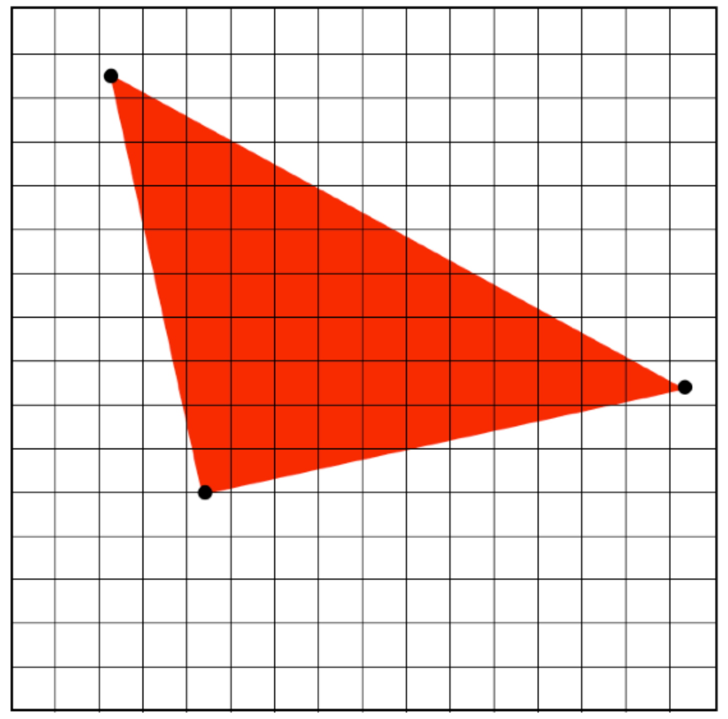
Lecture 4:

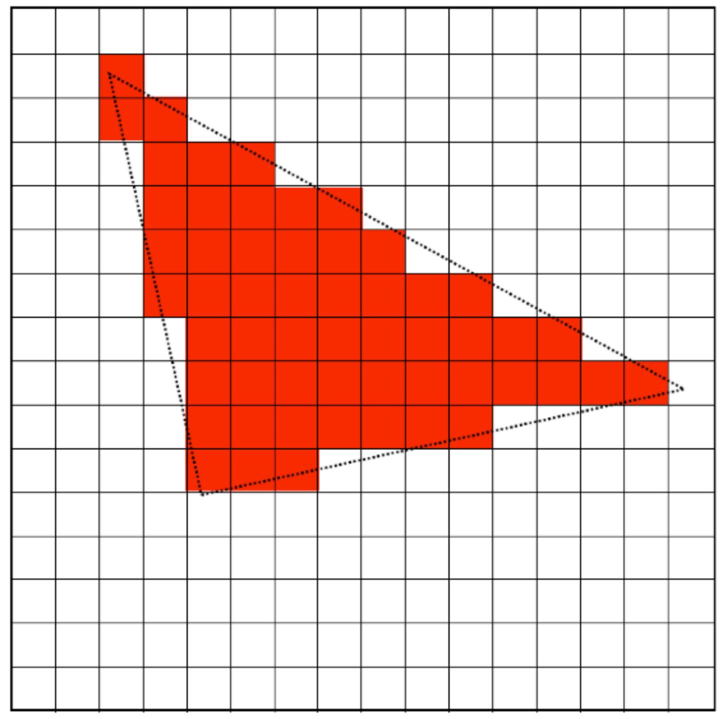
Drawing a Triangle (and an Intro to Sampling)

Computer Graphics
CMU 15-462/15-662

TODAY: Rasterization

- Two major techniques for "getting stuff on the screen"
- Rasterization (TODAY)
  - *for each primitive* (e.g., triangle), which pixels light up?
  - extremely fast (BILLIONS of triangles per second on GPU)
  - harder (but not impossible) to achieve photorealism
  - perfect match for 2D vector art, fonts, quick 3D preview, …
- Ray tracing (LATER)
  - *for each pixel,* which primitives are seen?
  - easier to get photorealism
  - generally slower
  - much more later in the semester!





3D Image Generation Pipeline(s)

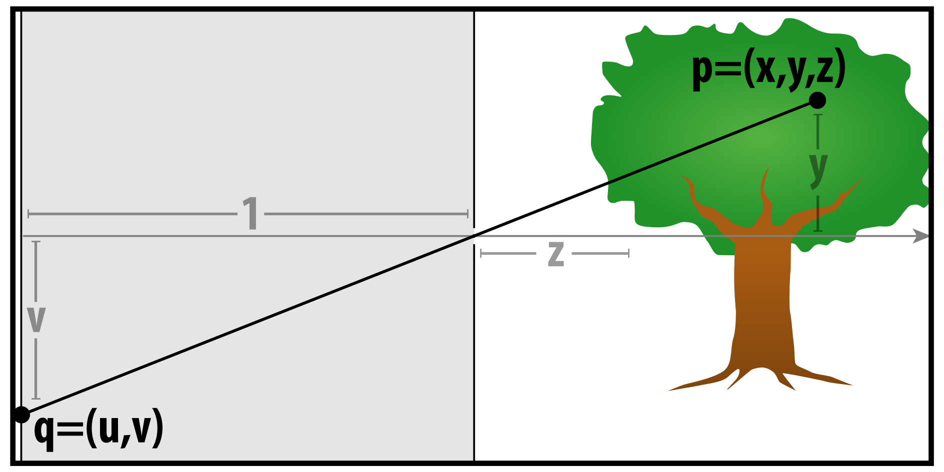
- Can talk about image generation in terms of a "pipeline":
  - INPUTS — what image do we want to draw?
  - STAGES — sequence of transformations from input → output
  - OUTPUTS — the final image

E.g., our pipeline from the first lecture:

```
VERTICES
A: ( 1, 1, 1 )    E: ( 1, 1,-1 )
B: (-1, 1, 1 )    F: (-1, 1,-1 )
C: ( 1,-1, 1 )    G: ( 1,-1,-1 )
D: (-1,-1, 1 )    H: (-1,-1,-1 )

EDGES
AB, CD, EF, GH,
AC, BD, EG, FH,
AE, CG, BF, DH
```

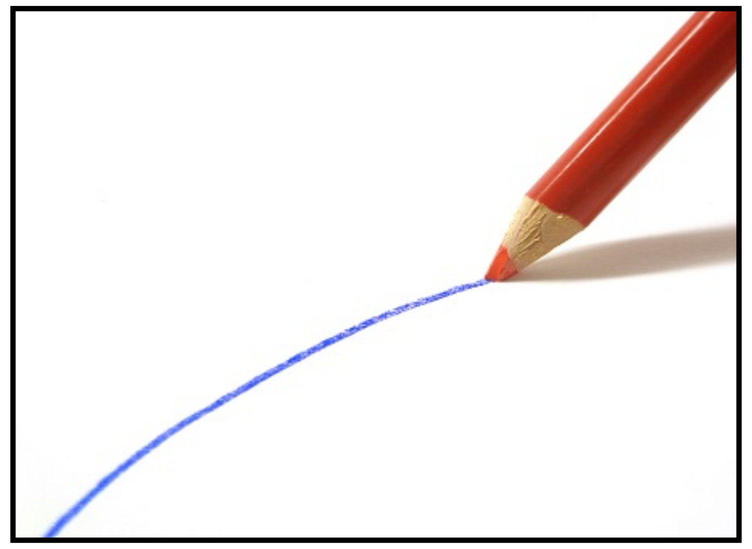
INPUT

→



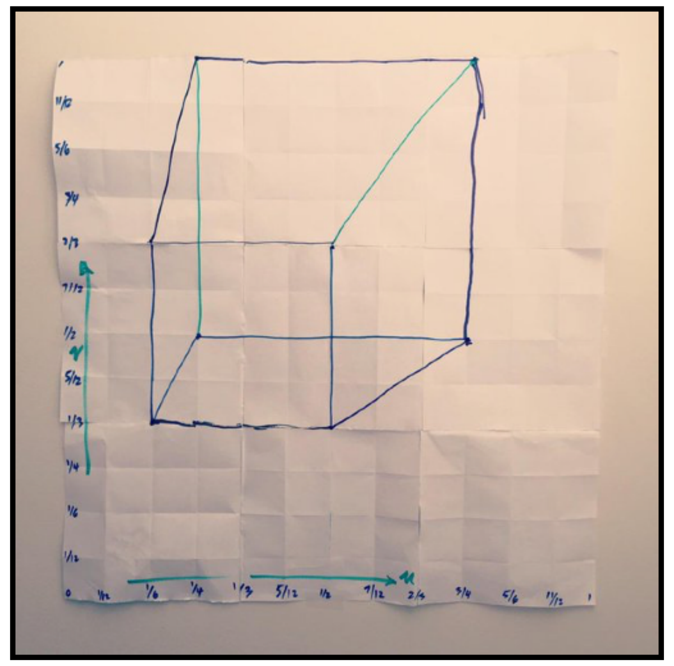
PERSPECTIVE PROJECTION STAGE

→



LINE DRAWING STAGE

→



OUTPUT

Rasterization Pipeline

- Modern real time image generation based on *rasterization*
  - INPUT: 3D "primitives"—essentially all triangles!
    - possibly with additional attributes (e.g., color)
  - OUTPUT: bitmap image (possibly w/ depth, alpha, ...)
- <u>Our goal:</u> understand the stages in between*

INPUT
(TRIANGLES)

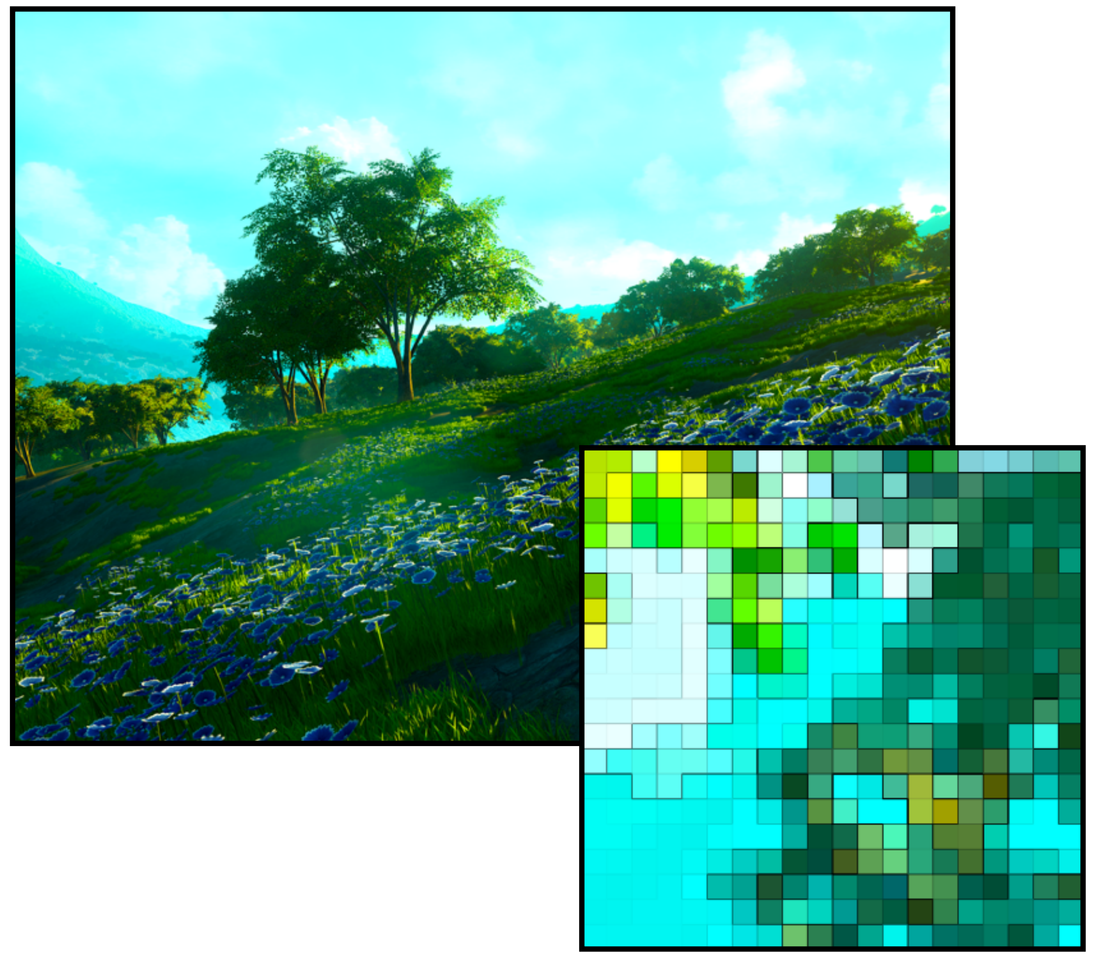
```
VERTICES
A: ( 1, 1, 1 )    E: ( 1, 1,-1 )
B: (-1, 1, 1 )    F: (-1, 1,-1 )
C: ( 1,-1, 1 )    G: ( 1,-1,-1 )
D: (-1,-1, 1 )    H: (-1,-1,-1 )

TRIANGLES
EHF, GFH, FGB, CBG,
GHC, DCH, ABD, CDB,
HED, ADE, EFA, BAF
```

→

RASTERIZATION
PIPELINE

→

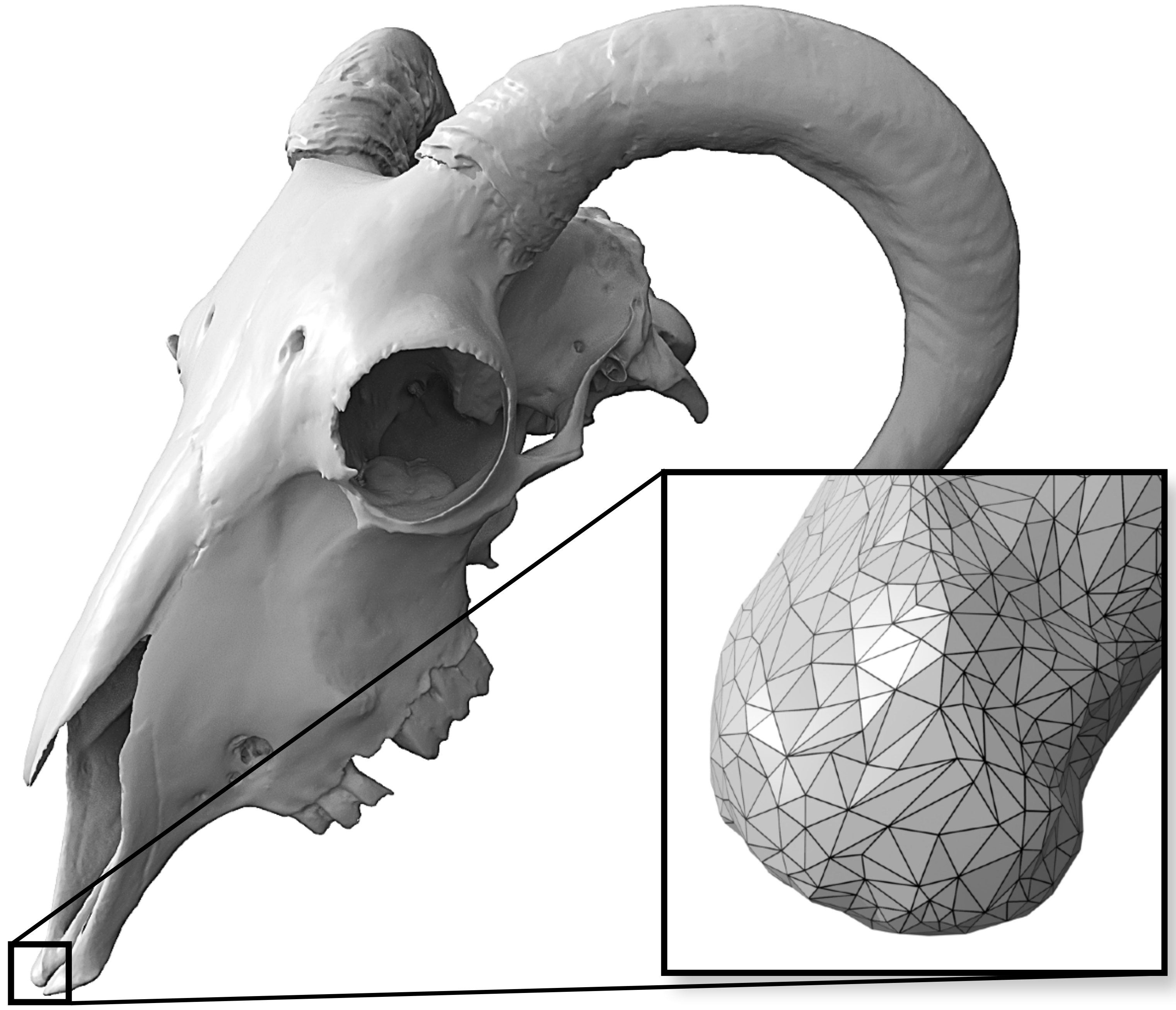
OUTPUT
(BITMAP IMAGE)



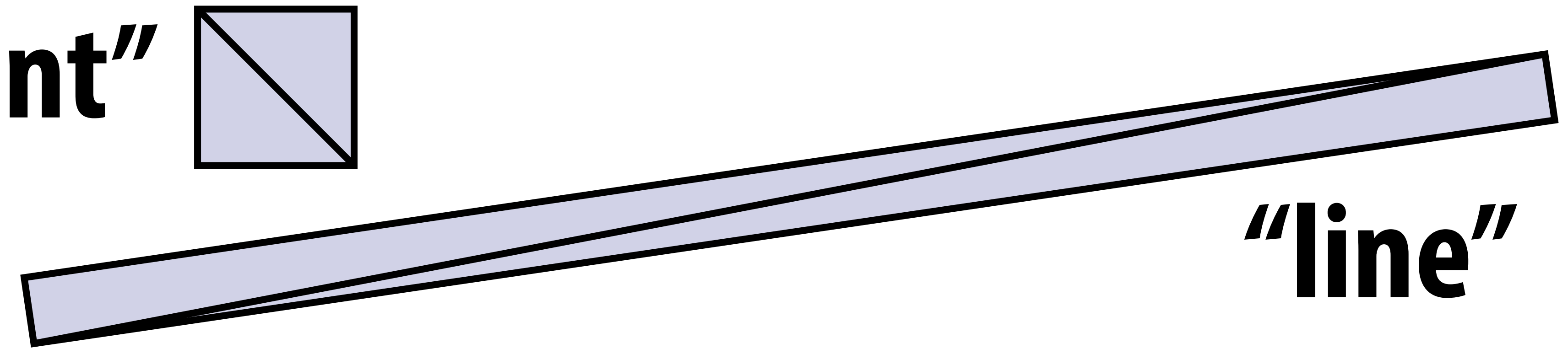
*In practice, usually executed by *graphics processing unit (GPU)*

Why triangles?

- **Can draw <u>all</u> primitives as triangles**
  - **even points and lines!***
- **Why?**
  - **can approximate any shape**
  - **always planar, well-defined normal**
  - **easy to interpolate data at corners**
    - ***"barycentric coordinates"***
- ***Key reason*: once everything is reduced to triangles, can focus on making an extremely well-optimized <u>pipeline</u> for drawing them**



"point"

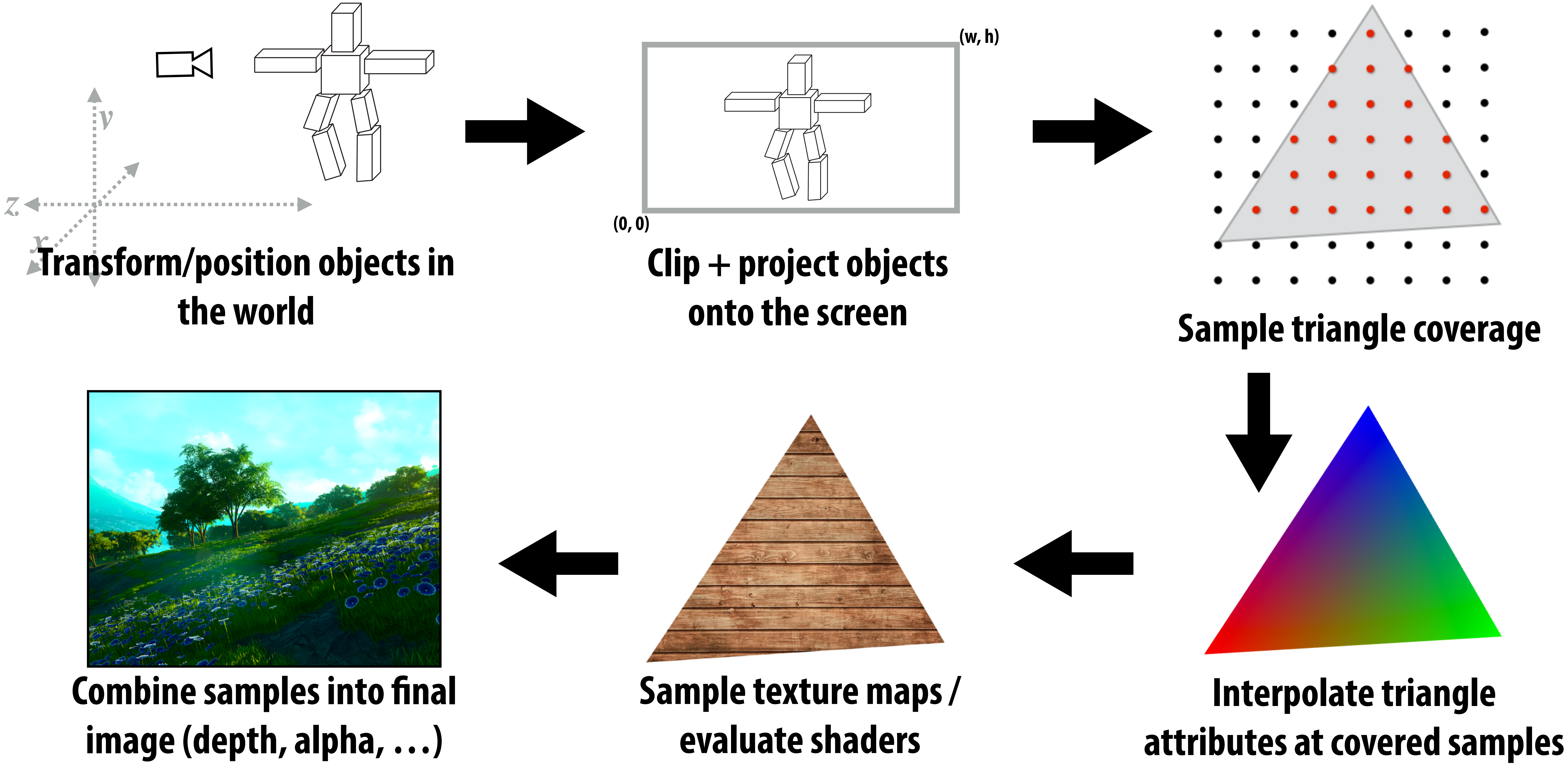


"line"

*** though "diamond-exit" lines and "triangle" lines don't cover the same pixels without some careful special-case handling.**

The Rasterization Pipeline

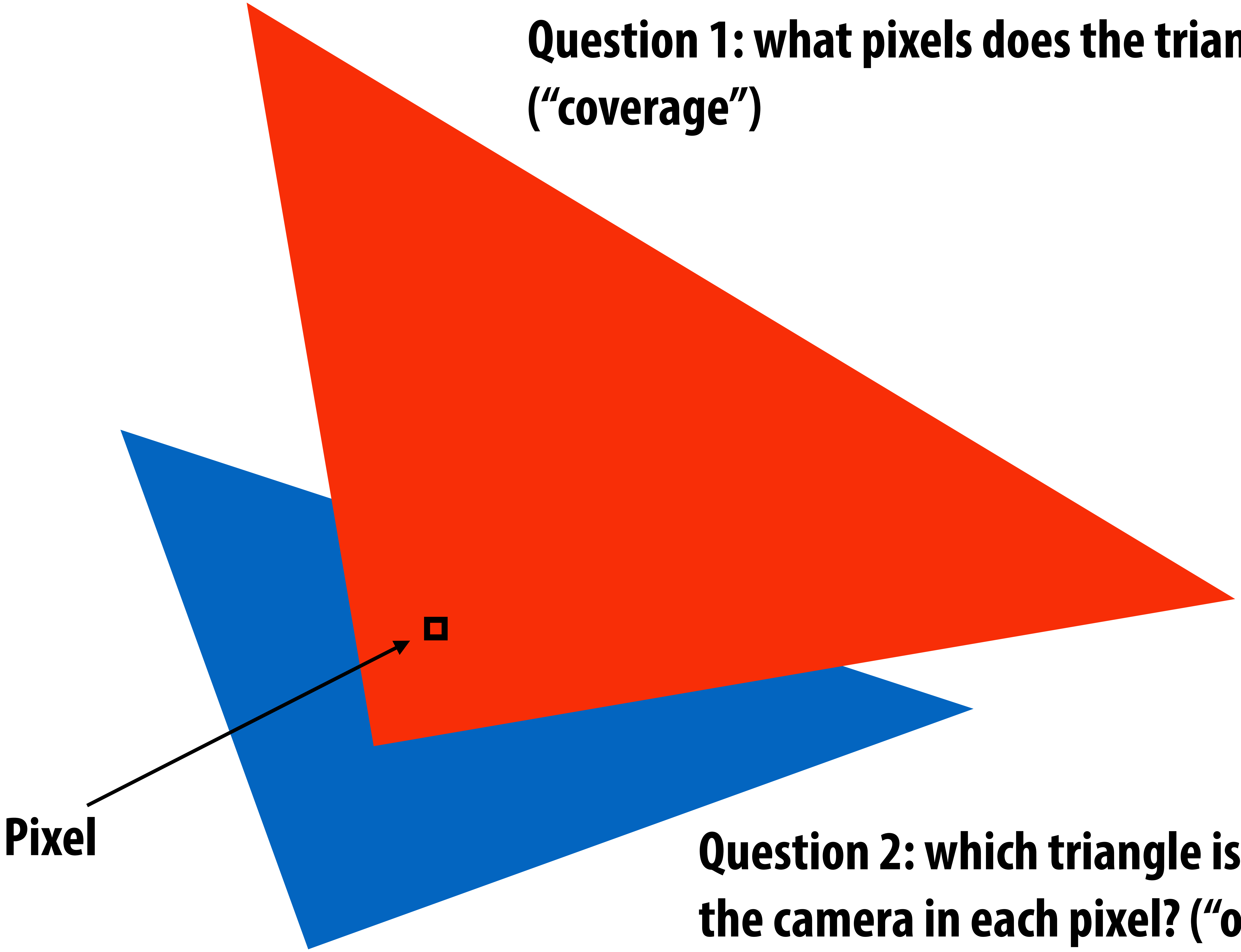
Rough sketch of rasterization pipeline:



- Reflects standard "real world" pipeline (OpenGL/Direct3D)
  – the rest is just details (e.g., API calls); will discuss in recitation

Let’s draw some triangles on the screen

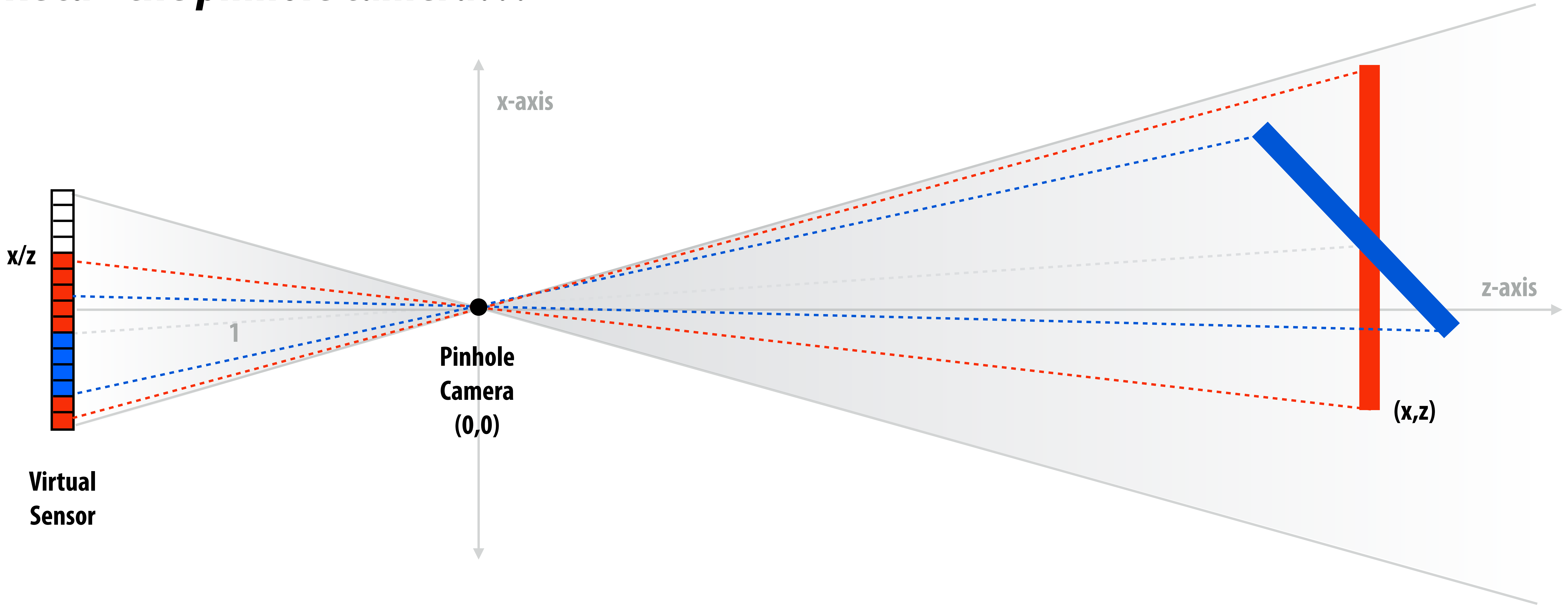
Question 1: what pixels does the triangle overlap? (“coverage”)



Question 2: which triangle is closest to the camera in each pixel? (“occlusion”)

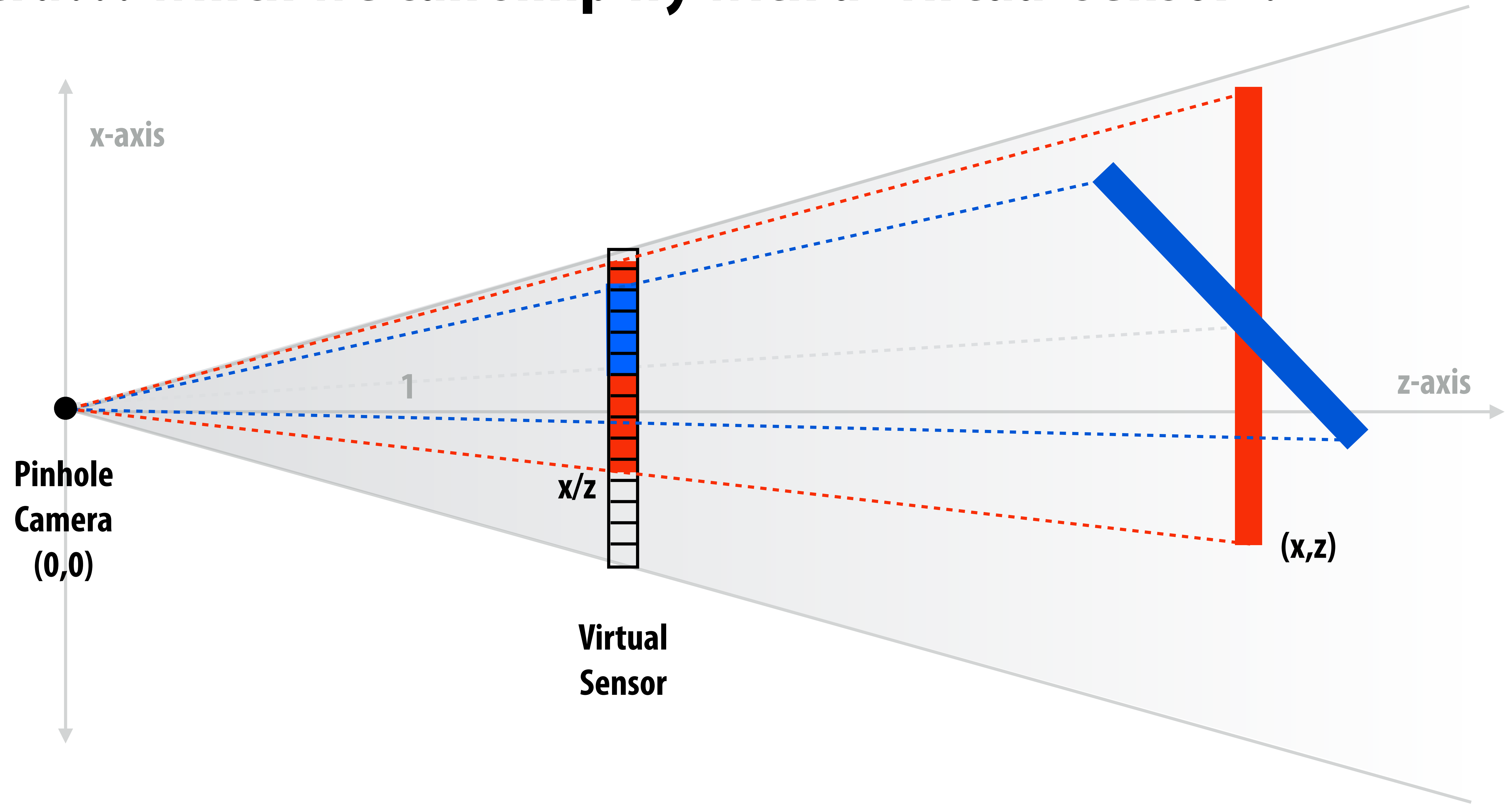
The visibility problem

Recall the *pinhole camera*…



The visibility problem

Recall the *pinhole camera*… which we can simplify with a "virtual sensor":

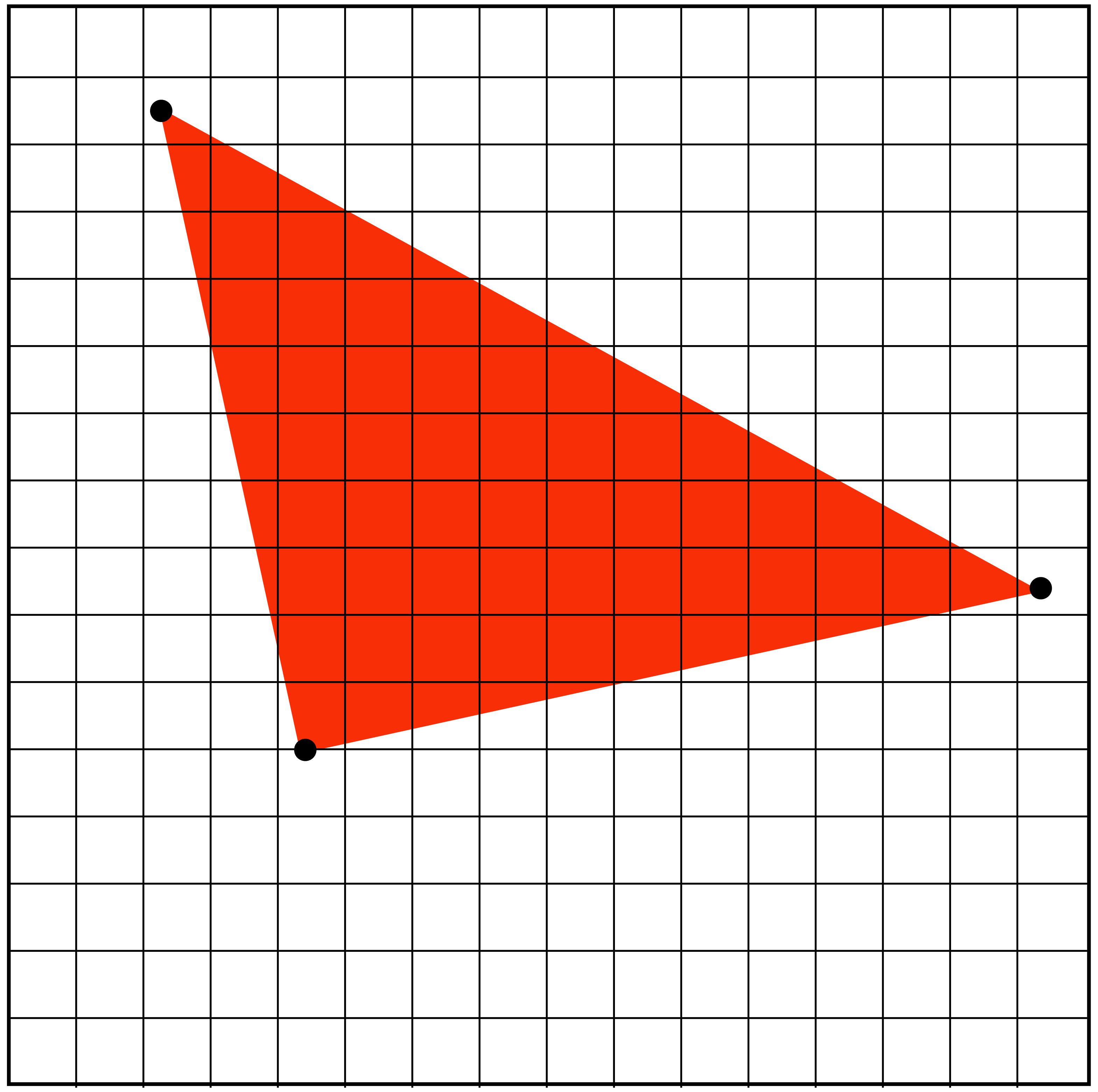


- **Visibility problem in terms of rays:**
  - **COVERAGE: What scene geometry is hit by a ray from a pixel through the pinhole?**
  - **OCCLUSION: Which object is the <u>first</u> hit along that ray?**

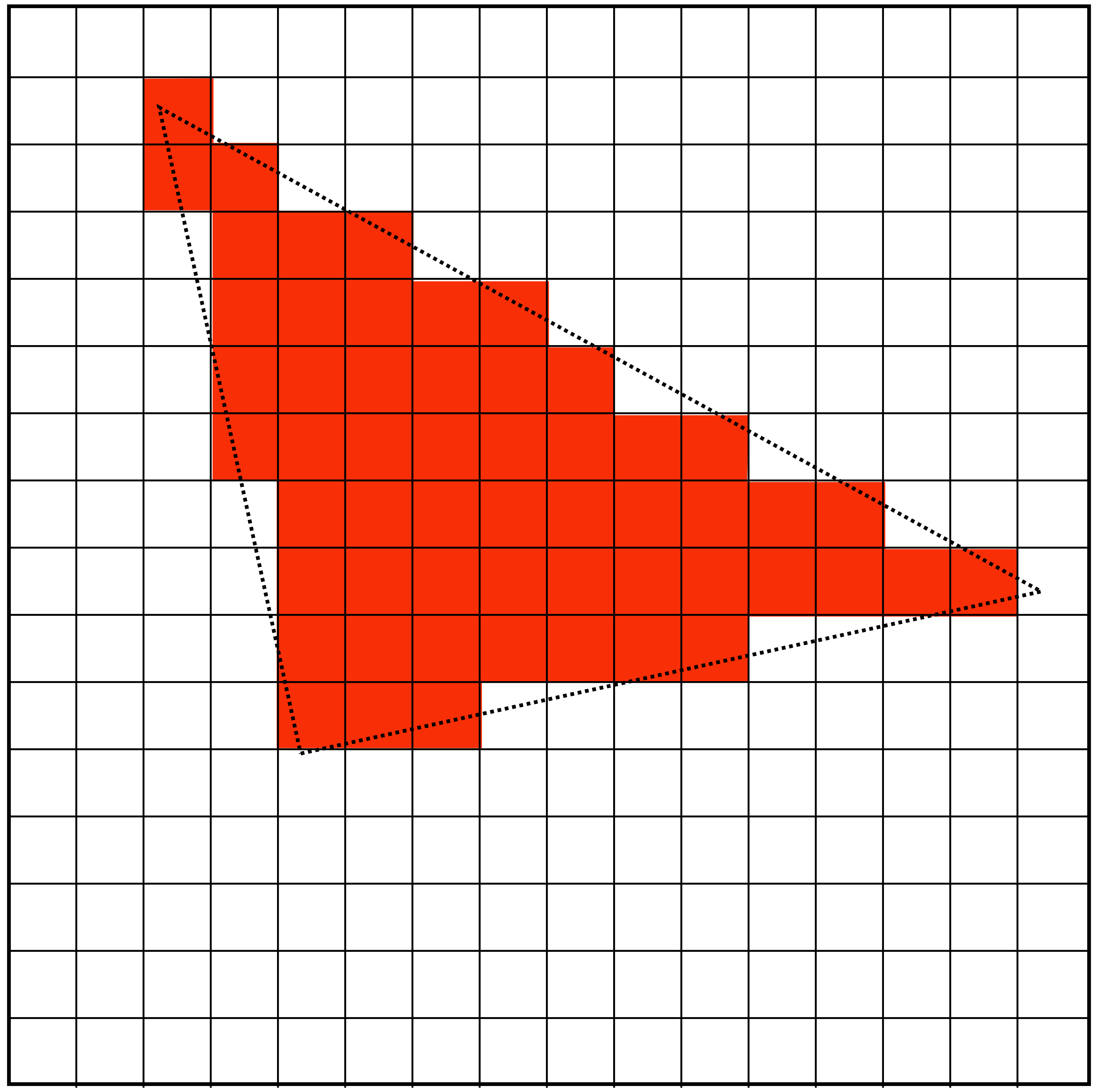
Computing triangle coverage

"Which pixels does the triangle overlap?"

