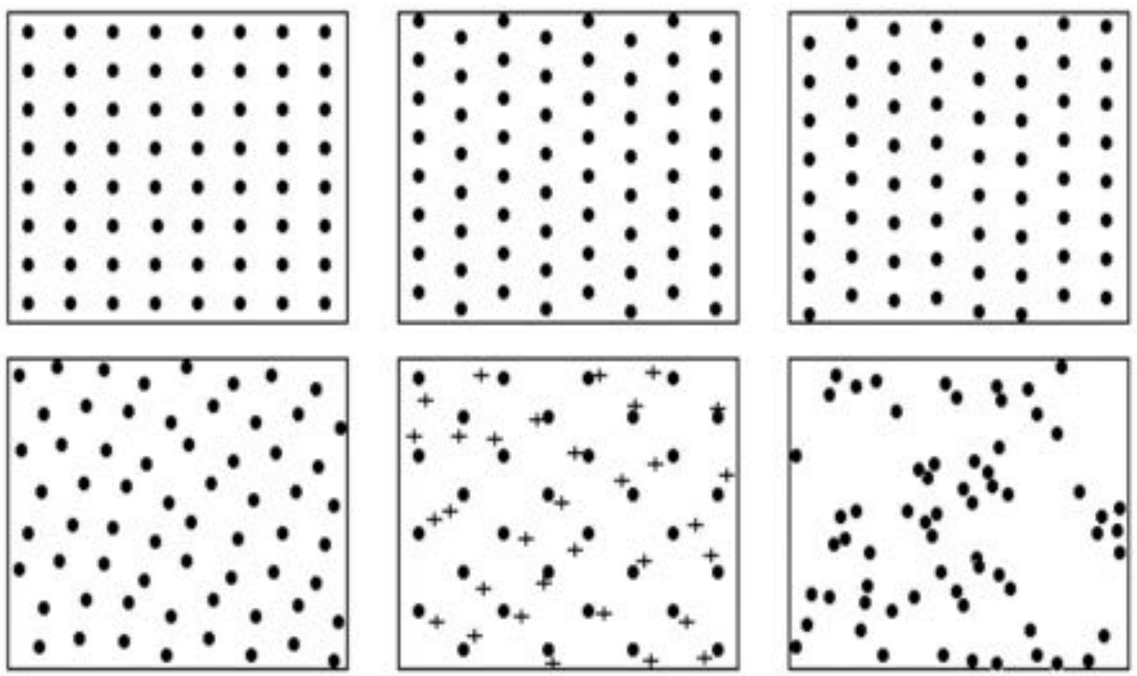
sample lights

Ok, so importance is important.

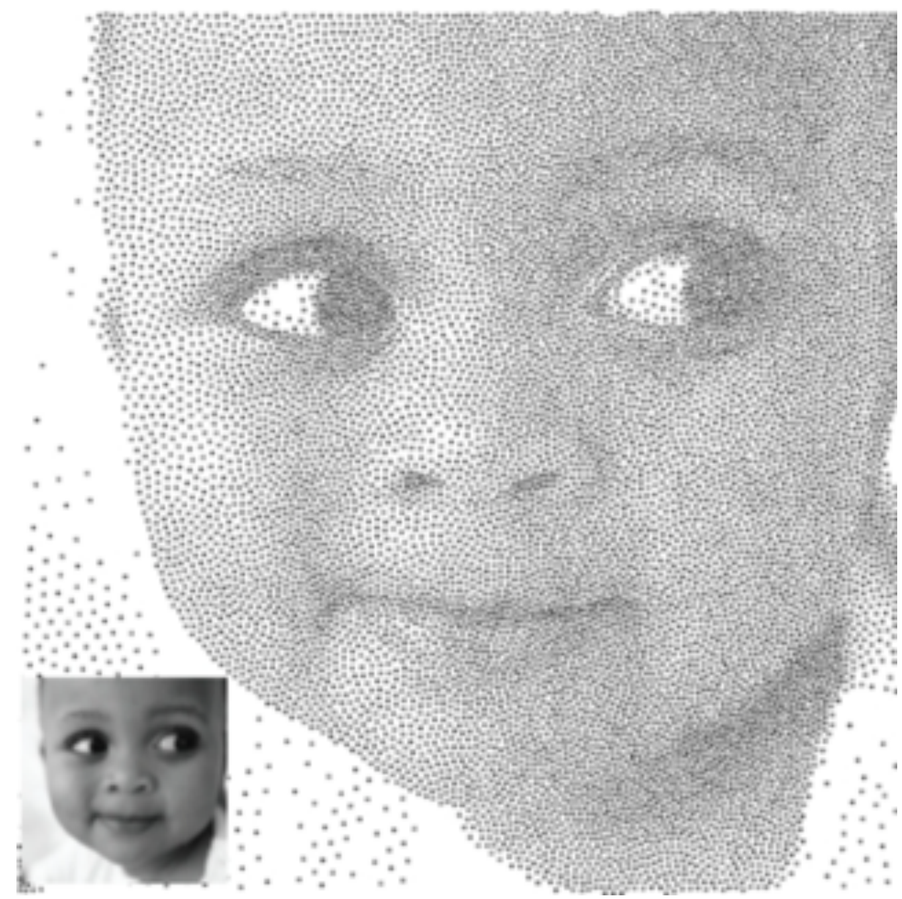
But how do we sample our function in the first place?

Sampling Patterns & Variance Reduction

- Want to pick samples according to a given density
- But even for uniform density, lots of possible sampling patterns
- Sampling pattern will affect variance (of estimator!)



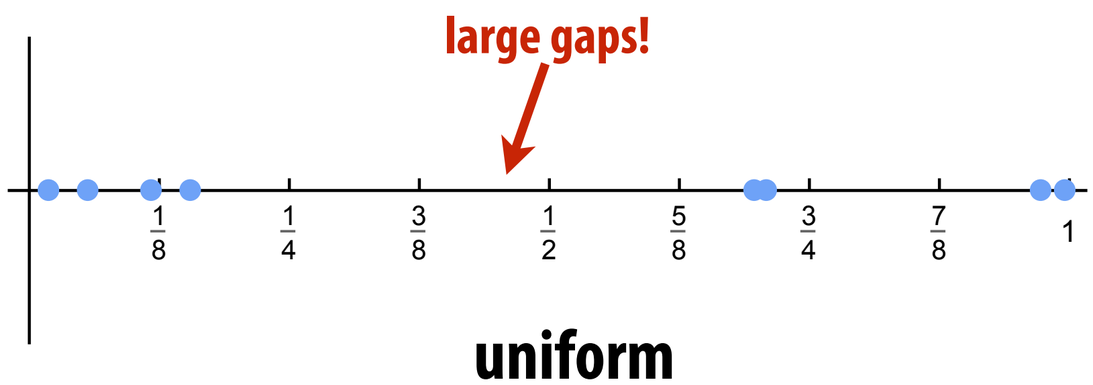
uniform sampling density

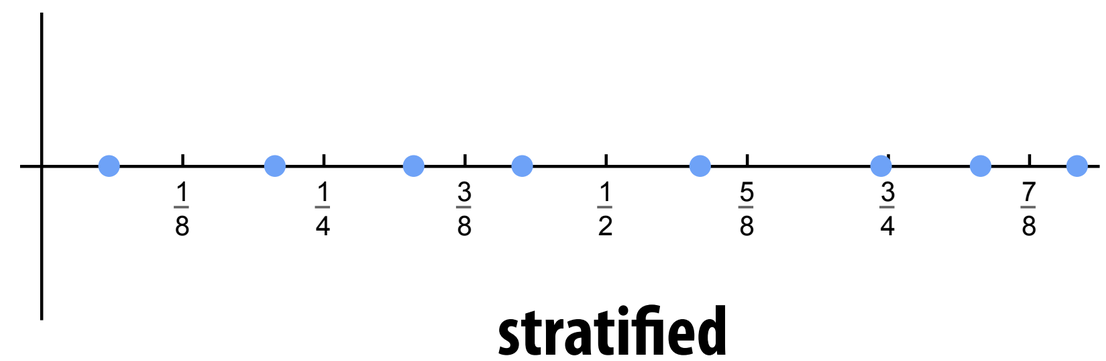


nonuniform sampling density

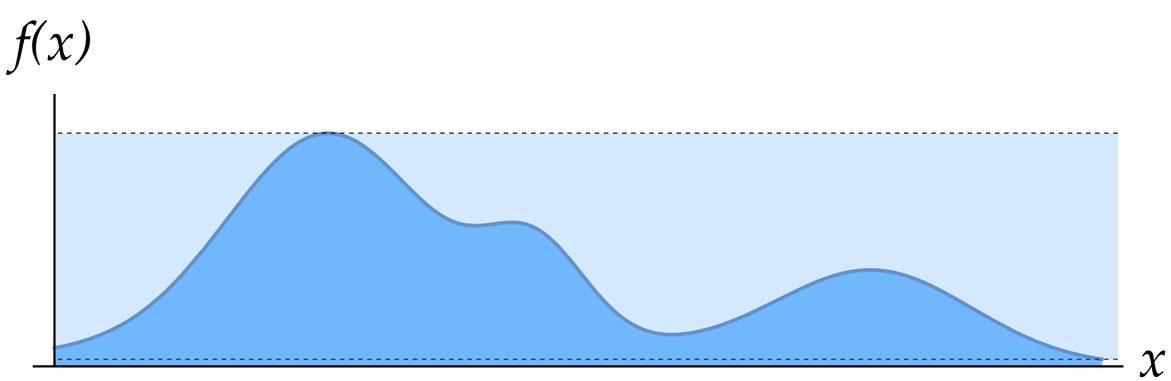
Stratified Sampling

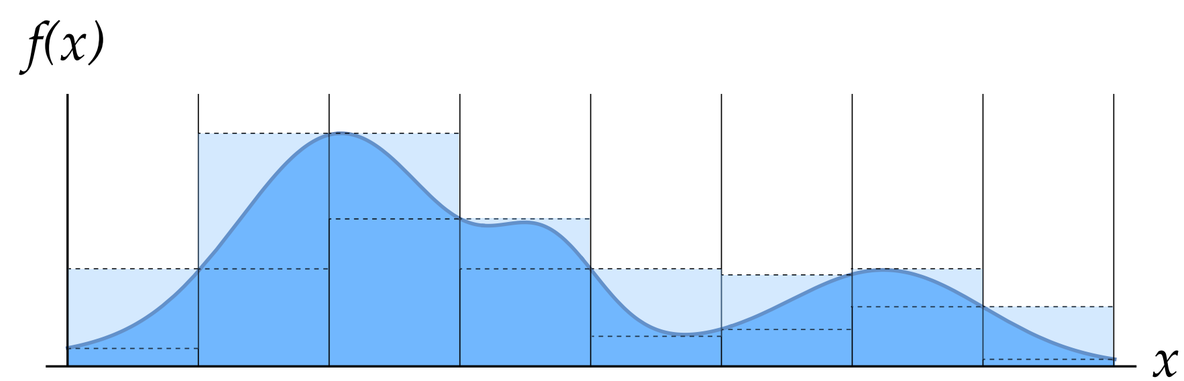
- How do we pick n values from [0,1]?
- Could just pick n samples uniformly at random
- Alternatively: split into n bins, pick uniformly in each bin





- FACT: stratified estimate never has larger variance (often lower)



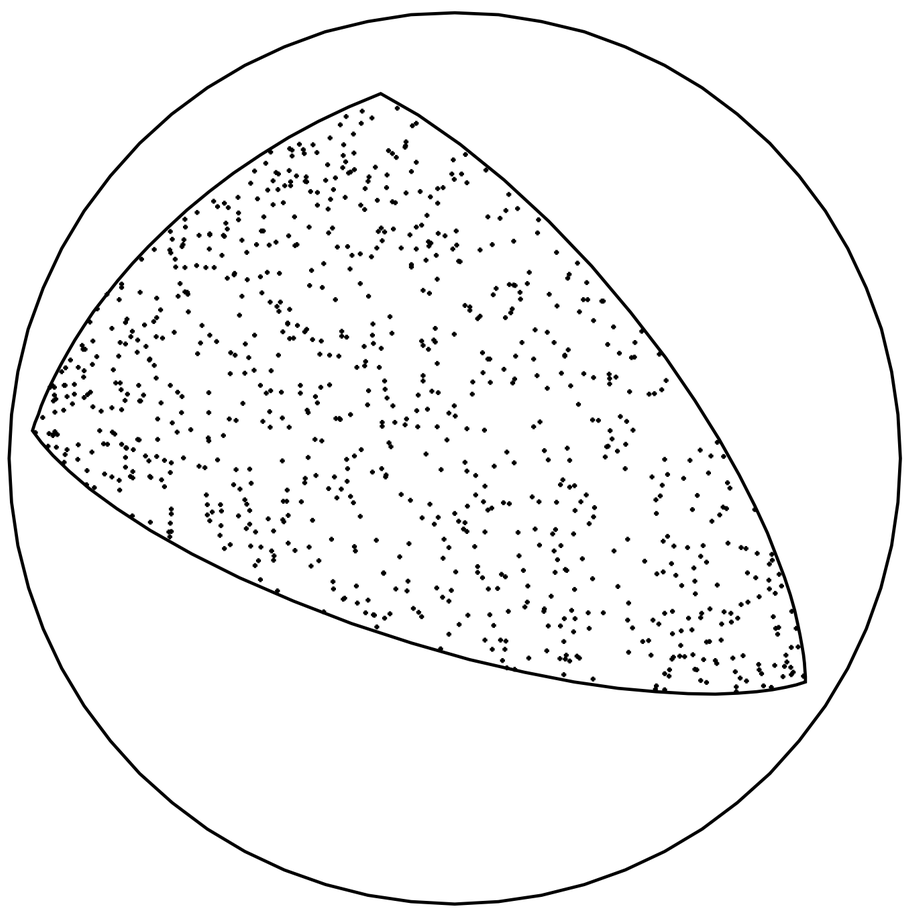


Intuition: each stratum has smaller variance. (Proof by linearity of expectation!)

