Depth and Transparency

Computer Graphics
CMU 15-462/15-662
Today: Wrap up the rasterization pipeline!

**Remember our goal:**

- Start with INPUTS (triangles)
  - possibly w/ other data (e.g., colors or texture coordinates)
- Apply a series of transformations: STAGES of pipeline
- Produce OUTPUT (final image)
What we know how to do so far...

- Position objects in the world (3D transformations)
- Project objects onto the screen (perspective projection)
- Sample triangle coverage (rasterization)
- Interpolate vertex attributes (barycentric coordinates)
- Sample texture maps (filtering, mipmaping)
- Put samples into frame buffer (depth & alpha)
Occlusion
Occlusion: which triangle is visible at each covered sample point?

Opaque Triangles

50% transparent triangles
Sampling Depth

Assume we have a triangle given by:
- the projected 2D coordinates \((x_i, y_i)\) of each vertex
- the “depth” \(d_i\) of each vertex (i.e., distance from the viewer)

Q: How do we compute the depth \(d\) at a given sample point \((x, y)\)?

A: Interpolate it using barycentric coordinates—just like any other attribute that varies linearly over the triangle
The depth-buffer (Z-buffer)

For each sample, *depth-buffer* stores the depth of the closest triangle seen so far.

Initialize all depth buffer values to “infinity” (max value)
Depth buffer example
Example: rendering three opaque triangles
Occlusion using the depth-buffer (Z-buffer)

Processing yellow triangle:
depth = 0.5

- Color buffer contents
- Depth buffer contents

- Sample passed depth test
Occlusion using the depth-buffer (Z-buffer)

After processing yellow triangle:

Color buffer contents

Depth buffer contents

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near — sample passed depth test
Occlusion using the depth-buffer (Z-buffer)

Processing blue triangle:
depth = 0.75

Color buffer contents

Depth buffer contents

---

near — sample passed depth test
Occlusion using the depth-buffer (Z-buffer)

After processing blue triangle:

Color buffer contents

Depth buffer contents

near — sample passed depth test

far
Occlusion using the depth-buffer (Z-buffer)

Processing red triangle:
depth = 0.25

Color buffer contents

Depth buffer contents

— sample passed depth test
Occlusion using the depth-buffer (Z-buffer)

After processing red triangle:

Color buffer contents

Depth buffer contents

- sample passed depth test
Occlusion using the depth buffer

```c
bool pass_depth_test(d1, d2) {
    return d1 < d2;
}
```

```c
draw_sample(x, y, d, c) //new depth d & color c at (x,y)
{
    if( pass_depth_test( d, zbuffer[x][y] ))
    {
        // triangle is closest object seen so far at this
        // sample point. Update depth and color buffers.
        zbuffer[x][y] = d; // update zbuffer
        color[x][y] = c; // update color buffer
    }
    // otherwise, we’ve seen something closer already;
    // don’t update color or depth
}
```
Q: Does depth-buffer algorithm handle interpenetrating surfaces?
A: Of course!

Occlusion test is based on depth of triangles at a given sample point. Relative depth of triangles may be different at different sample points.
Q: Does depth-buffer algorithm handle interpenetrating surfaces?
A: Of course!

Occlusion test is based on depth of triangles at a given sample point. Relative depth of triangles may be different at different sample points.
Depth + Supersampling

Q: Does depth buffer work with super sampling?  
A: Yes! If done per (super) sample.

(Here: green triangle occludes yellow triangle)
Depth + Supersampling

Color of super samples after rasterizing with depth buffer
Color buffer contents (4 samples per pixel)
Final resampled result

Note anti-aliasing of edge due to filtering of green and yellow samples
Summary: occlusion using a depth buffer

- Store one depth value per (super) sample—not one per pixel!
- Constant additional space per sample
  - Hence, constant space for depth buffer
  - Doesn’t depend on number of overlapping primitives!
- Constant time occlusion test per covered sample
  - Read-modify write of depth buffer if “pass” depth test
  - Just a read if “fail”
- Not specific to triangles: only requires that surface depth can be evaluated at a screen sample point

But what about semi-transparent surfaces?
Compositing
Representing opacity as alpha

An “alpha” value $0 \leq \alpha \leq 1$ describes the opacity of an object.