Input:
projected position of triangle vertices: P_0, P_1, P_2

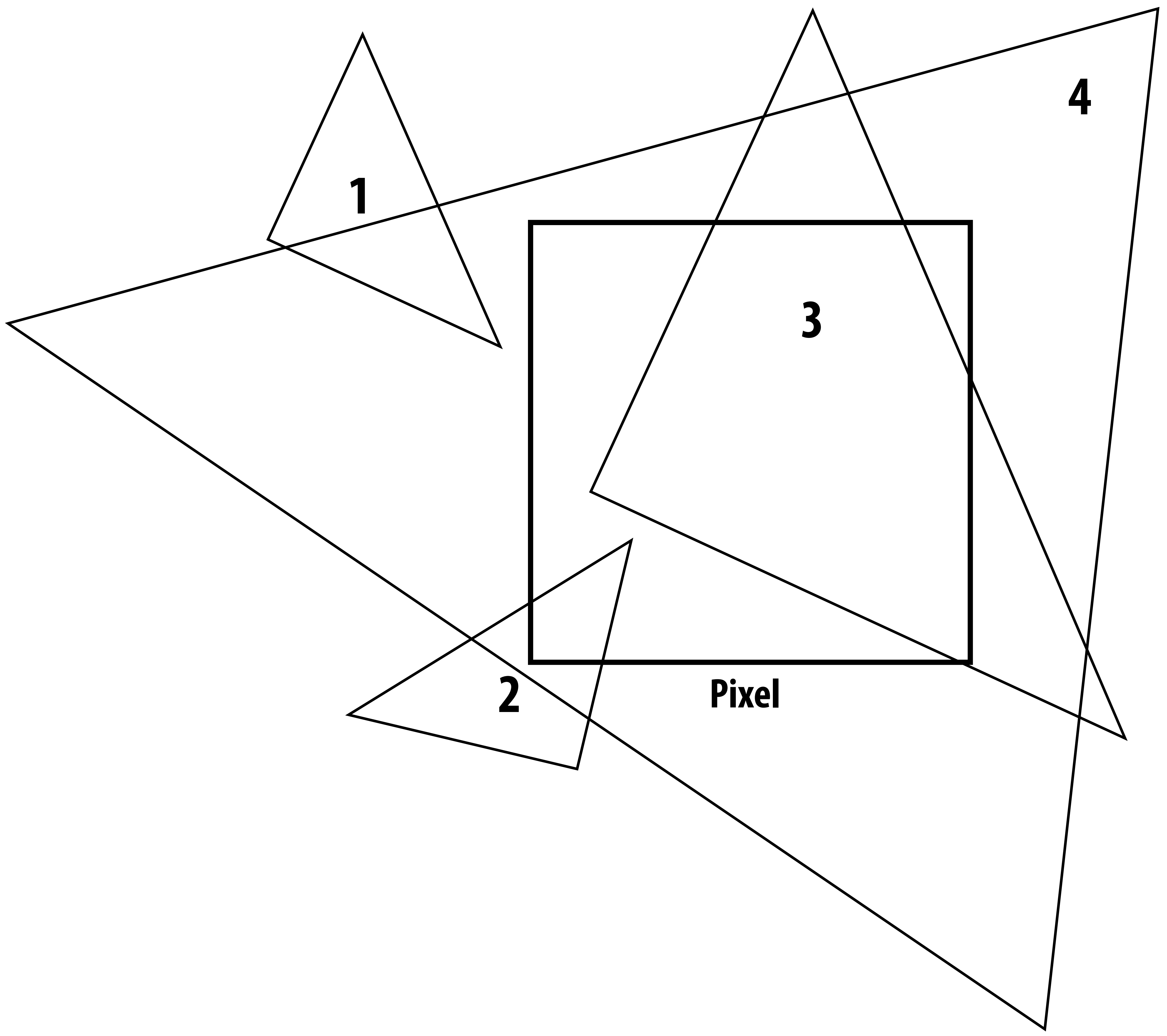


Output:
set of pixels "covered" by the triangle



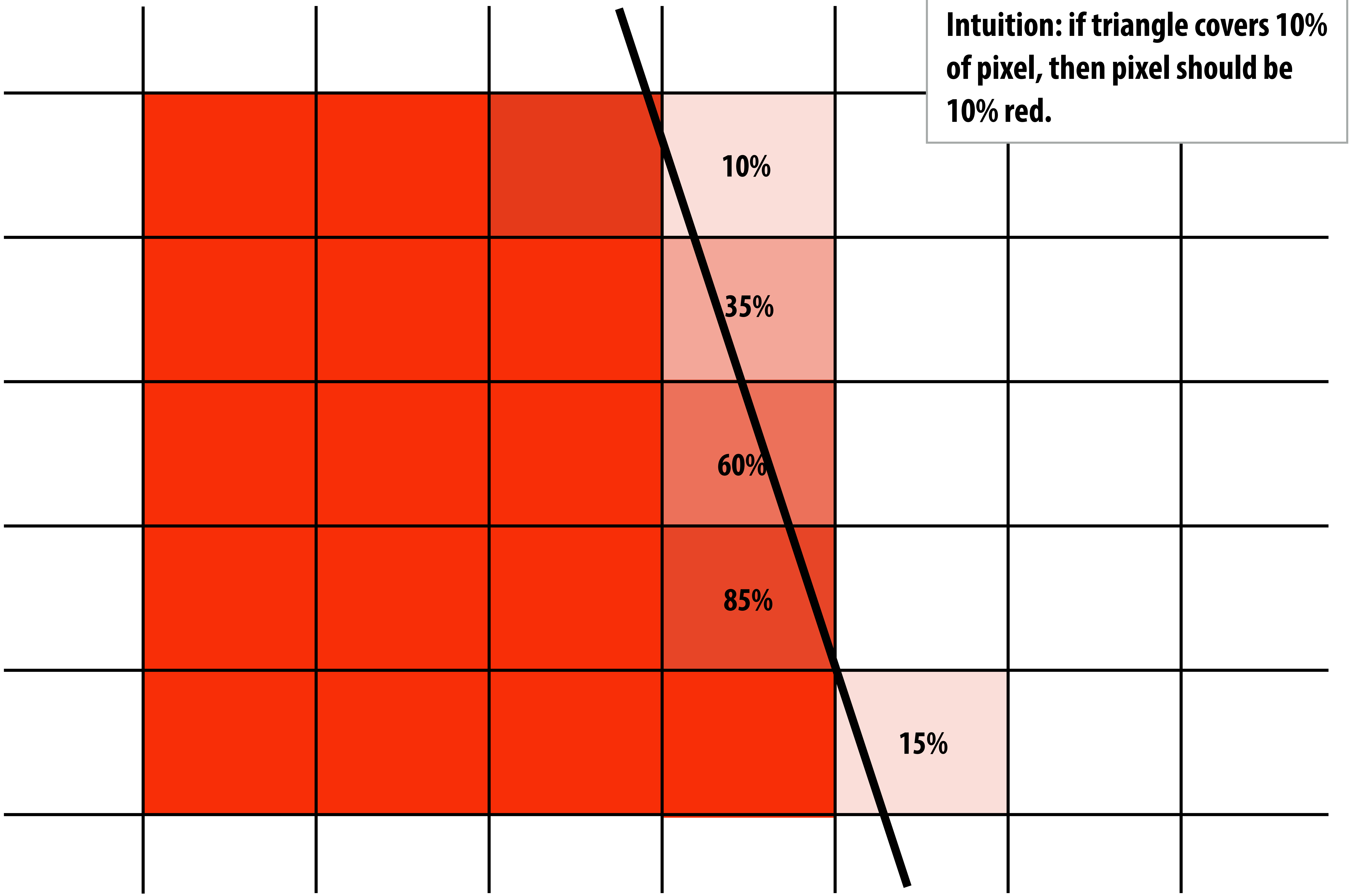
What does it mean for a pixel to be covered by a triangle?

Q: Which triangles “cover” this pixel?

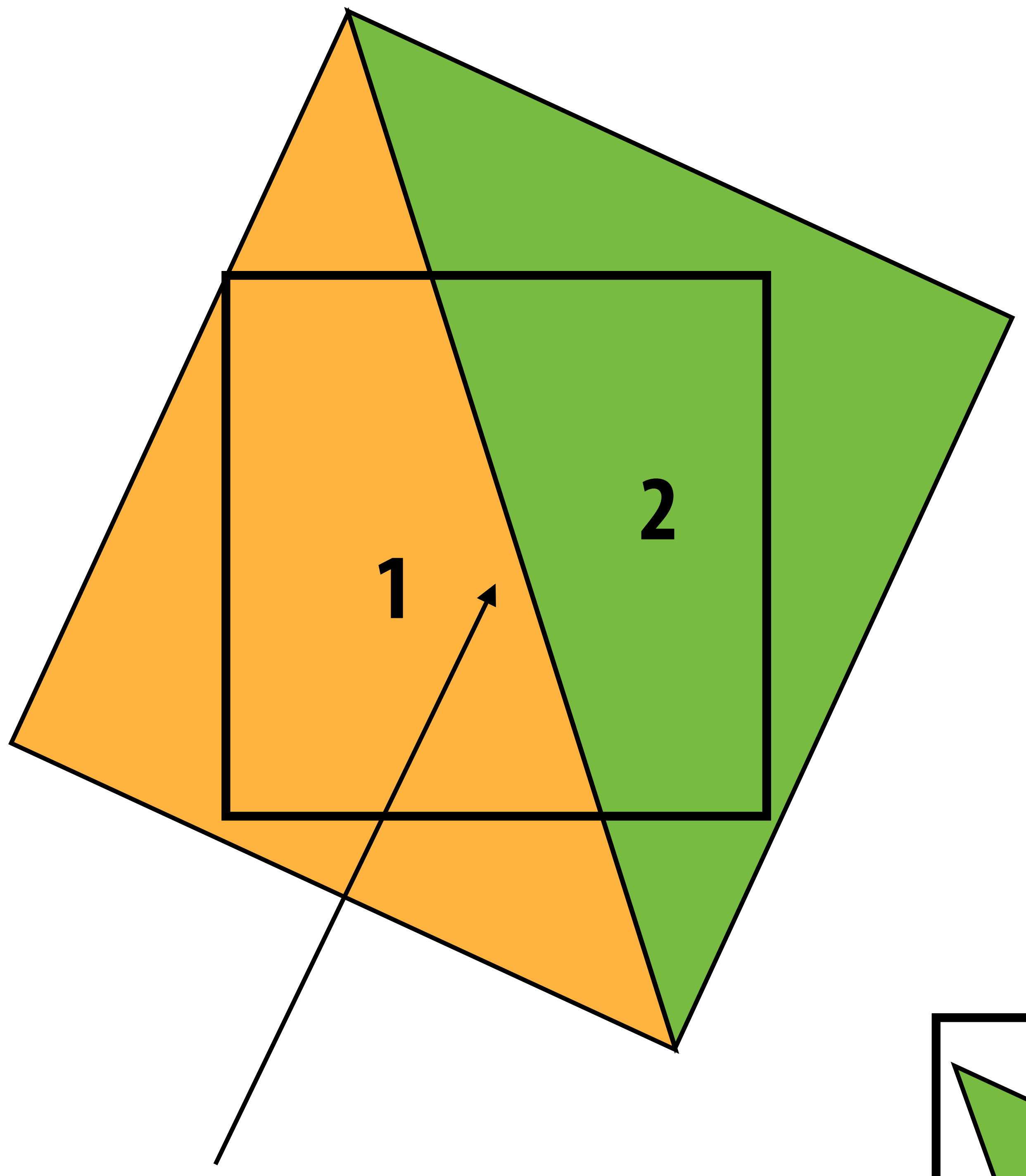


One option: compute fraction of pixel area covered by triangle, then color pixel according to this fraction.

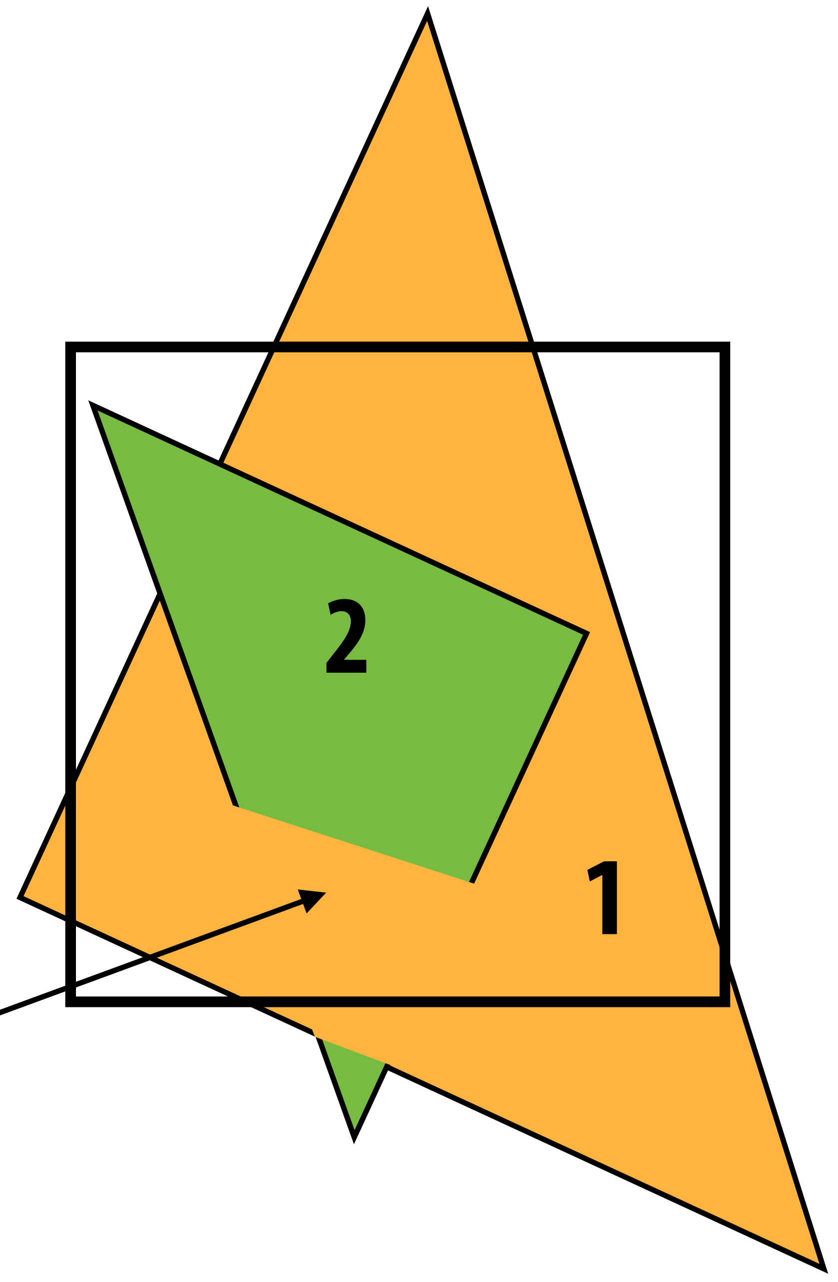
Intuition: if triangle covers 10% of pixel, then pixel should be 10% red.



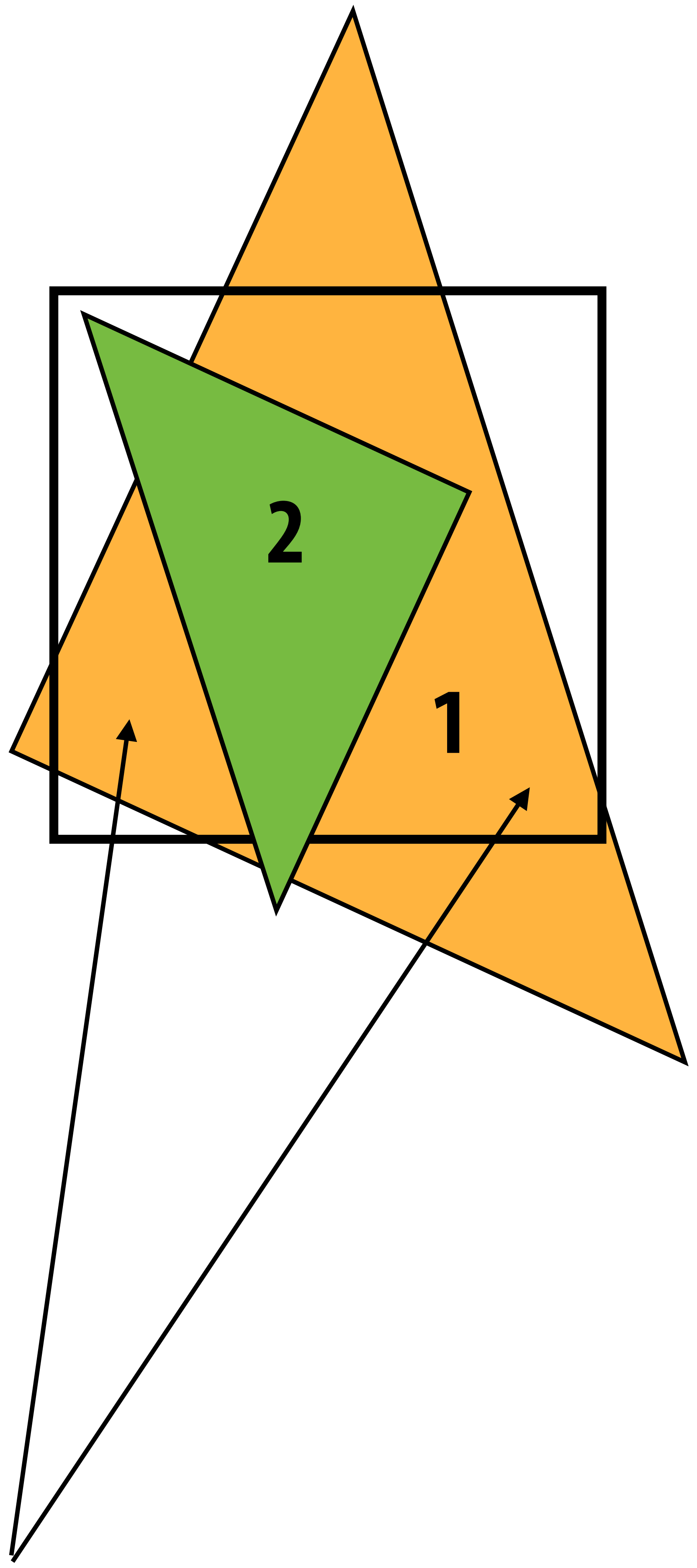
Coverage gets tricky when considering occlusion



Pixel covered by triangle 1, other half covered by triangle 2



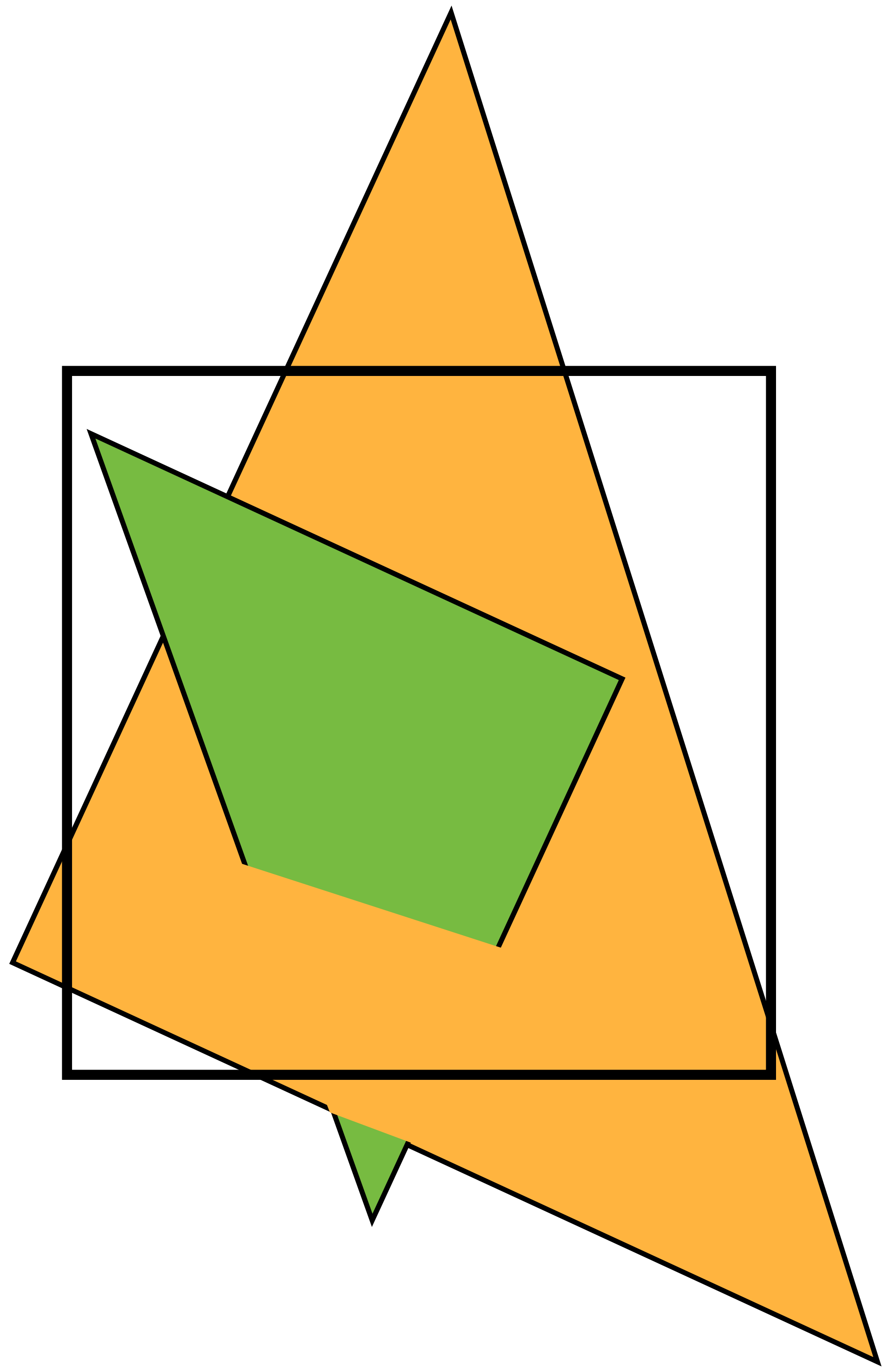
Interpenetration of triangles: even trickier

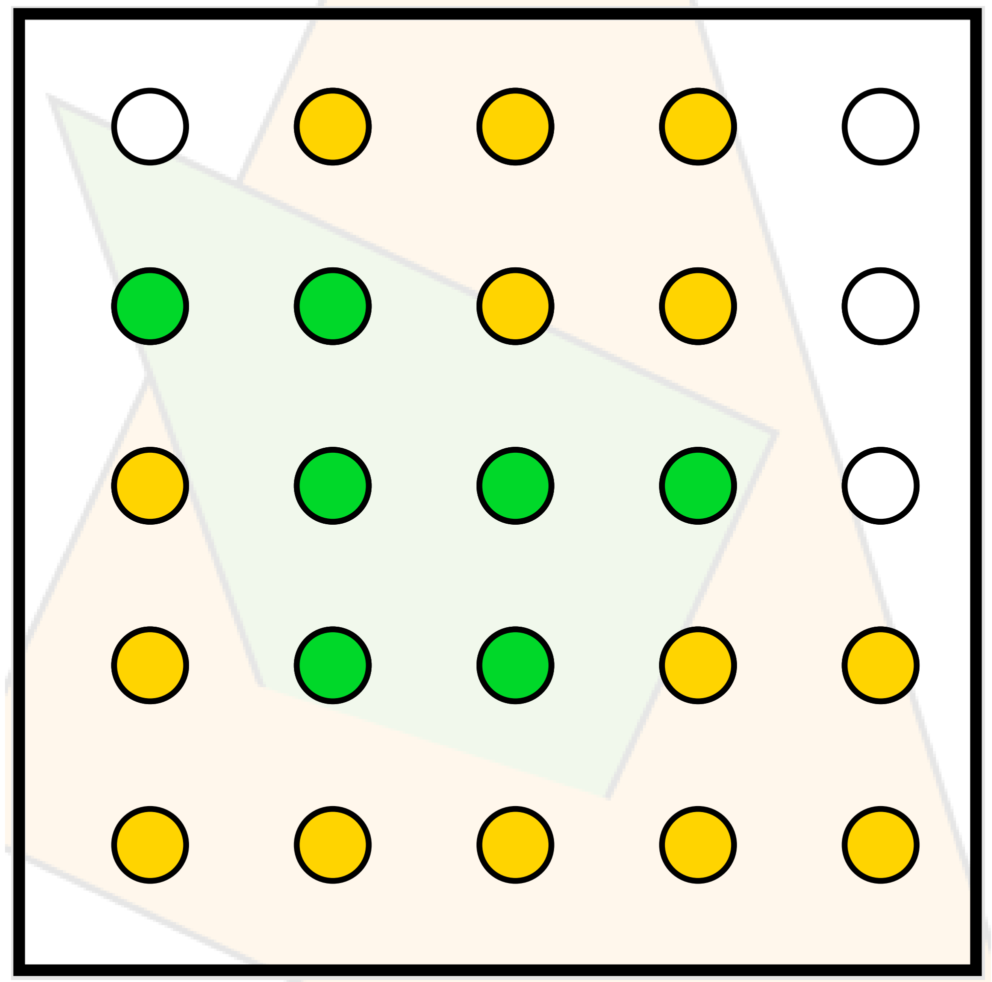


Two regions of triangle 1 contribute to pixel.
One of these regions is not even convex.

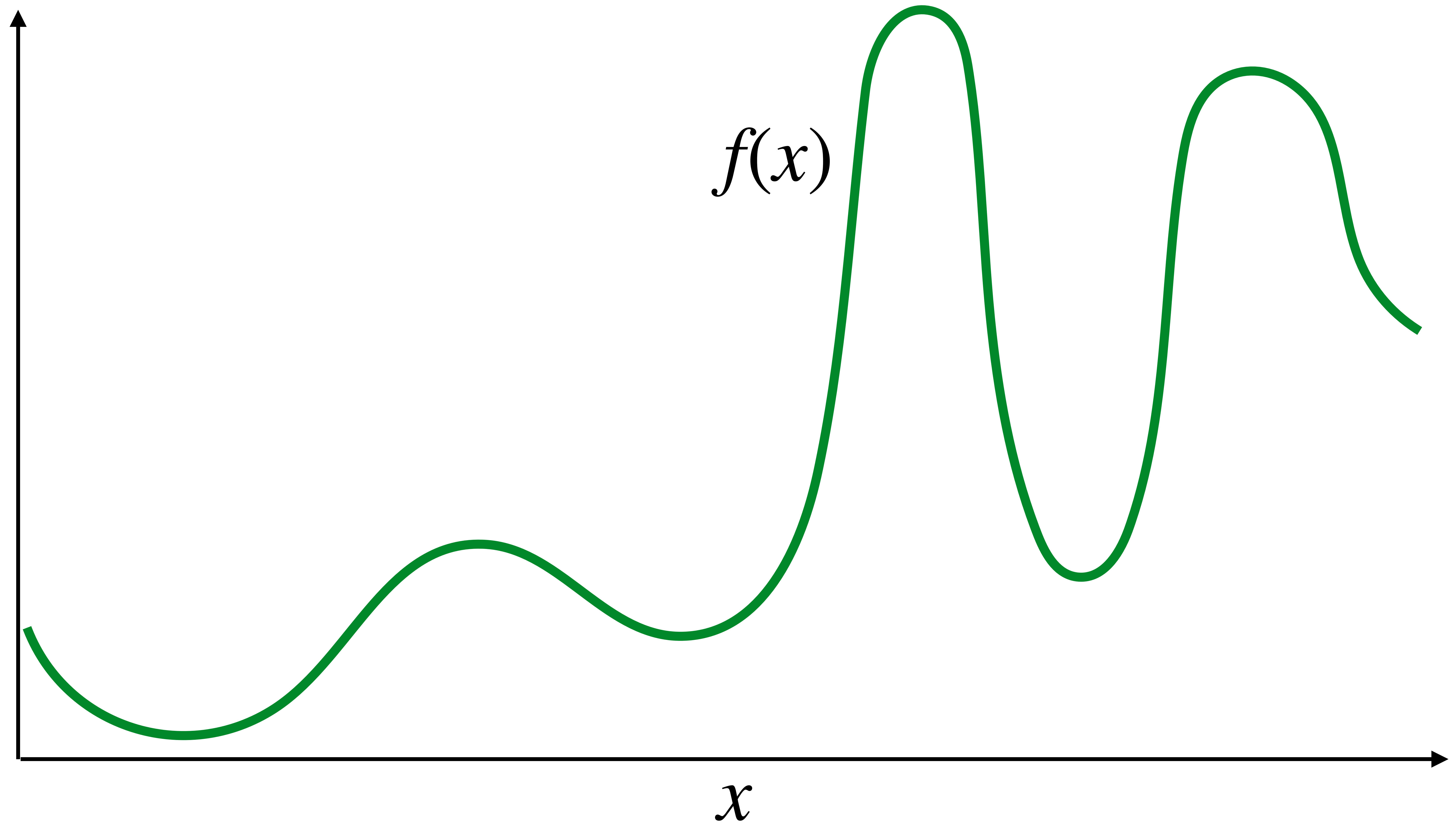
Coverage via sampling

- Real scenes are *complicated!*
  - occlusion, transparency, …
  - will talk about this more in a future lecture!
- Computing *exact* coverage is not practical
- Instead: view coverage as a sampling problem
  - don't compute exact/analytical answer
  - instead, test a collection of sample points
  - with enough points & smart choice of sample locations, can start to get a good estimate
- First, let's talk about sampling in general…