Stratified Sampling in Rendering/Graphics

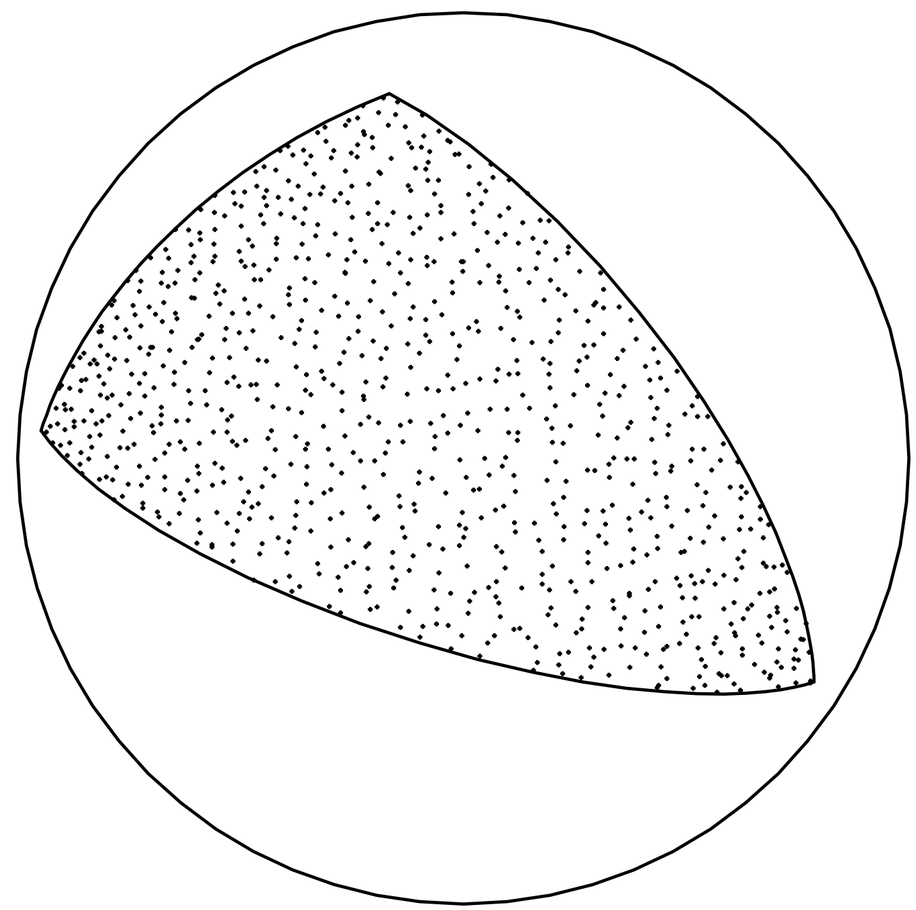
- Simply replacing uniform samples with stratified ones already improves quality of sampling for rendering (…and other graphics/visualization tasks!)

uniform



"more clumpy"

stratified



"more even"

See especially: Jim Arvo, "Stratified Sampling of Spherical Triangles" (SIGGRAPH 1995)

Low-Discrepancy Sampling

- "No clumps" hints at one possible criterion for a good sample:
- Number of samples should be proportional to area
- Discrepancy measures deviation from this ideal

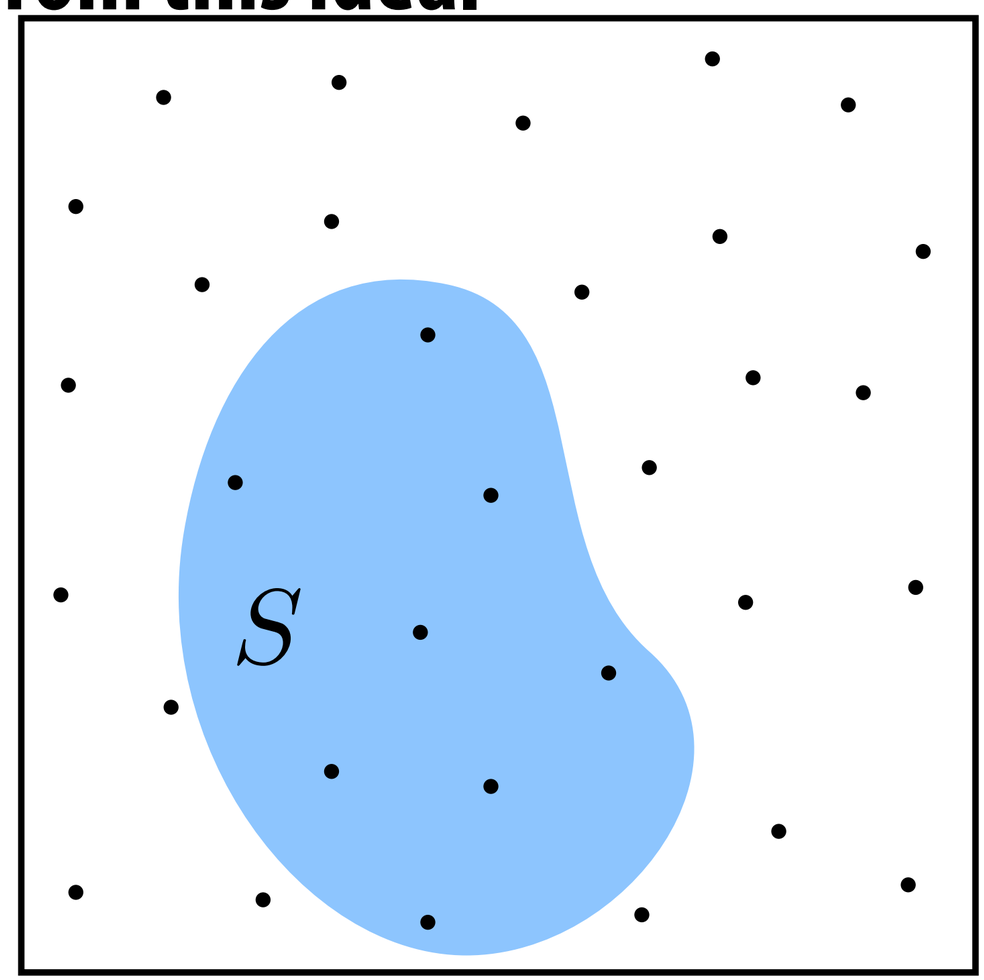
discrepancy of sample points X over a region S

number of samples in X covered by S

$$d_S(X) := \left| A(S) - \frac{n(S)}{|X|} \right|$$

area of S

total # of samples in X



overall discrepancy of X

$$D(X) := \max_{S \in F} d_S(X)$$

(ideally equal to zero!)

some family of regions S (e.g., boxes, disks, ...)

See especially: Dobkin et al, "Computing Discrepancy w/ Applications to Supersampling" (1996)

Quasi-Monte Carlo methods (QMC)

- Replace truly random samples with low-discrepancy samples
- Why? Koksma's theorem:

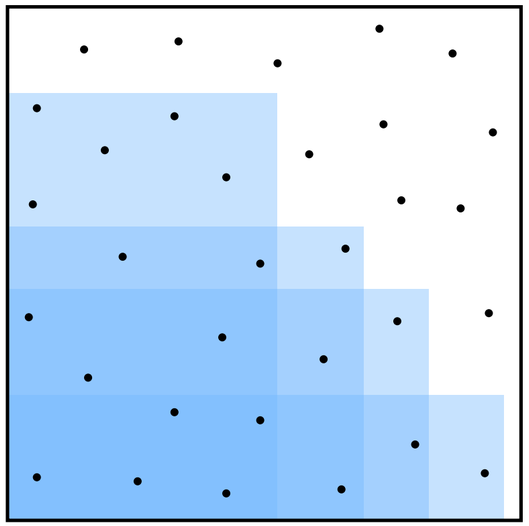
sample points in X

total variation of f (integral of |f'|)

discrepancy of sample X

$$\left| \frac{1}{n} \sum_{i=1}^{n} f(x_i) - \int_0^1 f(x)\ dx \right| \leq \mathcal{V}(f) D(X)$$

- I.e., for low-discrepancy X, estimate approaches integral
- Similar bounds can be shown in higher dimensions
- **WARNING:** total variation not always bounded!
- **WARNING:** only for family F of axis-aligned boxes S!
- Discrepancy still a very reasonable criterion in practice



F

Hammersley & Halton Points

- Can easy generate samples with near-optimal discrepancy
- First define radical inverse $\varphi_r(i)$
- Express integer i in base r, then reflect digits around decimal
- E.g., $\varphi_{10}(1234) = 0.4321$
- Can get n Halton points $x_1, \ldots, x_n$ in k-dimensions via

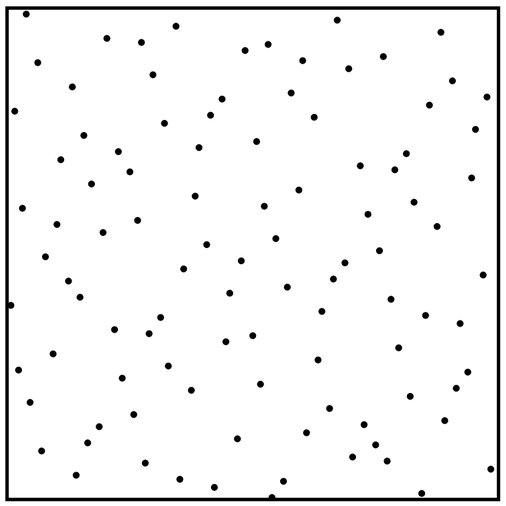
$$x_i = (\phi_{P_1}(i), \phi_{P_2}(i), \ldots, \phi_{P_k}(i))$$

kth prime number

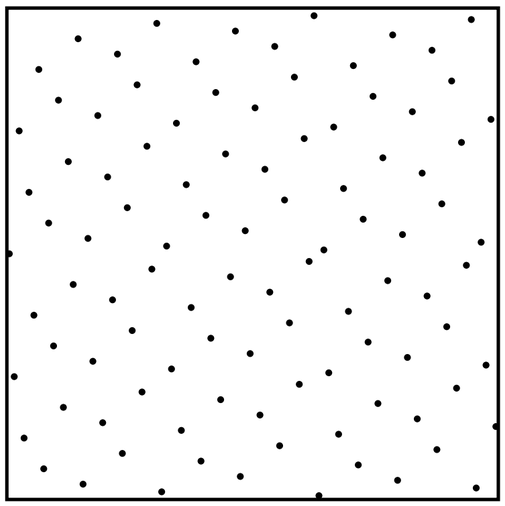
- Similarly, Hammersley sequence is

$$x_i = (i/n, \phi_{P_1}(i), \phi_{P_2}(i), \ldots, \phi_{P_{k-1}}(i))$$

n must be known ahead of time!

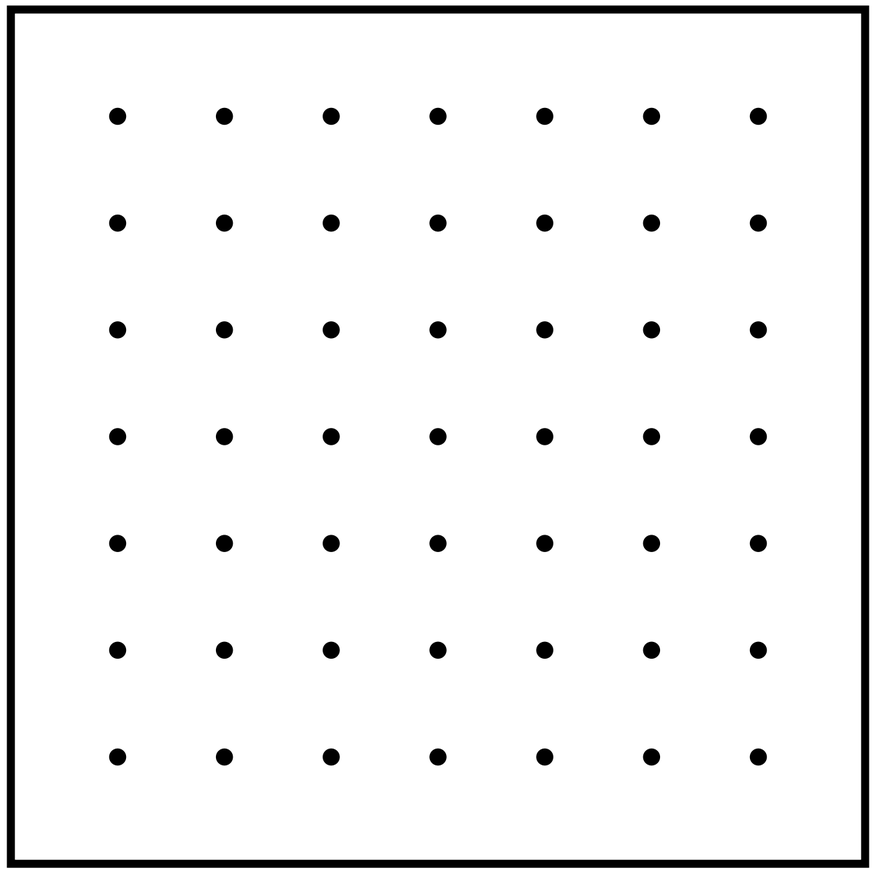


Halton



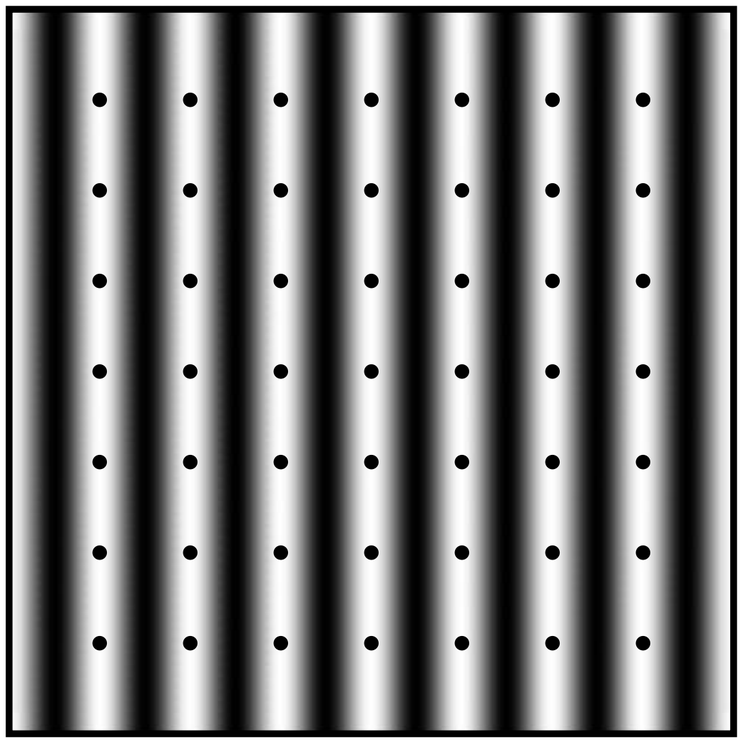
Hammersley

Wait, but doesn't a regular grid have really low discrepancy...?

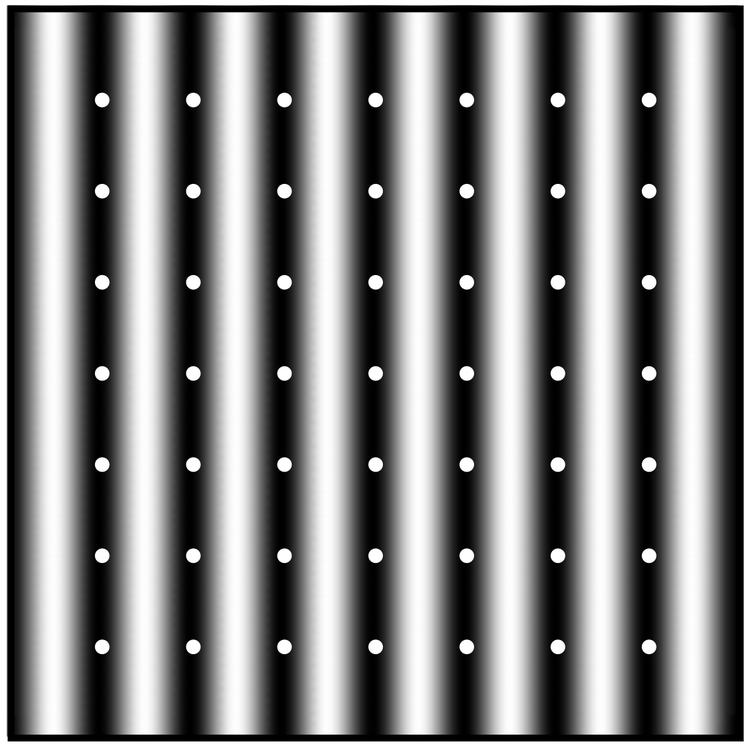


There's more to life than discrepancy

- Even low-discrepancy patterns can exhibit poor behavior:



$$\frac{1}{n}\sum_{i=1}^{n} f(x_i) = 1$$



$$\frac{1}{n}\sum_{i=1}^{n} f(x_i) = 0$$

- Want pattern to be anisotropic (no preferred direction)
- Also want to avoid any preferred frequency (see above!)

