Depth and Transparency

Computer Graphics
CMU 15-462/15-662
Today: Wrap up the rasterization pipeline!

**Remember our goal:**

- Start with INPUTS (triangles)
  - possibly w/ other data (e.g., colors or texture coordinates)
- Apply a series of transformations: STAGES of pipeline
- Produce OUTPUT (final image)

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**INPUT (TRIANGLES)**

<table>
<thead>
<tr>
<th>Vertices</th>
<th>Coordinates</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>(1, 1, 1)</td>
</tr>
<tr>
<td>B</td>
<td>(-1, 1, 1)</td>
</tr>
<tr>
<td>C</td>
<td>(1, -1, 1)</td>
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<tr>
<td>D</td>
<td>(-1, -1, 1)</td>
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<tr>
<td>E</td>
<td>(1, 1, -1)</td>
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<tr>
<td>F</td>
<td>(-1, 1, -1)</td>
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<tr>
<td>G</td>
<td>(1, -1, -1)</td>
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<tr>
<td>H</td>
<td>(-1, -1, -1)</td>
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</tbody>
</table>

**TRIANGLES**

- EHF, GFH, FGB, CBG,
- GHC, DCH, ABD, CDB,
- HED, ADE, EFA, BAF

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**RASTERIZATION PIPELINE**

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**OUTPUT (BITMAP IMAGE)**
What we know how to do so far...

- position objects in the world (3D transformations)
- project objects onto the screen (perspective projection)
- sample triangle coverage (rasterization)
- interpolate vertex attributes (barycentric coordinates)
- sample texture maps (filtering, mipmapping)
- put samples into frame buffer (depth & alpha)

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Occlusion
Occlusion: which triangle is visible at each covered sample point?

Opaque Triangles

50% transparent triangles
Sampling Depth

Assume we have a triangle given by:
- the projected 2D coordinates \((x_i, y_i)\) of each vertex
- the “depth” \(d_i\) of each vertex (i.e., distance from the viewer)

Q: How do we compute the depth \(d\) at a given sample point \((x, y)\)?

A: Interpolate it using barycentric coordinates (just like any other attribute that varies linearly over the triangle)
The depth-buffer (Z-buffer)

For each coverage sample point, depth-buffer stores the depth of the closest triangle seen so far.

Initial state of depth buffer before rendering any triangles (all samples store farthest distance)

Grayscale value of sample point used to indicate distance
Black = small distance
White = large distance ("infinity")
Depth buffer example
Example: rendering three opaque triangles
Occlusion using the depth-buffer (Z-buffer)

Processing yellow triangle:
depth = 0.5

Grayscale value of sample point used to indicate distance
White = large distance
Black = small distance
Red = sample passed depth test
# Occlusion using the depth-buffer (Z-buffer)

## After processing yellow triangle:

<table>
<thead>
<tr>
<th>Color buffer contents</th>
<th>Depth buffer contents</th>
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<tr>
<td>○ ○ ○ ○ ○ ○ ○ ○ ○ ○</td>
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Grayscale value of sample point used to indicate distance:
- White = large distance
- Black = small distance
- Red = sample passed depth test
Occlusion using the depth-buffer (Z-buffer)

Processing blue triangle:
depth = 0.75

Grayscale value of sample point used to indicate distance
White = large distance
Black = small distance
Red = sample passed depth test
## Occlusion using the depth-buffer (Z-buffer)

### After processing blue triangle:

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Grayscale value of sample point used to indicate distance:
- White = large distance
- Black = small distance
- Red = sample passed depth test
Occlusion using the depth-buffer (Z-buffer)

Processing red triangle:
depth = 0.25

Grayscale value of sample point used to indicate distance
White = large distance
Black = small distance
Red = sample passed depth test

Color buffer contents

Depth buffer contents
Occlusion using the depth-buffer (Z-buffer)

After processing red triangle:

Color buffer contents

Depth buffer contents

Grayscale value of sample point used to indicate distance
White = large distance
Black = small distance
Red = sample passed depth test
Occlusion using the depth buffer

bool pass_depth_test(d1, d2) {
    return d1 < d2;
}

depth_test(tri_d, tri_color, x, y) {
    if (pass_depth_test(tri_d, zbuffer[x][y])) {
        // triangle is closest object seen so far at this
        // sample point. Update depth and color buffers.
        zbuffer[x][y] = tri_d;       // update zbuffer
        color[x][y] = tri_color;    // update color buffer
    }
}
Does depth-buffer algorithm handle interpenetrating surfaces?