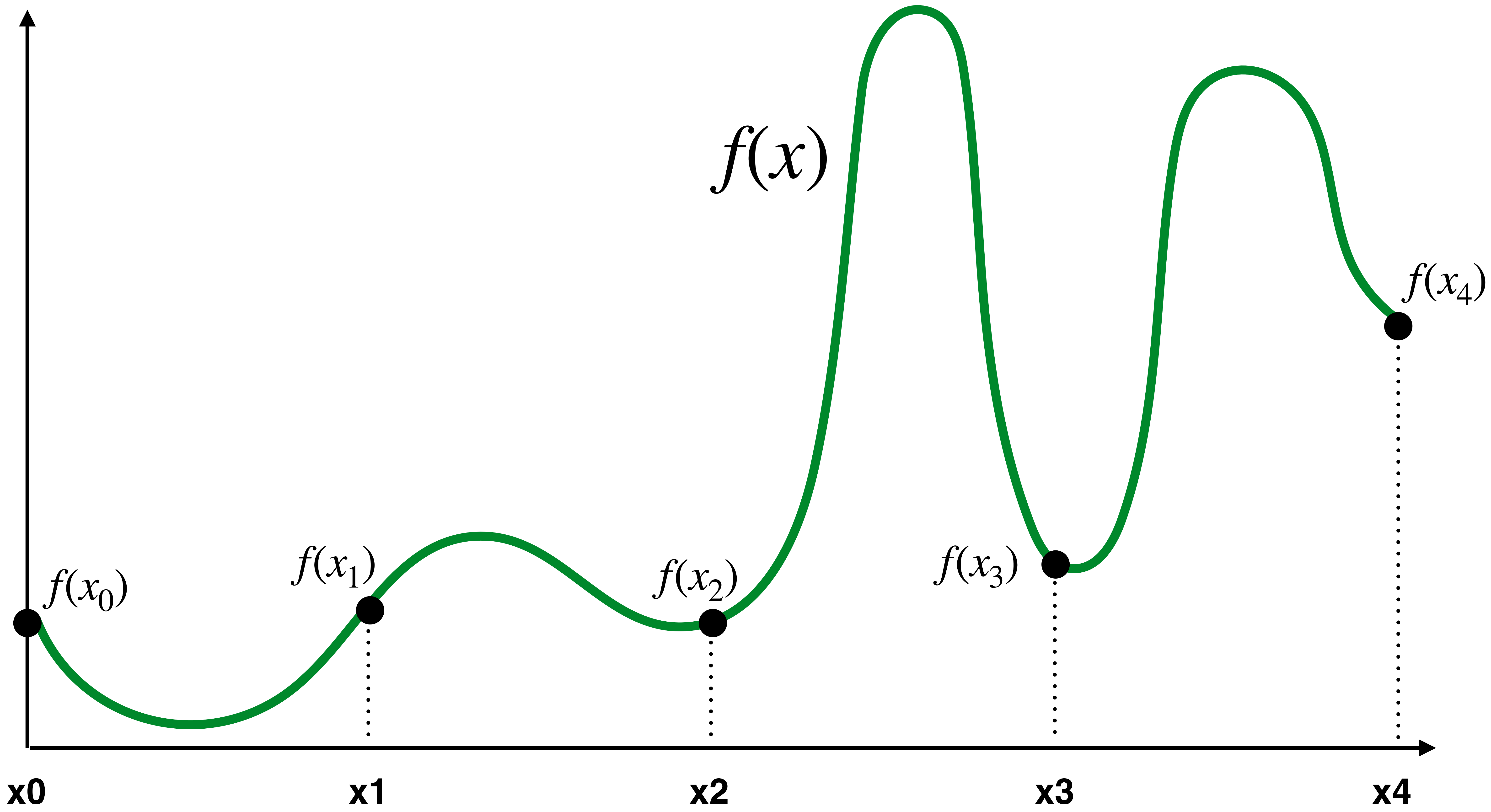


Sampling 101: Sampling a 1D signal

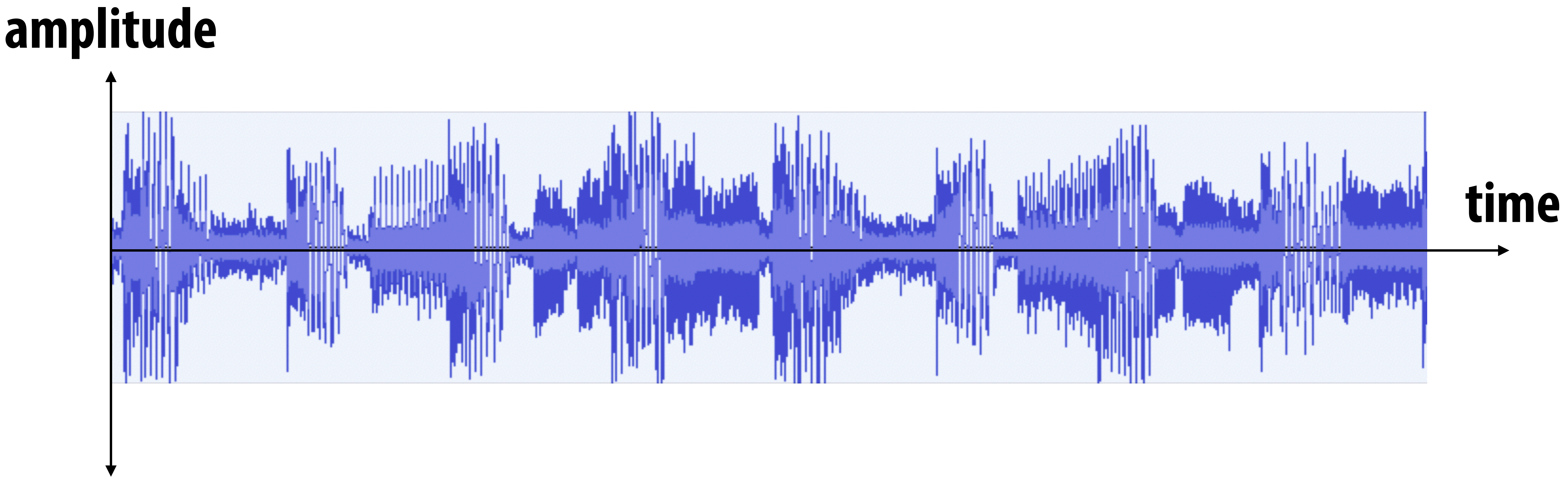


Sampling = taking measurements of a signal

Below: 5 measurements ("samples") of $f(x)$

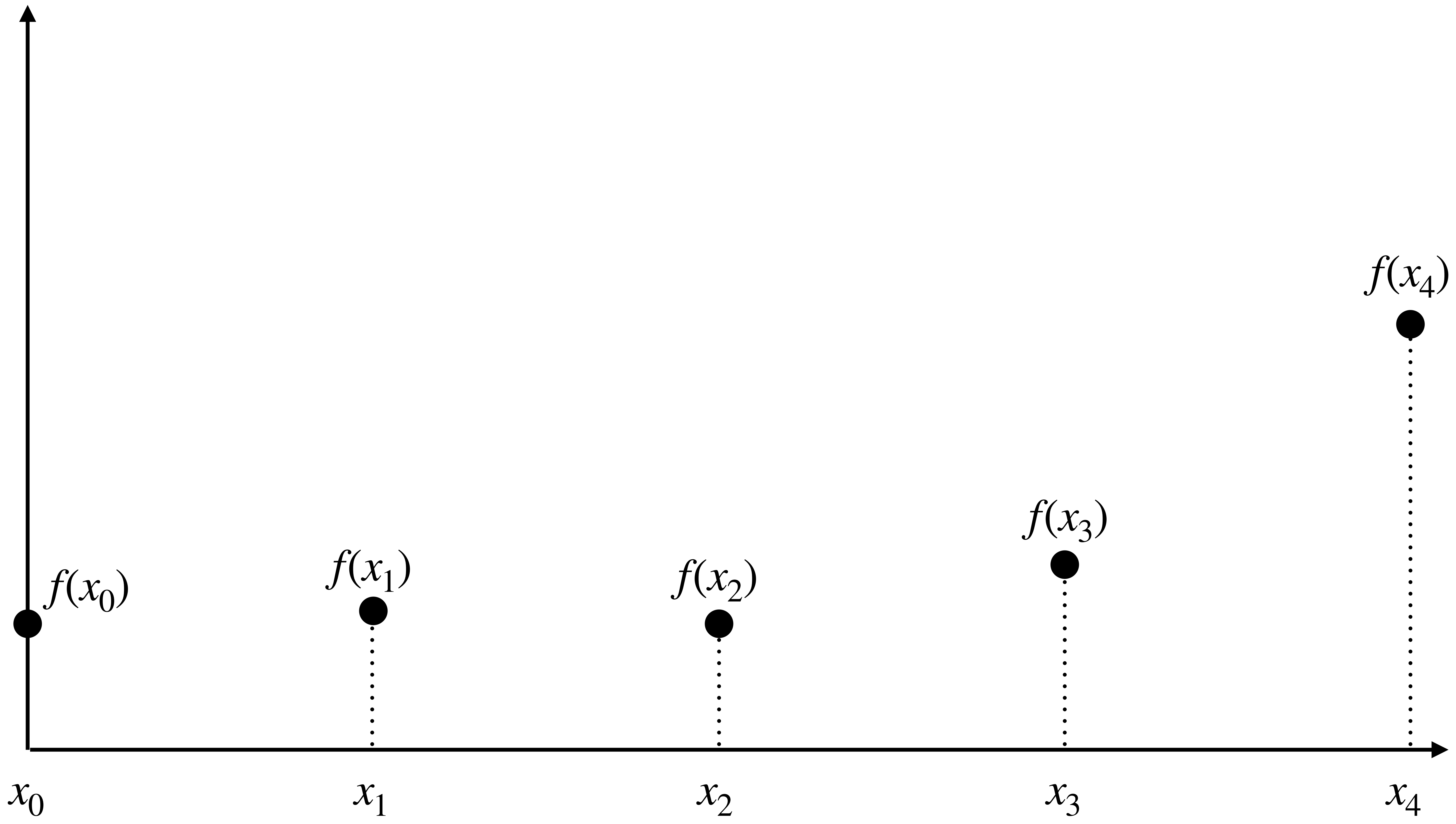


Audio file: stores samples of a 1D signal



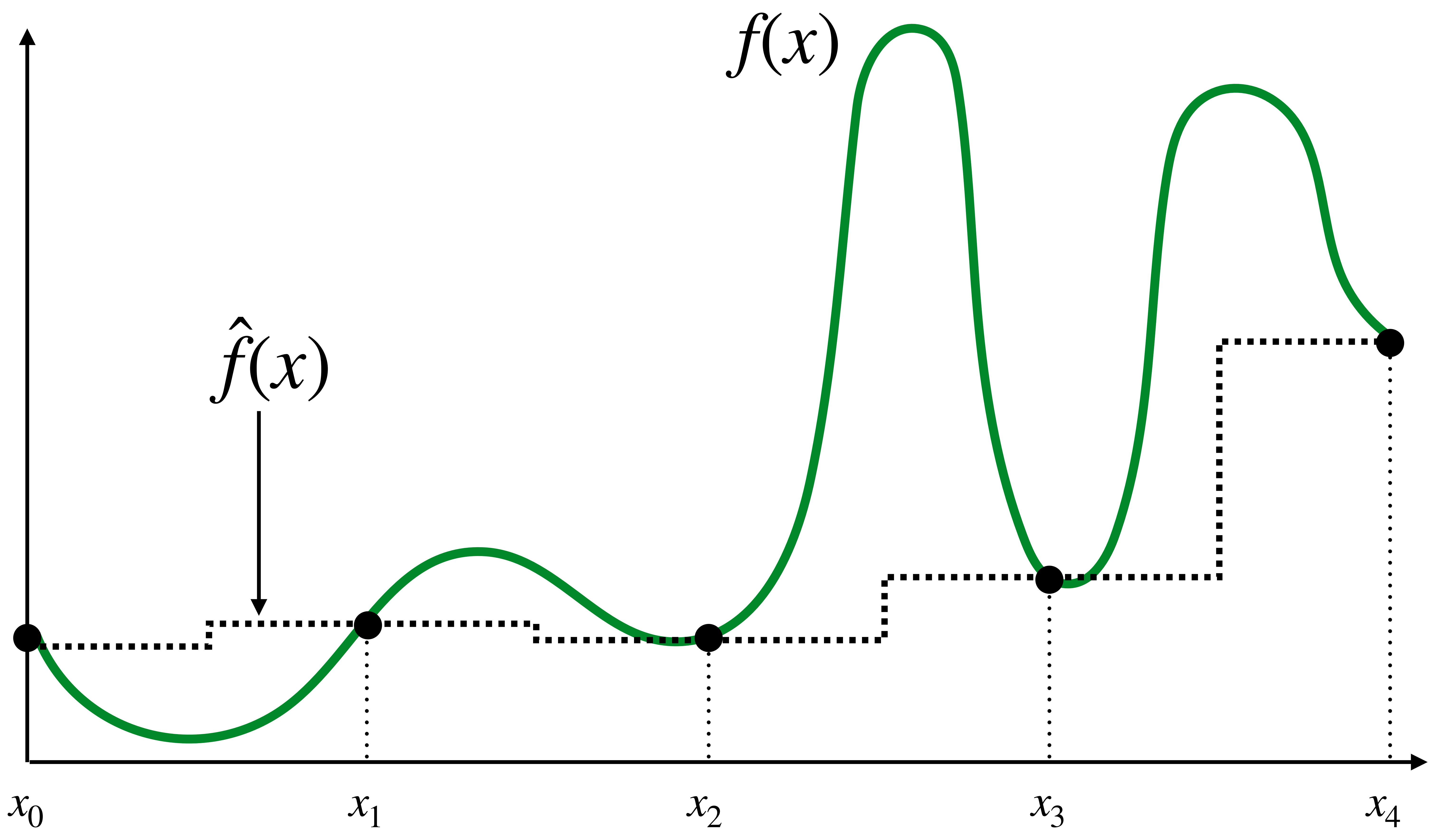
(most consumer audio is sampled 44,100 times per second, i.e., at *44.1 KHz*)

Reconstruction: given a set of samples, how might we attempt to reconstruct the original signal $f(x)$?



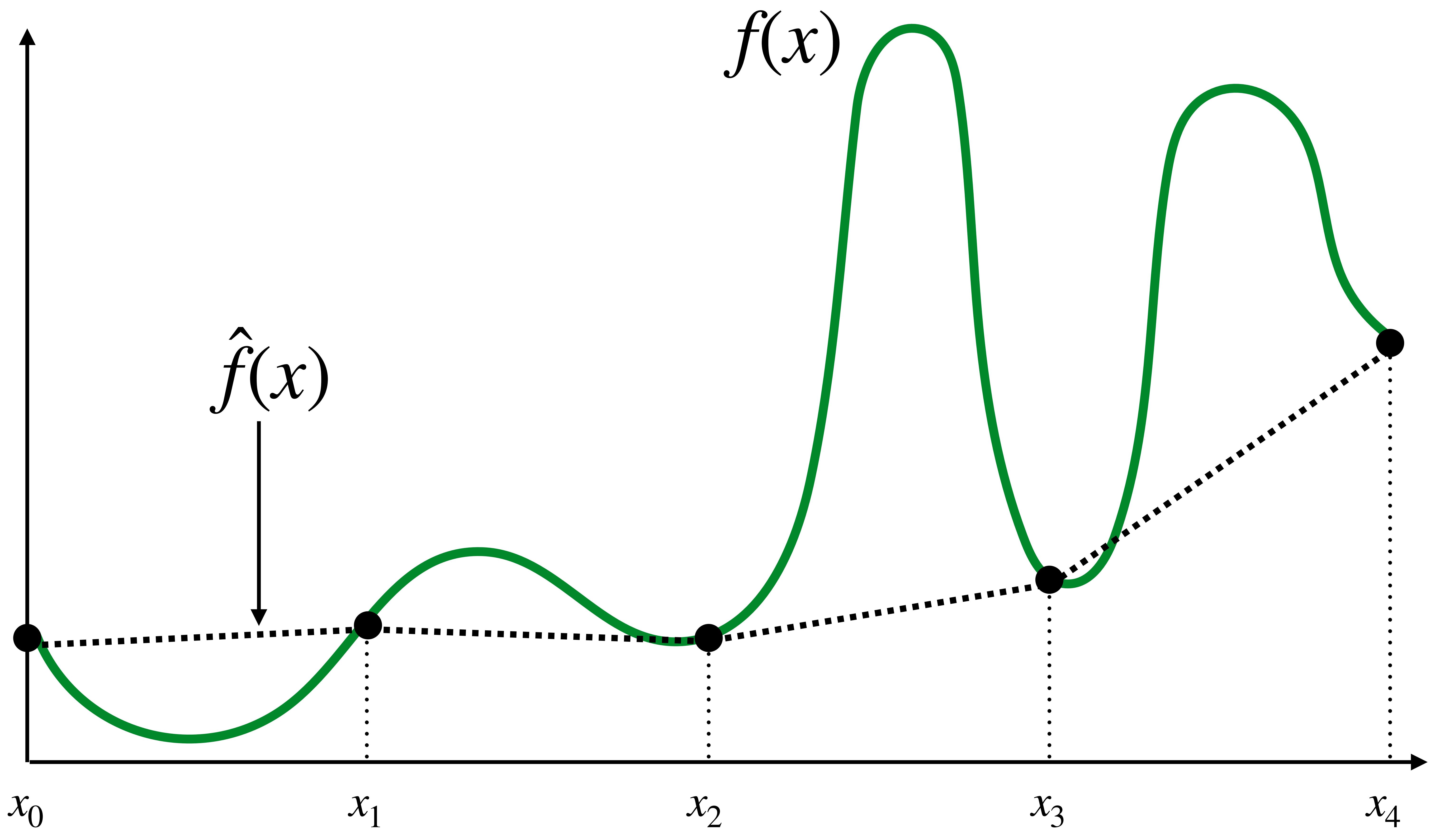
Piecewise constant approximation

$\hat{f}(x)$ = value of sample closest to x

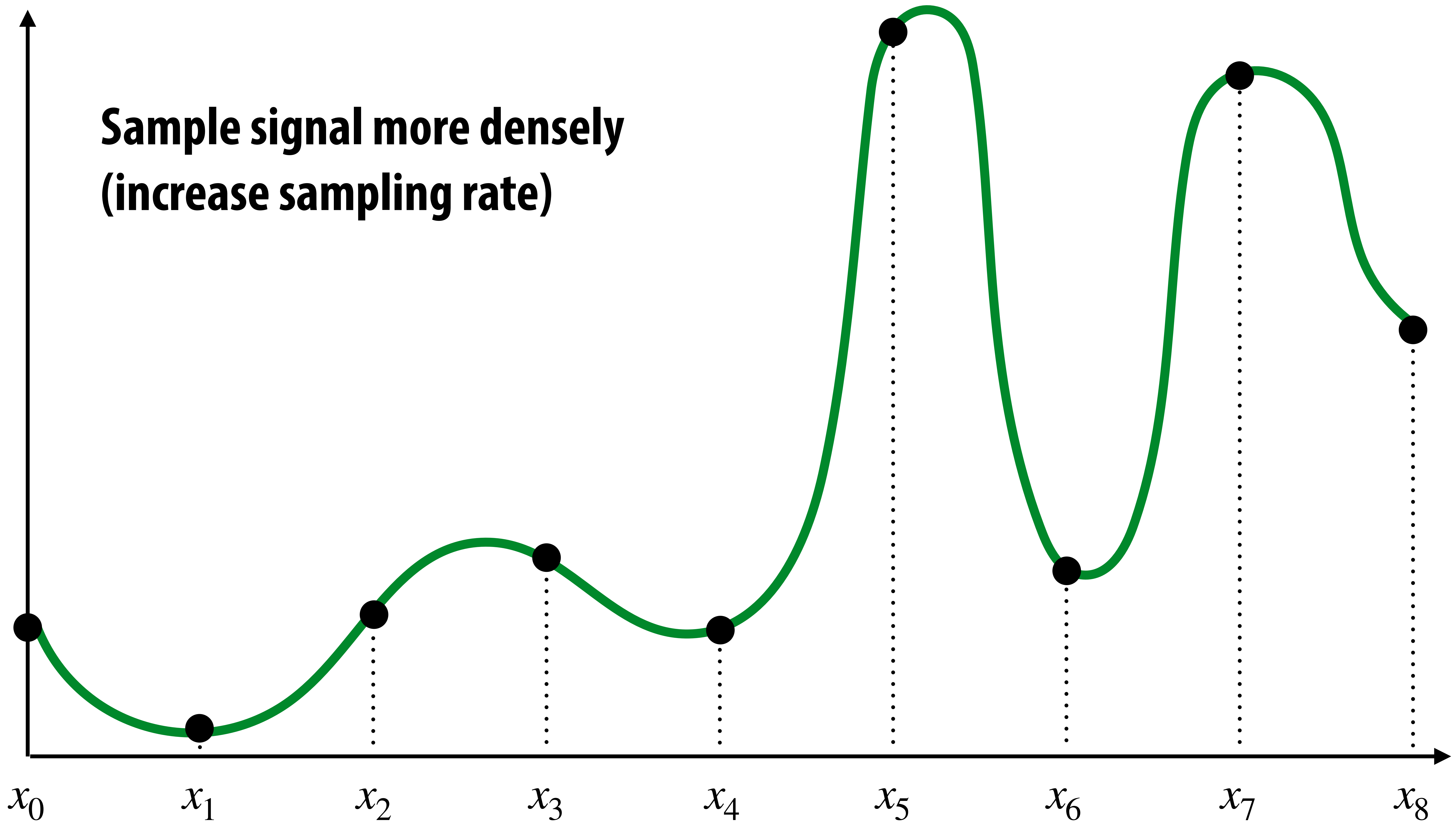


Piecewise linear approximation

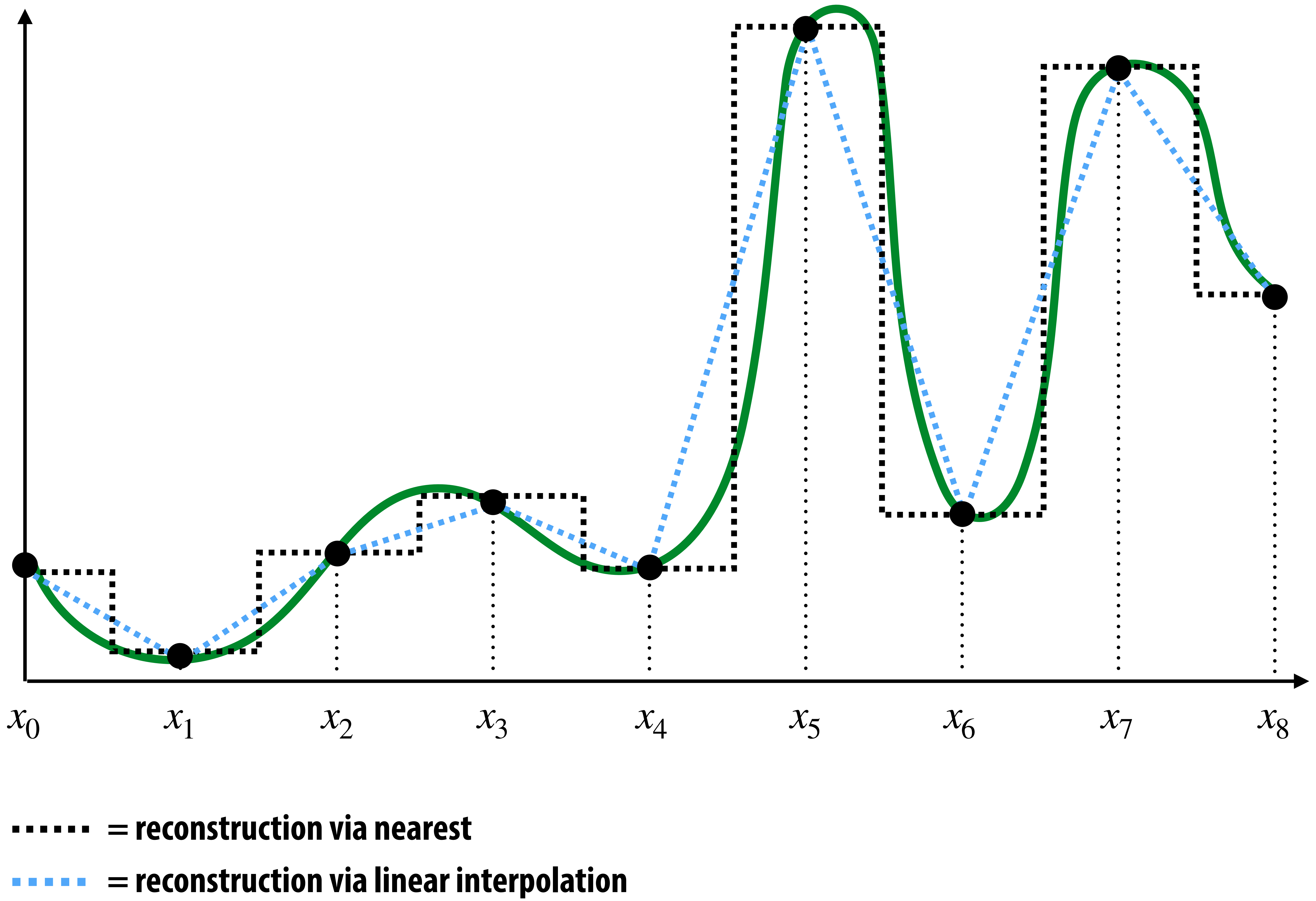
$\hat{f}(x)$ = linear interpolation between values of two closest samples to x



How can we represent the signal more accurately?



Reconstruction from denser sampling



2D Sampling & Reconstruction

- **Basic story doesn't change much for images:**
  - sample values measure image (i.e., signal) at sample points
  - apply interpolation/reconstruction filter to approximate image

original

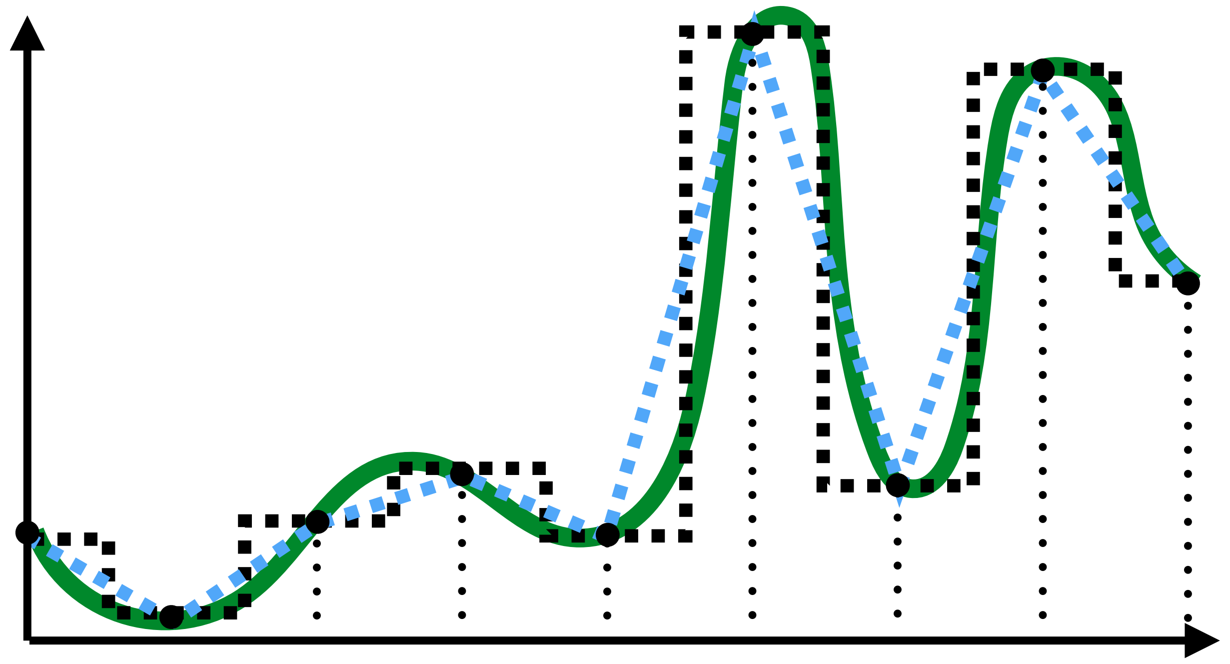
piecewise constant
("nearest neighbor")

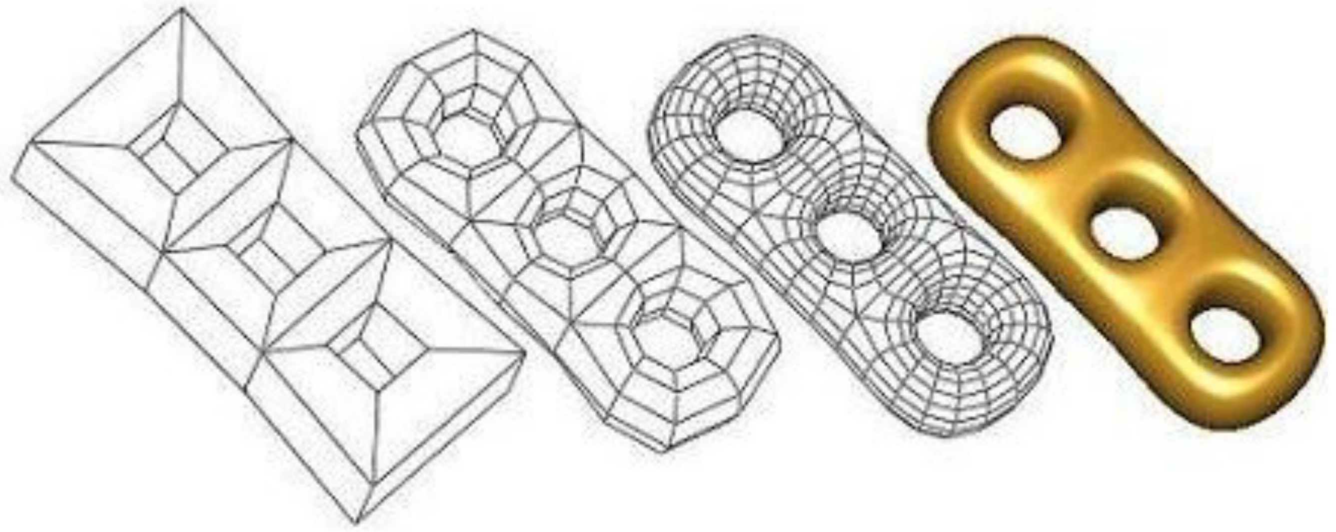
piecewise *bi*-linear

Sampling 101: Summary

- Sampling = measurement of a signal
  - Encode signal as discrete set of samples
  - In principle, represent values at specific points (though hard to measure in reality!)

- Reconstruction = generating signal from a discrete set of samples
  - Construct a function that interpolates or approximates function values
  - E.g., piecewise constant/“nearest neighbor”, or piecewise linear
  - Many more possibilities! For all kinds of signals (audio, images, geometry…)

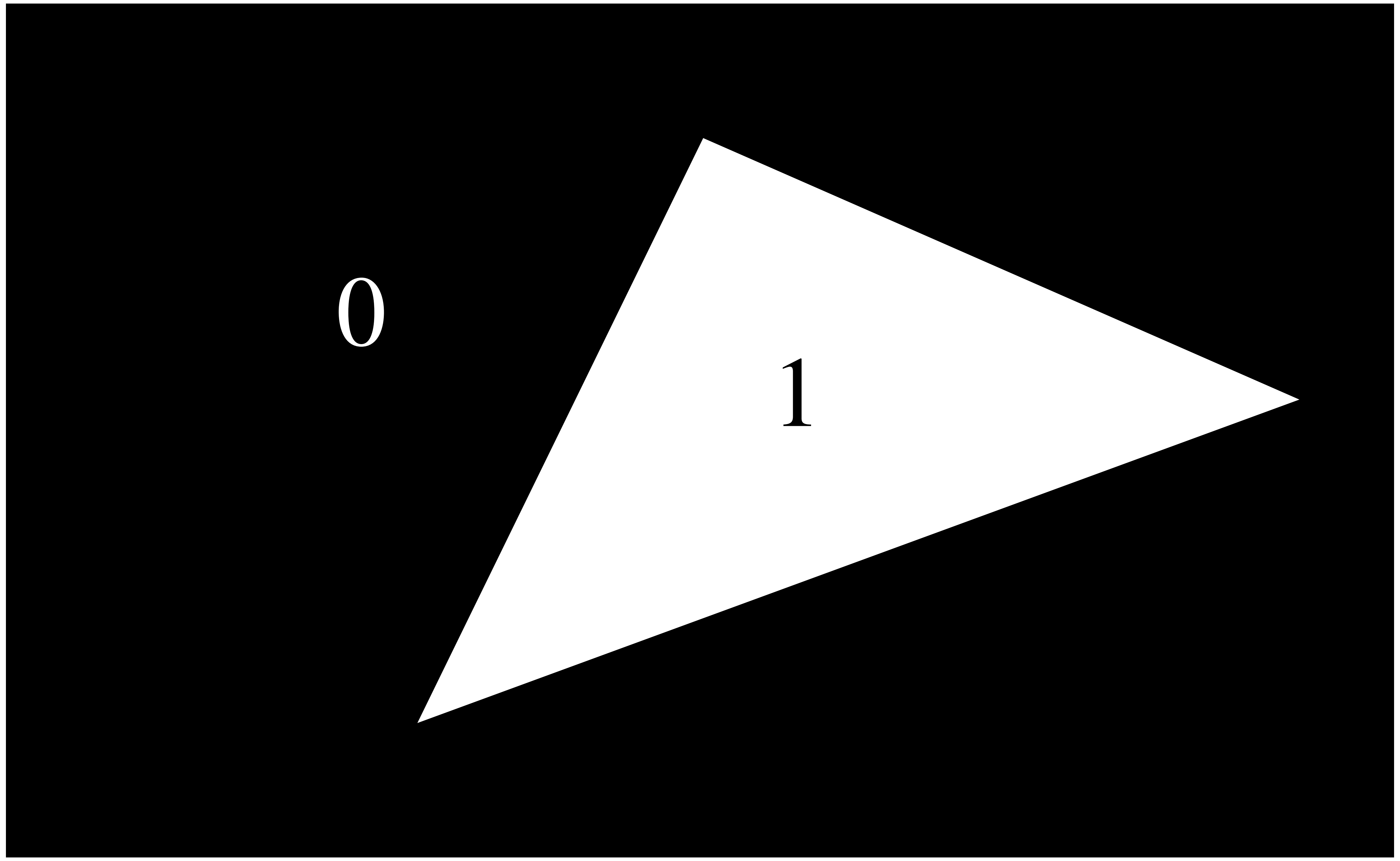




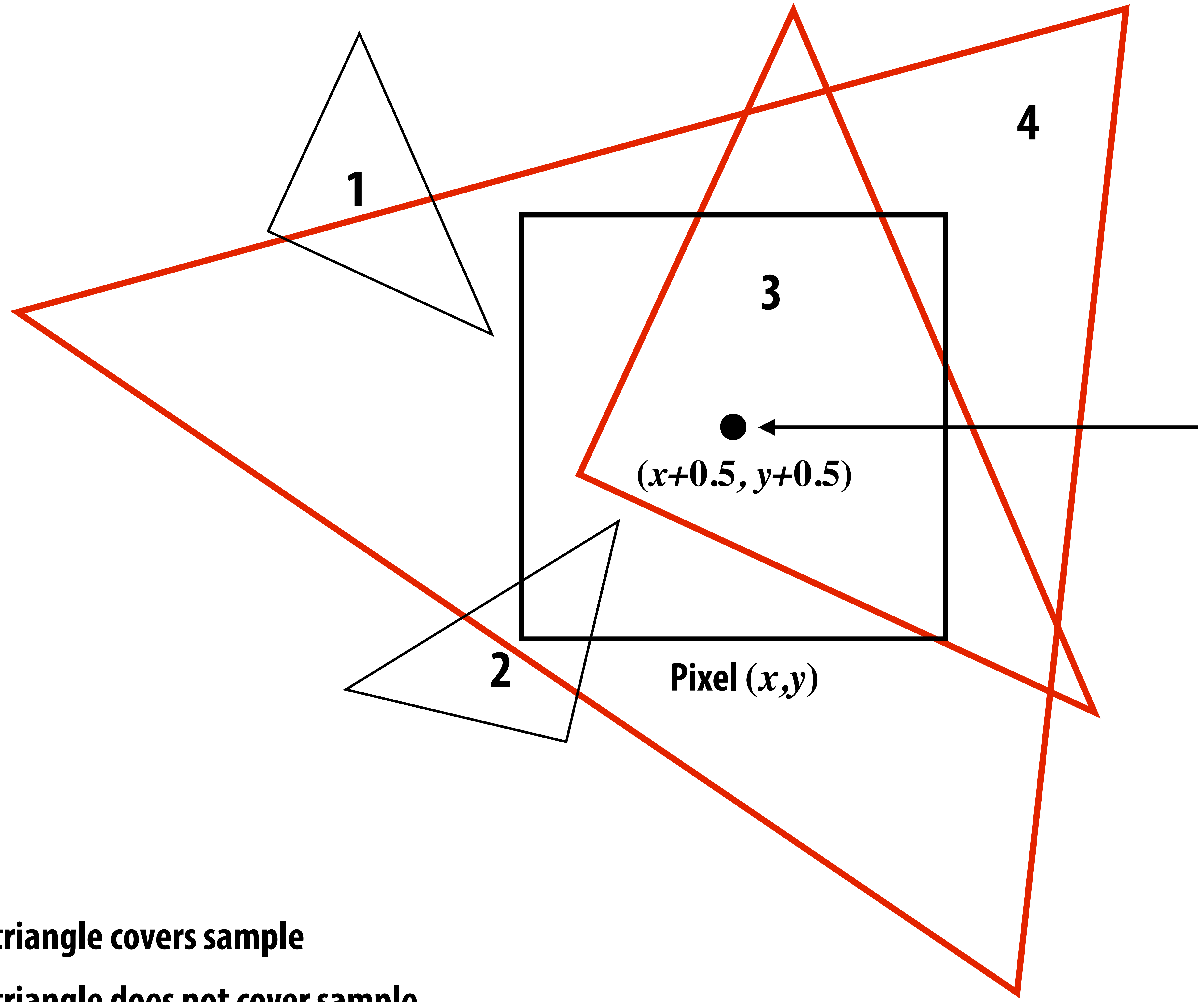
[Image credit: Wikipedia]

For rasterization, what function are we sampling?

$$\text{coverage}(x, y) := \begin{cases} 1, & \text{triangle contains point } (x, y) \\ 0, & \text{otherwise} \end{cases}$$



Simple rasterization: just <u>sample</u> the coverage function



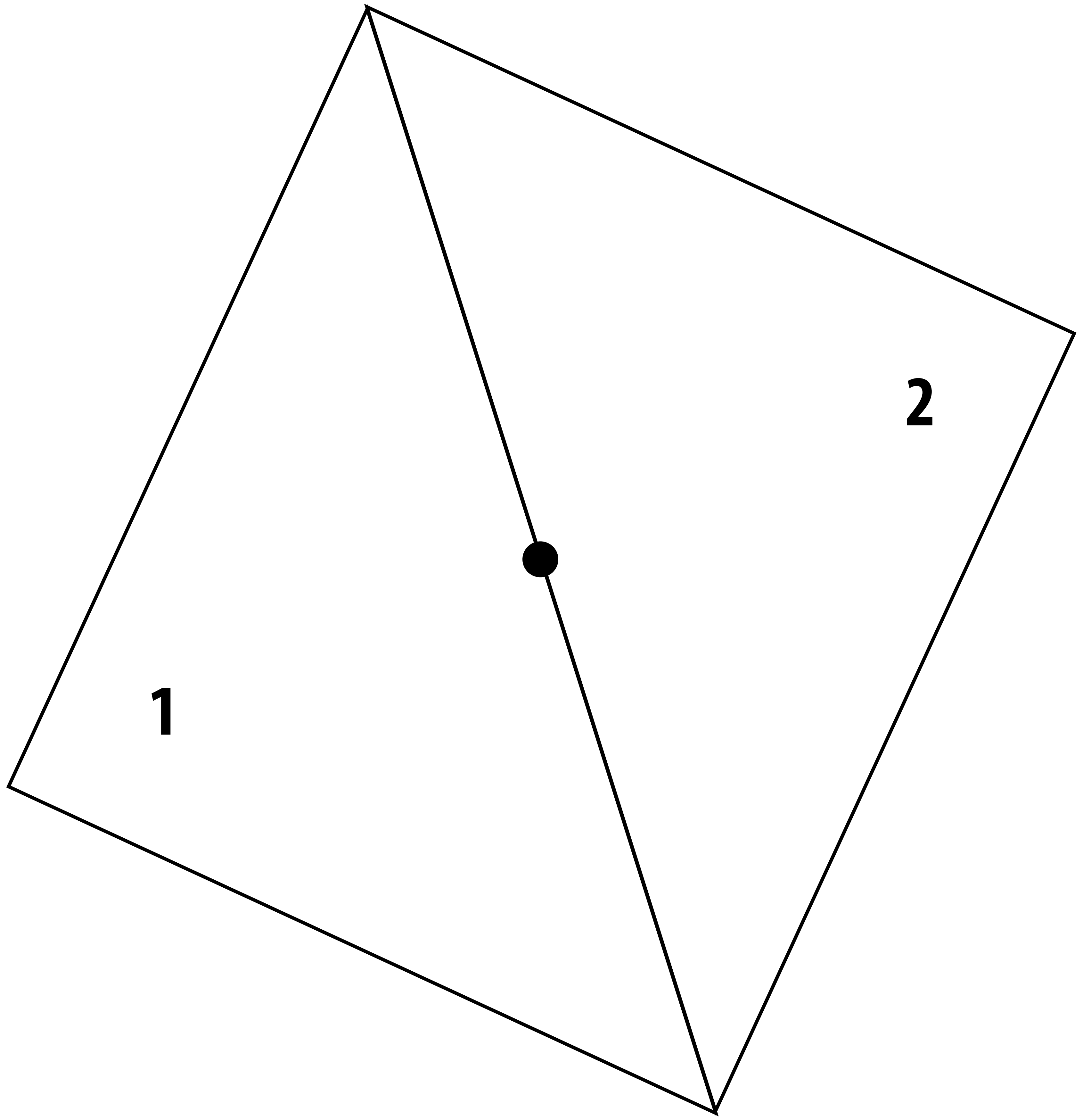
Example:
Here I chose the coverage sample point to be at a point corresponding to the pixel center.