Blue Noise - Motivation

- Can observe that monkey retina exhibits blue noise pattern [Yellott 1983]

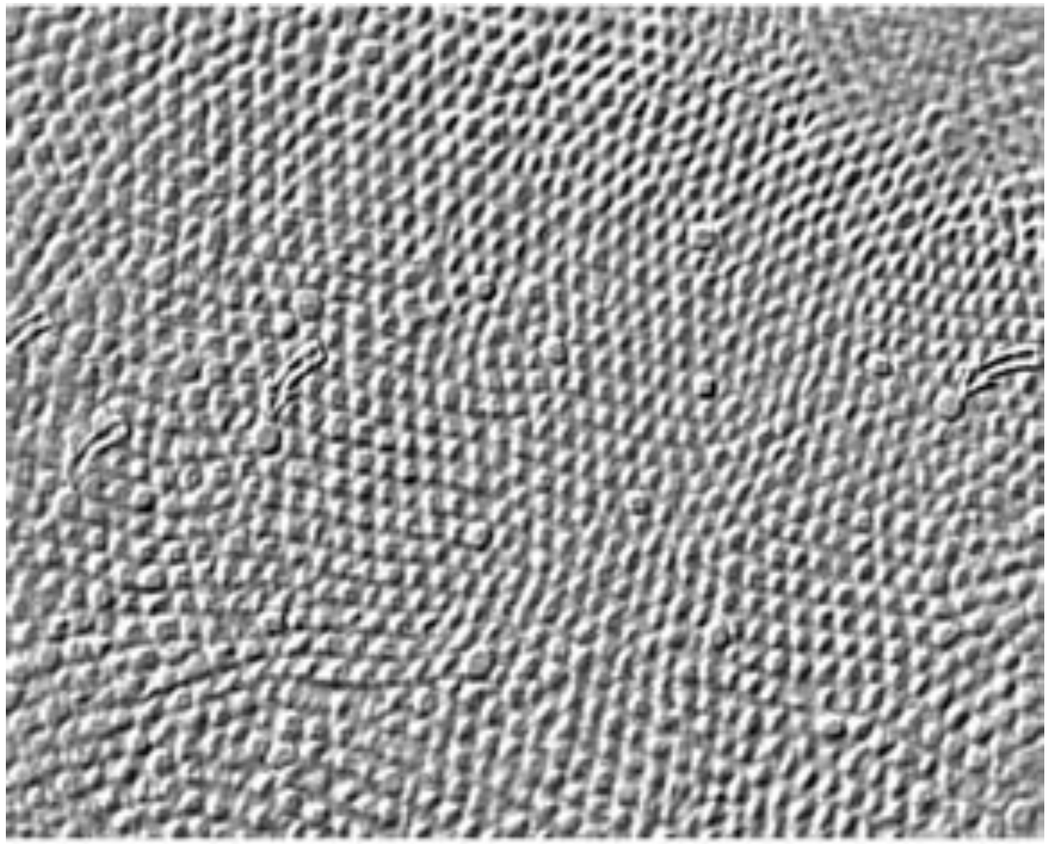
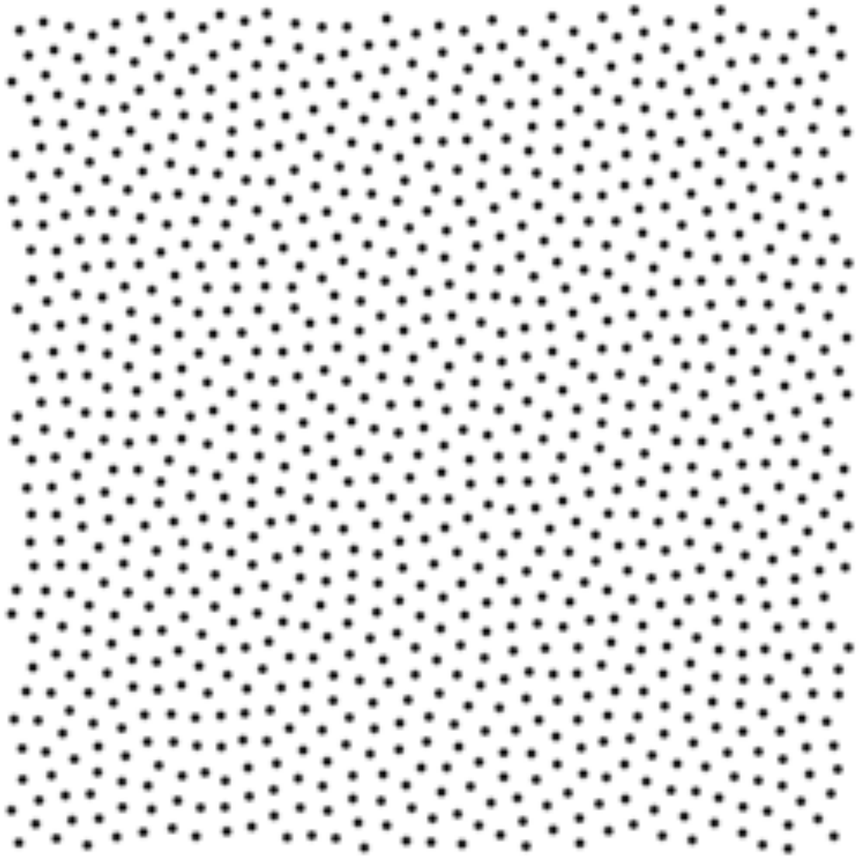


Fig. 13. Tangential section through the human fovea. Larger cones (arrows) are blue cones. From Ahnelt et al. 1987.

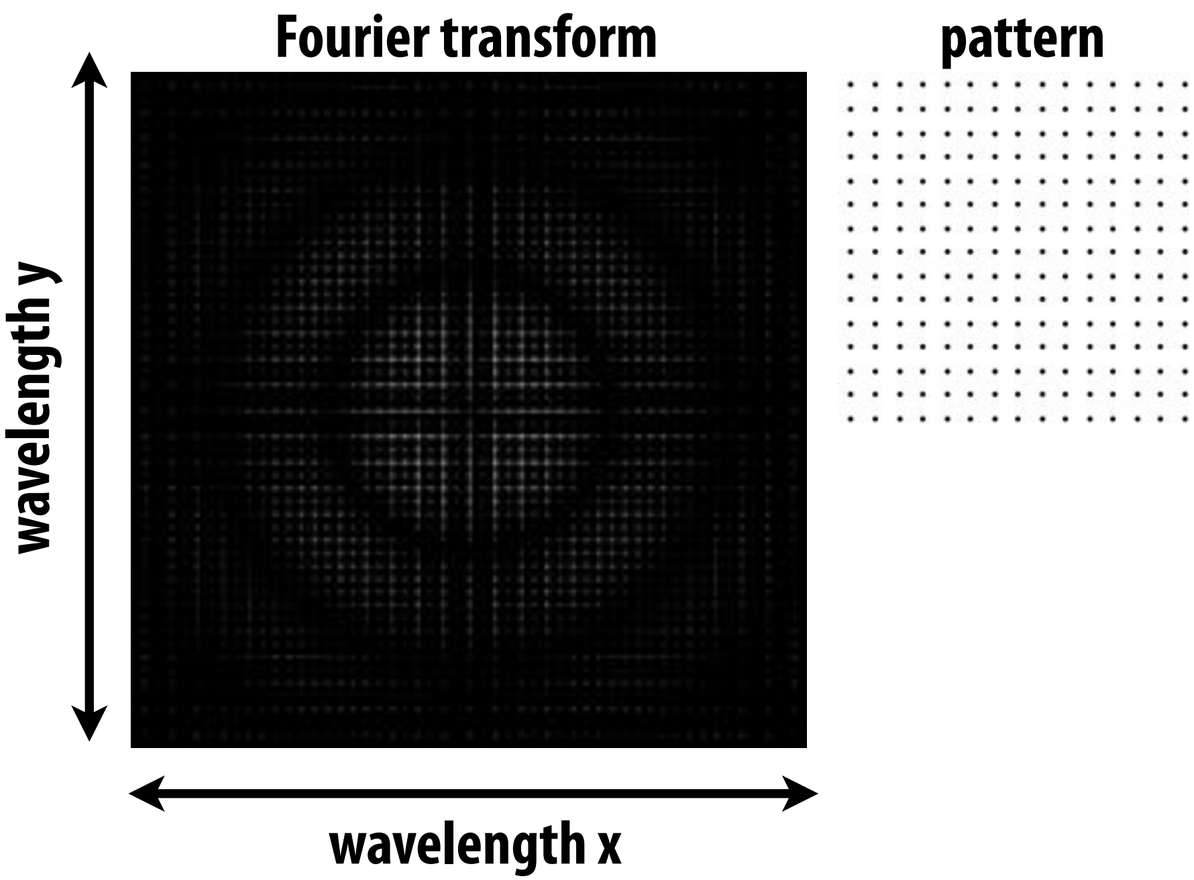


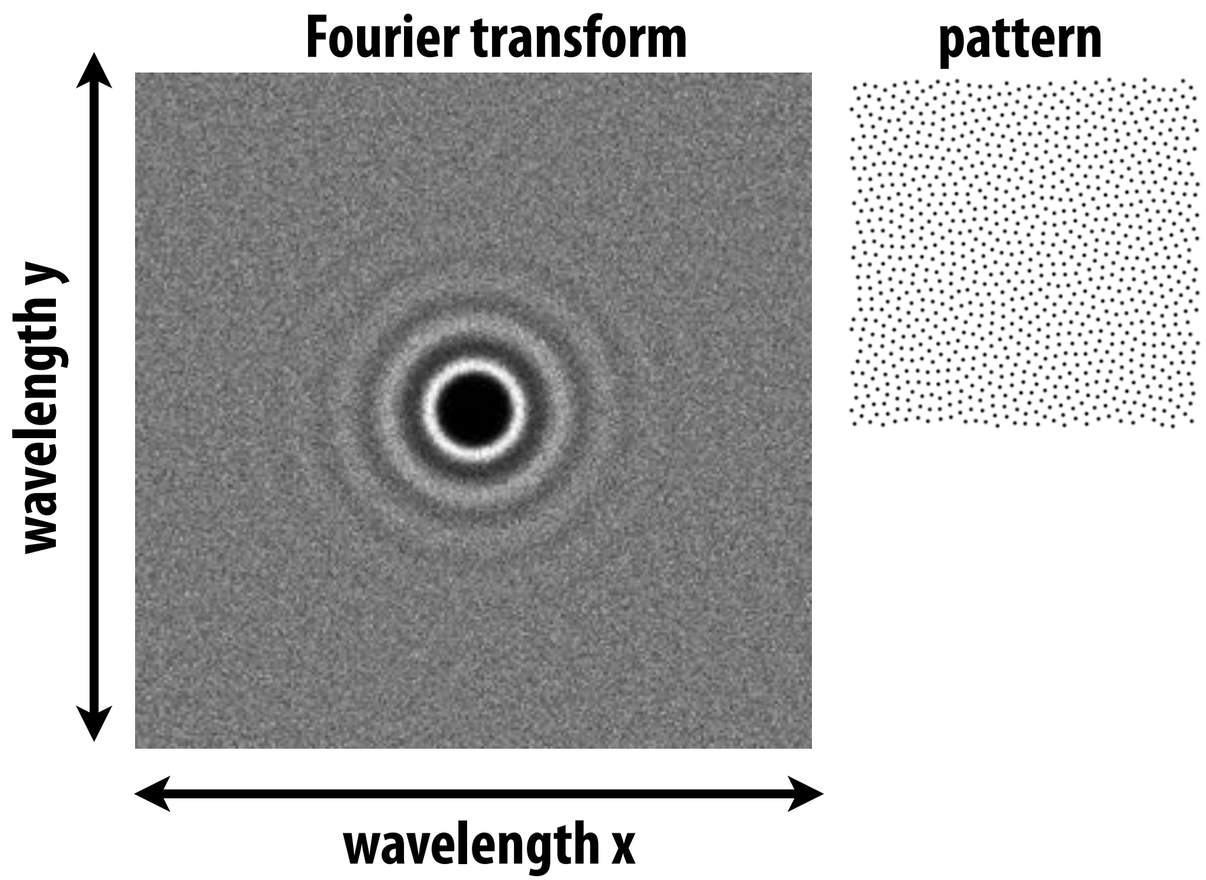
"blue noise"

- No obvious preferred directions (anisotropic)
- What about frequencies?

Blue Noise - Fourier Transform

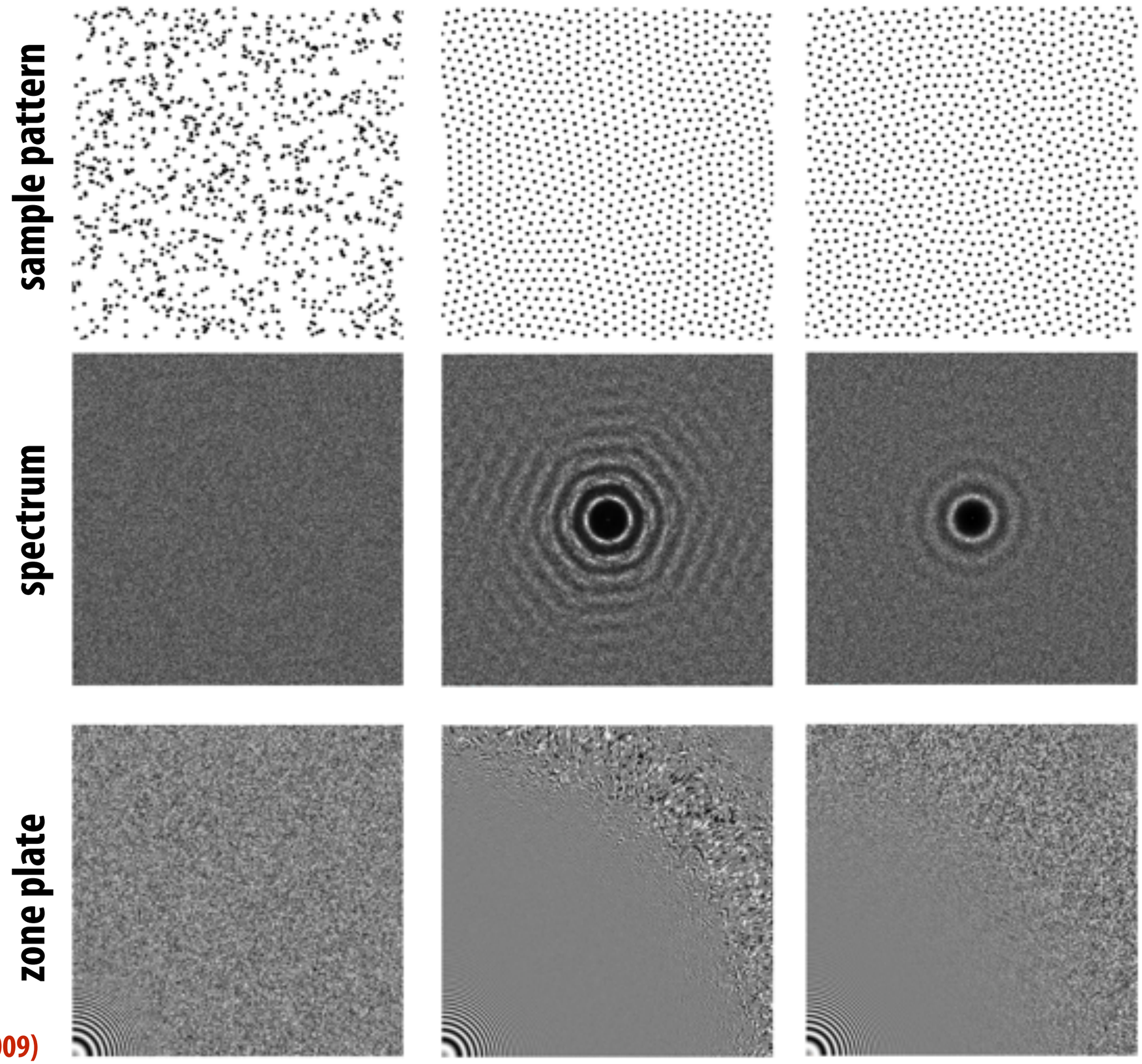
- Can analyze quality of a sample pattern in Fourier domain