Of course!

Occlusion test is based on depth of triangles at a given sample point. The relative depth of triangles may be different at different sample points.

Green triangle in front of yellow triangle

Yellow triangle in front of green triangle
Does depth-buffer algorithm handle interpenetrating surfaces?

Of course!

Occlusion test is based on depth of triangles at a given sample point. The relative depth of triangles may be different at different sample points.
Does depth buffer work with super sampling?

Of course! Occlusion test is per sample, not per pixel!

This example: green triangle occludes yellow triangle
Color buffer contents
Color buffer contents (4 samples per pixel)
Final resampled result

Note anti-aliasing of edge due to filtering of green and yellow samples.
Summary: occlusion using a depth buffer

- Store one depth value per coverage sample (not per pixel!)
- Constant space per sample
  - Implication: constant space for depth buffer
- Constant time occlusion test per covered sample
  - Read-modify write of depth buffer if “pass” depth test
  - Just a read if “fail”
- Not specific to triangles: only requires that surface depth can be evaluated at a screen sample point

But what about semi-transparent surfaces?
Compositing
Representing opacity as alpha

Alpha describes the opacity of an object
- Fully opaque surface: $\alpha = 1$
- 50% transparent surface: $\alpha = 0.5$
- Fully transparent surface: $\alpha = 0$

Red triangle with decreasing opacity

$\alpha = 1$  $\alpha = 0.75$  $\alpha = 0.5$  $\alpha = 0.25$  $\alpha = 0$
Alpha: additional channel of image (RGBA)

$\alpha$ of foreground object
Over operator:

Composite image B with opacity \( \alpha_B \) over image A with opacity \( \alpha_A \)

\[
\begin{align*}
\text{B over A} & \quad \text{A over B} \\
\text{A over B} & \quad \text{B over A}
\end{align*}
\]

A over B \( \neq \) B over A

“Over” is not commutative

Koala over NYC
Fringing

Poor treatment of color/alpha can yield dark “fringing”:

foreground color  foreground alpha  background color  fringing  no fringing
No fringing
Fringing (...why does this happen?)
Over operator: non-premultiplied alpha

Composite image B with opacity $\alpha_B$ over image A with opacity $\alpha_A$

A first attempt:

$$A = \begin{bmatrix} A_r & A_g & A_b \end{bmatrix}^T$$
$$B = \begin{bmatrix} B_r & B_g & B_b \end{bmatrix}^T$$

Composited color:

$$C = \alpha_B B + (1 - \alpha_B) \alpha_A A$$

Composite alpha:

$$\alpha_C = \alpha_B + (1 - \alpha_B) \alpha_A$$
Over operator: premultiplied alpha

Composite image B with opacity $\alpha_B$ over image A with opacity $\alpha_A$

Premultiplied alpha—multiply RGB by $\alpha$, then composite:

$$A' = [\alpha_A A_r \; \alpha_A A_g \; \alpha_A A_b \; \alpha_A]^T$$
$$B' = [\alpha_B B_r \; \alpha_B B_g \; \alpha_B B_b \; \alpha_B]^T$$
$$C' = B' + (1 - \alpha_B)A'$$

Notice premultiplied alpha composites alpha just like how it composites rgb. (Non-premultiplied alpha composites alpha differently than rgb.)

“Un-premultiply” to get final color:

$$[C_r \; C_g \; C_b \; \alpha_C]^T \implies [C_r/\alpha_C \; C_g/\alpha_C \; C_b/\alpha_C]^T$$

Q: By the way, does this division remind you of anything? :-)}
A problem with non-premultiplied alpha

- Suppose we upsample an image with an alpha mask, then composite it onto a background
- How should we compute the interpolated color/alpha values?
- If we interpolate color and alpha separately, then blend using the non-premultiplied "over" operator, here's what happens:

Notice black "fringe" that occurs because we're blending, e.g., 50% blue pixels using 50% alpha, rather than, say, 100% blue pixels with 50% alpha.
Eliminating fringe w/ premultiplied “over”

- If we instead use the premultiplied “over” operation, we get the correct alpha:

$$\text{upsampled color} + (1-\alpha) \times \text{background} = \text{composite image w/ no fringe}$$

(Every pixel is a “pure” combination of blue & yellow)
Eliminating fringe w/ premultiplied “over”

- If we instead use the premultiplied “over” operation, we get the correct alpha:

![Composite image with fringe](image)