= triangle covers sample

= triangle does not cover sample

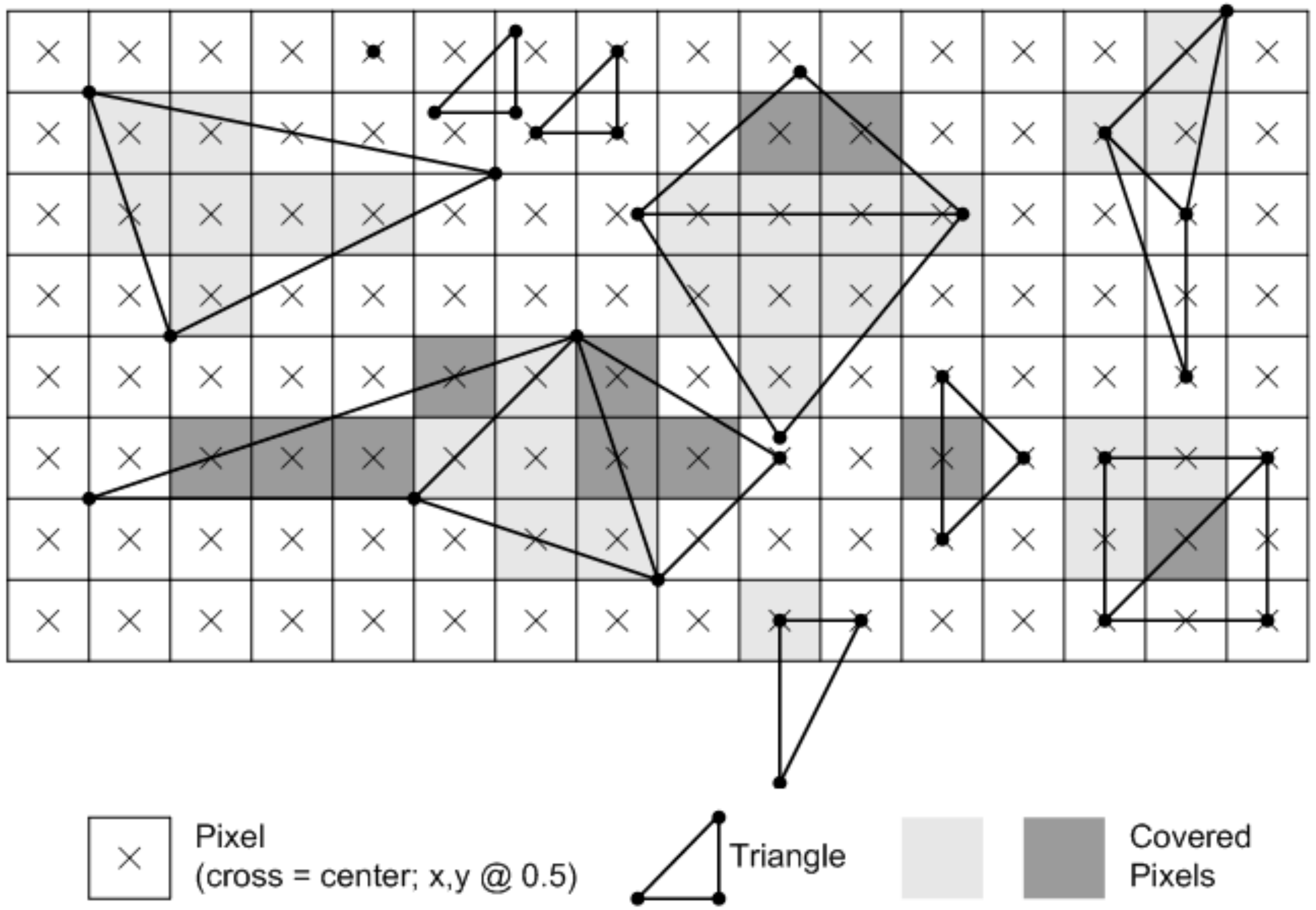
Edge cases (literally)

Is this sample point covered by triangle 1? or triangle 2? or both?



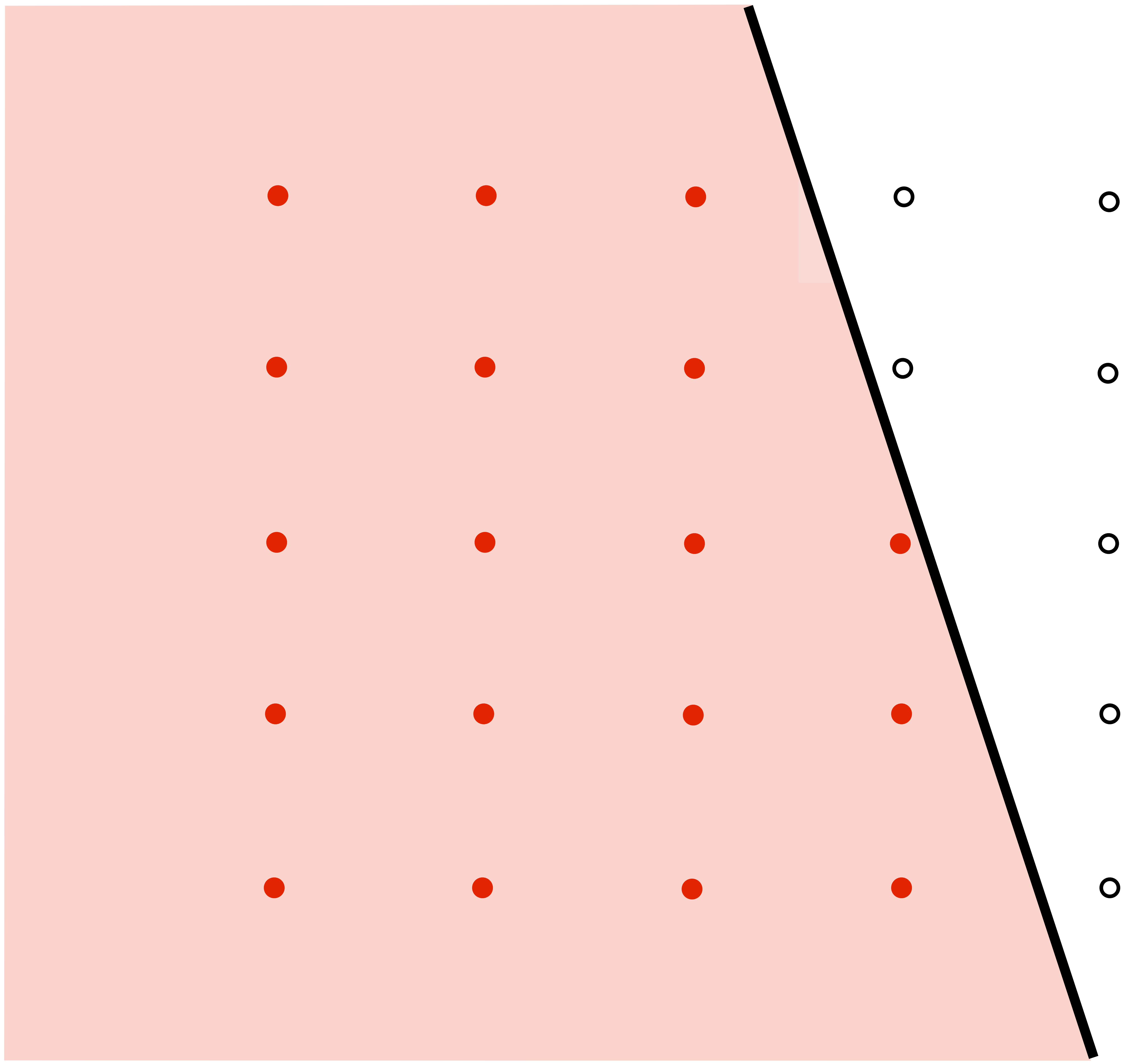
Breaking Ties*

- When edge falls directly on a screen sample point, the sample is classified as within triangle if the edge is a "top edge" or "left edge"
    - Top edge: horizontal edge that is above all other edges
    - Left edge: an edge that is not exactly horizontal and is on the left side of the triangle. (triangle can have one or two left edges)

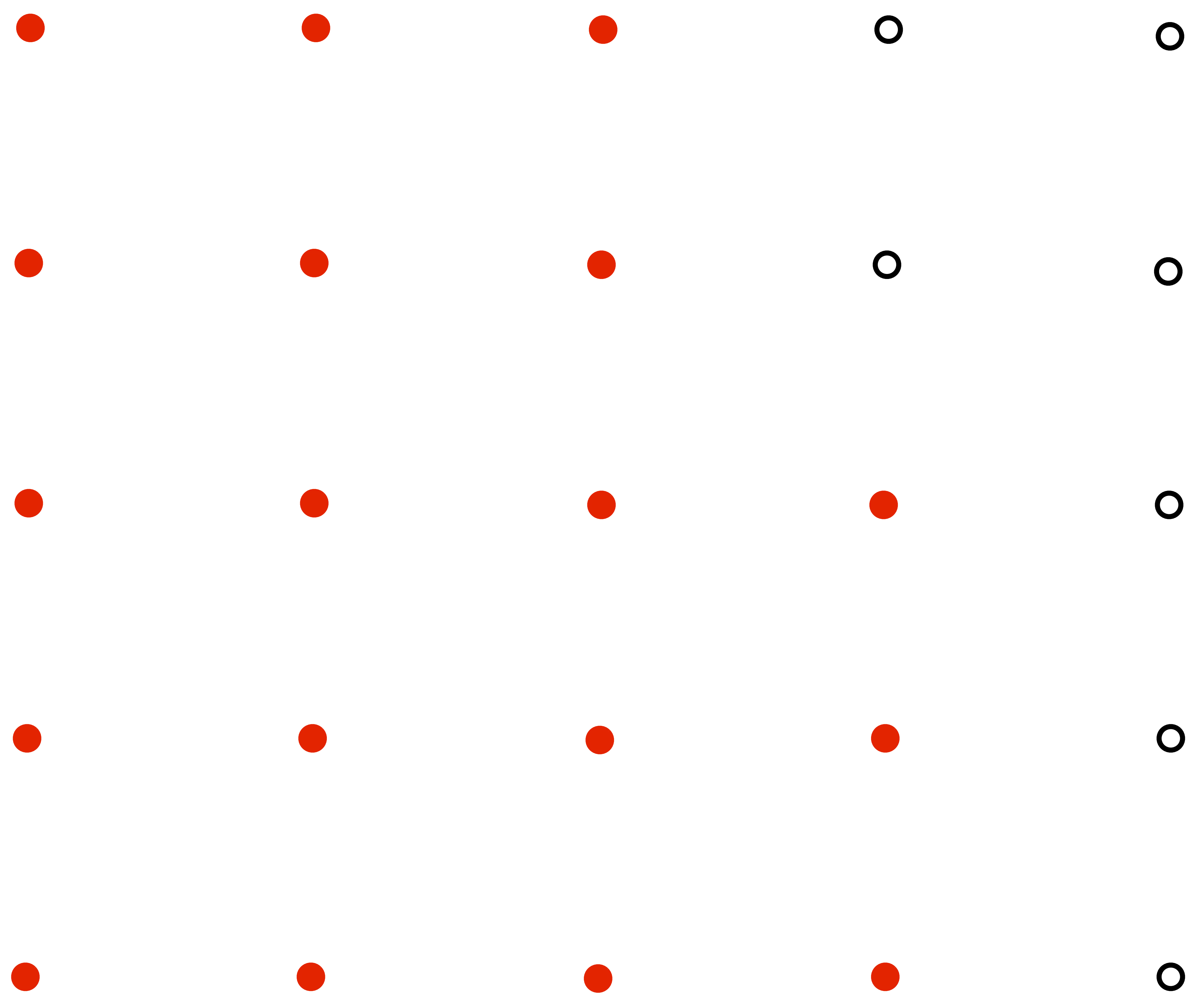


*These are the rules used in OpenGL/Direct3D, i.e., in modern GPUs. Source: Direct3D Programming Guide, Microsoft

Results of sampling triangle coverage



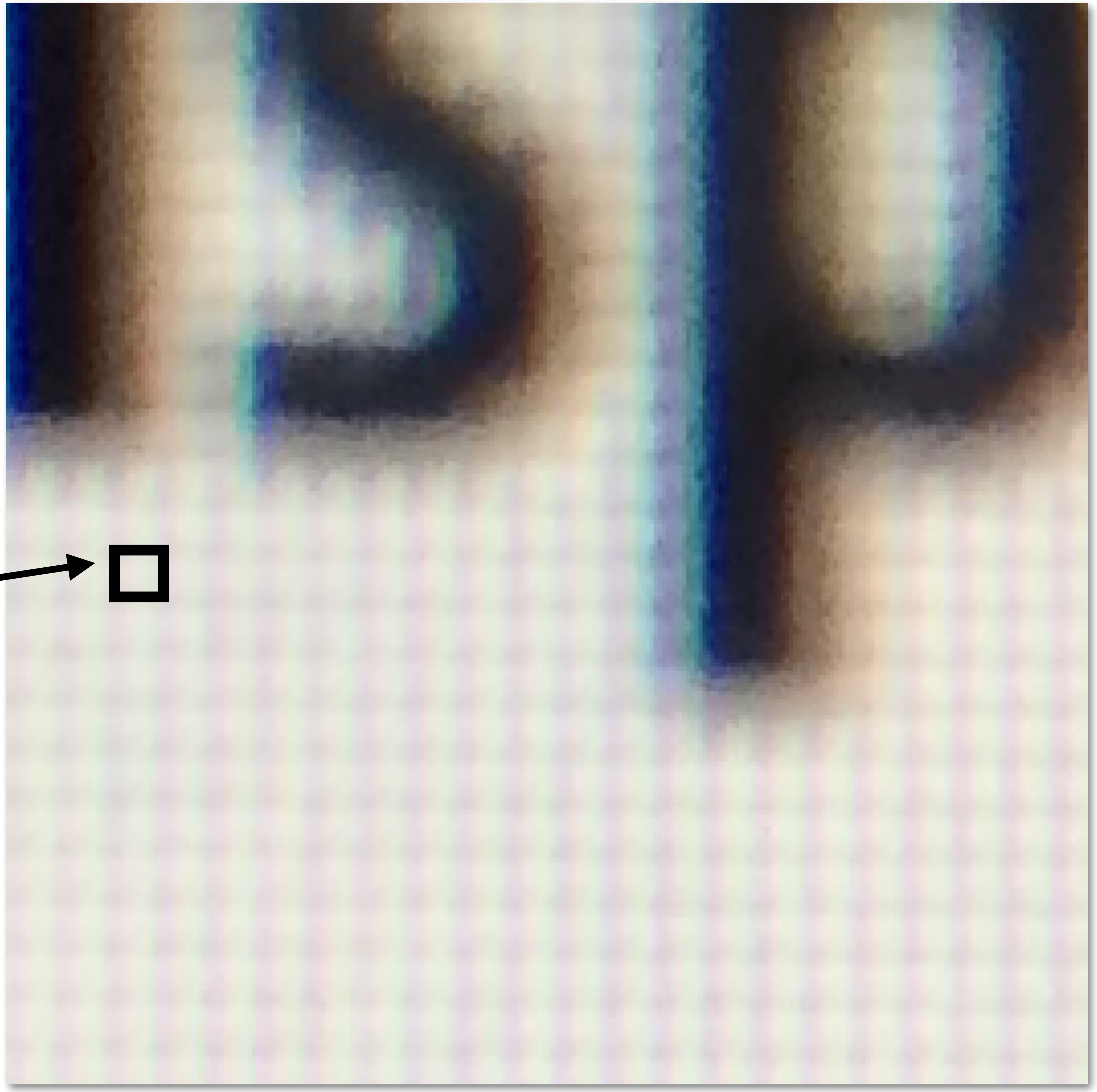
I have a sampled signal, now I want to display it on a screen



Pixels on a screen

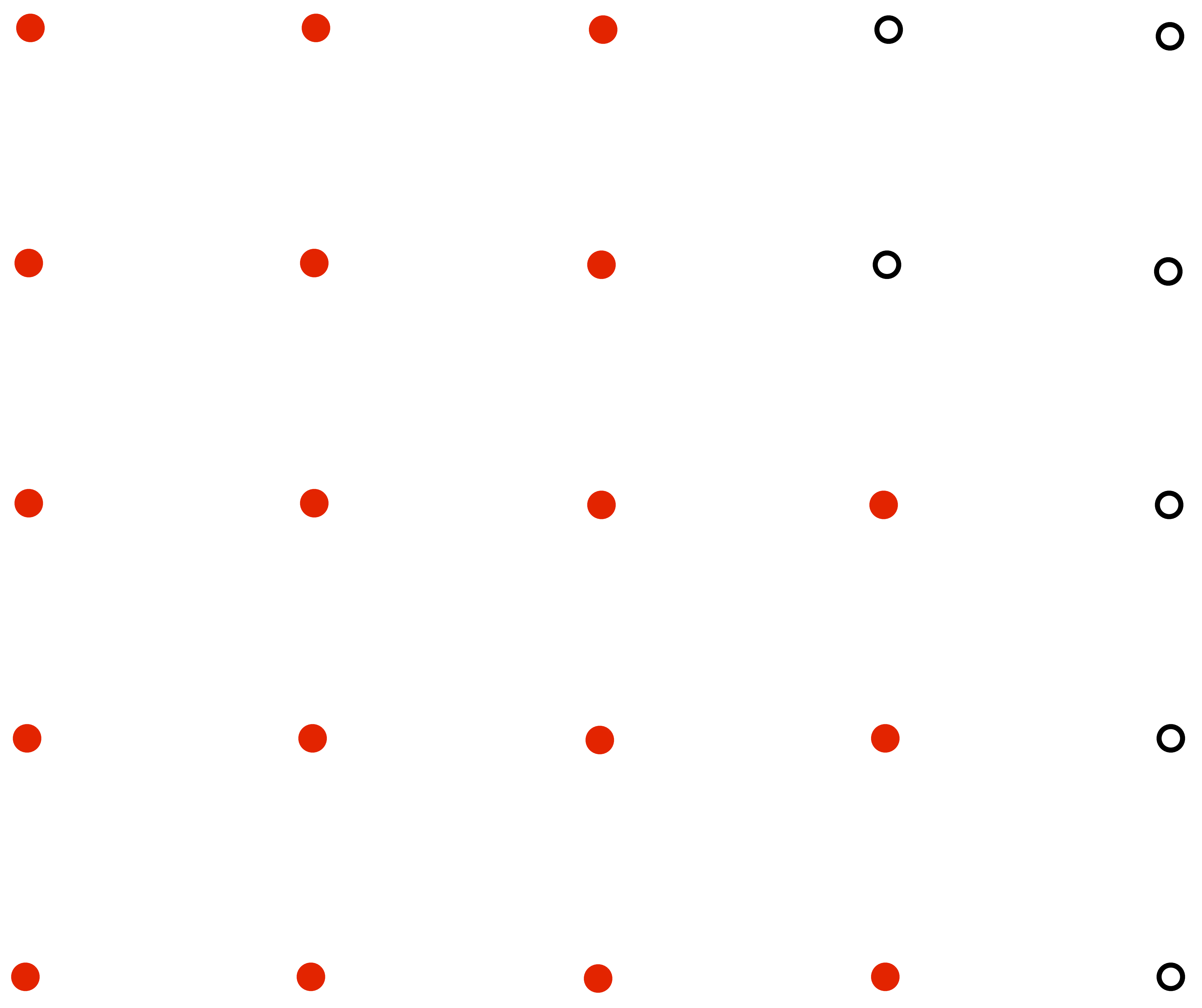
Each image sample sent to the display is converted into a little square of light of the appropriate color:
(a pixel = picture element)

LCD display pixel on my laptop



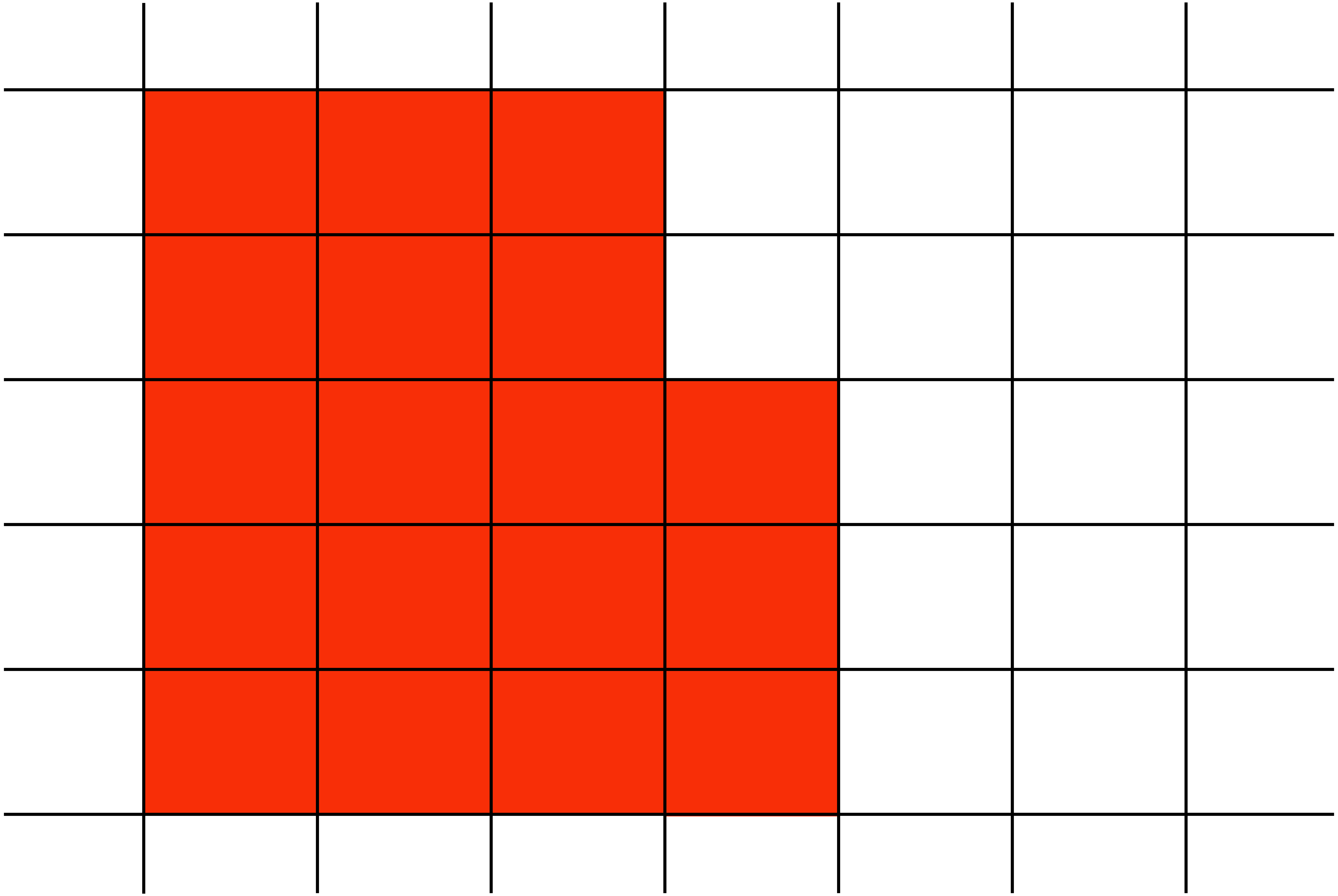
*** Thinking of each LCD pixel as emitting a square of uniform intensity light of a single color is a bit of an approximation to how real displays work, but it will do for now.**

So if we send the display this:



We see this when we look at the screen

(assuming a screen pixel emits a square of perfectly uniform intensity of light)

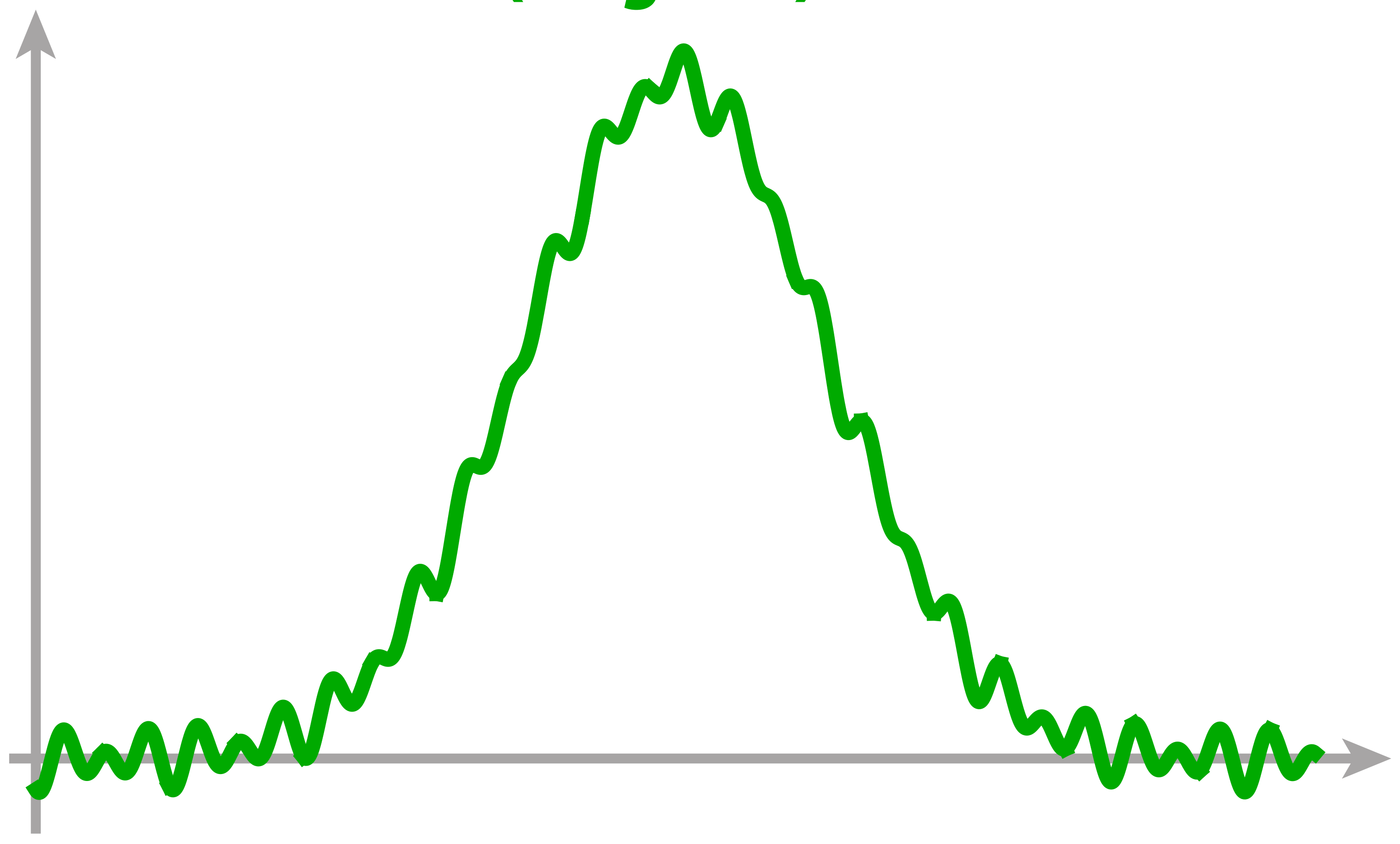


But the real coverage signal looked like this!

Aliasing

Sampling & Reconstruction

continuous signal
(original)



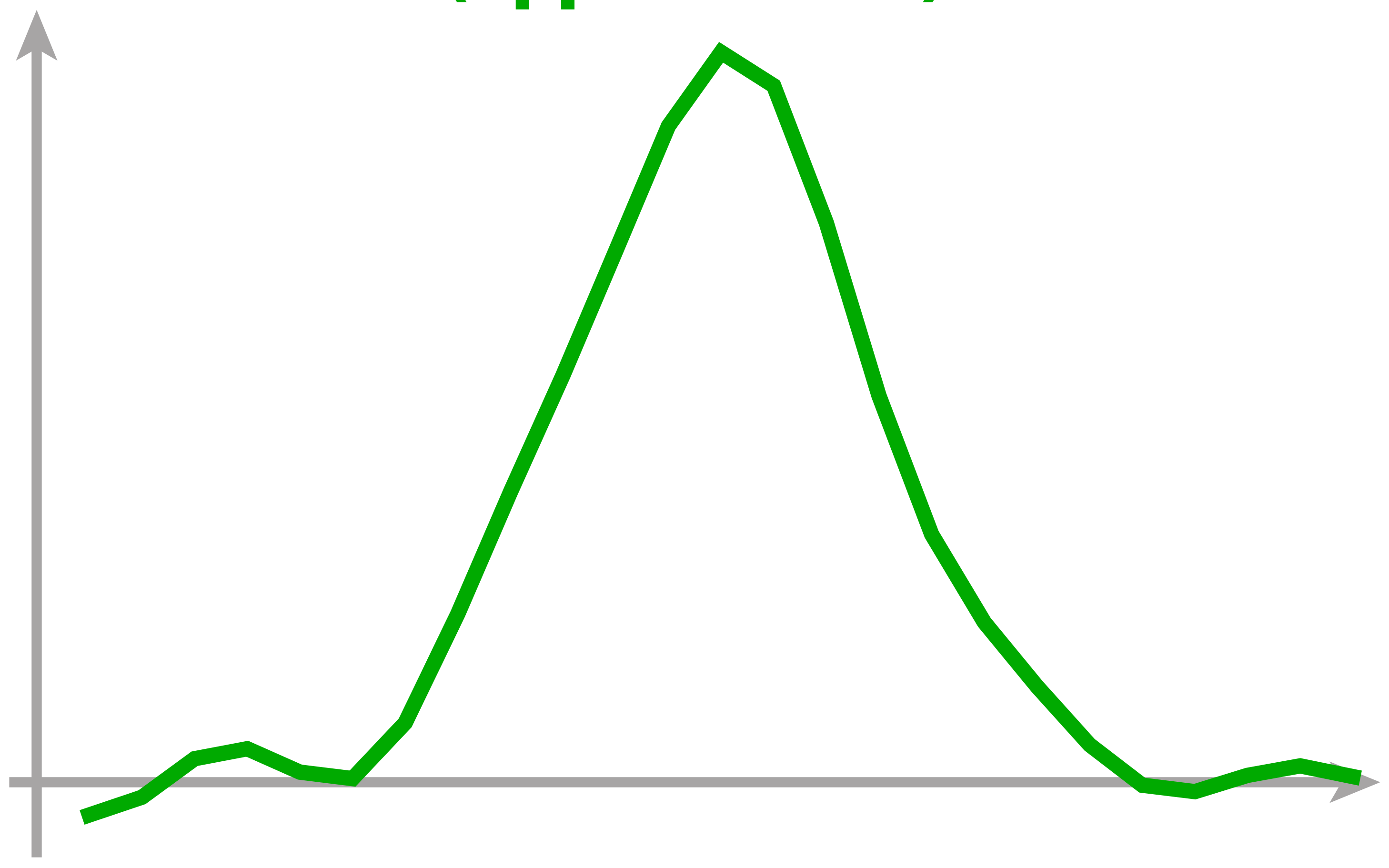
sample

-0.0457447
-0.0209434
0.0328632
0.0468068
0.0141212
0.00506562
0.0829571
0.235369
0.405682
0.569204
0.742511
0.916946
1.02
0.973226
0.781528
0.539974
0.3464
0.223501
0.134011
0.0523279
-0.00416744
-0.0131345
0.00959633
0.0228676
0.00789505

digital information

reconstruct

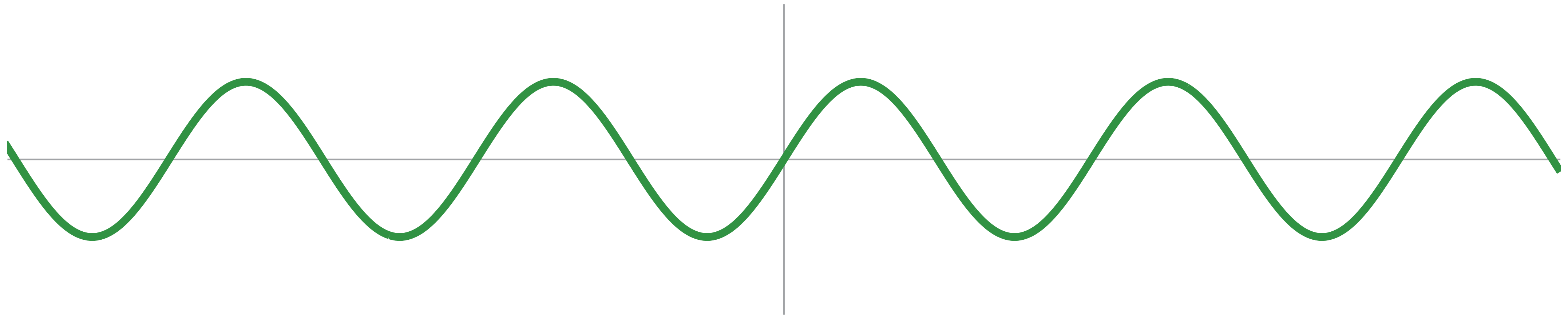
continuous signal
(approximate)



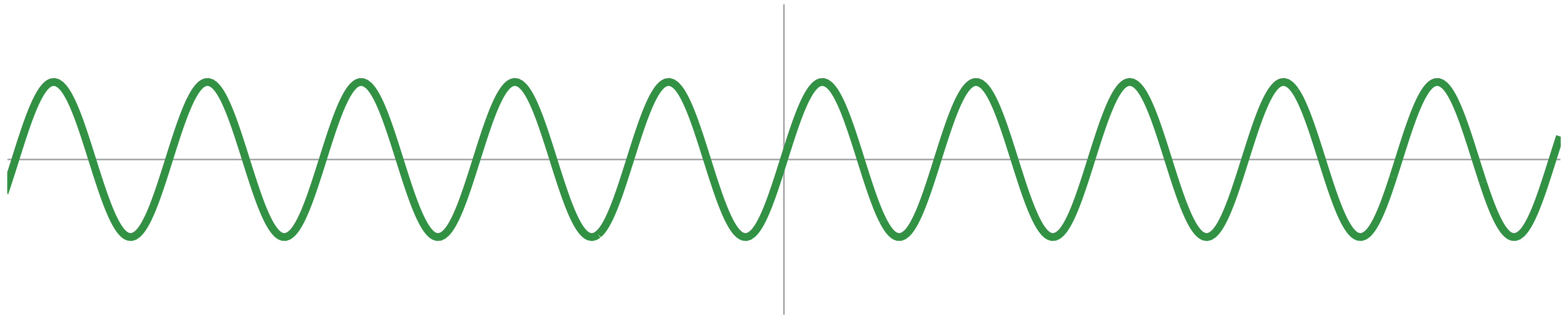
Goal: reproduce original signal as accurately as possible.