- Regular pattern has "spikes" at regular intervals
- Blue noise is spread evenly over all frequencies in all directions
- bright center "ring" corresponds to sample spacing

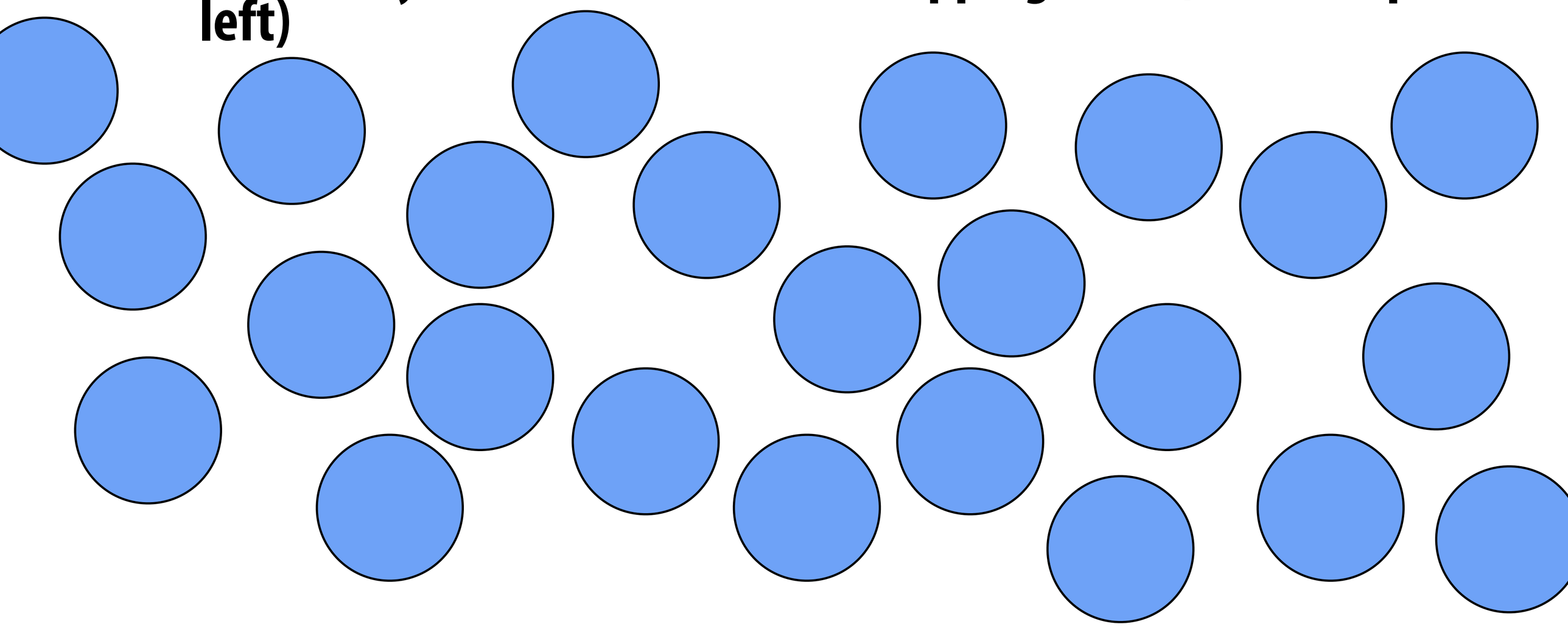
Spectrum affects reconstruction quality



(from Balzer et al 2009)

Poisson Disk Sampling

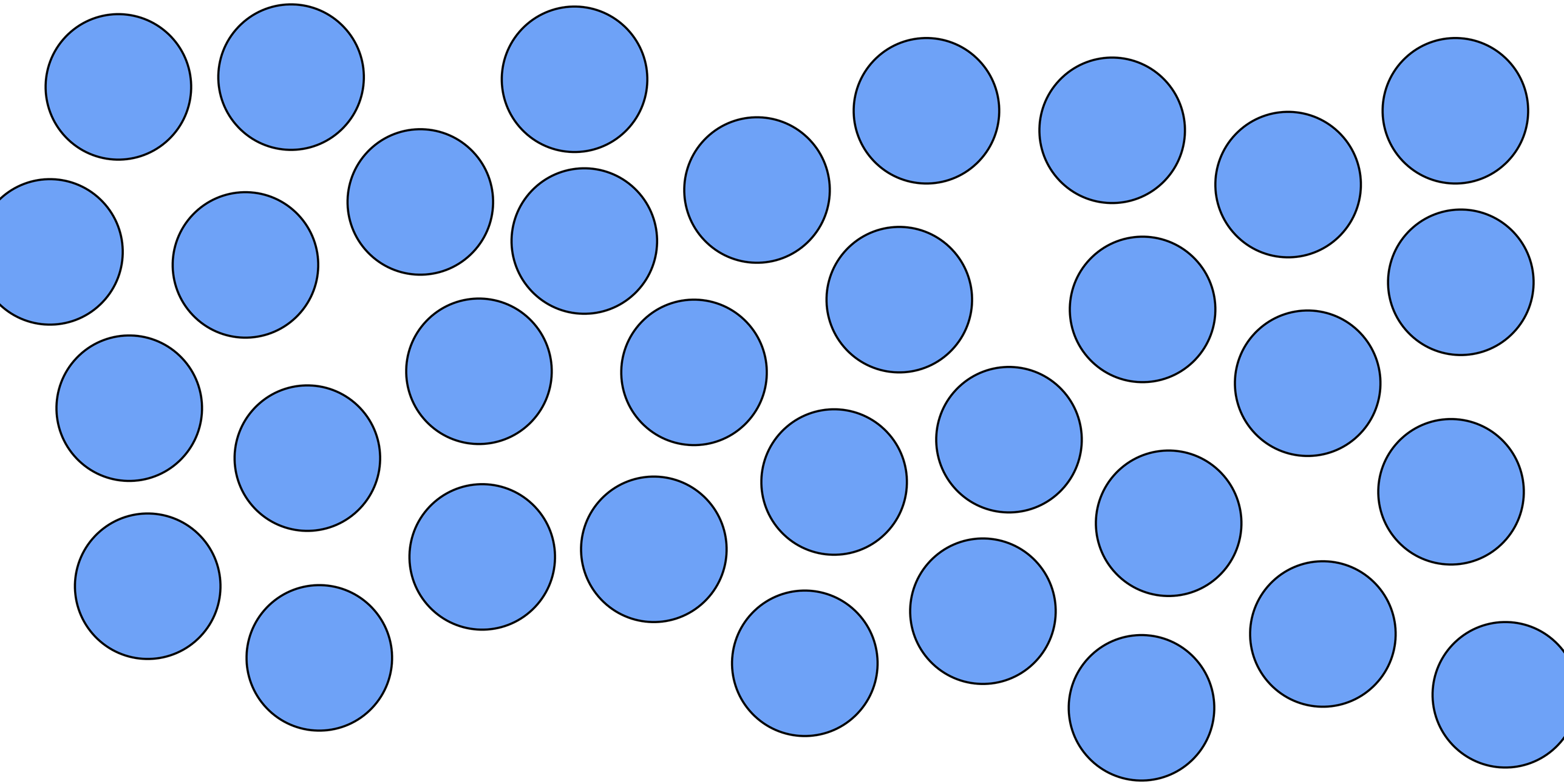
- How do you generate a "nice" sample?
- One of the earliest algorithms: Poisson disk sampling
- Iteratively add random non-overlapping disks (until no space left)



Decent spectral quality, but we can do better.

Lloyd Relaxation

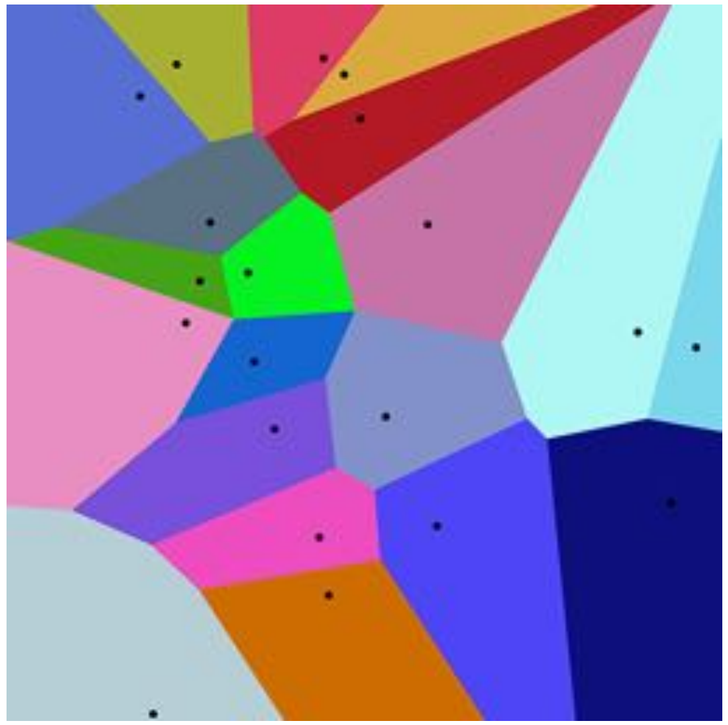
- Iteratively move each disk to the center of its neighbors



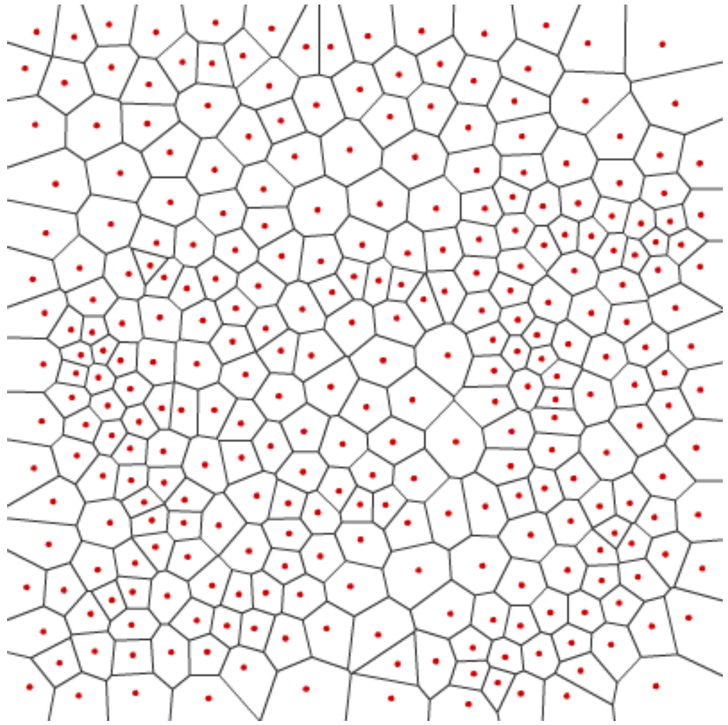
Better spectral quality, slow to converge. Can do better yet...

Voronoi-Based Methods

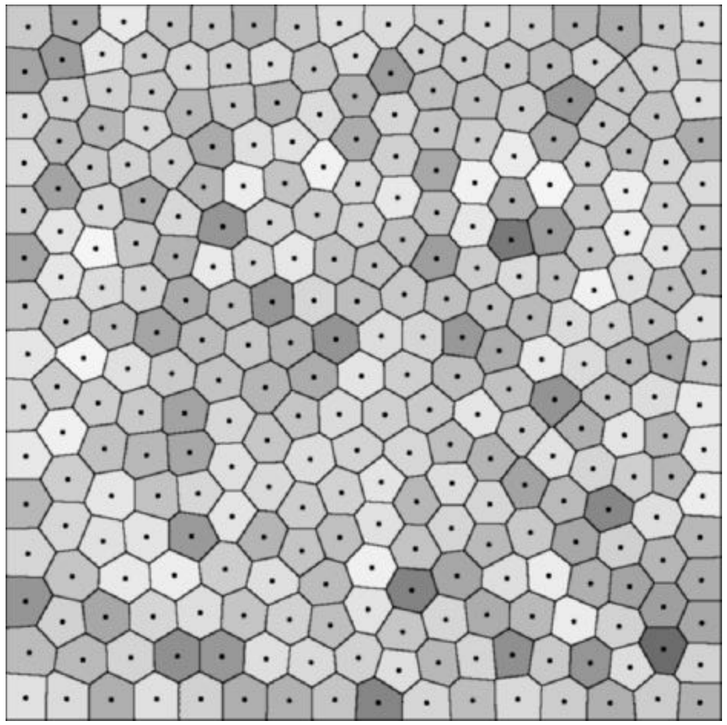
- Natural evolution of Lloyd
- Associate each sample with set of closest points (Voronoi cell)
- Optimize qualities of this Voronoi diagram
- E.g., sample is at cell's center of mass, cells have same area, etc.