- $\alpha = 1$: fully opaque
- $\alpha = 3/4$
- $\alpha = 1/2$
- $\alpha = 1/4$
- $\alpha = 0$: fully transparent
Alpha channel of an image

Key idea: can use $\alpha$ channel to composite one image on top of another.
Fringing

Poor treatment of color/alpha can yield dark “fringing”:

foreground color

foreground alpha

background color

fringing

no fringing
No fringing
Fringing (…why does this happen?)
**Over operator:**

Composites image $B$ with opacity $\alpha_B$ over image $A$ with opacity $\alpha_A$

Informally, captures behavior of “tinted glass”

Notice: “over” is not commutative

$A$ over $B \neq B$ over $A$
Over operator: non-premultiplied alpha

Composite image $B$ with opacity $\alpha_B$ over image $A$ with opacity $\alpha_A$

A first attempt:

$$A = (A_r, A_g, A_b)$$
$$B = (B_r, B_g, B_b)$$

Composite color:

$$C = \alpha_B B + (1 - \alpha_B)\alpha_A A$$

Composite alpha:

$$\alpha_C = \alpha_B + (1 - \alpha_B)\alpha_A$$
Over operator: premultiplied alpha

Composite image $B$ with opacity $\alpha_B$ over image $A$ with opacity $\alpha_A$

Premultiplied alpha—multiply color by $\alpha$, then composite:

$A' = (\alpha_A A_r, \alpha_A A_g, \alpha_A A_b, \alpha_A)$

$B' = (\alpha_B B_r, \alpha_B B_g, \alpha_B B_b, \alpha_B)$

$C' = B' + (1 - \alpha_B)A'$

Notice premultiplied alpha composites alpha just like how it composites rgb.
(Non-premultiplied alpha composites alpha differently than rgb.)

"Un-premultiply" to get final color:

$$(C_r, C_g, C_b, \alpha_C) \implies (C_r/\alpha_C, C_g/\alpha_C, C_b/\alpha_C)$$

Q: Does this division remind you of anything?
Compositing with & without premultiplied $\alpha$

Suppose we upsample an image with an $\alpha$ channel, then composite it onto a background:

$\alpha_B B + (1 - \alpha_B)A$

$B' + (1 - \alpha_B)A'$

Q: Why do we get the “green fringe” when we don’t premultiply?
Similar problem with non-premultiplied $\alpha$

Consider pre-filtering (downsampling) a texture with an alpha matte

![Diagram of leaf color and alpha](image)

<table>
<thead>
<tr>
<th>input color</th>
<th>input $\alpha$</th>
<th>filtered color</th>
<th>filtered $\alpha$</th>
<th>composited over white</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Image of input color]</td>
<td>![Image of input alpha]</td>
<td>![Image of filtered color]</td>
<td>![Image of filtered alpha]</td>
<td>![Image of composited over white]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>premultiplied color</th>
<th>premultiplied $\alpha$</th>
<th>filtered color</th>
<th>filtered $\alpha$</th>
<th>composited over white</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Image of premultiplied color]</td>
<td>![Image of premultiplied alpha]</td>
<td>![Image of filtered color]</td>
<td>![Image of filtered alpha]</td>
<td>![Image of composited over white]</td>
</tr>
</tbody>
</table>
More problems: applying “over” repeatedly

Composite image $C$ with opacity $\alpha_C$ over $B$ with opacity $\alpha_B$ over image $A$ with opacity $\alpha_A$

Premultiplied alpha is closed under composition; non-premultiplied alpha is not!

Example: composite 50% bright red over 50% bright red (where “bright red” = $(1,0,0)$, and $\alpha = 0.5$)

non-premultiplied

\begin{align*}
\text{color} & \quad \frac{.5(1,0,0) + (1-.5).5(1,0,0)}{\alpha} \\
& \quad (0.75,0,0) \\
\text{alpha} & \quad .5 + (1-.5).5 = .75
\end{align*}

premultiplied

\begin{align*}
\text{color} & \quad \frac{(.5,0,0,.5)+(1-.5)(.5,0,0,.5)}{\alpha} \\
& \quad (.75,0,0.75) \\
& \quad \text{divide by} \alpha \\
& \quad \text{bright red} \quad (1,0,0) \\
\text{alpha} & \quad \alpha = 0.75
\end{align*}
Summary: advantages of premultiplied alpha

- Compositing operation treats all channels the same (color and $\alpha$)
- Fewer arithmetic operations for “over” operation than with non-premultiplied representation
- Closed under composition (repeated “over” operations)
- Better representation for filtering (upsampling/downsampling) images with alpha channel
- Fits naturally into rasterization pipeline (homogeneous coordinates)
Strategy for drawing semi-transparent primitives

Assuming all primitives are semi-transparent, and color values are encoded with premultiplied alpha, here’s a strategy for rasterizing an image:

```c
over(c1, c2)
{
    return c1.rgba + (1-c1.a) * c2.rgba;
}
```

```c
update_color_buffer( x, y, sample_color, sample_depth )
{
    if (pass_depth_test(sample_depth, zbuffer[x][y])
    {
        // (how) should we update depth buffer here??
        color[x][y] = over(sample_color, color[x][y]);
    }
}
```

Q: What is the assumption made by this implementation?