*Some pixels are a combination of blue, yellow, and BLACK*
Similar problem with non-premultiplied alpha

Consider pre-filtering (downsampling) a texture with an alpha matte

Desired filtered result

input color

input \( \alpha \)

filtered color

filtered \( \alpha \)

Downsampling non-premultiplied alpha image results in 50% opaque brown

\[
0.25 \times ((0, 1, 0, 1) + (0, 1, 0, 1) + (0, 0, 0, 0) + (0, 0, 0, 0)) = (0, 0.5, 0, 0.5)
\]

Result of filtering premultiplied image

filtered result compositied over white

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More problems: applying “over” repeatedly

Composite image $C$ with opacity $\alpha_C$ over $B$ with opacity $\alpha_B$ over image $A$ with opacity $\alpha_A$

Premultiplied alpha is closed under composition; non-premultiplied alpha is not!

Example: compositing 50% red over 50% red (where “red” = $[1 \ 0 \ 0]$)

\[
\text{NON-PREMULTIPLIED} \\
\begin{bmatrix} 0.5 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} + (1 - 0.5) \cdot 0.5 \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \Downarrow \quad \begin{bmatrix} 0.75 & 0 & 0 \end{bmatrix}
\]

\[
\text{PREMULTIPLIED} \\
\begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0.5 \end{bmatrix} + (1 - 0.5) \cdot \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 \end{bmatrix} \Downarrow \quad \begin{bmatrix} 0.75 & 0 & 0 \\ 0.75 \end{bmatrix} \Downarrow \quad \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}
\]

(Alpha is .5 in both cases)
Summary: advantages of premultiplied alpha

- Simple: compositing operation treats all channels (RGB and A) the same
- More efficient than non-premultiplied representation: “over” requires fewer math ops
- Closed under composition
- Better representation for filtering (upsampling/downsampling) textures with alpha channel
Strategy for drawing semi-transparent primitives

Assuming all primitives are semi-transparent, and RGBA values are encoded with premultiplied alpha, here’s one strategy for creating a correctly rasterized image:

```c
over(c1, c2) {
    return c1.rgba + (1-c1.a) * c2.rgba;
}
```

```c
update_color_buffer( x, y, sample_color, sample_depth )
{
    if (pass_depth_test(sample_depth, zbuffer[x][y]) {
        // (how) should we update depth buffer here??
        color[x][y] = over(sample_color, color[x][y]);
    }
}
```

Q: What is the assumption made by this implementation?

Triangles must be rendered in back to front order!
Putting it all together

Now what if we have a mixture of opaque and transparent triangles?

Step 1: render opaque primitives (in any order) using depth-buffered occlusion
   If pass depth test, triangle overwrites value in color buffer at sample

Step 2: disable depth buffer update, render semi-transparent surfaces in back-to-front order.
   If pass depth test, triangle is composited OVER contents of color buffer at sample
End-to-end rasterization pipeline
("real-time graphics pipeline")
Goal: turn these inputs into an image!

Inputs:

list_of_positions = { v0x, v0y, v0z, v1x, v1y, v1x, v2x, v2y, v2z, v3x, v3y, v3x, v4x, v4y, v4z, v5x, v5y, v5x };

list_of_texcoords = { v0u, v0v, v1u, v1v, v2u, v2v, v3u, v3v, v4u, v4v, v5u, v5v };

Object-to-camera-space transform: \( T \)

Perspective projection transform \( P \)

Size of output image \((W, H)\)

At this point we should have all the tools we need, but let’s review...
Step 1:
Transform triangle vertices into camera space
Step 2:
Apply perspective projection transform to transform triangle vertices into normalized coordinate space
Step 3: clipping

- Discard triangles that lie completely outside the unit cube (culling)
  - They are off screen, don’t bother processing them further
- Clip triangles that extend beyond the unit cube to the cube
  - (possibly generating new triangles)
Step 4: transform to screen coordinates

Perform homogeneous divide, transform vertex xy positions from normalized coordinates into screen coordinates (based on screen w,h)
Step 5: setup triangle (triangle preprocessing)

Before rasterizing triangle, can compute a bunch of data that will be used by all fragments, e.g.,

- triangle edge equations
- triangle attribute equations
- etc.

\[
\begin{align*}
E_{01}(x, y) & \quad U(x, y) \\
E_{12}(x, y) & \quad V(x, y) \\
E_{20}(x, y) & \\
\frac{1}{w}(x, y) & \\
Z(x, y) &
\end{align*}
\]
Step 6: sample coverage