Voronoi



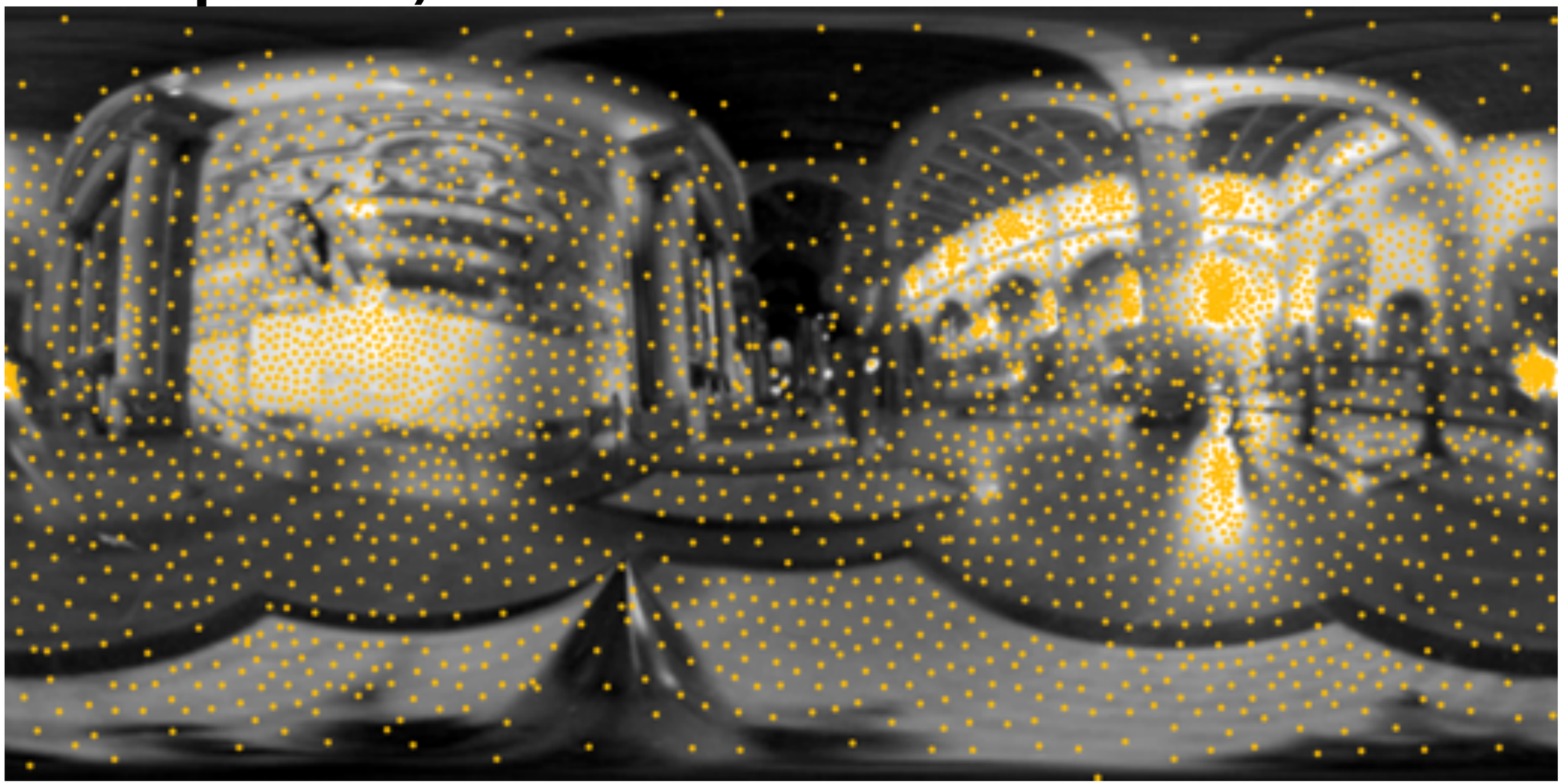
centroidal



equal area

Adaptive Blue Noise

- Can adjust cell size to sample a given density (e.g., importance)



Computational tradeoff: expensive* precomputation / efficient sampling.

*But these days, not that expensive...

How do we efficiently sample from a large distribution?

Sampling from the CDF

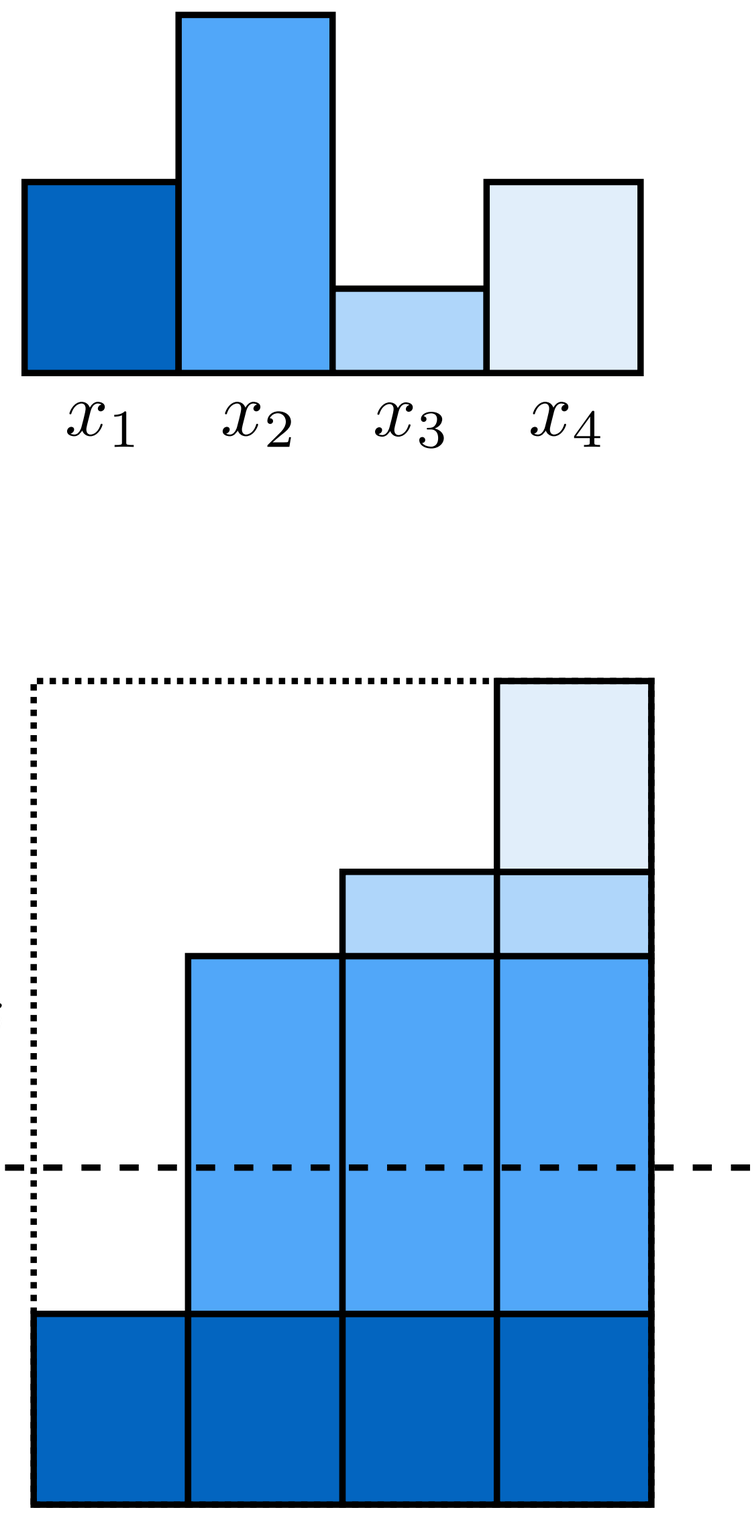
To randomly select an event, select x_i if

$$P_{i-1} < \xi < P_i$$

Uniform random variable $\in [0, 1]$

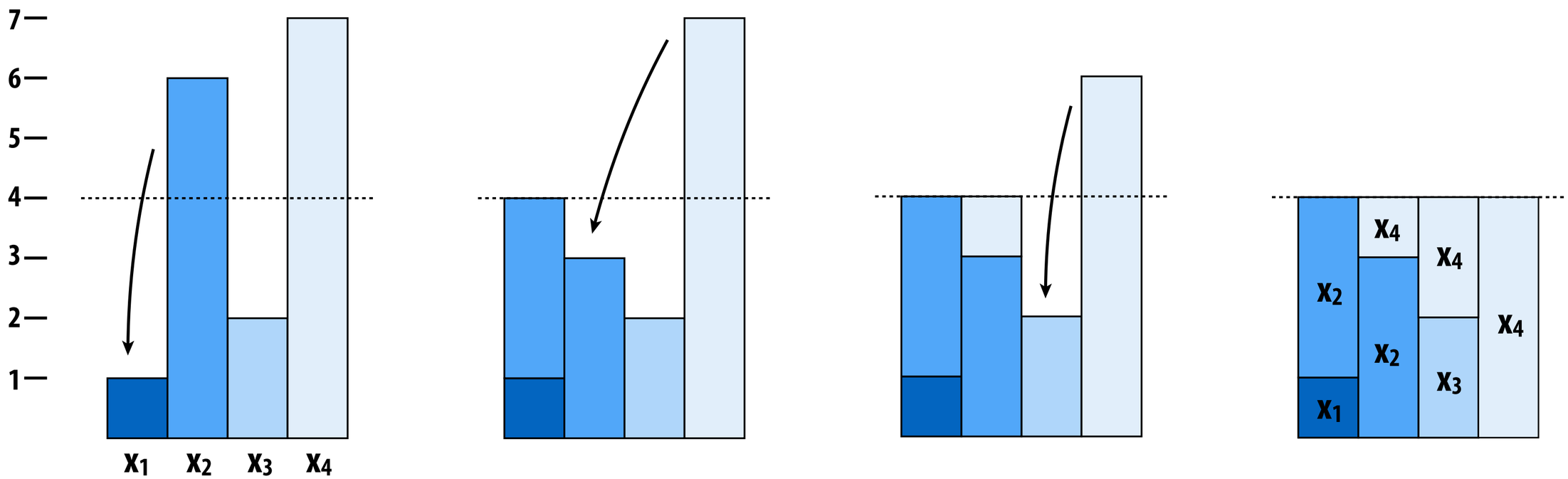
e.g., # of pixels in an environment map (big!)

Cost? O(n log n)



Alias Table

- Get amortized O(1) sampling by building "alias table"
- Basic idea: rob from the rich, give to the poor (O(n)):

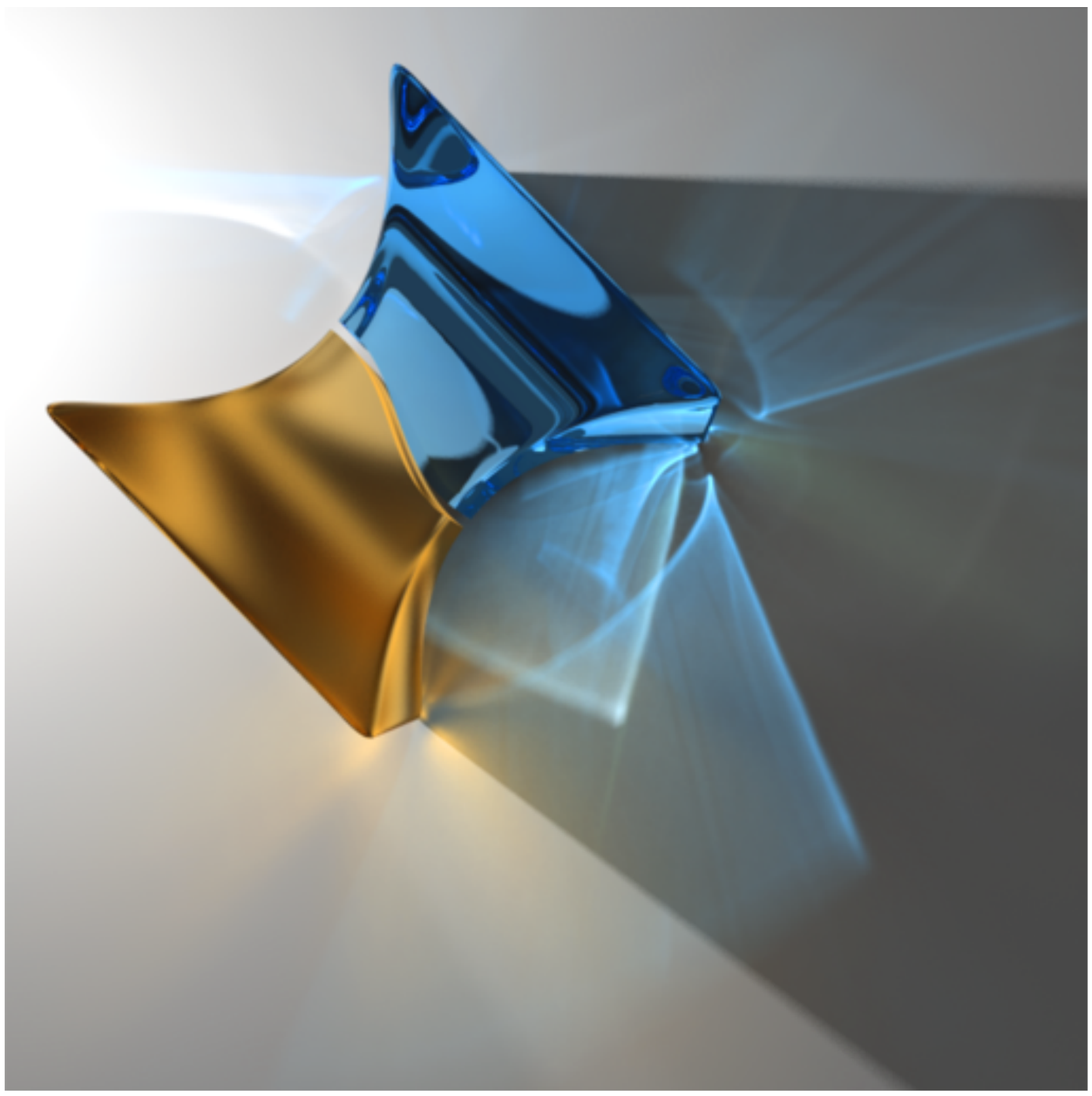


- Table just stores two identities & ratio of heights per column
- To sample:
  - pick uniform # between 1 and n
  - biased coin flip to pick one of the two identities in nth column

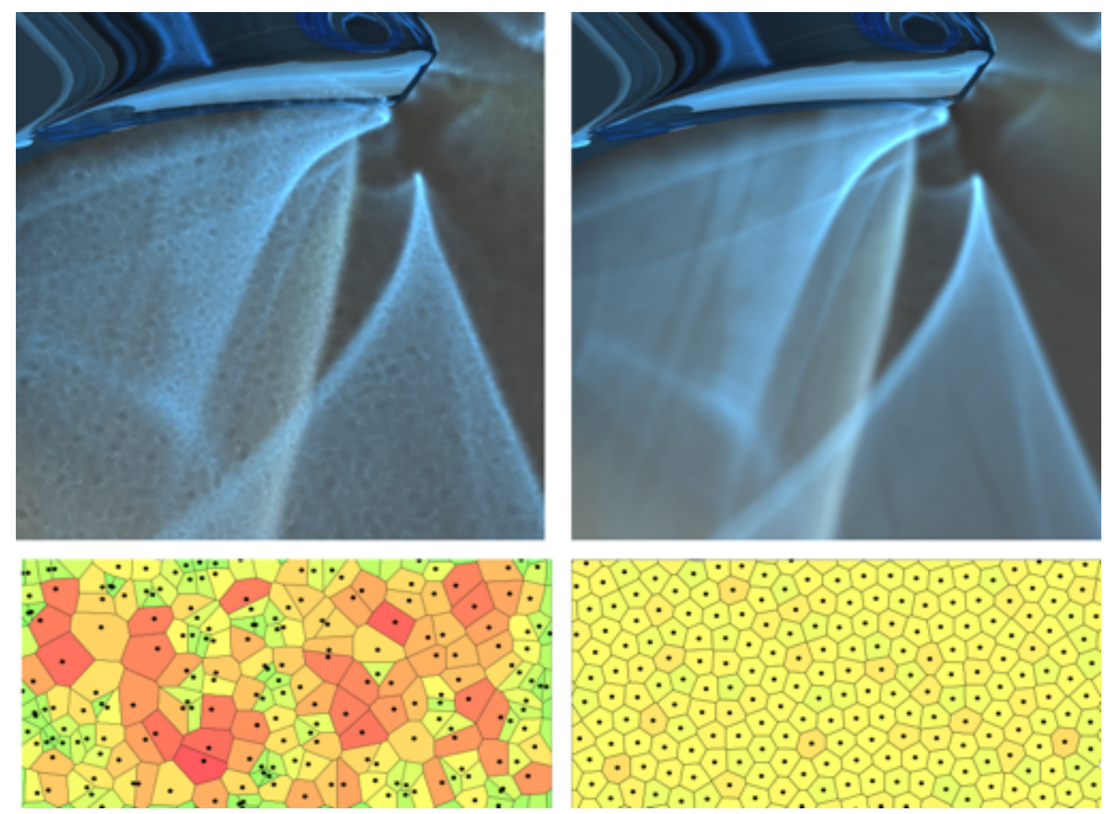
Ok, great!
Now that we’ve mastered Monte Carlo rendering, what other techniques are there?