1D signal can be expressed as a superposition of frequencies

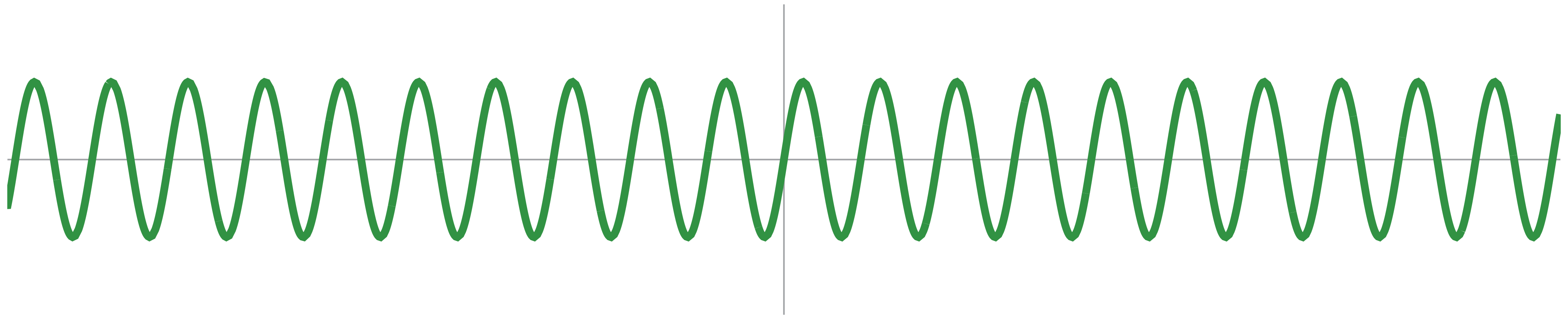
$f_1(x) = sin(\pi x)$



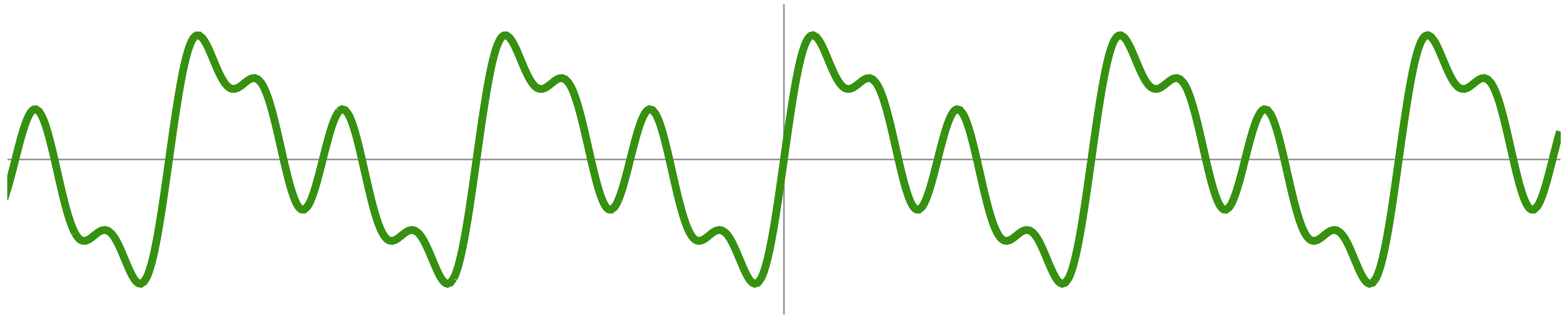
$f_2(x) = sin(2\pi x)$



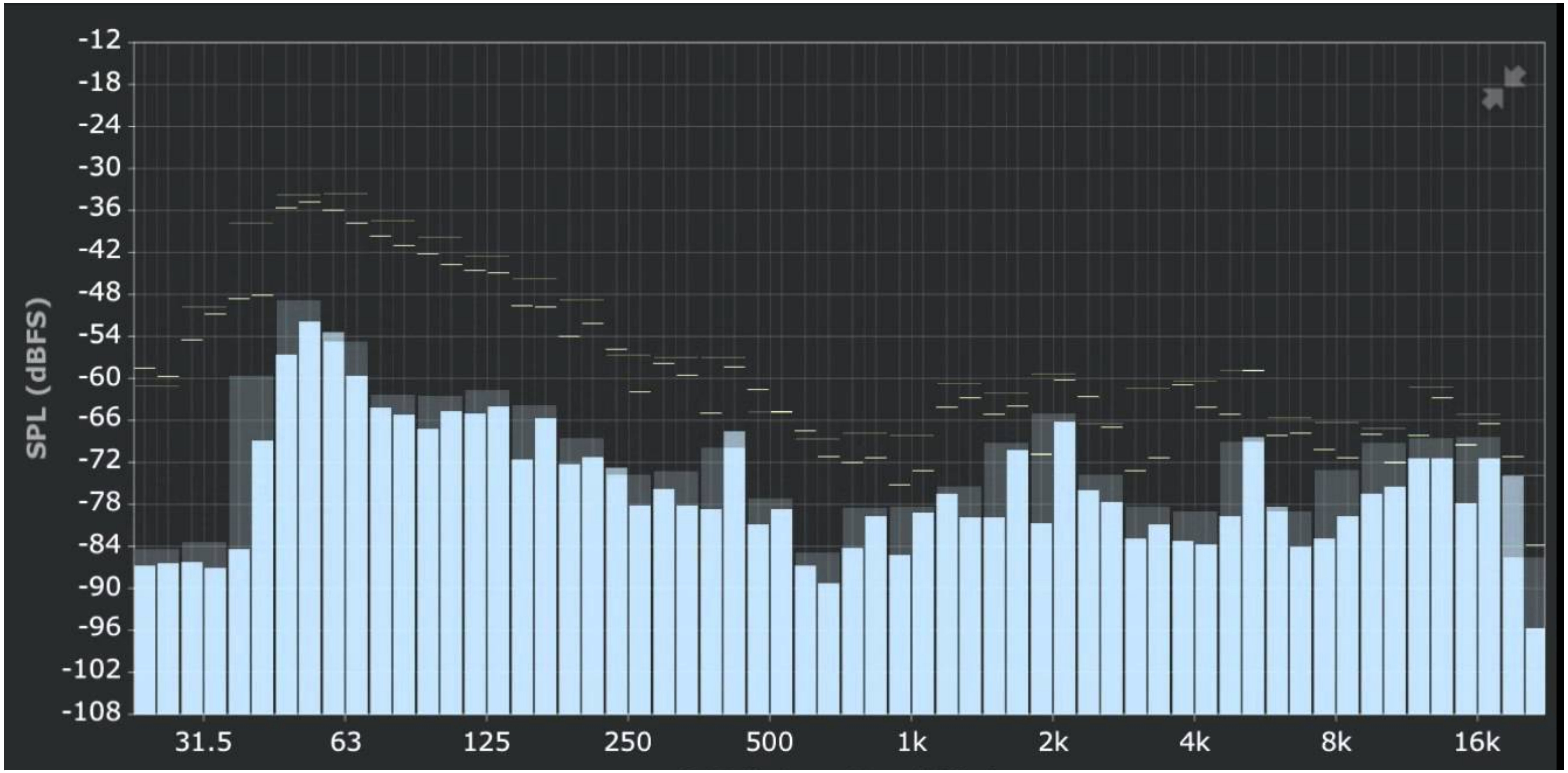
$f_4(x) = sin(4\pi x)$



$f(x) = f_1(x) + 0.75\, f_2(x) + 0.5\, f_4(x)$



E.g., audio spectrum analyzer shows the amplitude of each frequency



Intensity of low-frequencies (bass)

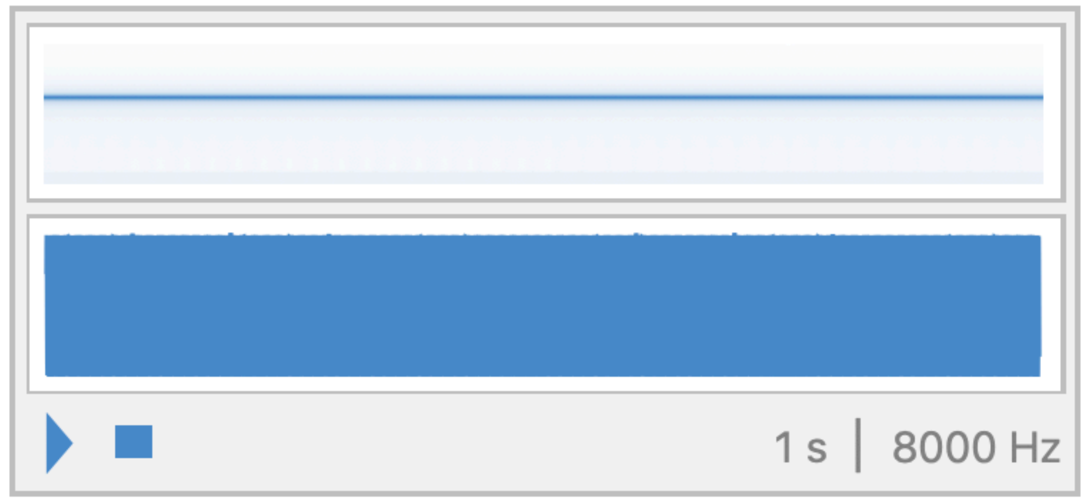
Intensity of high frequencies

Image credit: ONYX Apps

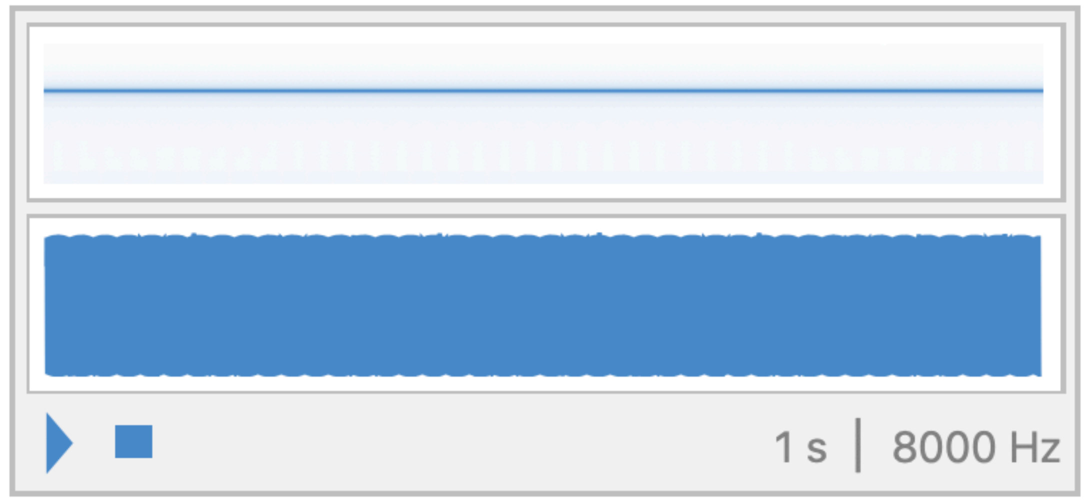
Aliasing in Audio

Get a constant tone by playing a sinusoid of frequency ω:

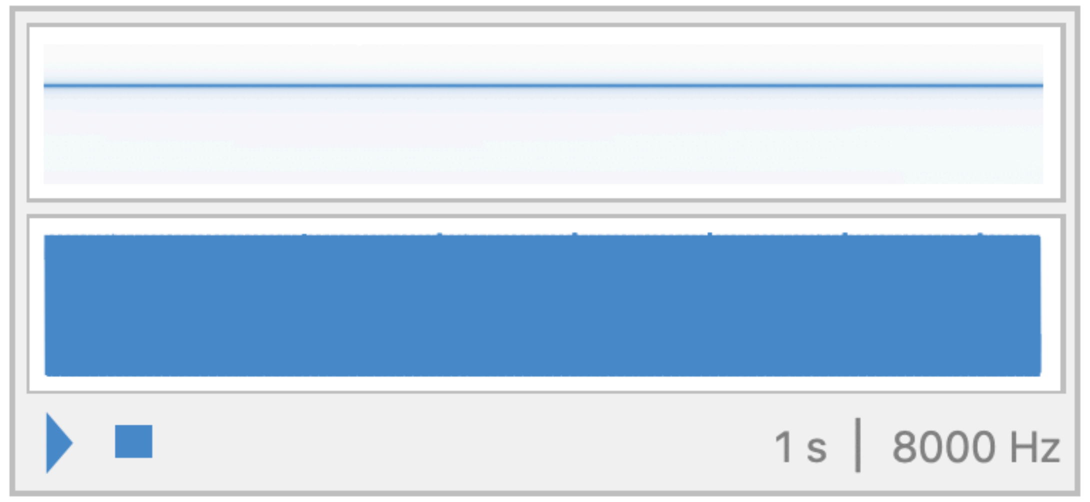
```
Play[Sin[4000 t], {t, 0, 1}]
```




```
Play[Sin[5000 t], {t, 0, 1}]
```

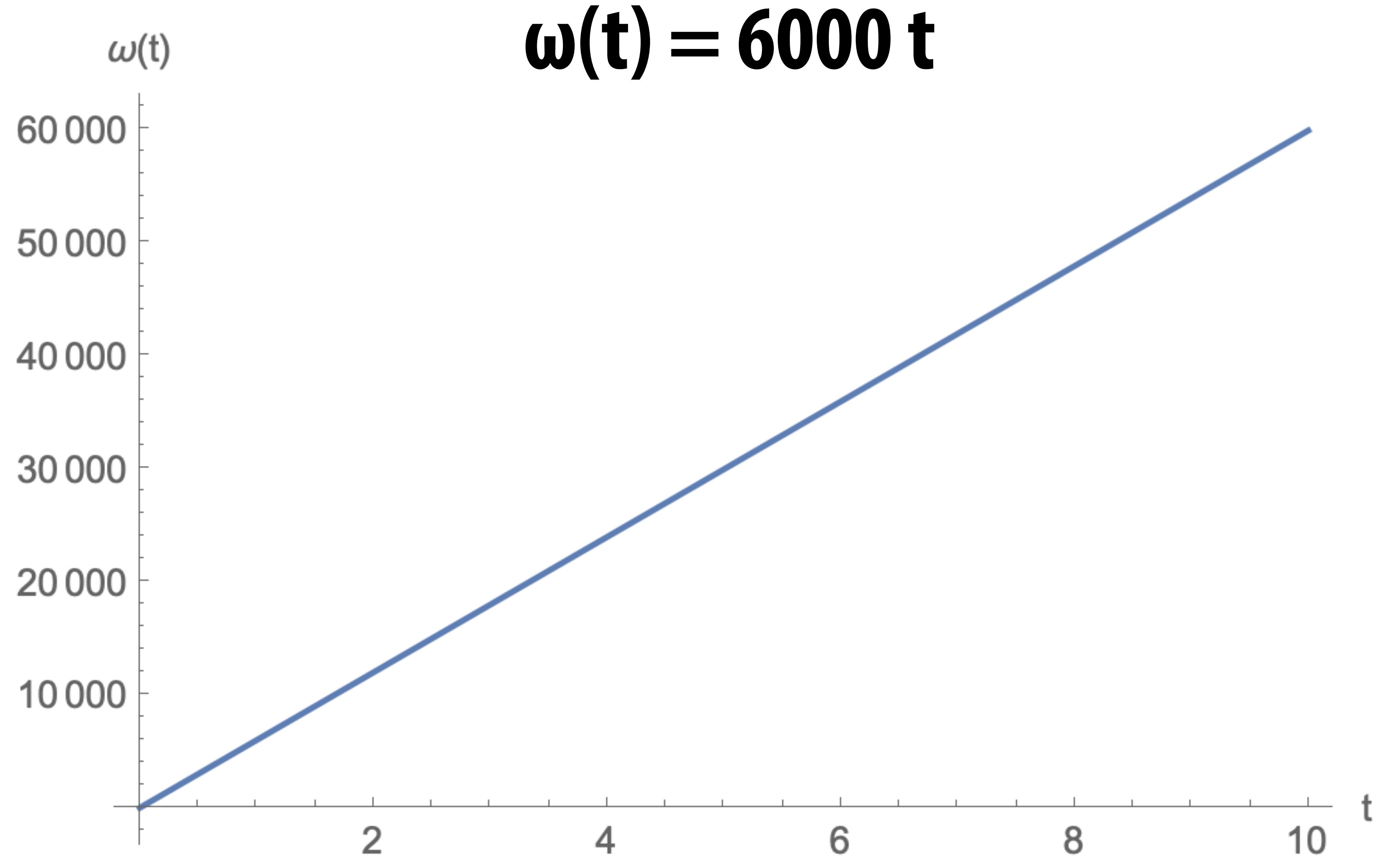



```
Play[Sin[6000 t], {t, 0, 1}]
```

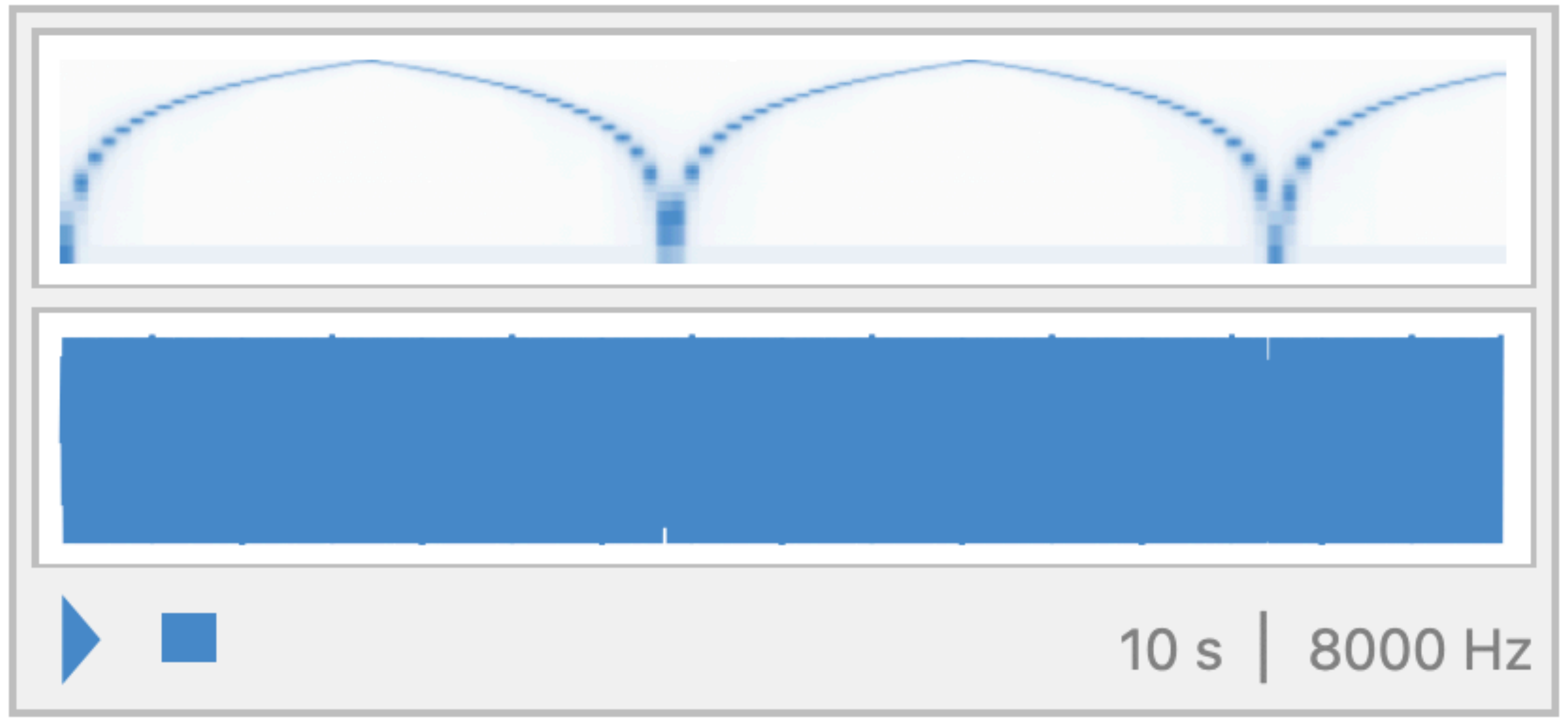


Q: What happens if we <u>increase</u> ω over time?

$\omega(t) = 6000\,t$

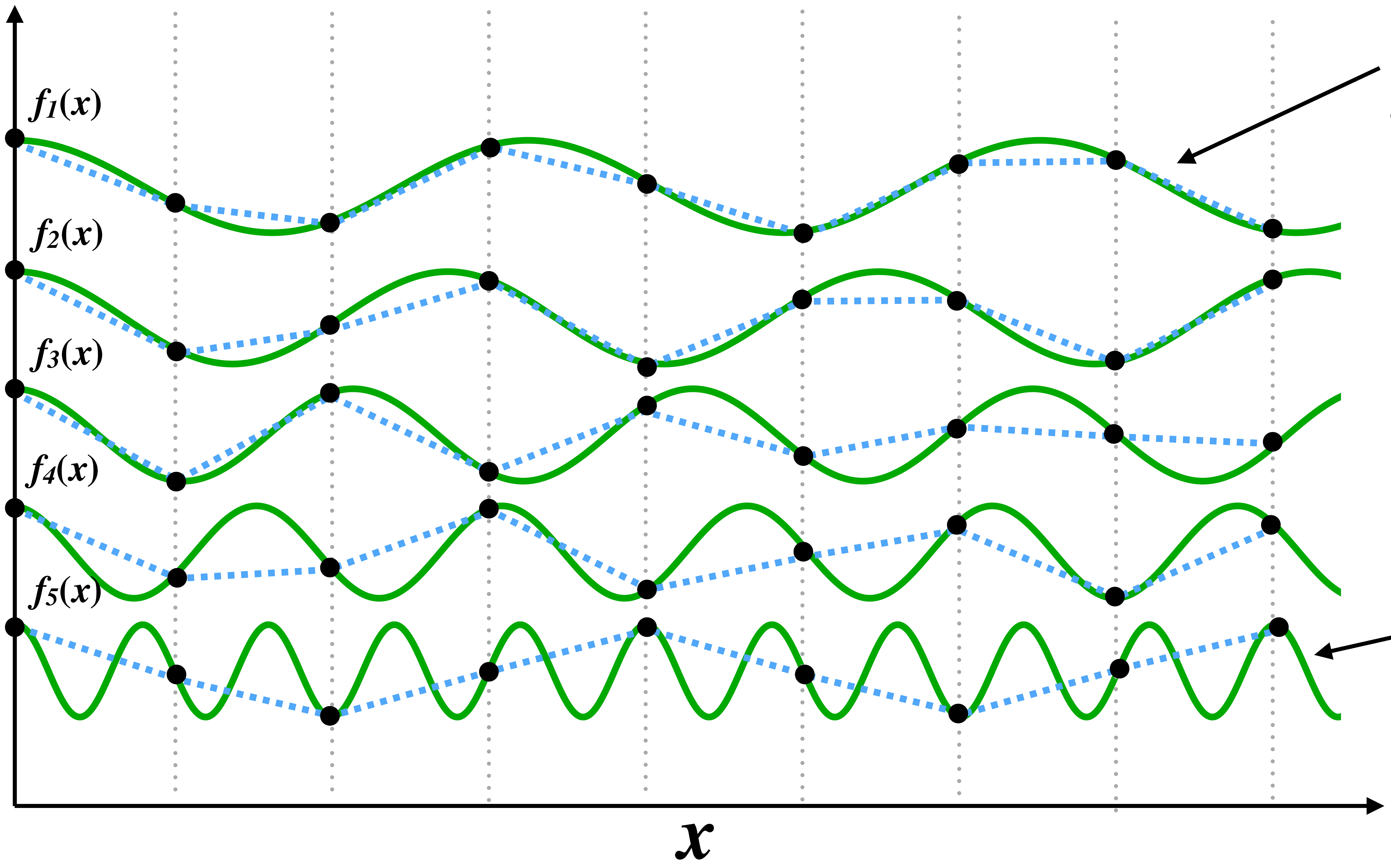



```
Play[Sin[ω t], {t, 0, 10}]
```



Why did that happen?

Undersampling high-frequency signals results in <u>aliasing</u>



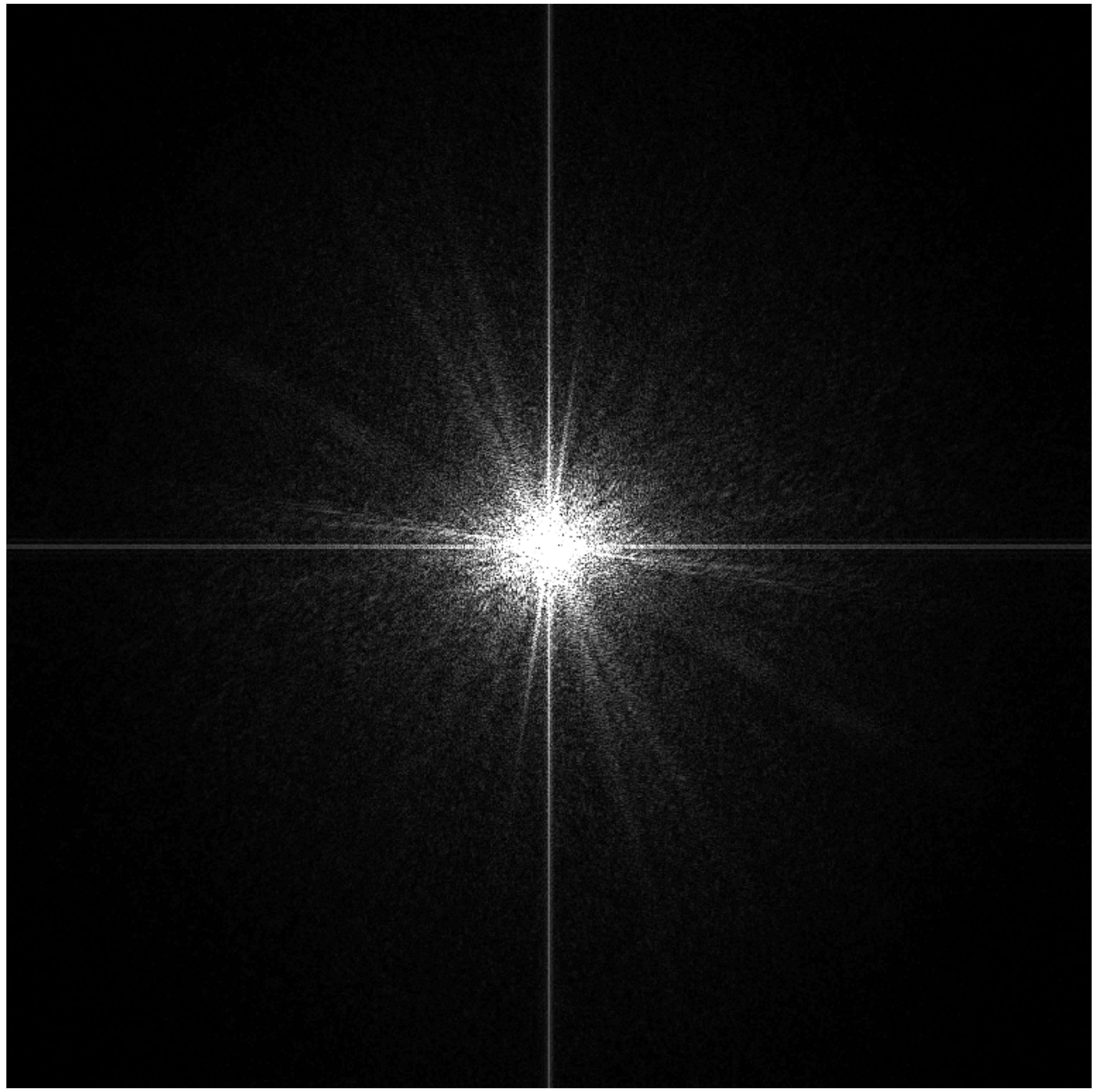
Low-frequency signal: sampled adequately for accurate reconstruction

High-frequency signal is insufficiently sampled: reconstruction appears to be from a low frequency signal

"Aliasing": high frequencies in the original signal masquerade as low frequencies after reconstruction (due to undersampling)

Images can also be decomposed into "frequencies"

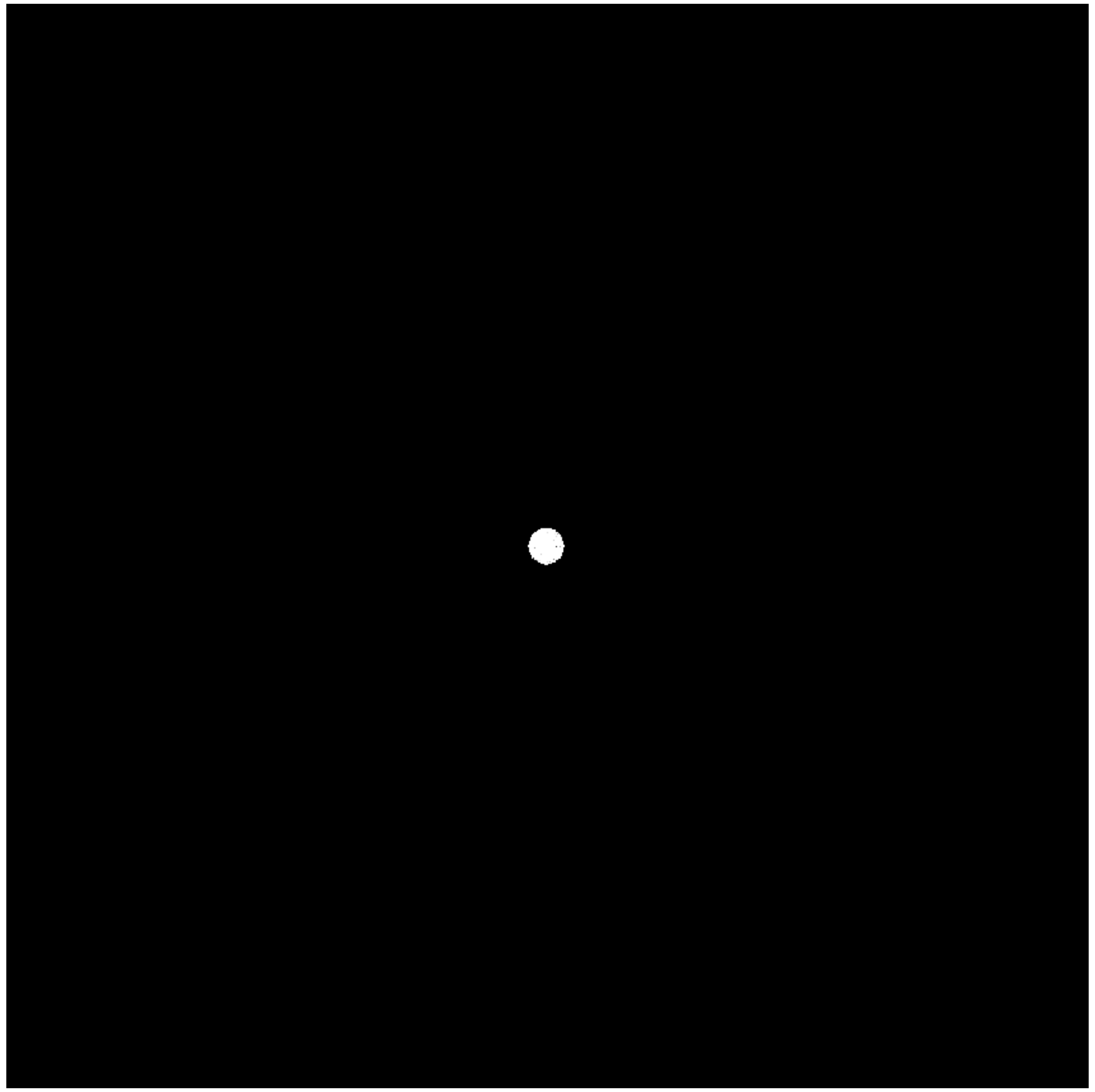
Spatial domain result



Spectrum

Low frequencies only (smooth gradients)