Evaluate attributes $z$, $u$, $v$ at all covered samples
Step 6: compute triangle color at sample point

e.g., sample texture map *

* So far, we’ve only described computing triangle’s color at a point by interpolating per-vertex colors, or by sampling a texture map. Later in the course, we’ll discuss more advanced algorithms for computing its color based on material properties and scene lighting conditions.
Step 7: perform depth test (if enabled)

Also update depth value at covered samples (if necessary)
Step 8: update color buffer* (if depth test passed)

* Possibly using OVER operation for transparency; in general there are more compositing/raster operations that can be used to compute the new color value.
OpenGL/Direct3D graphics pipeline *

Structures rendering computation as a series of operations on vertices, primitives, fragments, and screen samples

* Several stages of the modern OpenGL pipeline are omitted
OpenGL/Direct3D graphics pipeline *

Pipeline inputs:
- Input vertex data (positions, colors, UVs)
- Parameters needed to compute vertex coordinates in 3D space (e.g., transform matrices)
- Parameters needed to compute color of fragments (e.g., textures)
- “Shader” programs that define behavior of vertex and fragment stages

* several stages of the modern OpenGL pipeline are omitted
Programmable Shaders

Define behavior of vertex processing and fragment processing stages
Describe operation on a single vertex (or single fragment)

Example GLSL fragment shader program

```glsl
uniform sampler2D myTexture;
uniform vec3 lightDir;
varying vec2 uv;
varying vec3 norm;
void diffuseShader()
{
    vec3 kd;
    kd = texture2d(myTexture, uv);
    kd *= clamp(dot(-lightDir, norm), 0.0, 1.0);
    gl_FragColor = vec4(kd, 1.0);
}
```

Shader function executes once per fragment.

Outputs color of surface at sample point corresponding to fragment.
(this shader performs a texture lookup to obtain the surface's material color at this point, then performs a simple lighting computation)

Constant parameters

Per-fragment attributes (interpolated by rasterizer)

Grab color from texture

Output surface color at 100% opacity (1.0)

Modulate color according to light/normal direction
Goal: render very high complexity 3D scenes

- 100’s of thousands to millions of triangles in a scene
- Complex vertex and fragment shader computations
- High resolution screen outputs (~10Mpixel + supersampling)
- 30-120 fps
Graphics pipeline implementation: GPUs

Specialized processors for executing graphics pipeline computations

Discrete GPU card
(NVIDIA GeForce Titan X)

Integrated GPU: part of modern Intel CPU die
Modern GPUs offer ~13 TFLOPs of performance for generic vertex/fragment programs ("compute")

Still enormous amount of fixed-function compute over here

This part (mostly) not used by CUDA/OpenCL; raw graphics horsepower still greater than compute!
Modern Rasterization Pipeline

- Trend toward more generic (but still highly parallel!) computation:
  - more programmable stages
  - replace fixed function vertex, fragment processing
  - add geometry, tessellation shaders
  - generic “compute” shaders (whole other story…)
  - more flexible scheduling of stages

(DirectX 12 Pipeline)
Ray Tracing in Graphics Pipeline

- New pipelines coming down the pipe...

GPU Ray Tracing Demo ("Reflections")

Great video on how this was done: https://youtu.be/JAKXySYfLWo
Summary

- Occlusion resolved independently at each screen sample using the depth buffer

- Alpha compositing for semi-transparent surfaces
  - Premultiplied alpha forms simply repeated composition
  - “Over” compositing operations is not commutative: requires triangles to be processed in back-to-front (or front-to-back) order

- Graphics pipeline:
  - Structures rendering computation as a sequence of operations performed on vertices, primitives (e.g., triangles), fragments, and screen samples
  - Behavior of parts of the pipeline is application-defined using shader programs.
  - Pipeline operations implemented by highly, optimized parallel processors and fixed-function hardware (GPUs)
What else do we need to know to generate images like these?

**GEOMETRY**

How do we describe complex shapes (so far just triangles...)

**RENDERING**

How does light interact w/ materials to produce color?

**ANIMATION**

How do we describe the way things move?

(“Moana”, Disney 2016)
Course roadmap

Drawing Things

Introduction
- Drawing a triangle (by sampling)
- Transforms and coordinate spaces
- Perspective projection and texture sampling
- Today: putting it all together: end-to-end rasterization pipeline

Geometry

Next time!

Materials and Lighting