Photon Mapping

- Trace particles from light, deposit "photons" in kd-tree
- Especially useful for, e.g., caustics, participating media (fog)



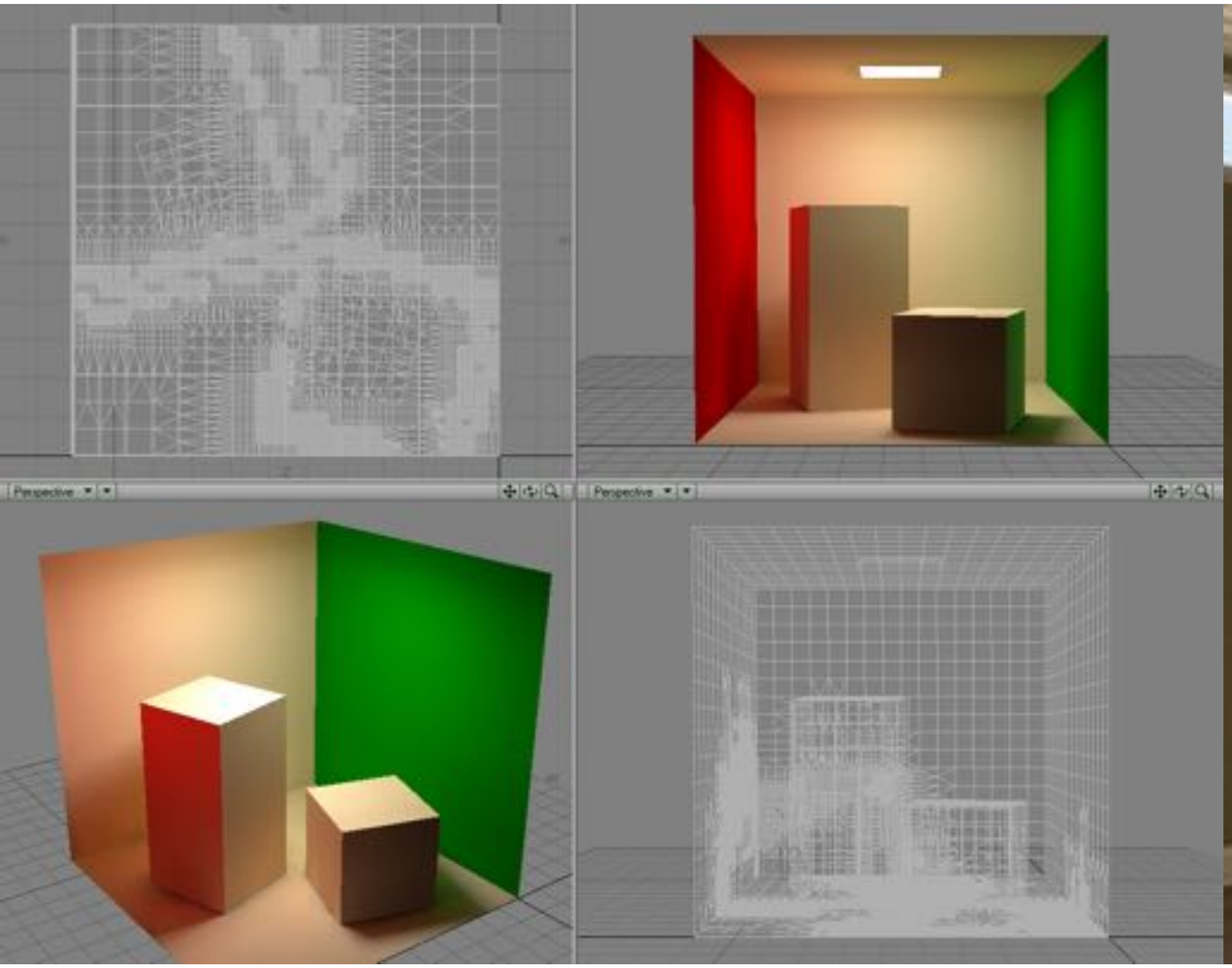
Voronoi diagrams can be used to improve photon distribution



(from Spencer & Jones 2013)

Finite Element Radiosity

- Very different approach: transport between patches in scene
- Solve large linear system for equilibrium distribution
- Good for diffuse lighting; hard to capture other light paths

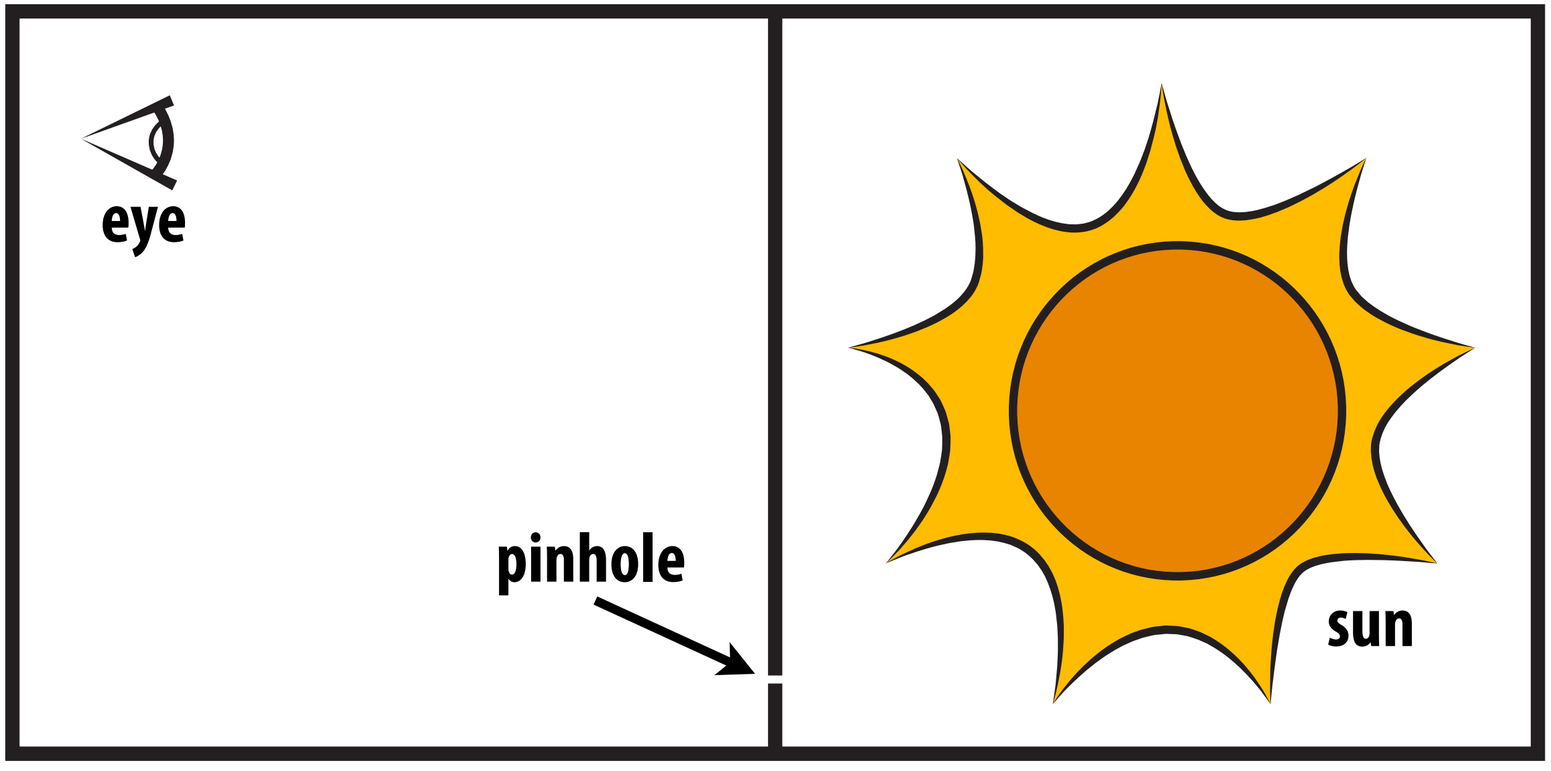


Consistency & Bias in Rendering Algorithms

method	consistent?	unbiased?
rasterization	NO	NO
path tracing	ALMOST	ALMOST
bidirectional path tracing	YES	YES
Metropolis light transport	YES	YES
photon mapping	YES	NO
radiosity	NO	NO

Can you certify a renderer?

- Grand challenge: write a renderer that comes with a certificate (i.e., provable, formally-verified guarantee) that the image produced represents the illumination in a scene.
- Harder than you might think!
- Inherent limitation of sampling: you can never be 100% certain that you didn't miss something important.



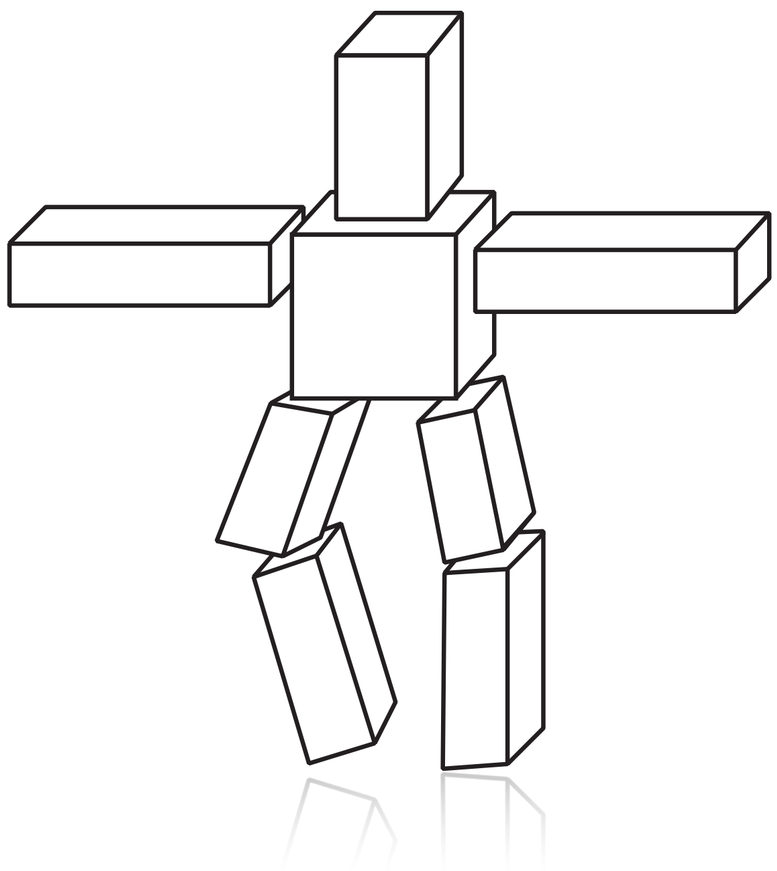
Can always make sun brighter, hole smaller...!

Introduction to Animation

Computer Graphics
CMU 15-462/15-662

Increasing the complexity of our models

Transformations



Geometry

Materials, lighting, ...

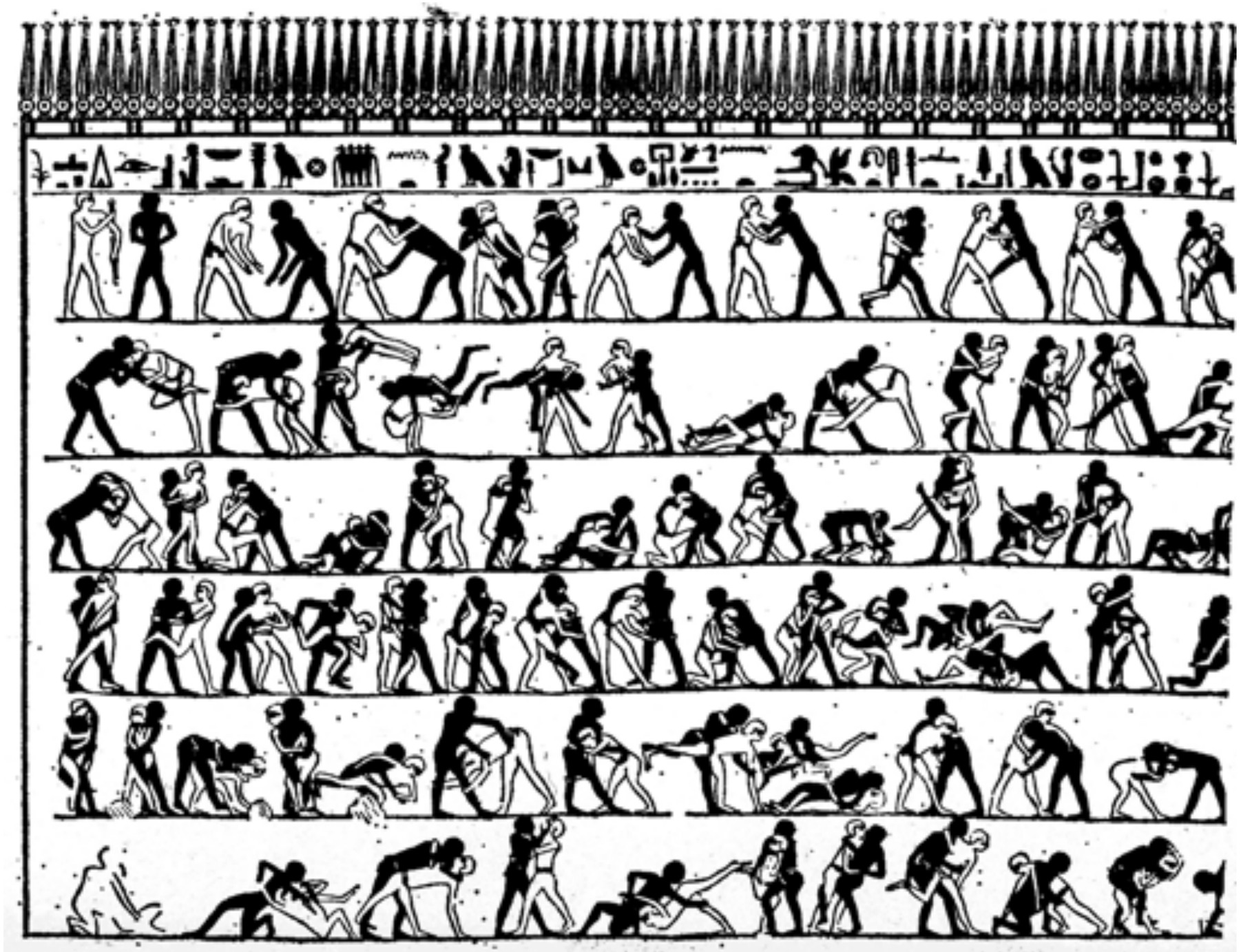
Increasing the complexity of our models

...but what about motion?