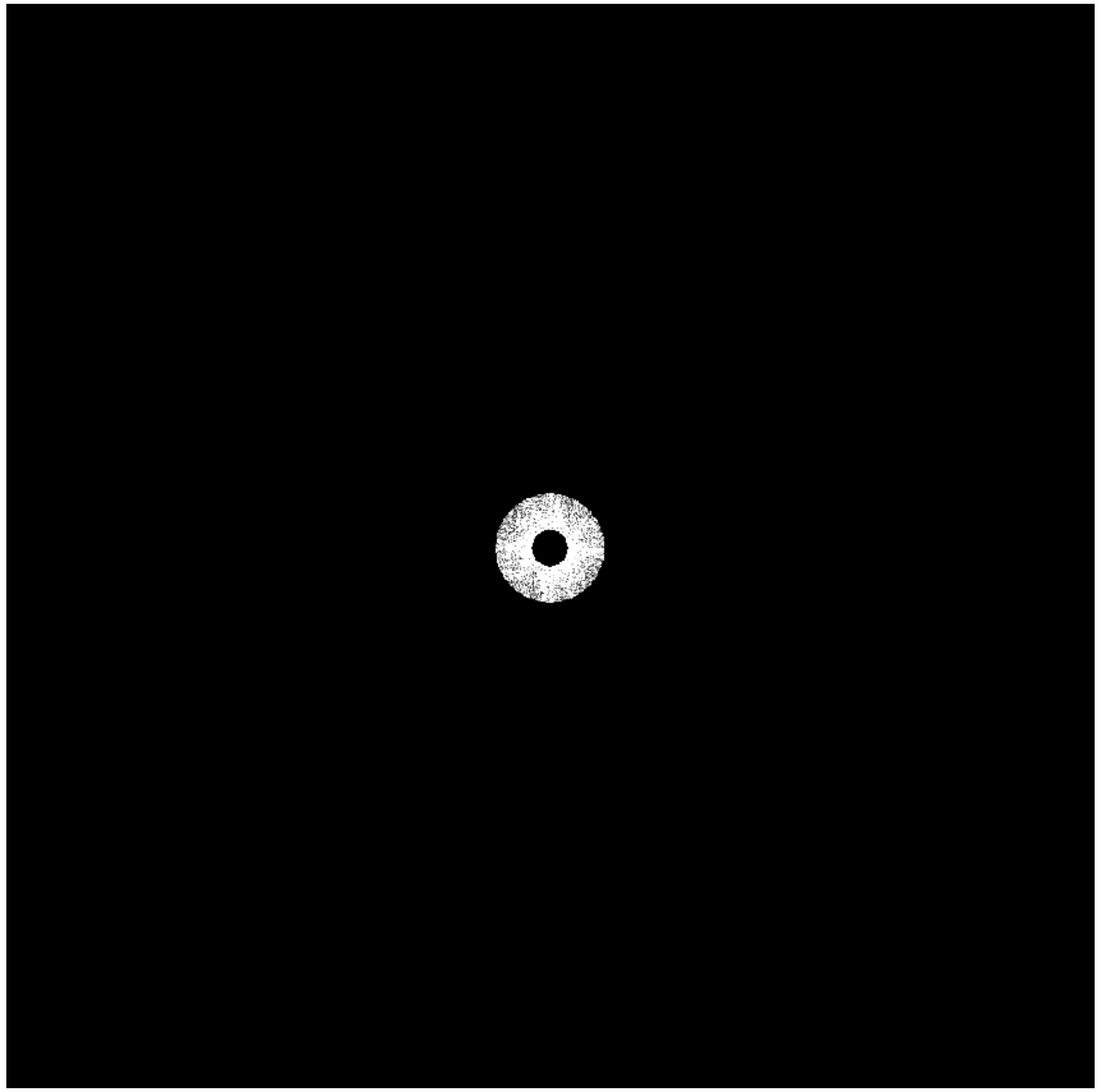
Spatial domain result



Spectrum (after low-pass filter)
All frequencies above cutoff have 0 magnitude

Mid-range frequencies

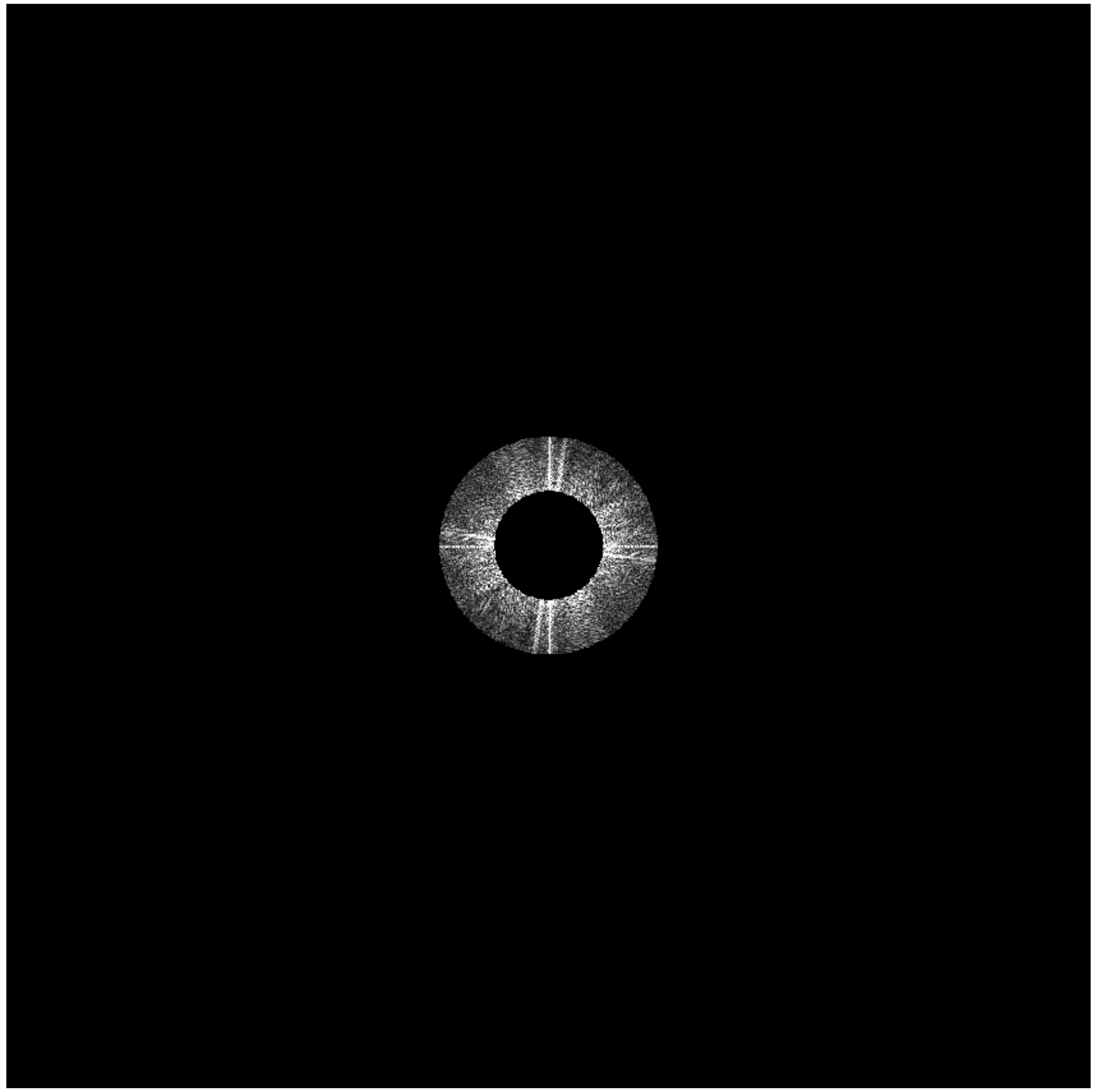
Spatial domain result



Spectrum (after band-pass filter)

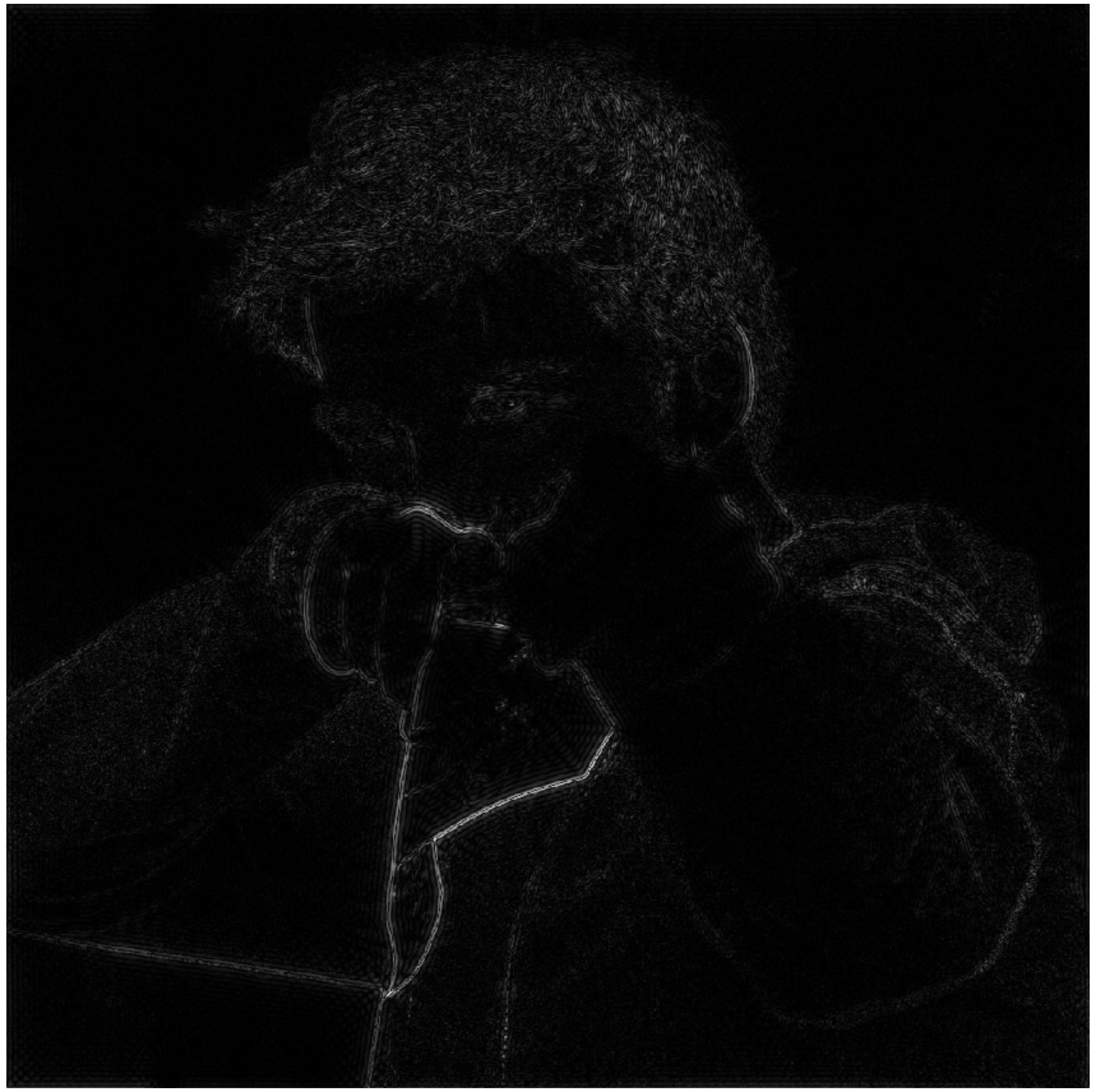
Mid-range frequencies

Spatial domain result

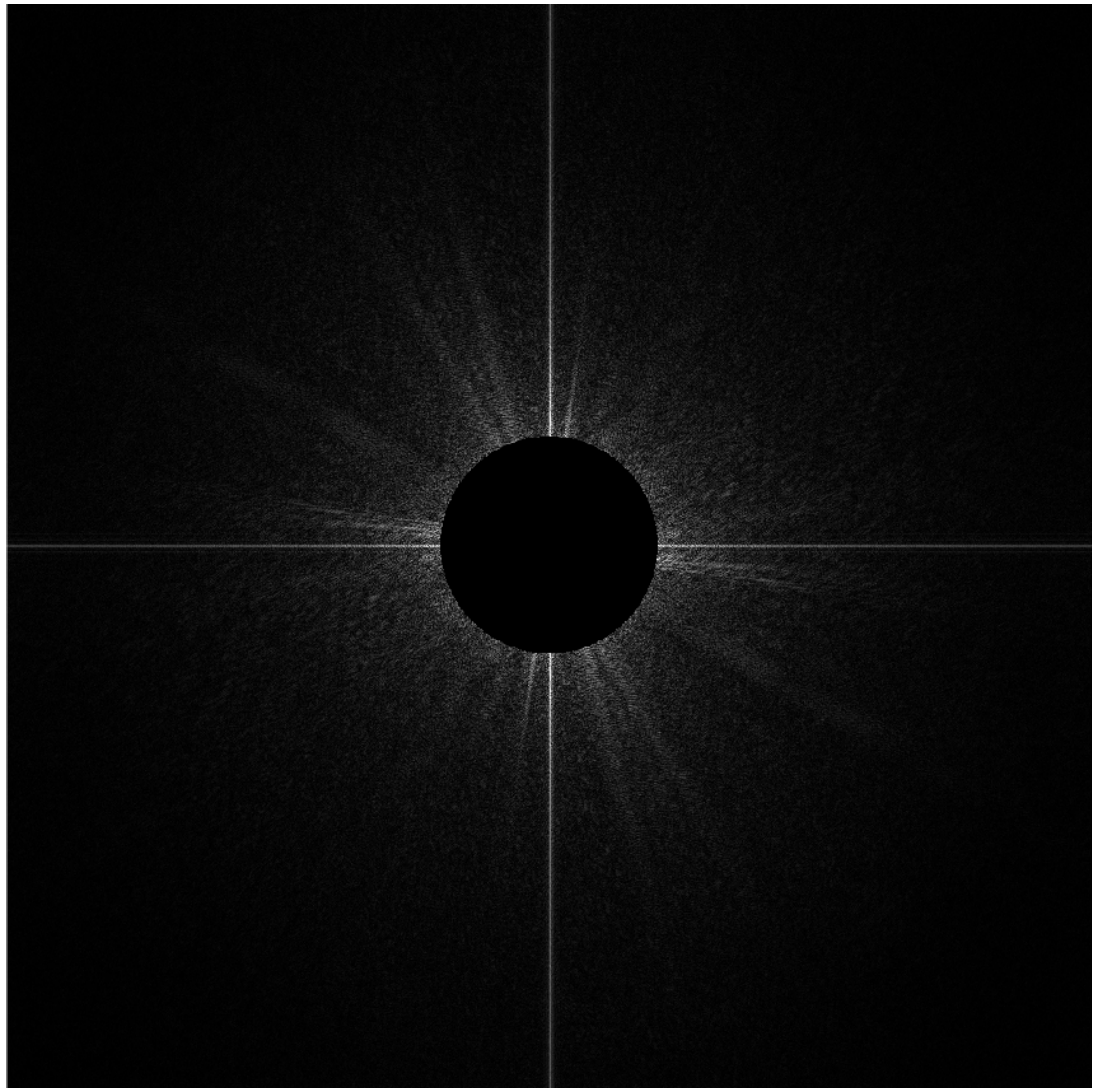


Spectrum (after band-pass filter)

High frequencies (edges)

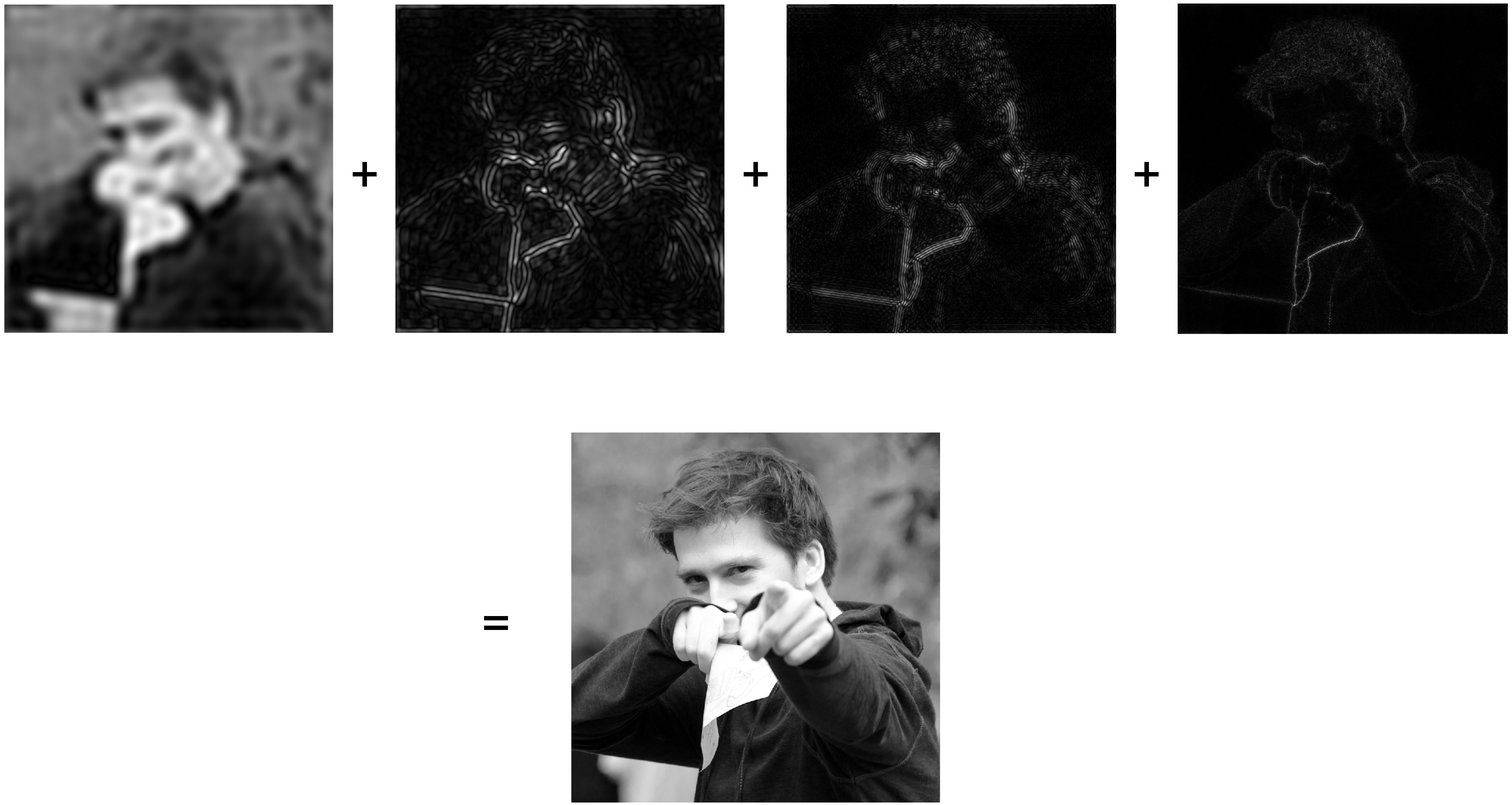


Spatial domain result
(strongest edges)



Spectrum (after high-pass filter)
All frequencies below threshold
have 0 magnitude

An image as a sum of its frequency components



Spatial aliasing: the function $\sin(x^2 + y^2)$

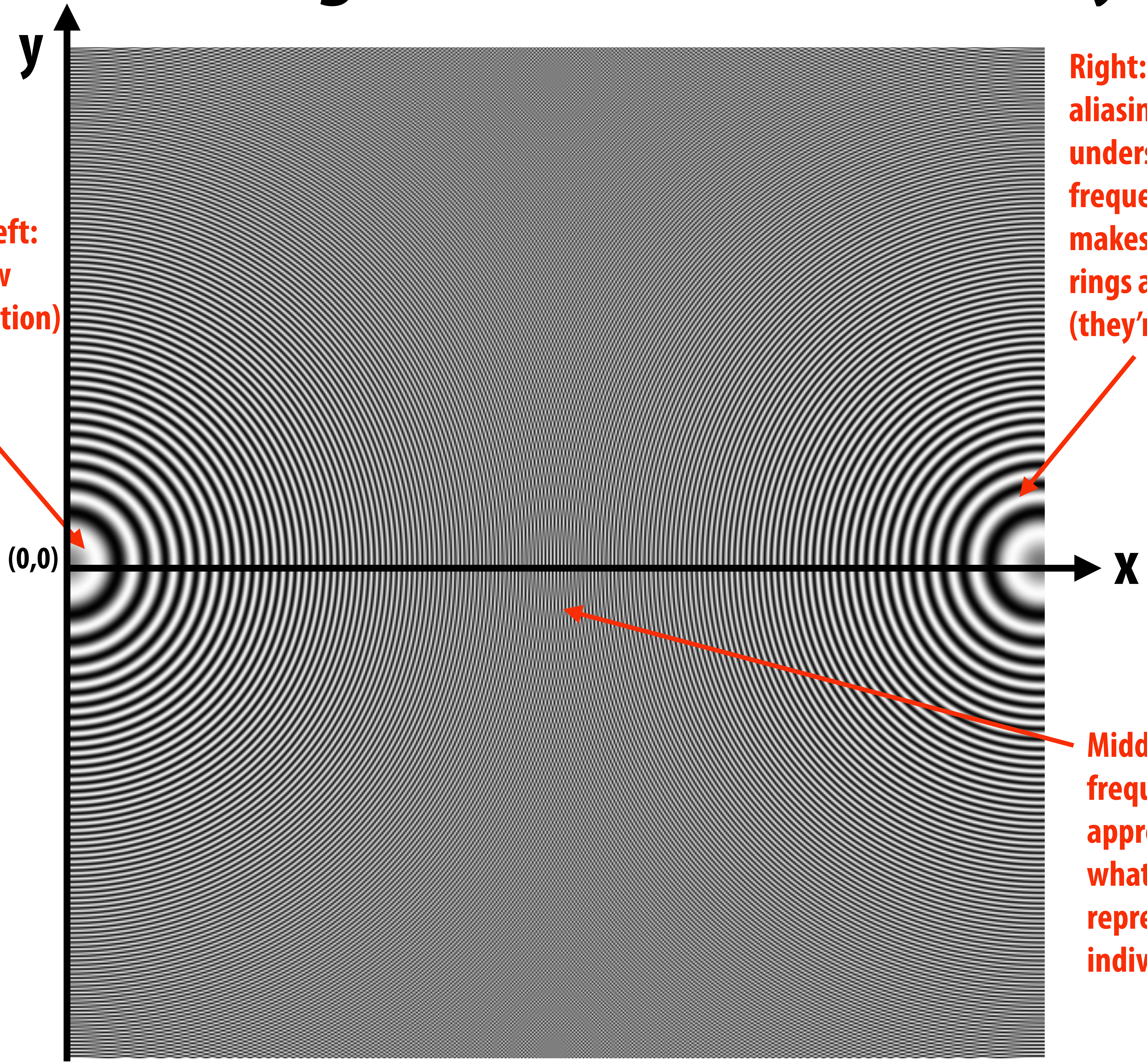
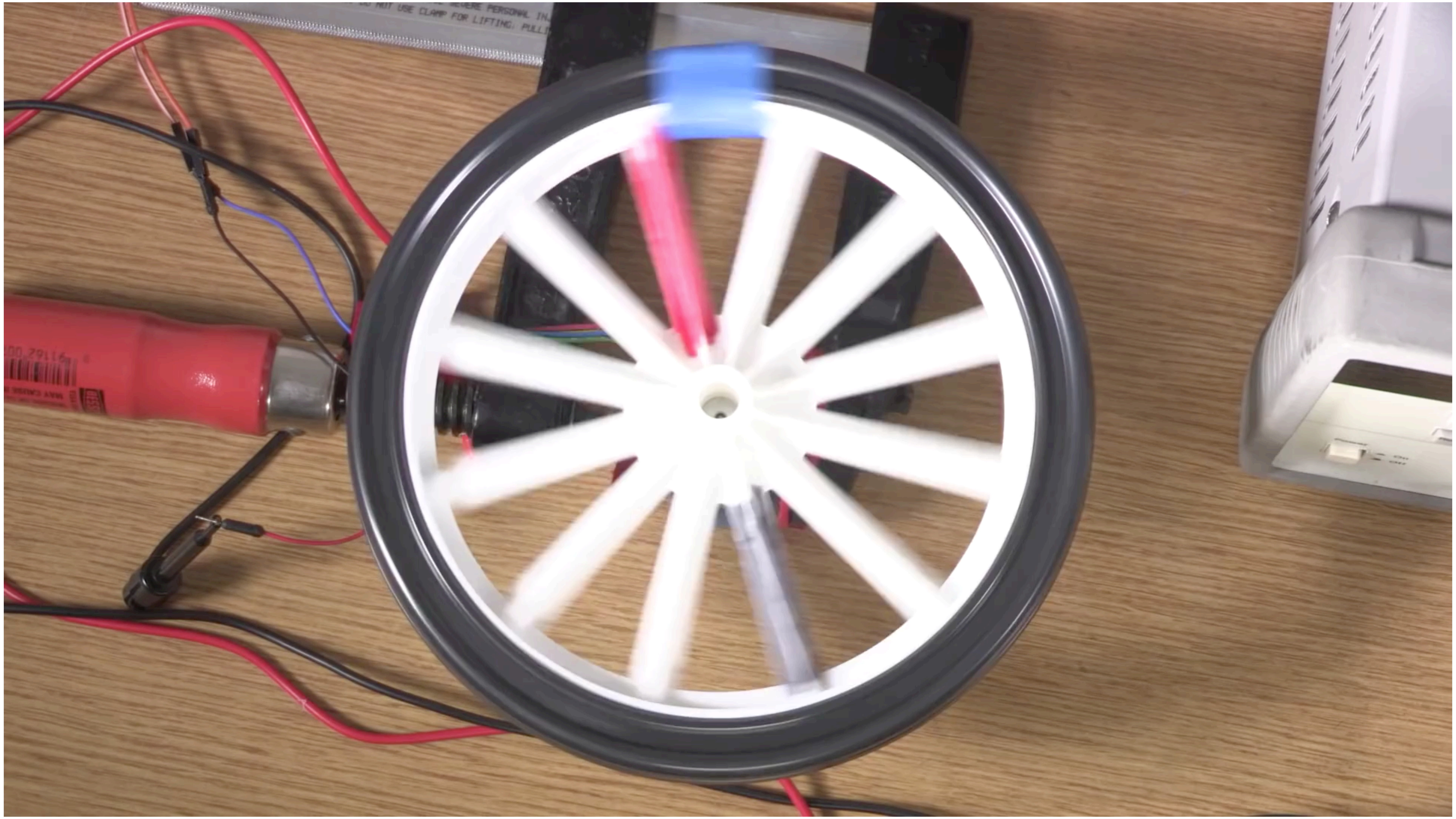


Figure credit: Pat Hanrahan and Bryce Summers

Temporal aliasing: wagon wheel effect

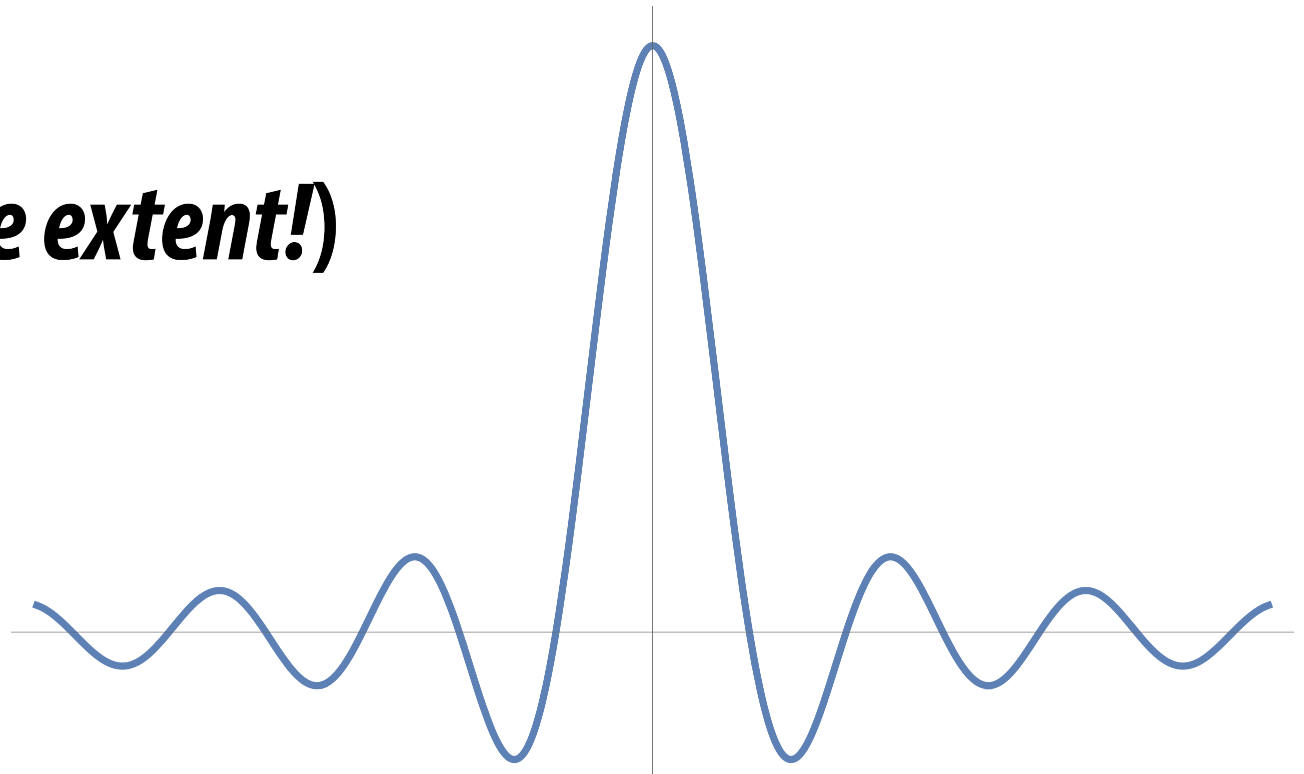


Camera's frame rate (temporal sampling rate) is too low for rapidly spinning wheel.

Nyquist-Shannon theorem

- Consider a *band-limited* signal: has no frequencies above some threshold ω_0
  - 1D example: low-pass filtered audio signal
  - 2D example: blurred image example from a few slides ago

- The signal can be perfectly reconstructed if sampled with period $T = 1 / 2\omega_0$
- …and if interpolation is performed using a "*sinc filter*"
  - ideal filter with no frequencies above cutoff (*infinite extent!*)

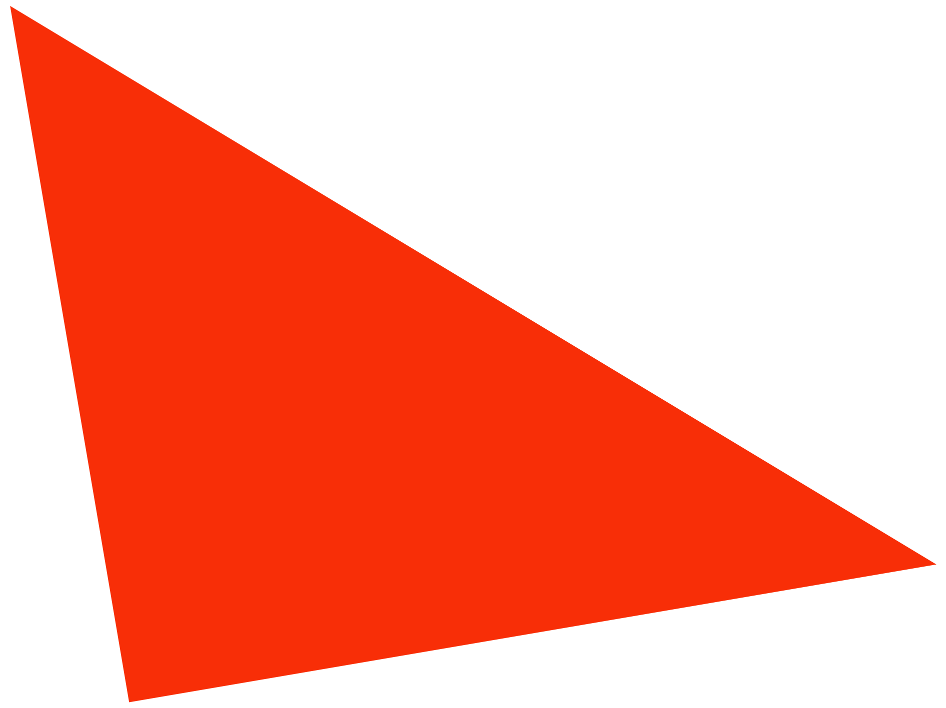


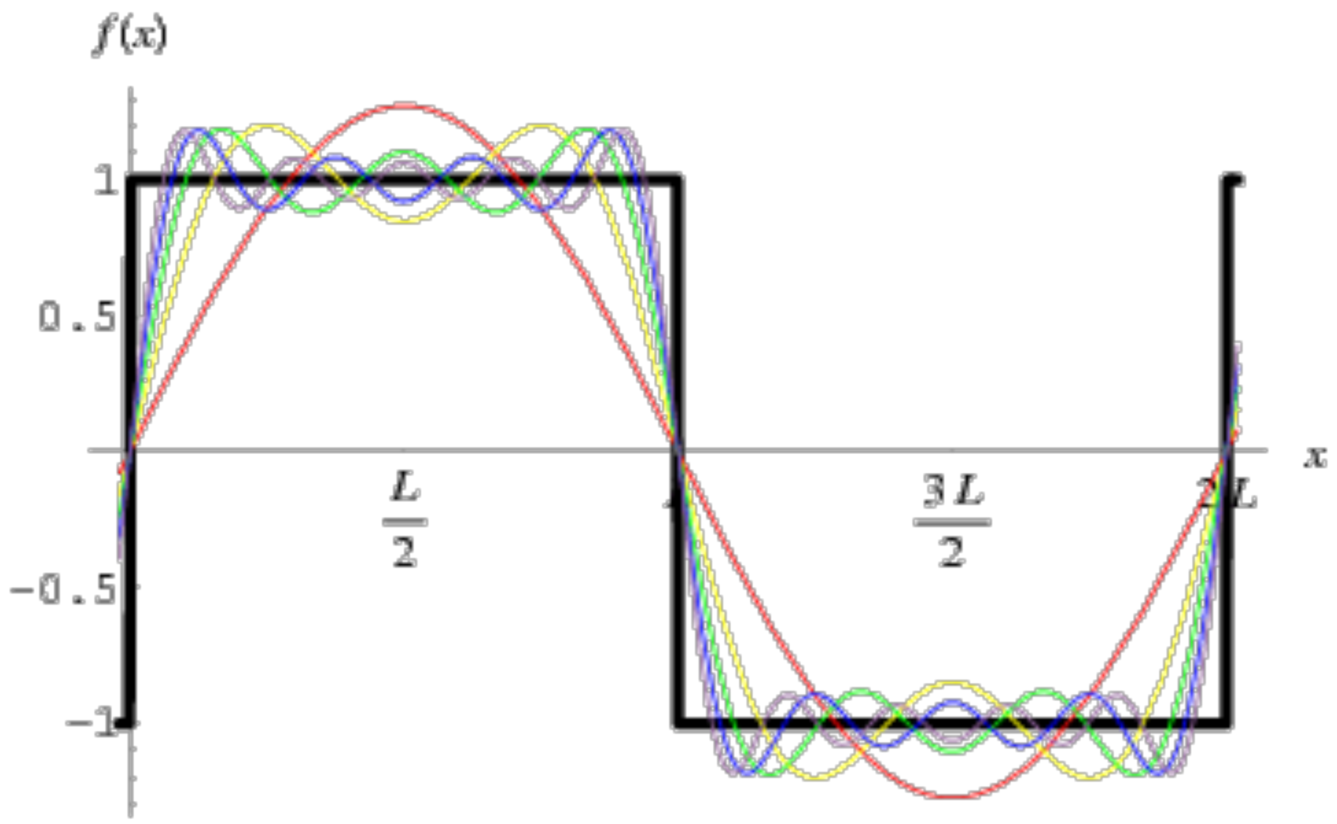
$$\mathrm{sinc}(x) = \frac{1}{\pi x}\sin(\pi x)$$

Challenges of sampling in computer graphics

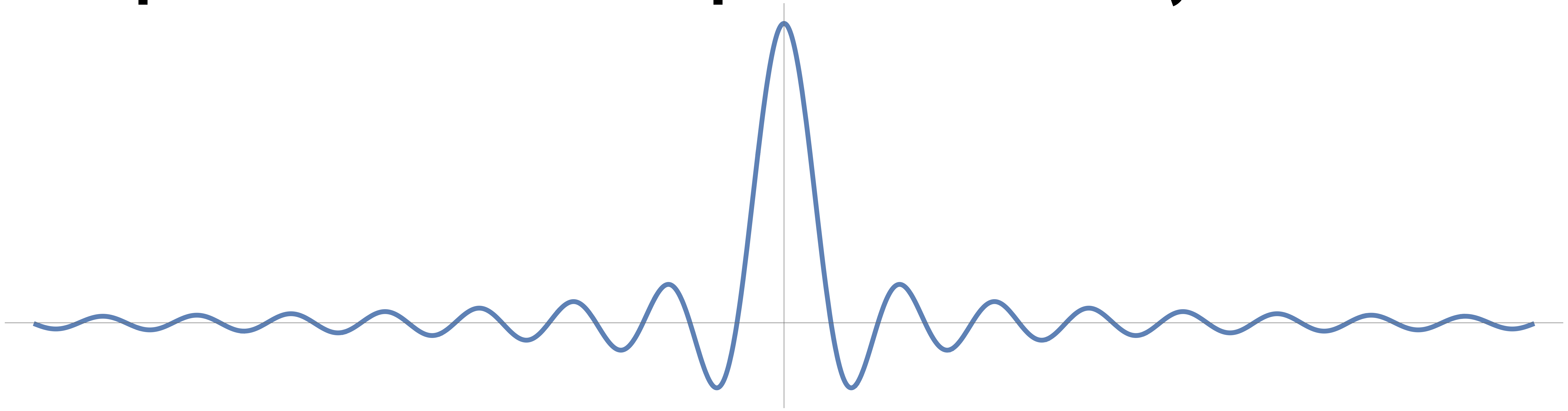
- Signals are often not band-limited in computer graphics. Why?