Triangles must be rendered in back to front order!
Putting it all together

What if we have a mixture of opaque and transparent triangles?

**Step 1:** render opaque primitives (in any order) using depth-buffered occlusion
   If pass depth test, triangle overwrites value in color buffer at sample

**Step 2:** disable depth buffer update, render semi-transparent surfaces in back-to-front order.
   If pass depth test, triangle is composited OVER contents of color buffer at sample
End-to-end rasterization pipeline
Goal: turn inputs into an image!

**Inputs:**

```javascript
positions = {
    v0x, v0y, v0z,
    v1x, v1y, v1x,
    v2x, v2y, v2z,
    v3x, v3y, v3x,
    v4x, v4y, v4z,
    v5x, v5y, v5x
};
texcoords = {
    v0u, v0v,
    v1u, v1v,
    v2u, v2v,
    v3u, v3v,
    v4u, v4v,
    v5u, v5v
};

Object-to-camera-space transform $T \in \mathbb{R}^{4 \times 4}$

Perspective projection transform $P \in \mathbb{R}^{4 \times 4}$

Size of output image $(W, H)$

At this point we have all the tools we need to make an image... Let's review!
Step 1:
Transform triangle vertices into camera space
Step 2:
Apply perspective projection transform to transform triangle vertices into normalized coordinate space.
Step 3: clipping

- Discard triangles that lie complete outside the unit cube (culling)
  - They are off screen, don’t bother processing them further
- Clip triangles that extend beyond the unit cube to the cube
  - (possibly generating new triangles)

Triangles before clipping

Triangles after clipping
Step 4: transform to screen coordinates

Perform homogeneous divide, transform vertex \(xy\) positions from normalized coordinates into screen coordinates (based on screen \(w,h\))
Step 5: setup triangle (triangle preprocessing)

Before rasterizing triangle, can compute a bunch of data that will be used by all fragments, e.g.,

- triangle edge equations
- triangle attribute equations
- etc.

\[
\begin{align*}
E_{01}(x, y) & \quad U(x, y) \\
E_{12}(x, y) & \quad V(x, y) \\
E_{20}(x, y) & \\
\frac{1}{w}(x, y) & \\
Z(x, y) &
\end{align*}
\]
Step 6: sample coverage

Evaluate attributes \( z, u, v \) at all covered samples
Step 6: compute triangle color at sample point

e.g., sample texture map *

*Not the only way to get a color! Later we’ll talk about more general models of materials...
Step 7: perform depth test (if enabled)
Also update depth value at covered samples (if necessary)
Step 8: update color buffer* (if depth test passed)

* Possibly using OVER operation for transparency
OpenGL/Direct3D graphics pipeline

Our rasterization pipeline doesn’t look much different from “real” pipelines used in modern APIs / graphics hardware

- **Operations on vertices**
  - **Vertex Processing**
    - Vertex stream

- **Operations on primitives (triangles, lines, etc.)**
  - **Primitive Processing**
    - Primitive stream

- **Operations on fragments**
  - **Fragment Generation (Rasterization)**
    - Fragment stream
  - **Fragment Processing**
    - Shaded fragment stream

- **Operations on screen samples**
  - Screen sample operations (depth and color)

Input: vertices in 3D space

* Several stages of the modern OpenGL pipeline are omitted*
Goal: render very high complexity 3D scenes

- 100's of thousands to millions of triangles in a scene
- Complex vertex and fragment shader computations
- High resolution screen outputs (~10Mpixel + supersampling)
- 30-120 fps
Graphics pipeline implementation: GPUs
Specialized processors for executing graphics pipeline computations

discrete GPU card

smartphone GPU (integrated)

integrated GPU: part of modern CPU die
GPU: heterogeneous, multi-core processor

Modern GPUs offer ~35 TFLOPs of performance for generic vertex/fragment programs ("compute")

still enormous amount of fixed-function compute over here

This part (mostly) not used by CUDA/OpenCL; raw graphics horsepower still greater than compute!
Modern Rasterization Pipeline

- Trend toward more generic (but still highly parallel!) computation:
  - make stages programmable
  - replace fixed function vertex, fragment processing
  - add geometry, tessellation shaders
  - generic “compute” shaders (whole other story…)
  - more flexible scheduling of stages
Ray Tracing in Graphics Pipeline

- More recently: specialized pipeline for ray tracing (NVIDIA RTX)

GPU Ray Tracing Demo ("Marbles at Night")
What else do we need to know to generate images like these?

**GEOMETRY**
How do we describe complex shapes (so far just triangles...)

**RENDERING**
How does light interact w/ materials to produce color?

**ANIMATION**
How do we describe the way things move?

(“Moana”, Disney 2016)
Today: putting it all together: end-to-end rasterization pipeline

Next time!