First Animation

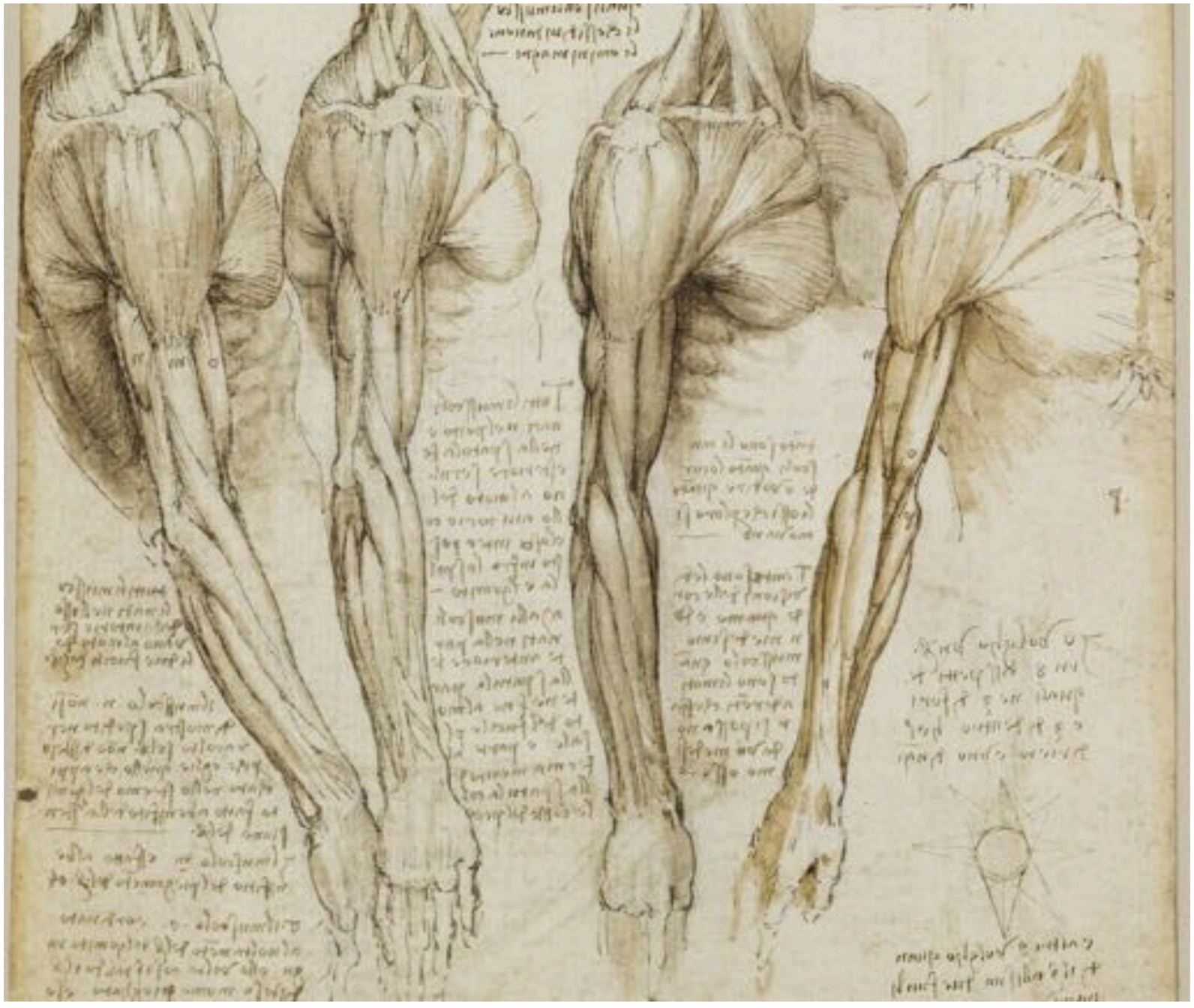
(Shahr-e Sukhteh, Iran 3200 BCE)

History of Animation



(tomb of Khnumhotep, Egypt 2400 BCE)

History of Animation



Leonardo da Vinci (1510)

History of Animation

Claude Monet, “Woman with a Parasol” (1875)

History of Animation

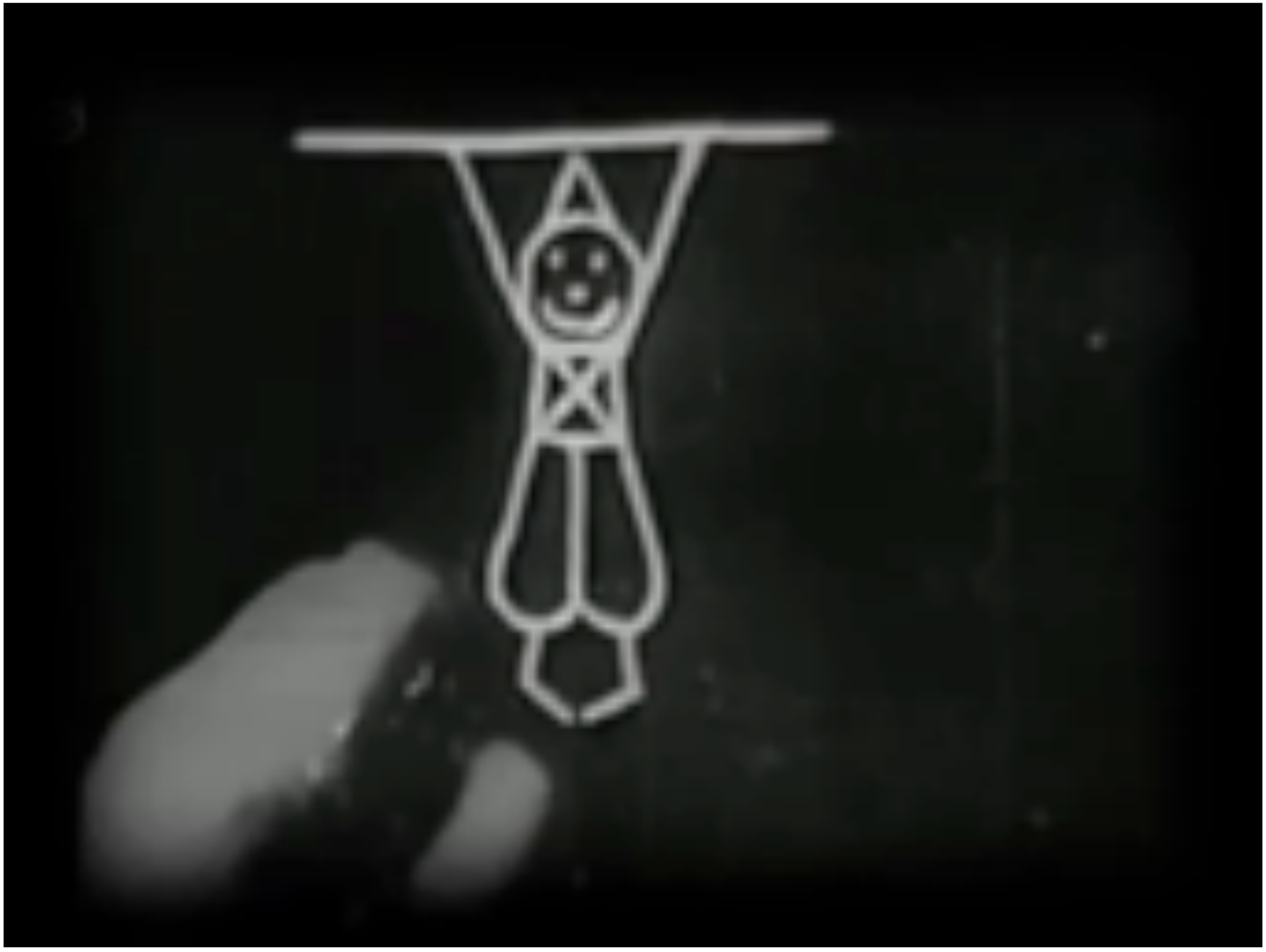
(Phenakistoscope, 1831)

First Film

- Originally used as scientific tool rather than for entertainment
- Critical technology that accelerated development of animation

Eadweard Muybridge, “Sallie Gardner” (1878)

First Animation on Film



Emile Cohl, “Fantasmagorie” (1908)

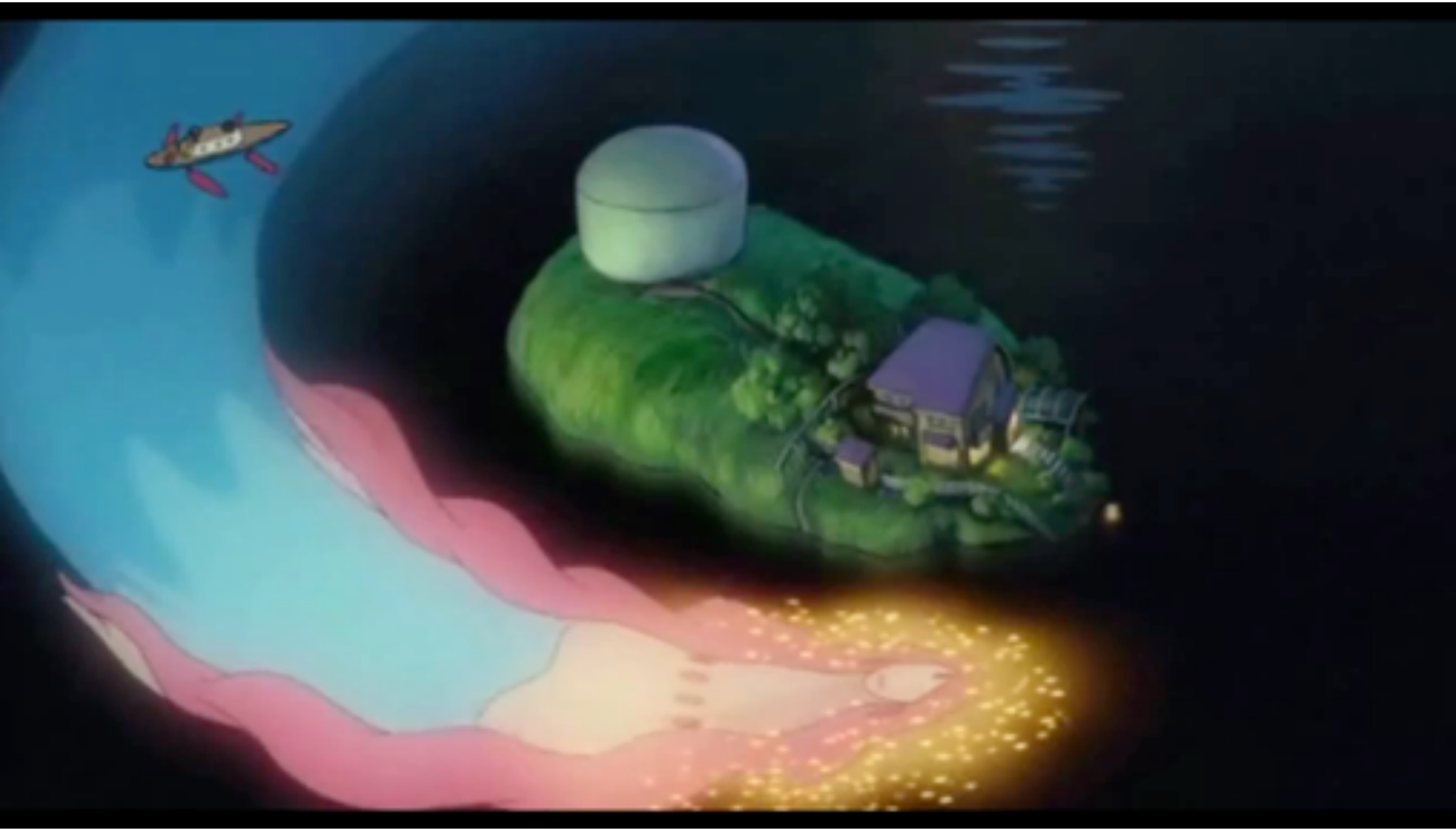
First Feature-Length Animation

Lotte Reiniger, “Die Abenteuer des Prinzen Achmed” (1926)

First Hand-Drawn Feature-Length Animation

Disney, “Snow White and the Seven Dwarves” (1937)

Hand-Drawn Animation - Present Day



Studio Ghibli, “Ponyo” (2008)

First Computer-Generated Animation

- New technology, also developed as a scientific tool
- Again turbo-charged the development of animation

John Whitney, “Catalog” (1961)

First Digital-Computer-Generated Animation

Ivan Sutherland, “Sketchpad” (1963)

First 3D Computer Animation

William Fetter, “Boeing Man” (1964)

Early Computer Animation

Nikolay Konstantinov, “Kitty” (1968)

Early Computer Animation

Ed Catmull & Fred Park, “Computer Animated Faces” (1972)