Hint:



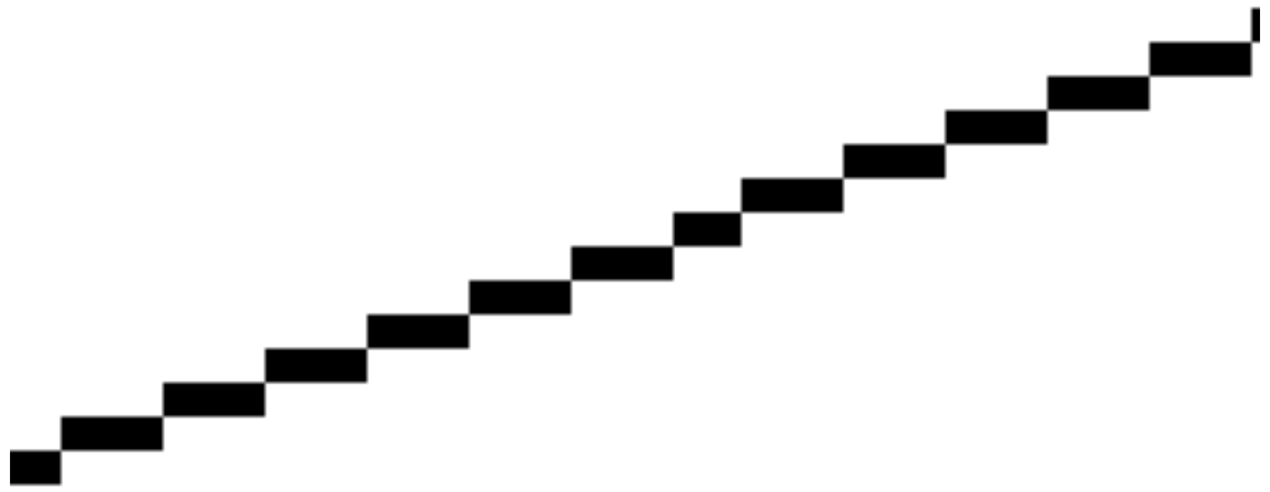


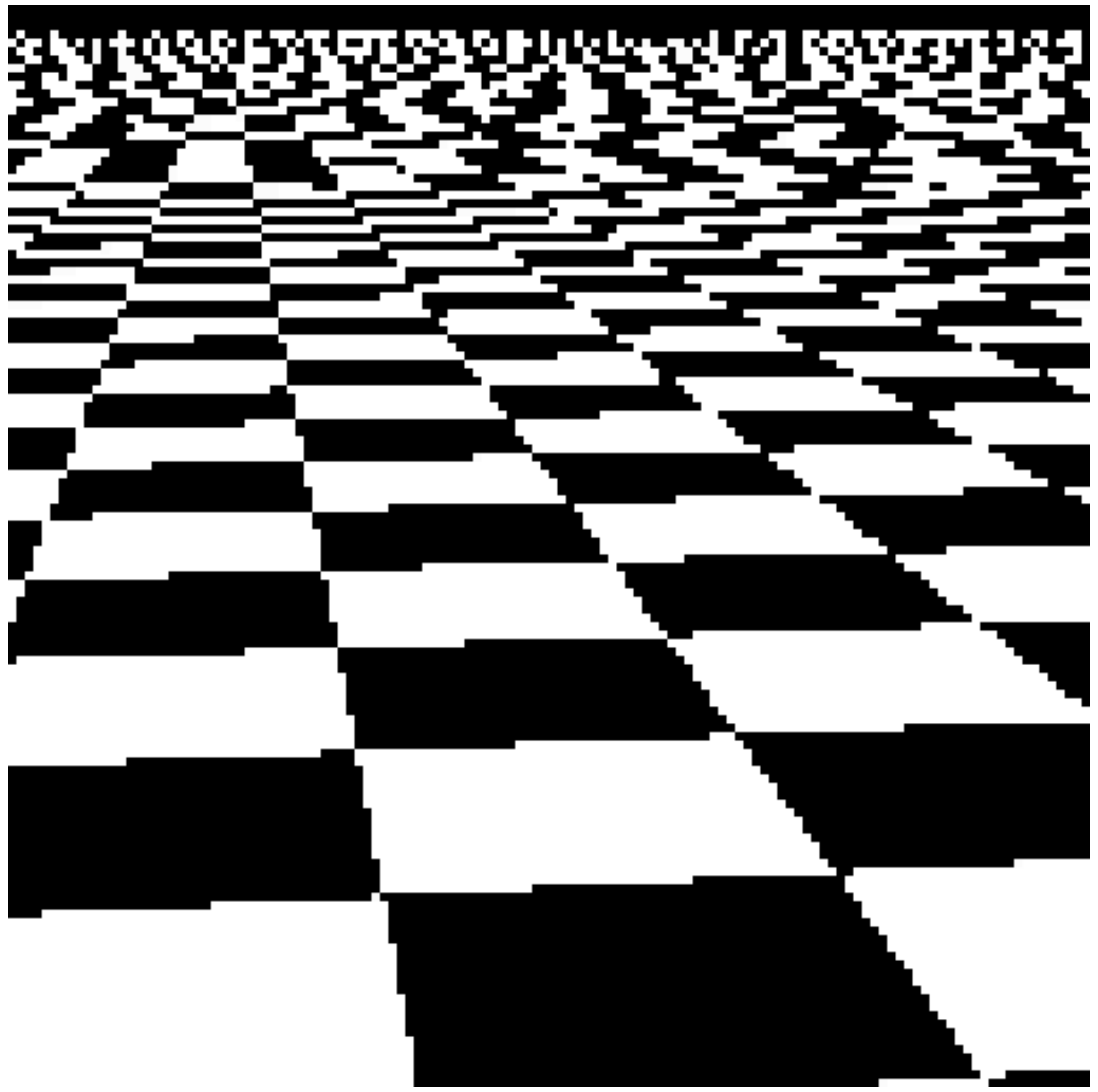
- Also, infinite extent of "ideal" reconstruction filter (sinc) is impractical for efficient implementations. Why?



Aliasing artifacts in images

- **Imperfect sampling + imperfect reconstruction leads to image artifacts**
  - "Jaggies" in a static image
  - "Roping" or "shimmering" of images when animated
  - Moiré patterns in high-frequency areas of images



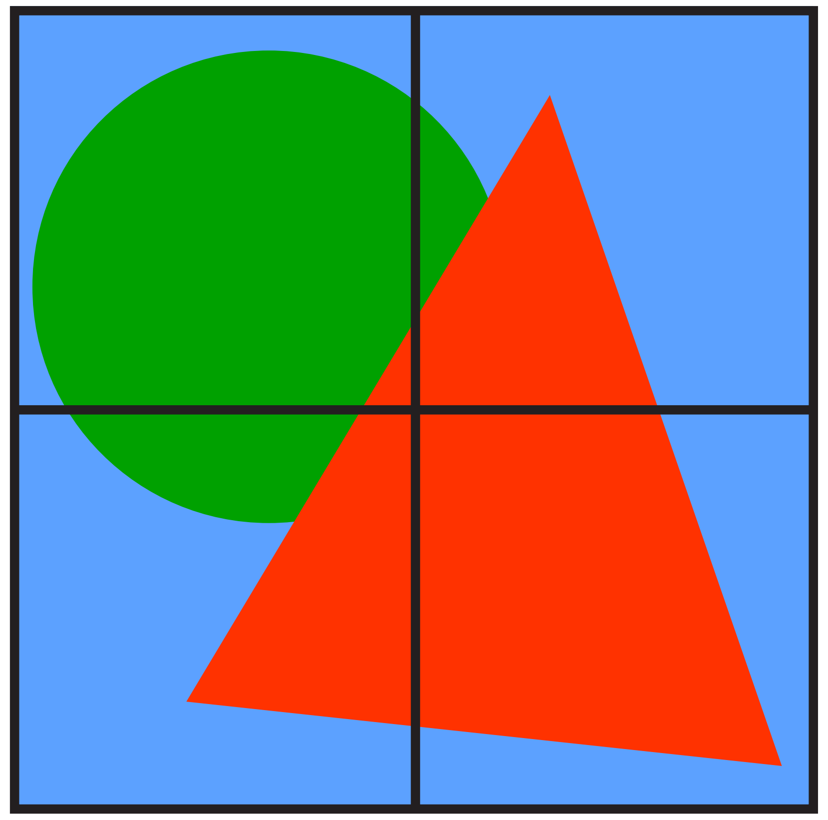


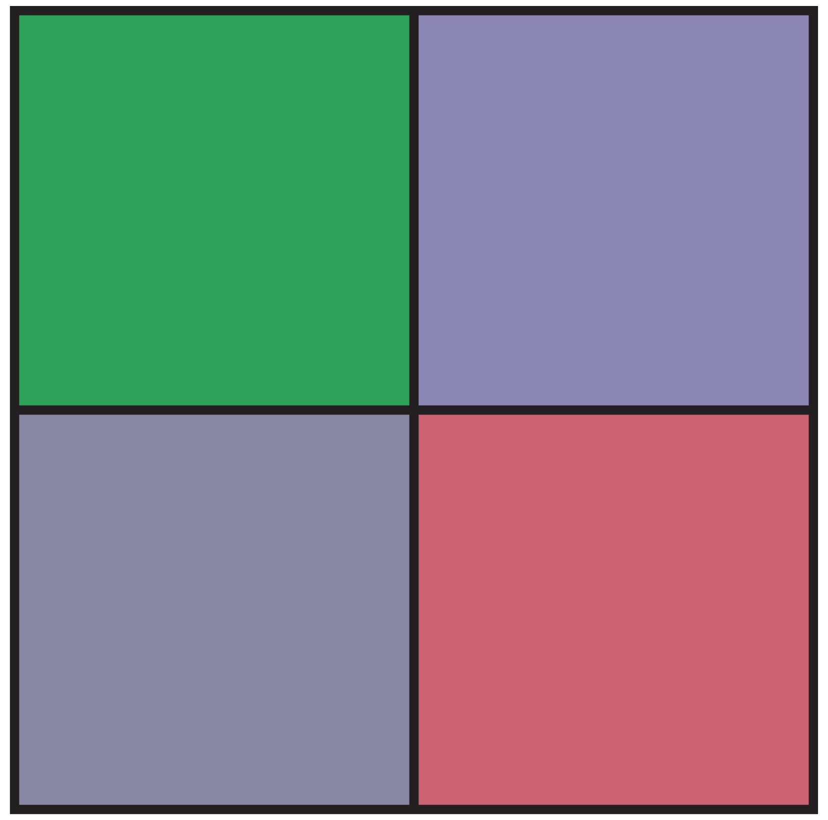
How can we reduce aliasing?

- No matter what we do, aliasing is a fact of life: any sampled representation eventually fails to capture frequencies that are too high.

- But we can still do our best to try to match sampling and reconstruction so that the signal we reproduce looks as much as possible like the signal we acquire

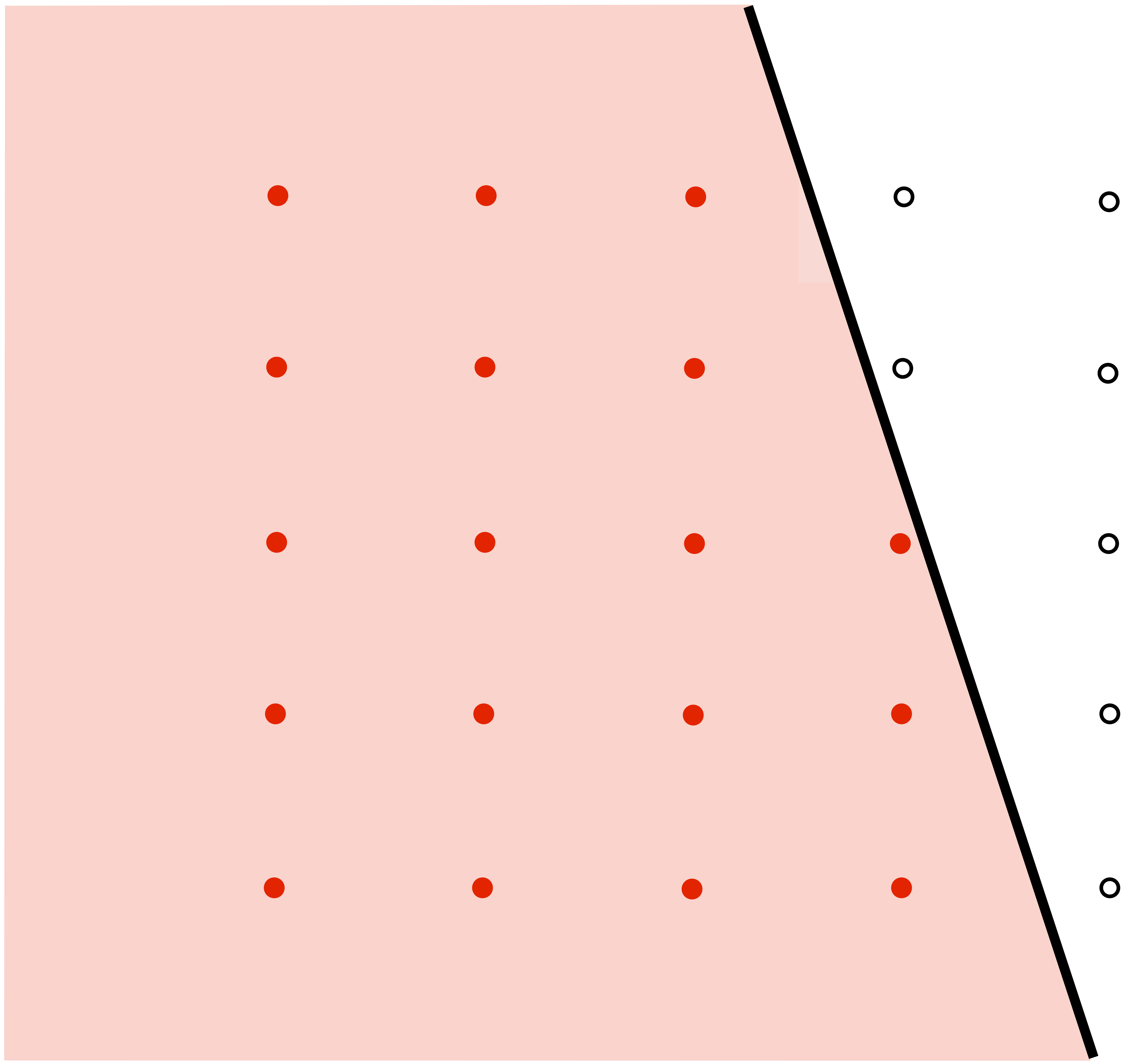
- For instance, if we think of a pixel as a "little square" of light, then we want the total light emitted to be the same as the total light in that pixel
  - I.e., we want to *integrate* the signal over the pixel ("box filter")



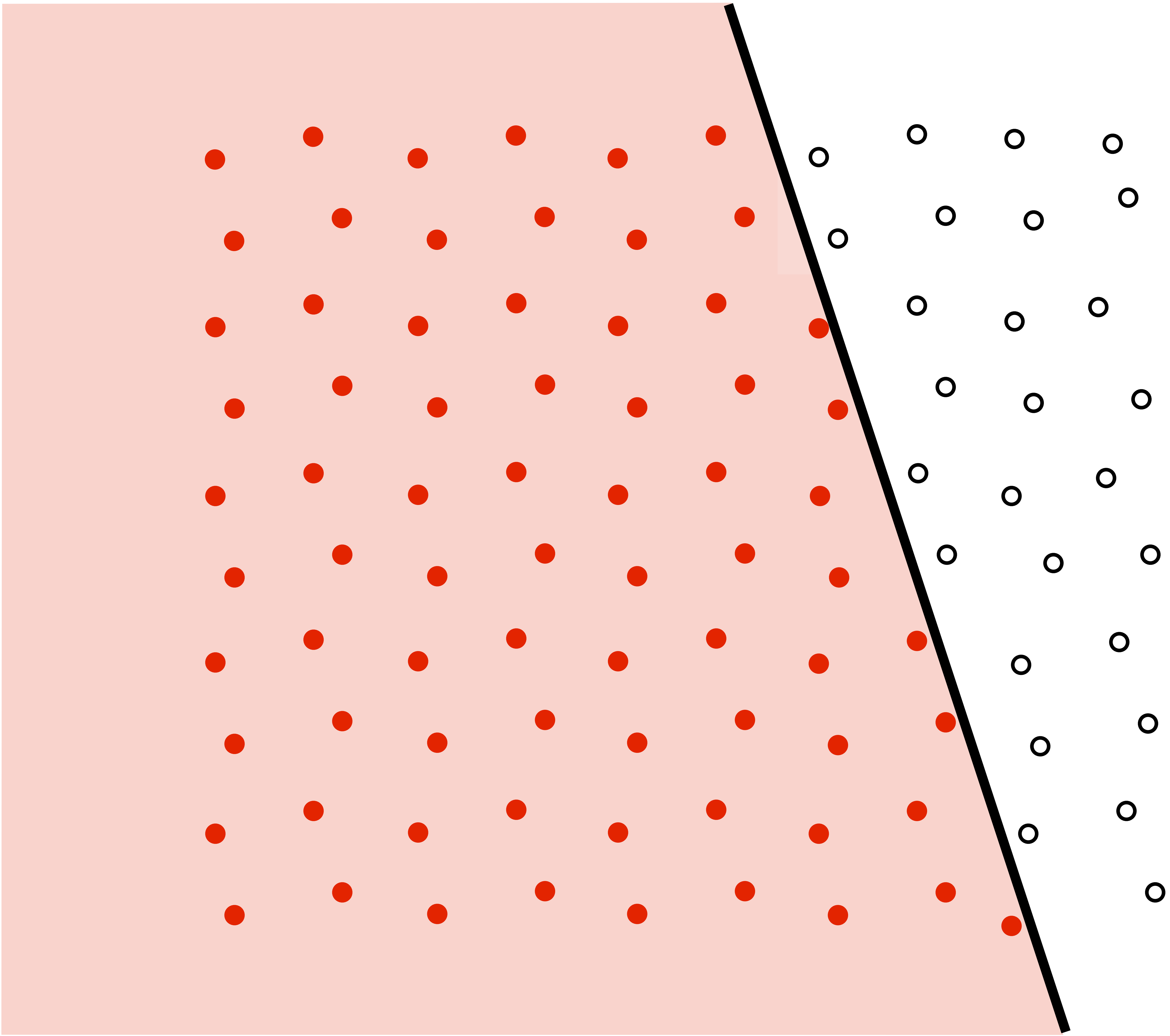


Let's (approximately) integrate the signal coverage (x,y) by *sampling*…

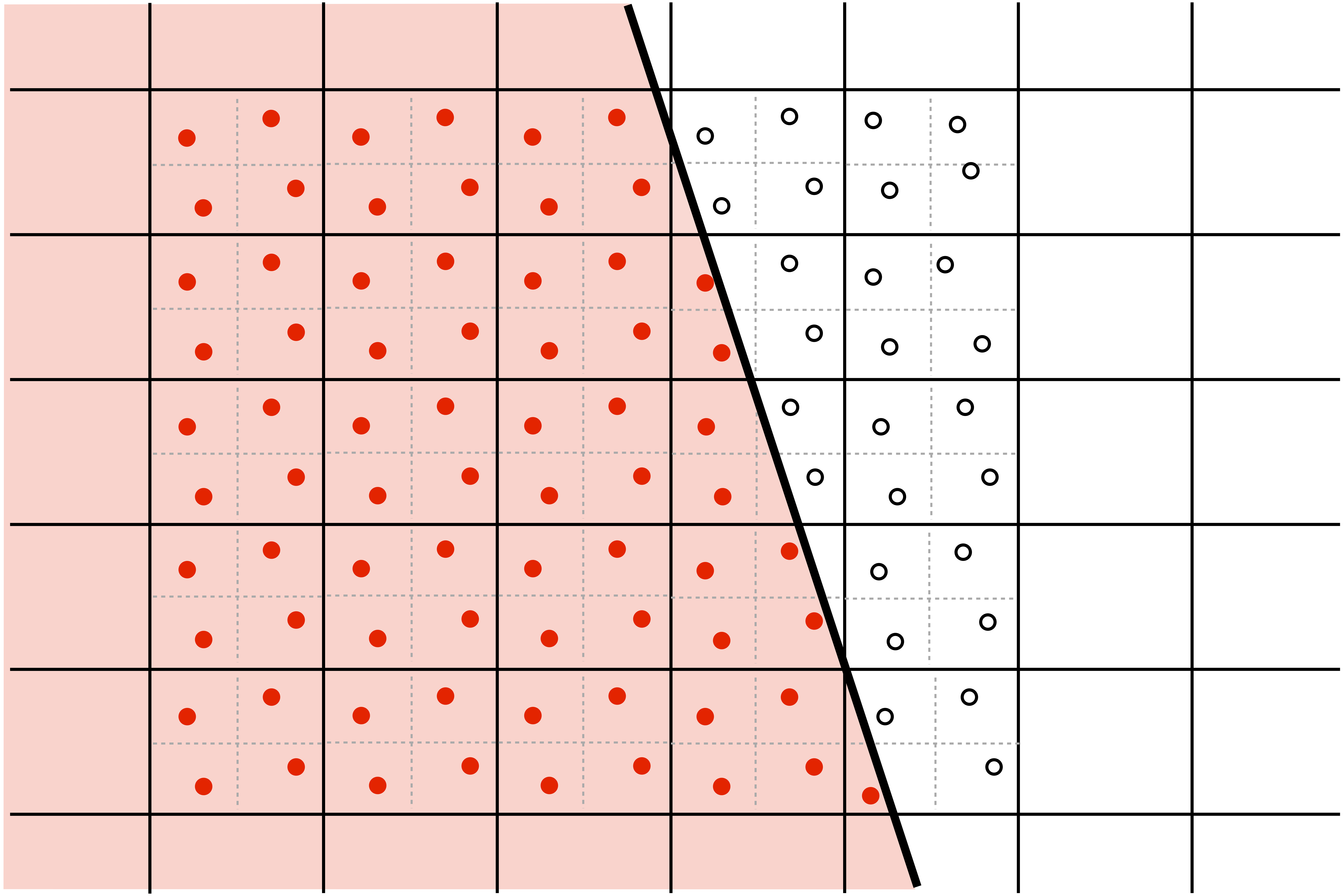
Initial coverage sampling rate (1 sample per pixel)



Increase frequency of sampling coverage signal

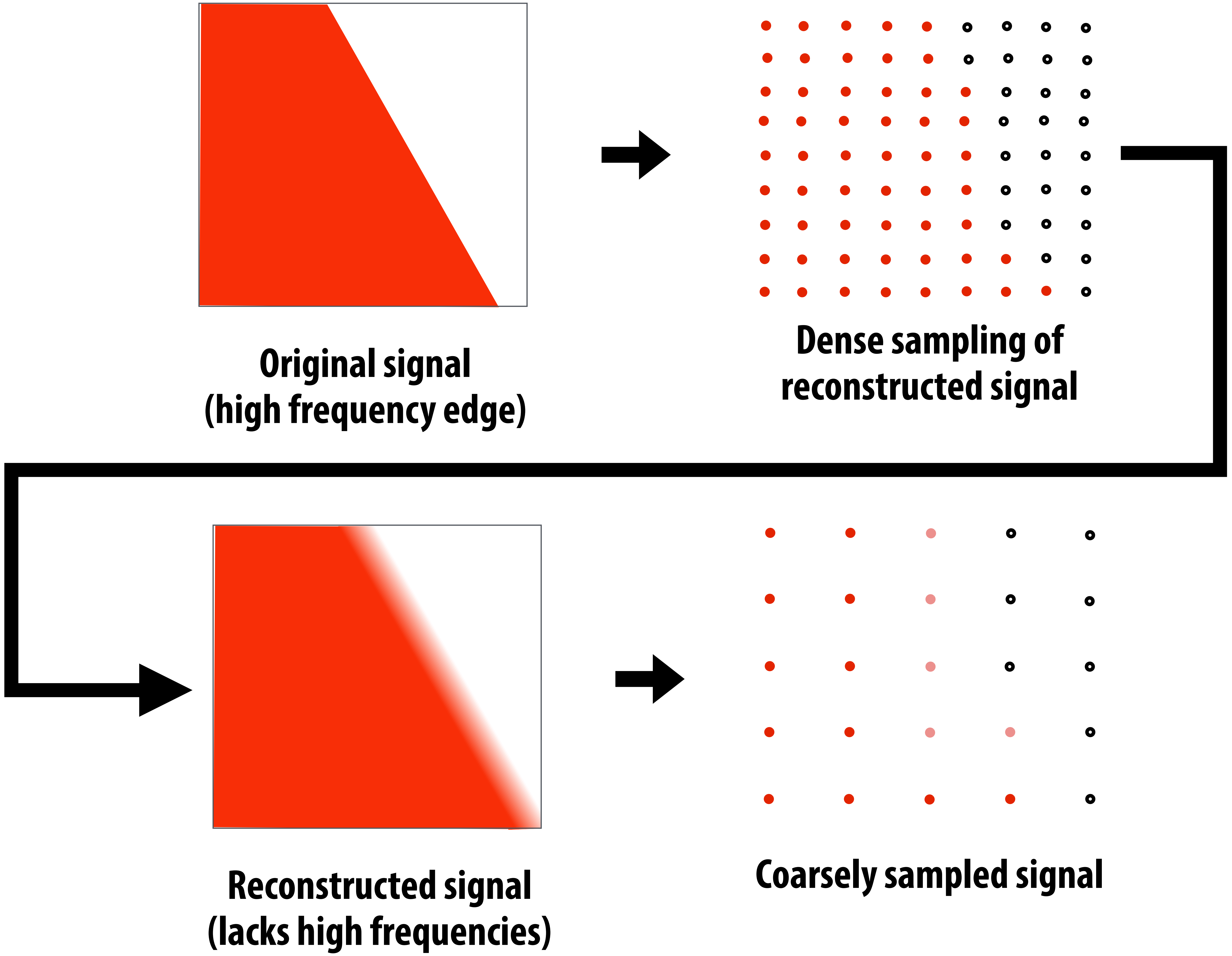


Supersampling



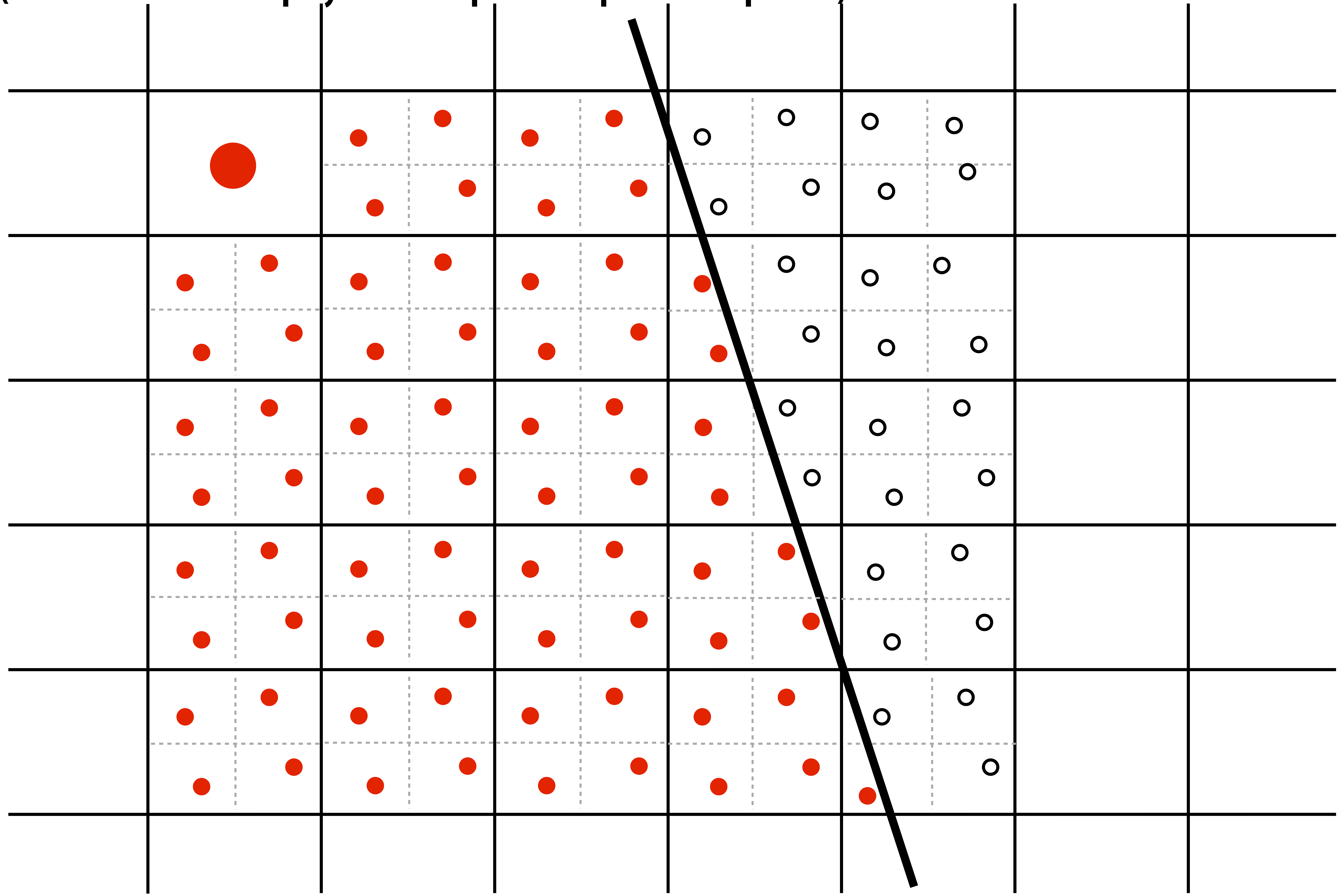
Resampling

Converting from one discrete sampled representation to another

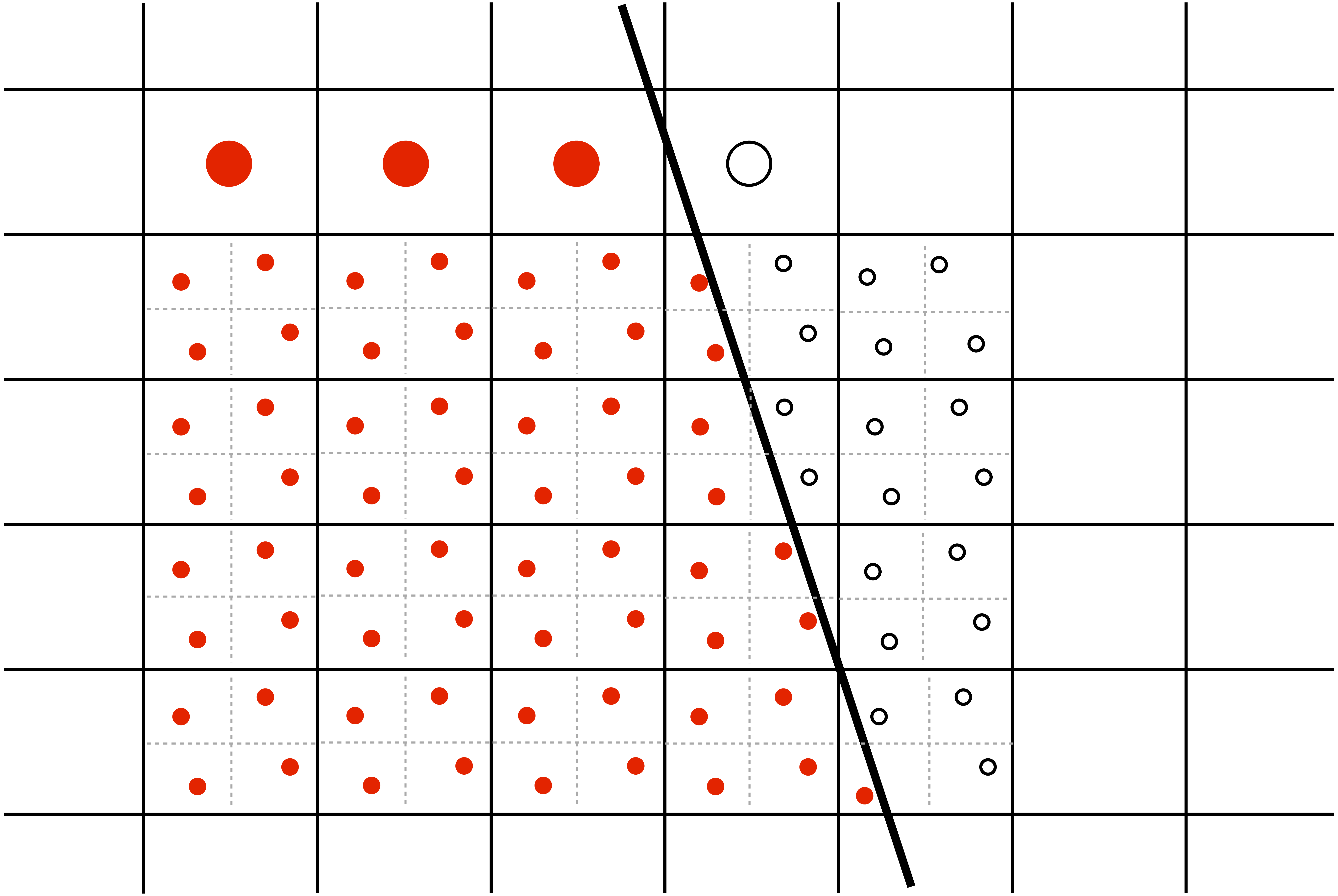


Resample to display's pixel resolution

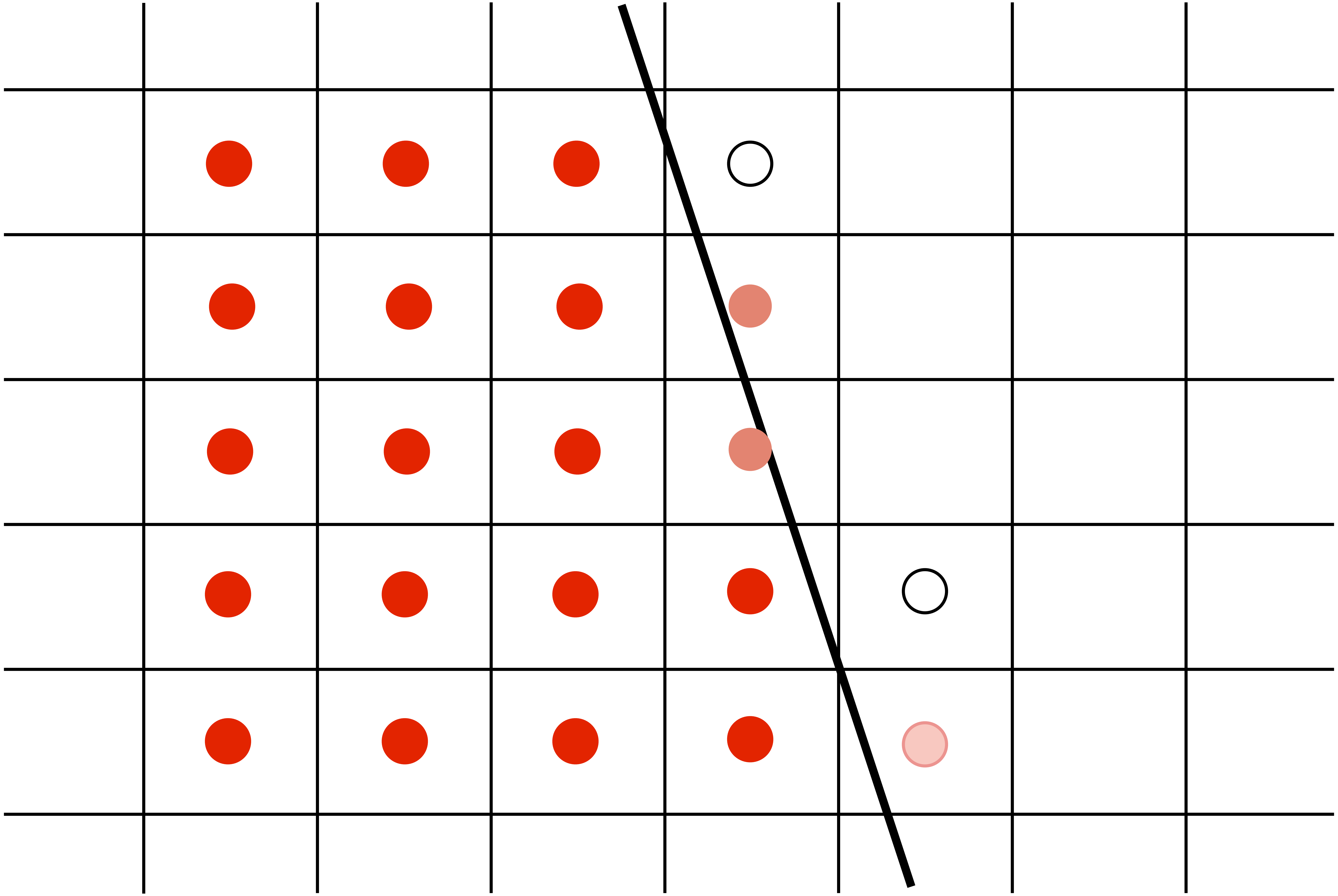
(Because a screen displays one sample value per screen pixel...)