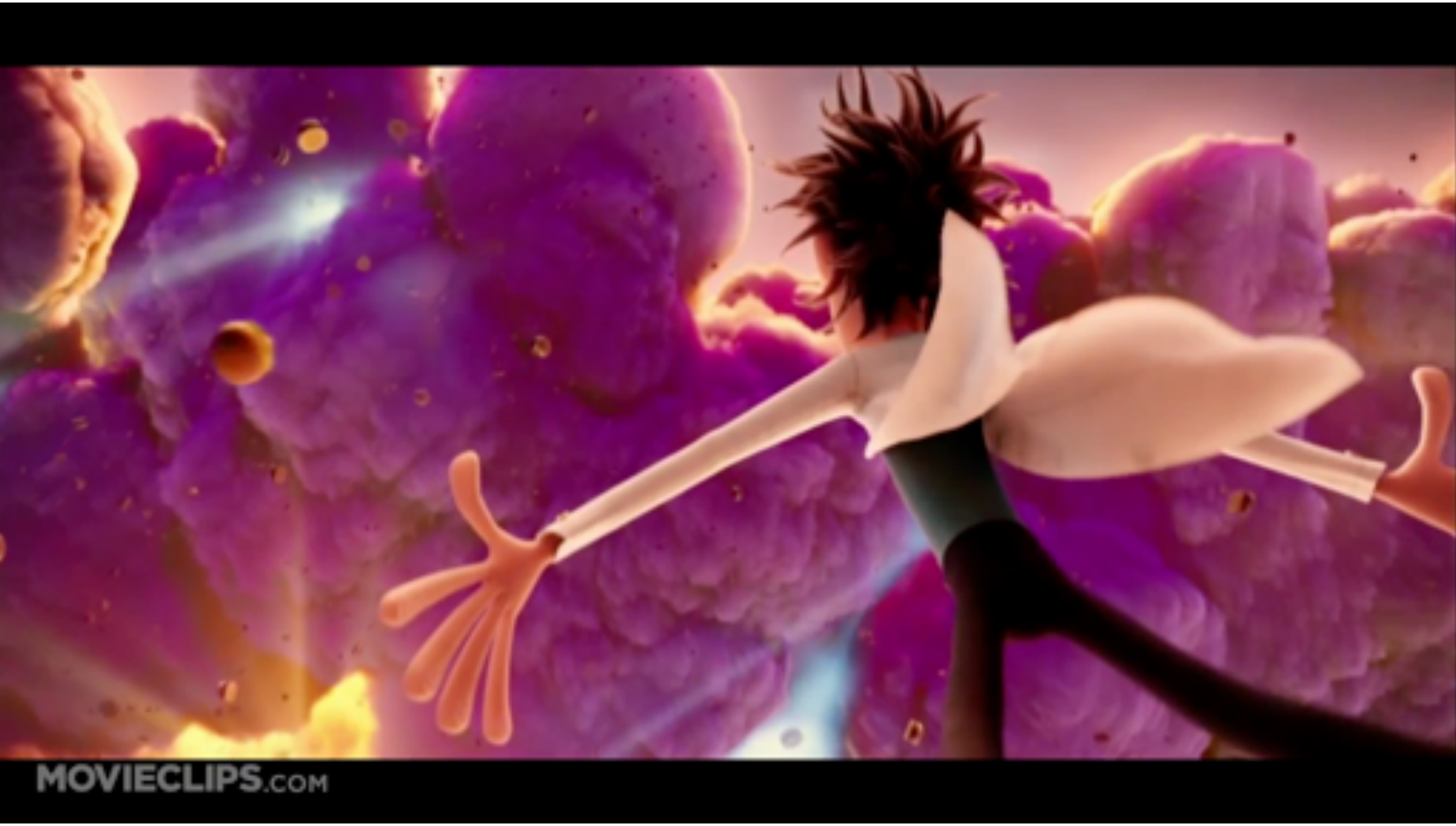
First Attempted CG Feature Film

NYIT [Williams, Heckbert, Catmull, ...], “The Works” (1984)

First CG Feature Film

Pixar, “Toy Story” (1995)

Computer Animation - Present Day



Sony Pictures Animation, “Cloudy With a Chance of Meatballs” (2009)

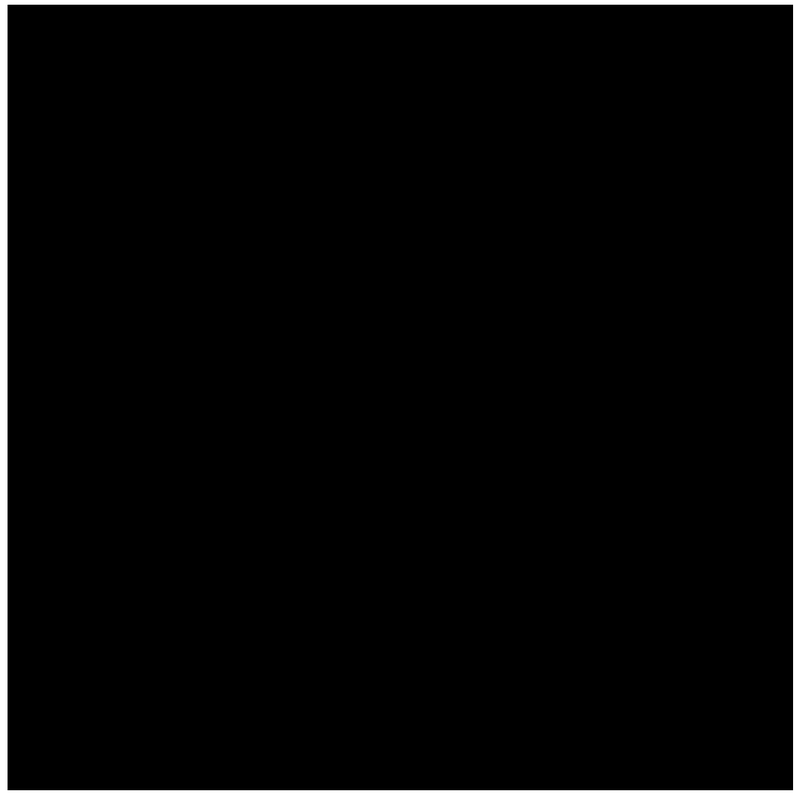
Zoetrope - Solid Animation

Zoetrope - 3D Printed Animation

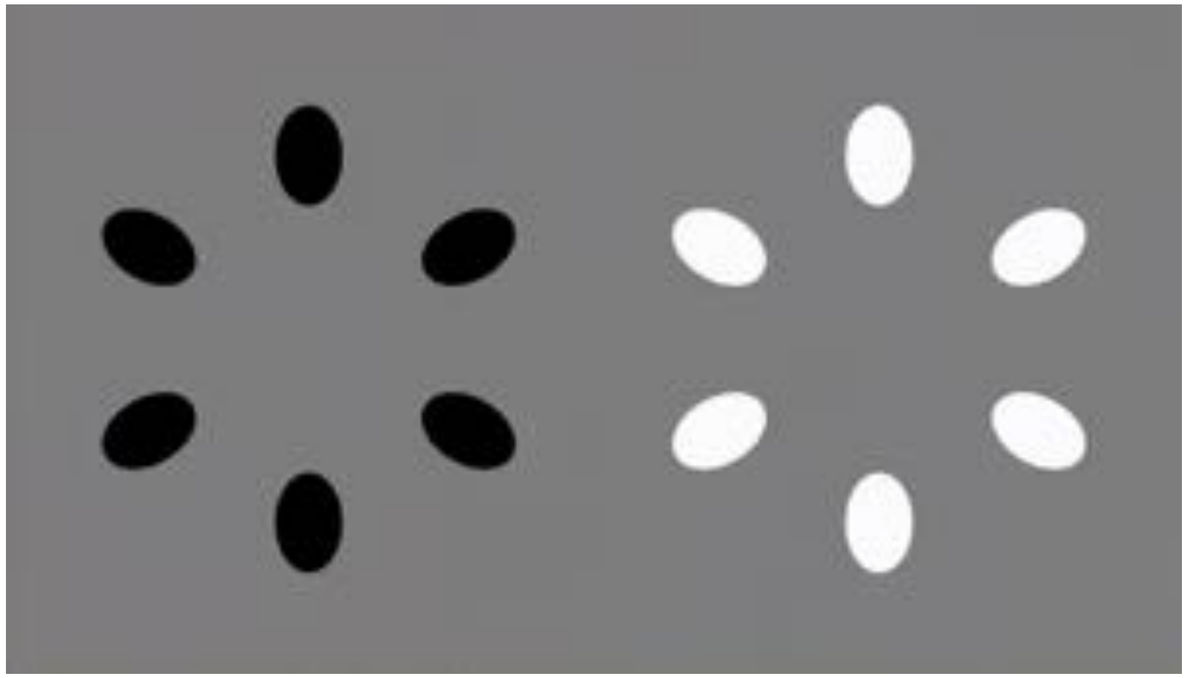
John Edmark — BLOOMS

Perception of Motion

- Original (but debunked) theory: persistence of vision ("streaking")
- The eye is not a camera! More modern explanation:
  - beta phenomenon: visual memory in brain—not eyeball
  - phi phenomenon: brain anticipates, giving sense of motion



beta

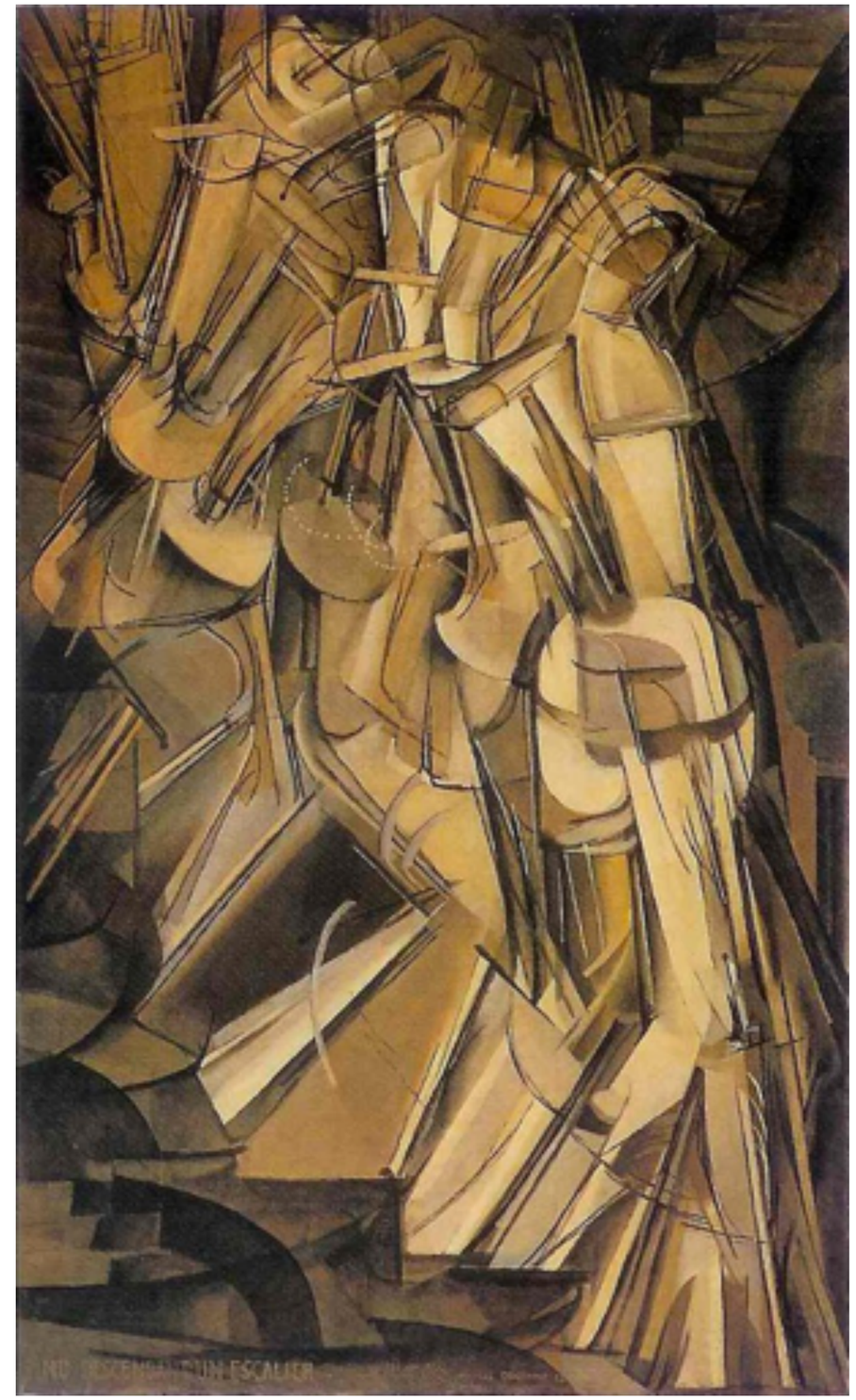


phi

credit: Akiyoshi Kitaoka

Depiction of Motion

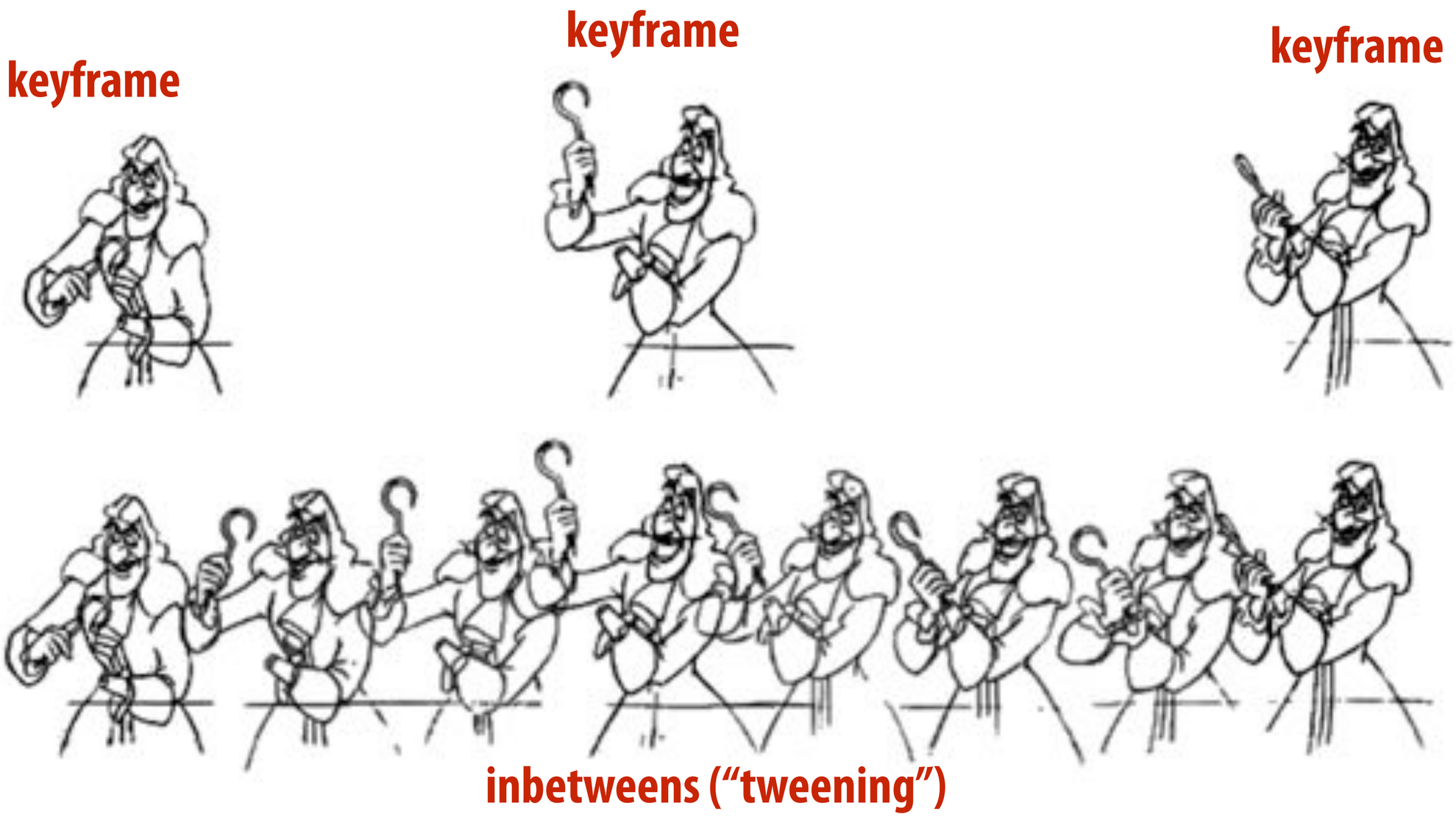
beta (Muybridge, 1887)



phi (Duchamp, 1912)

Generating Motion (Hand-Drawn)

- Senior artist draws keyframes
- Apprentice draws inbetweens
- Tedious / labor intensive (opportunity for technology!)



Next time:
How do we describe motion on a computer?