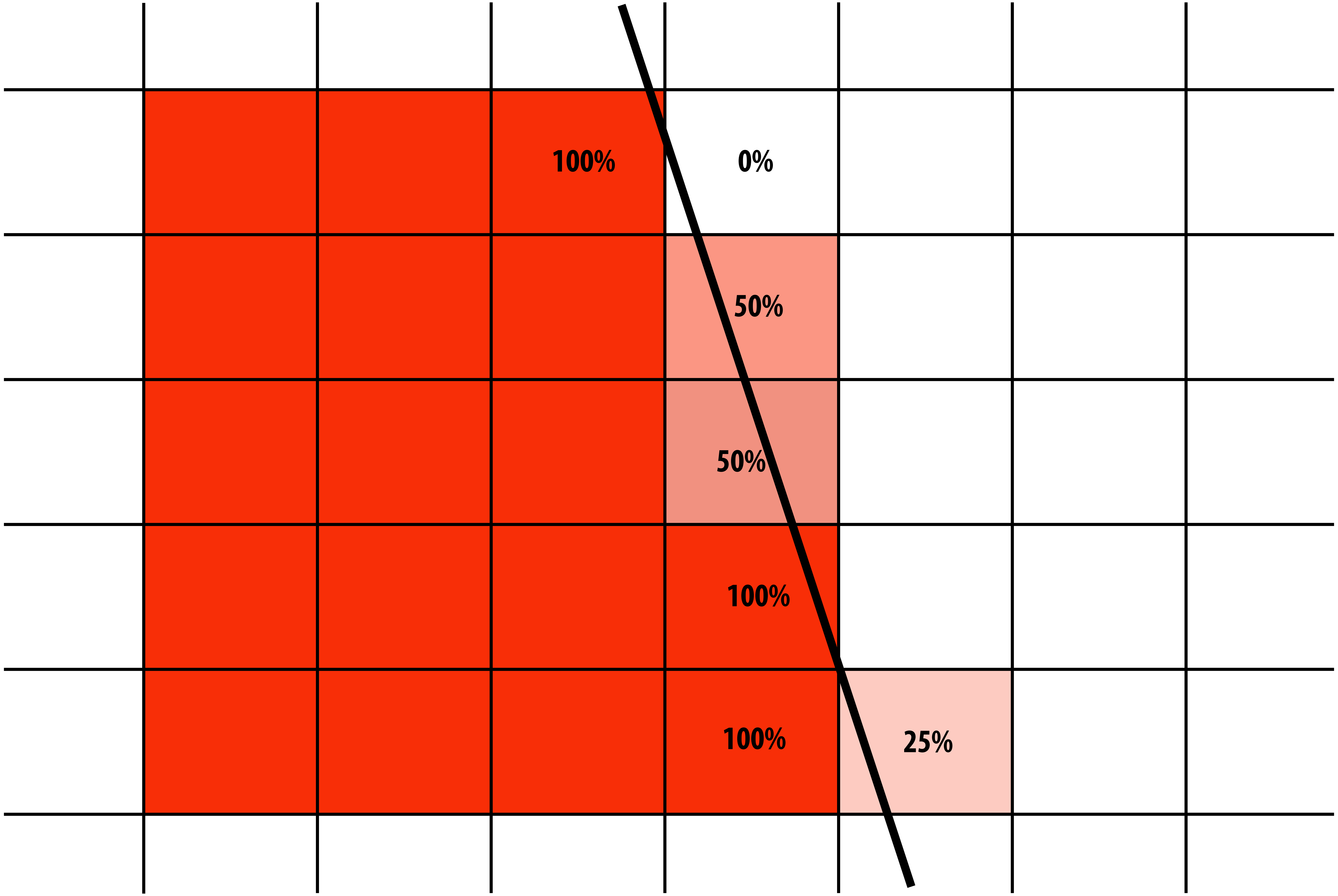
Resample to display's pixel rate (box filter)



Resample to display's pixel rate (box filter)

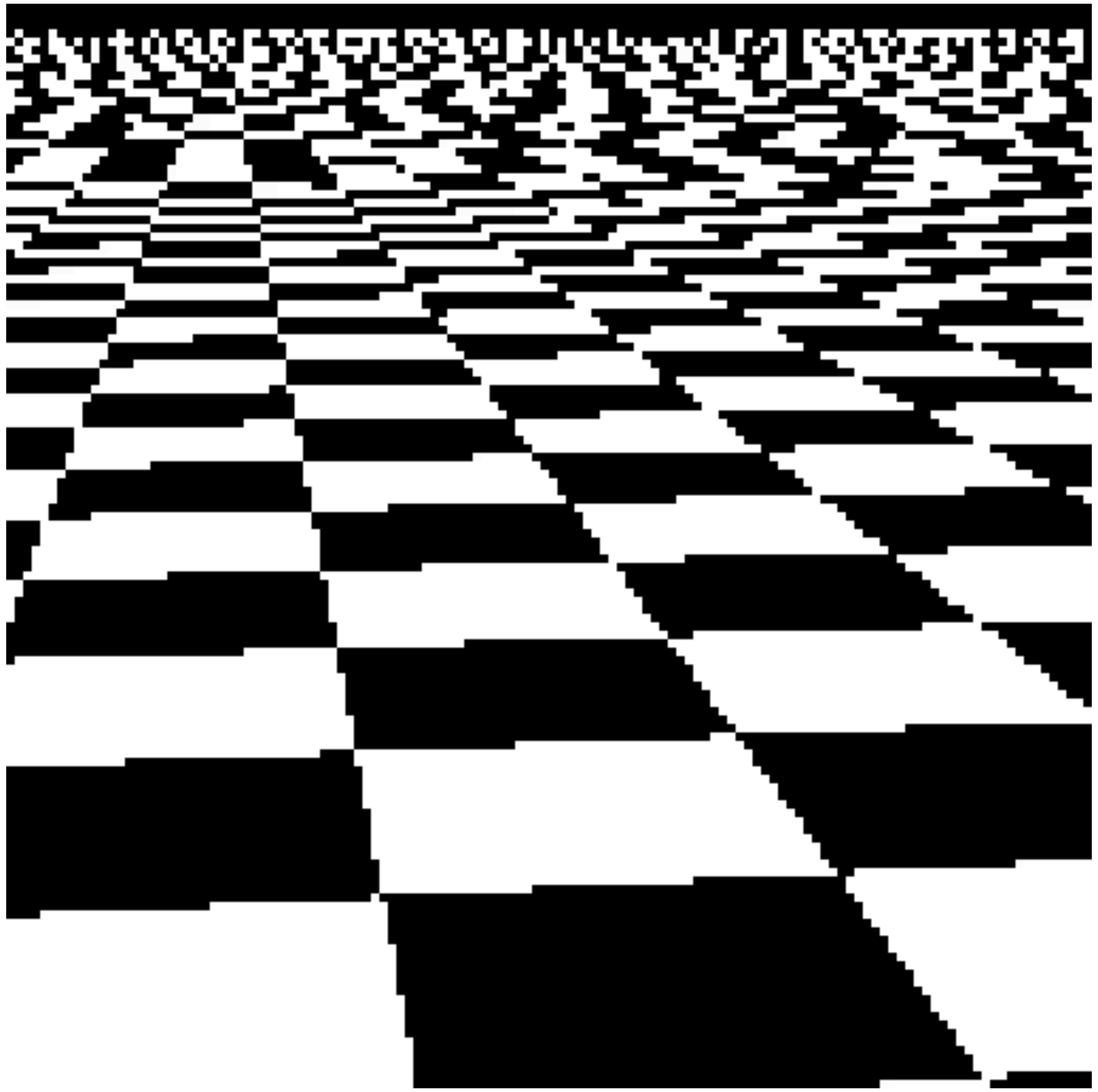


Displayed result (note anti-aliased edges)

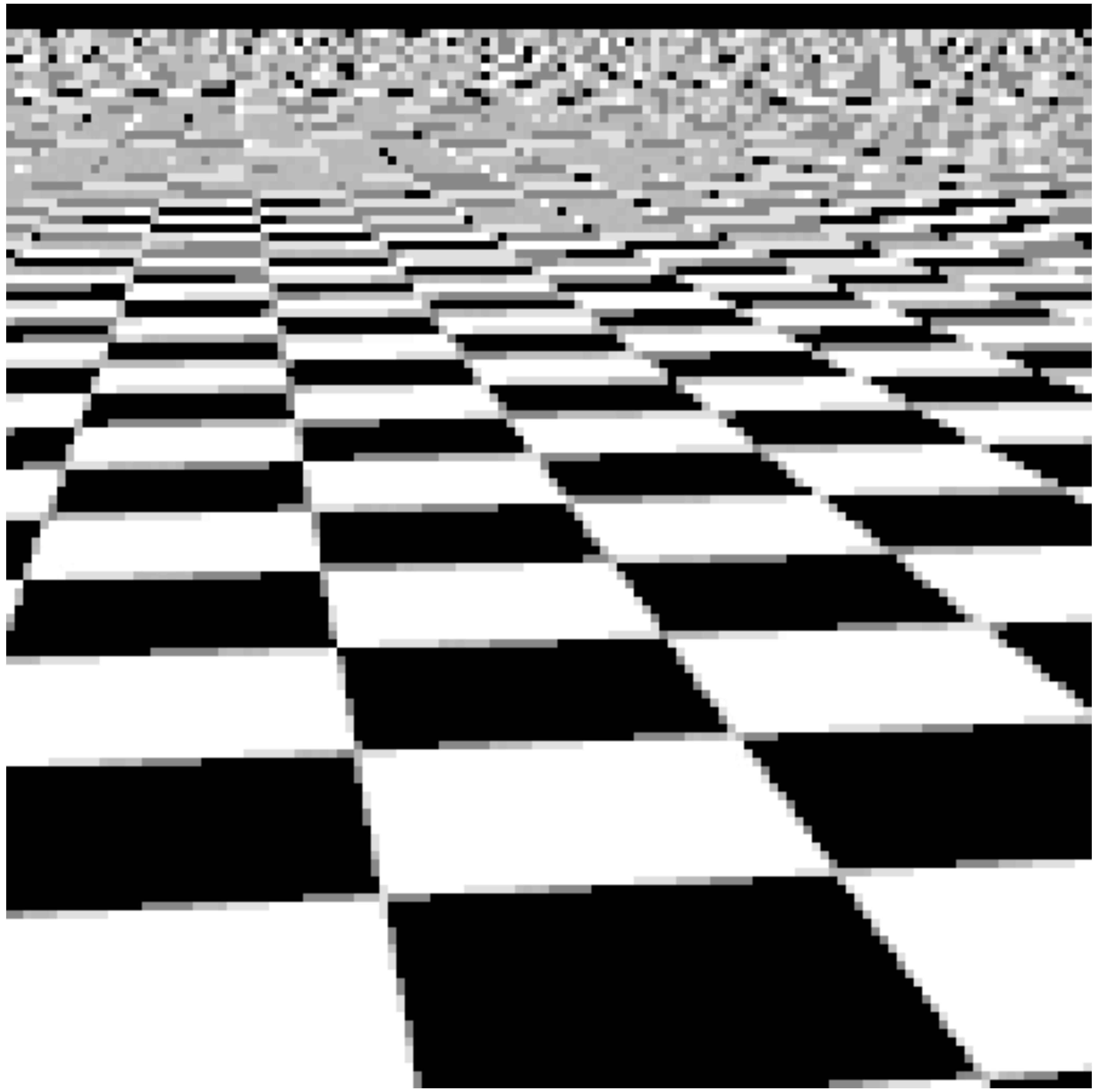


Recall: the real coverage signal was this

Single Sample vs. Supersampling

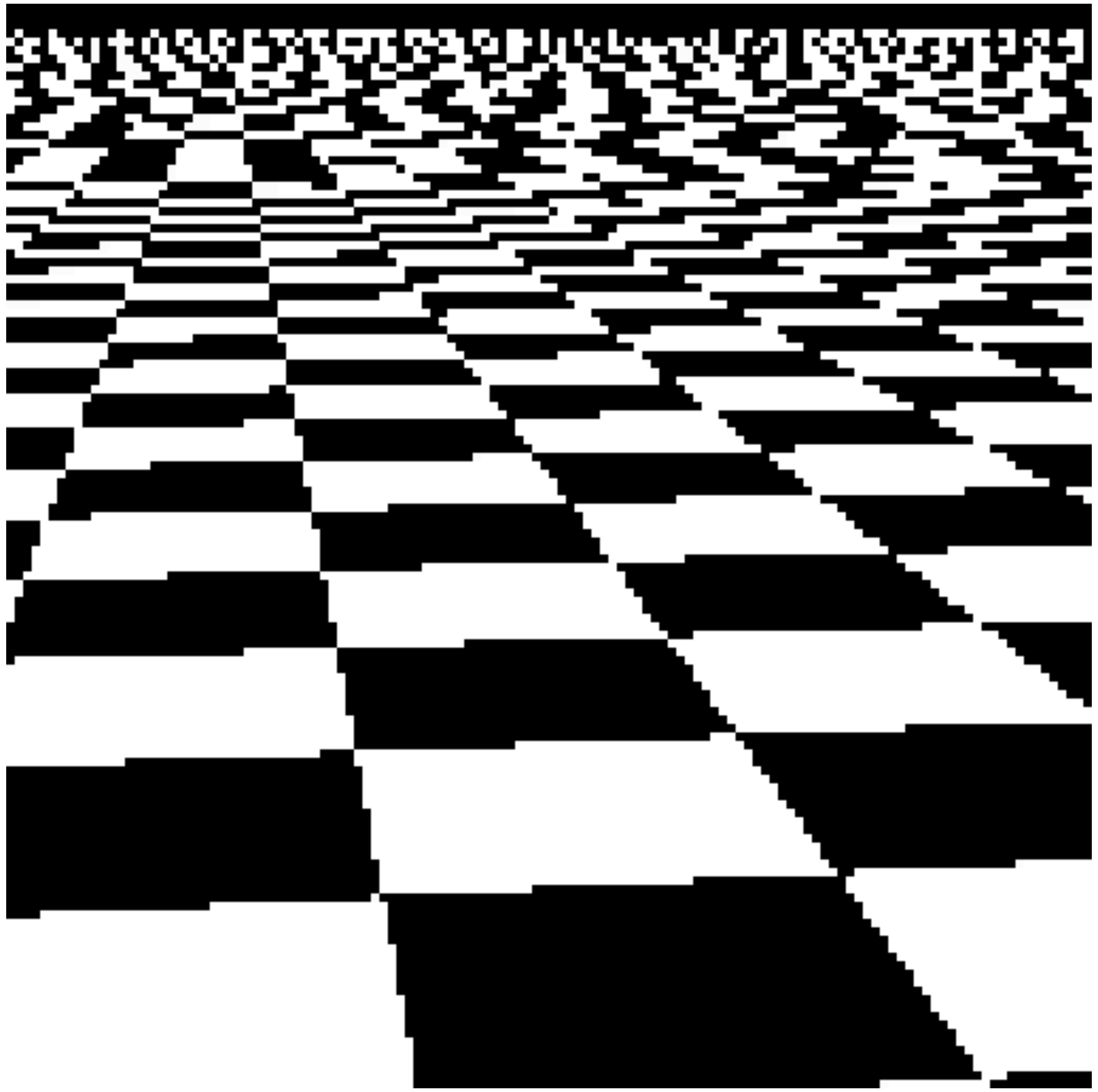


single sampling

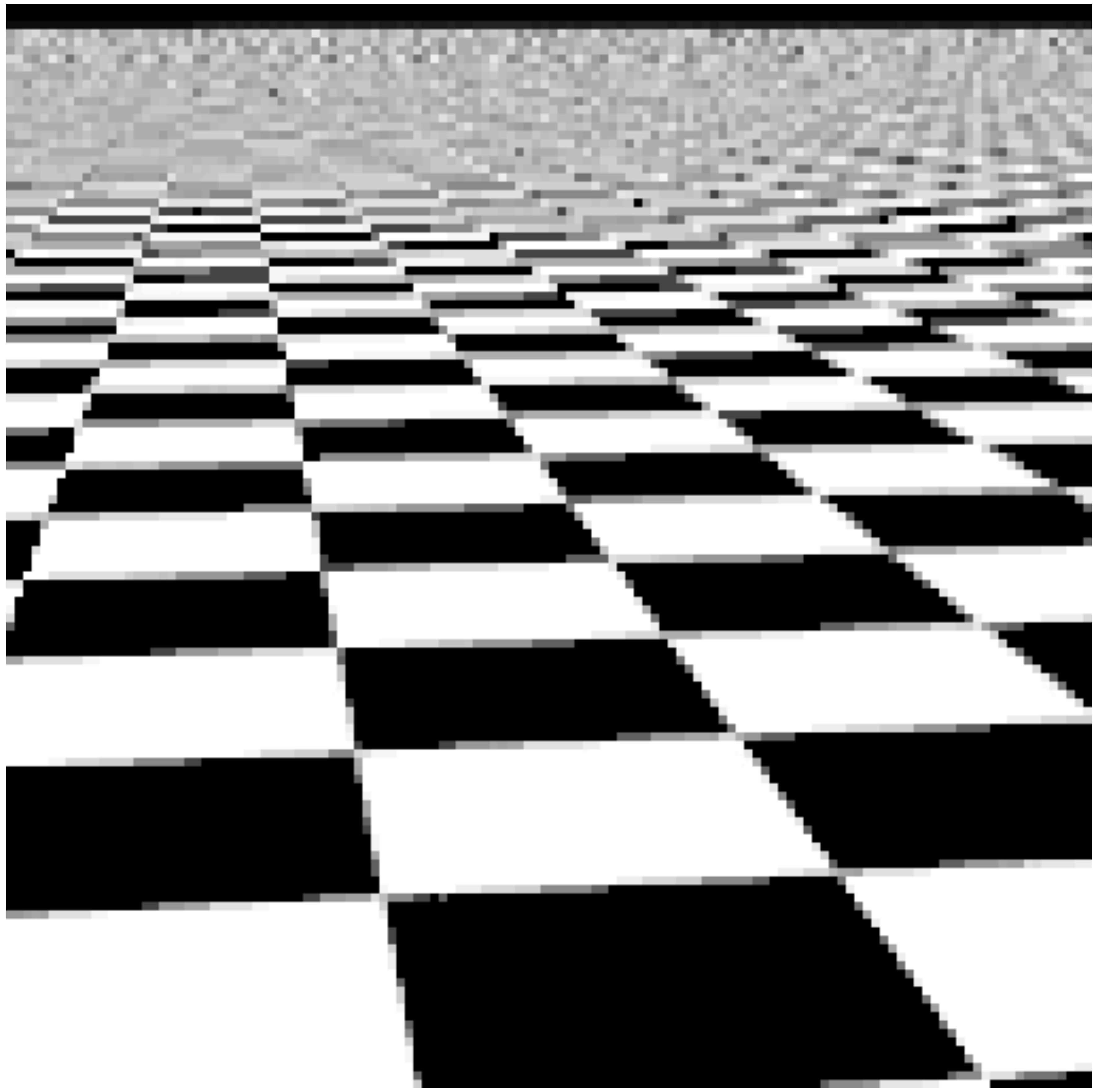


2x2 supersampling

Single Sample vs. Supersampling

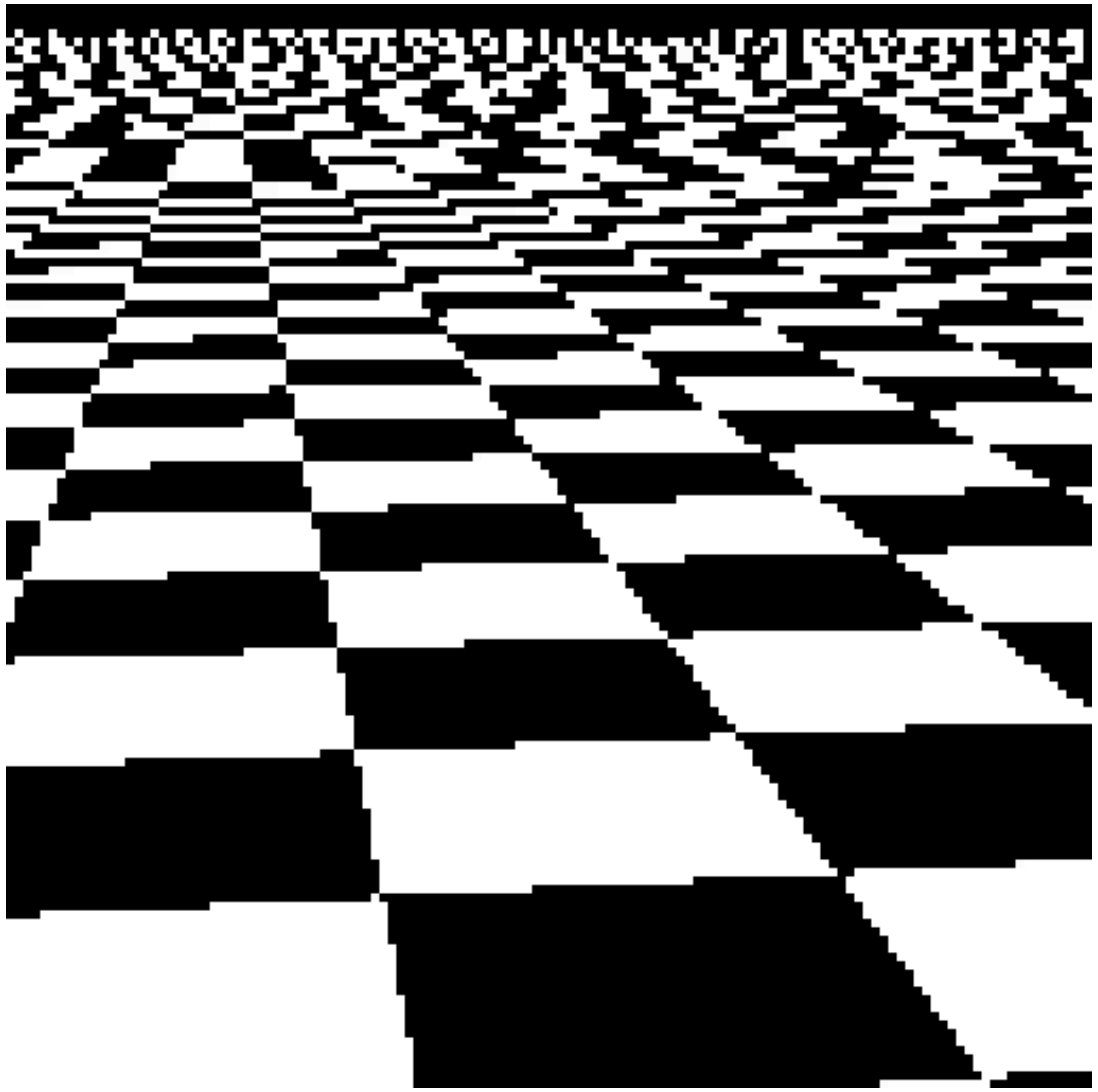


single sampling

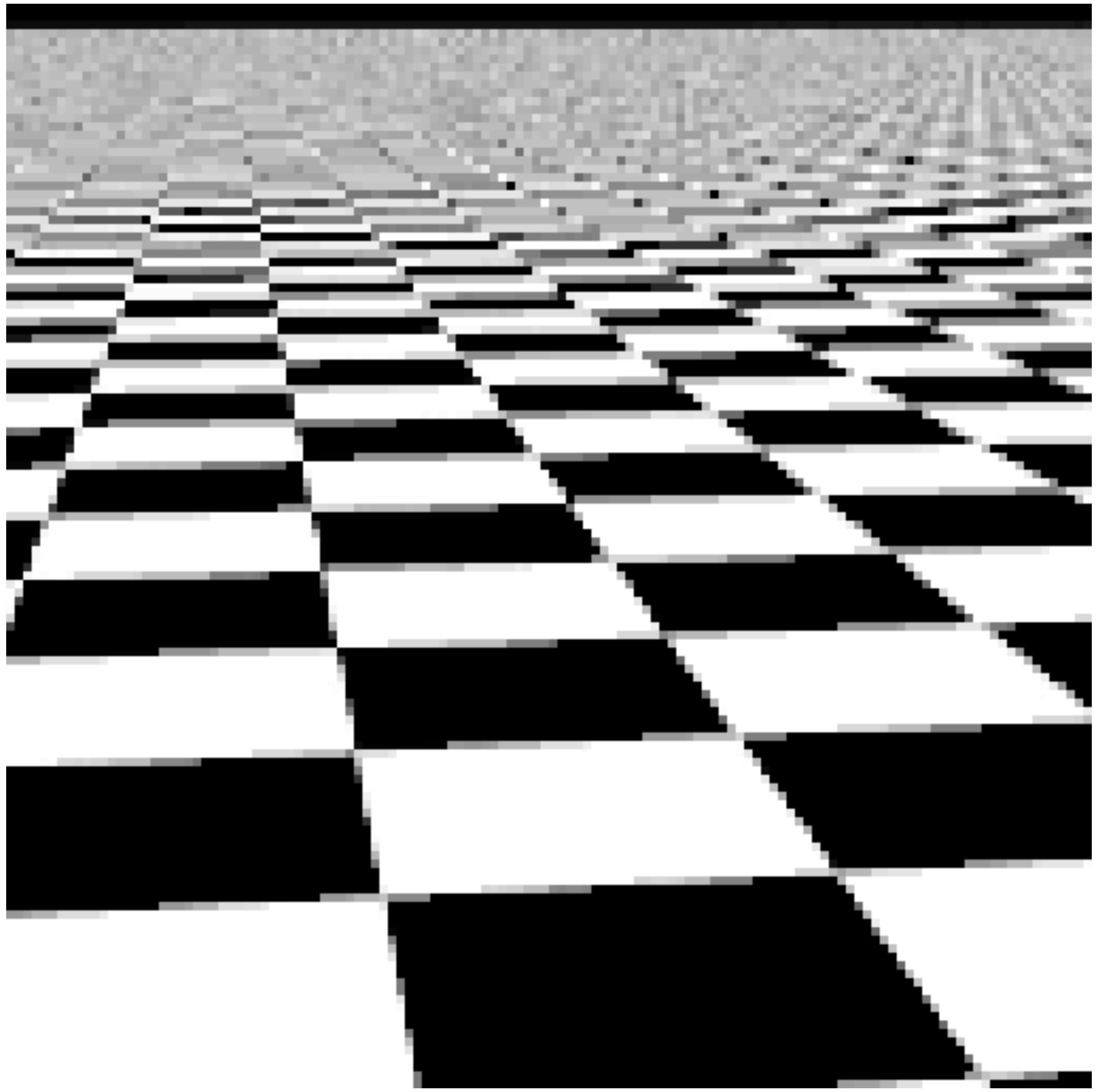


4x4 supersampling

Single Sample vs. Supersampling



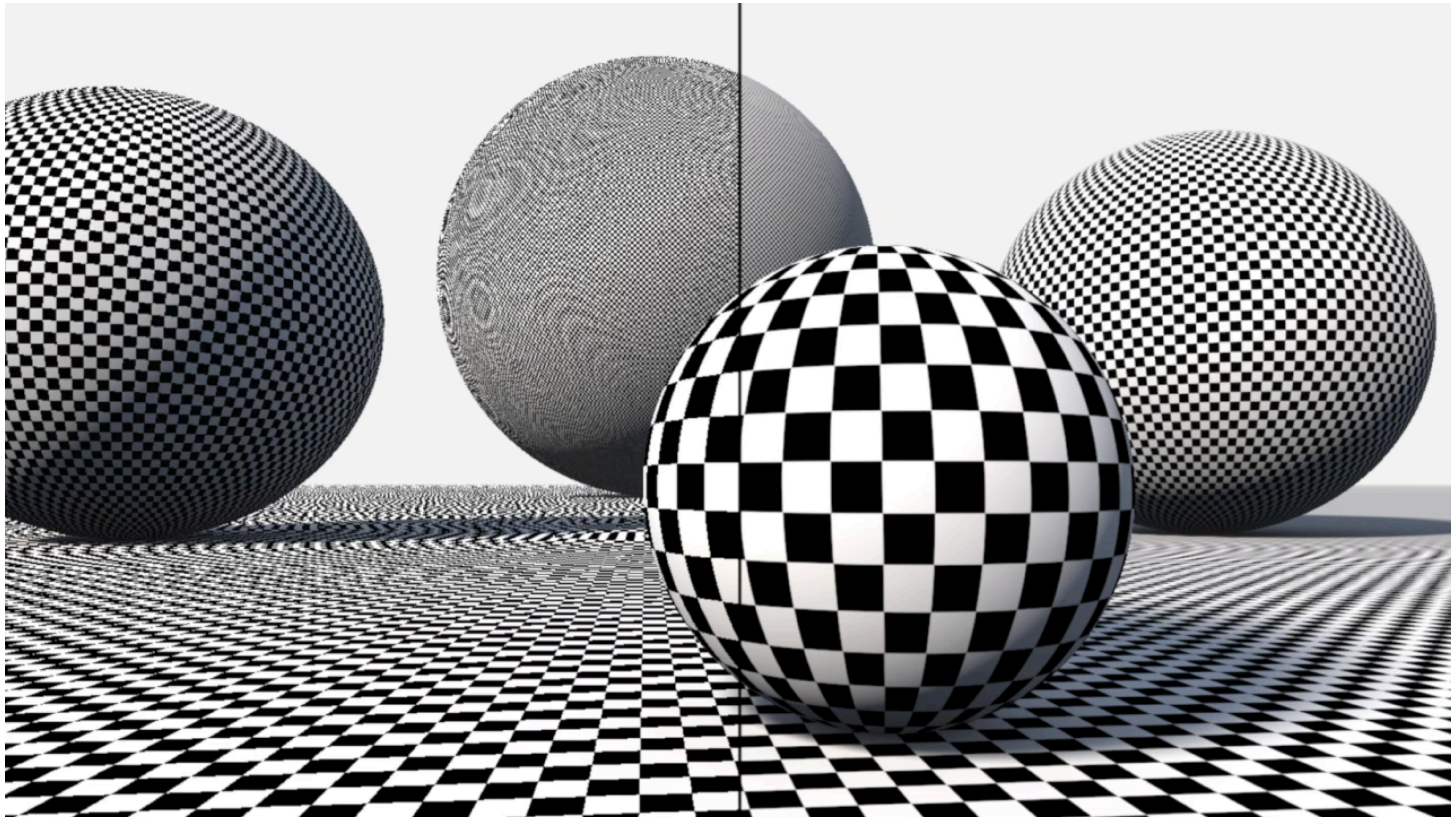
single sampling



32x32 supersampling

Checkerboard — Exact Solution

In <u>very</u> special cases we can compute the *exact* coverage:



Such cases are <u>extremely</u> rare—want solutions that will work in the general case!

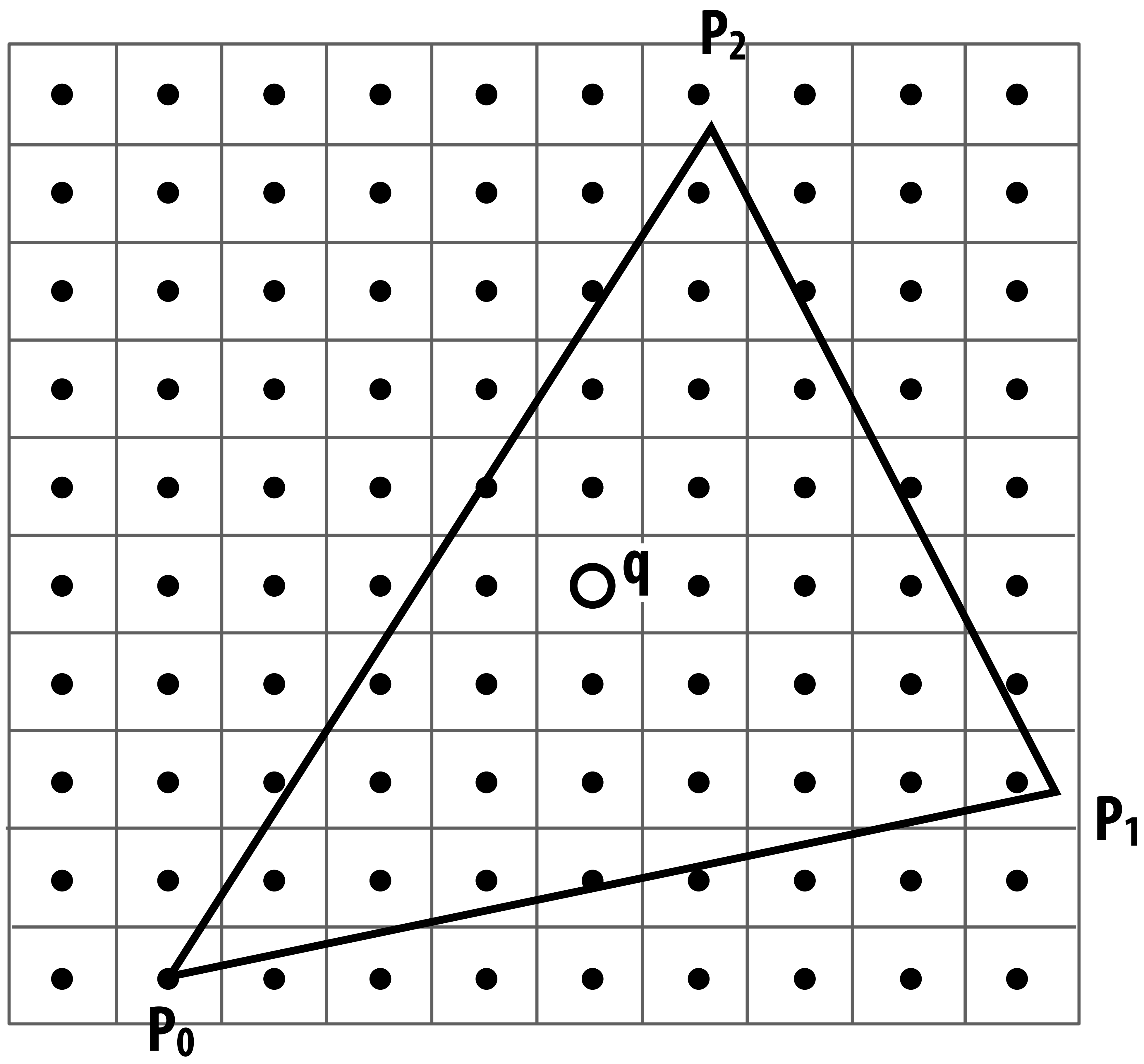
See: Inigo Quilez, ***"Filtering the Checkerboard Pattern"*** & Apodaca et al, ***"Advanced Renderman"*** (p. 273)

How do we actually evaluate coverage(x,y) for a triangle?

Point-in-triangle test

Q: How do we check if a given point q is inside a triangle?

A: Check if it's contained in three <u>half planes</u> associated with the edges.



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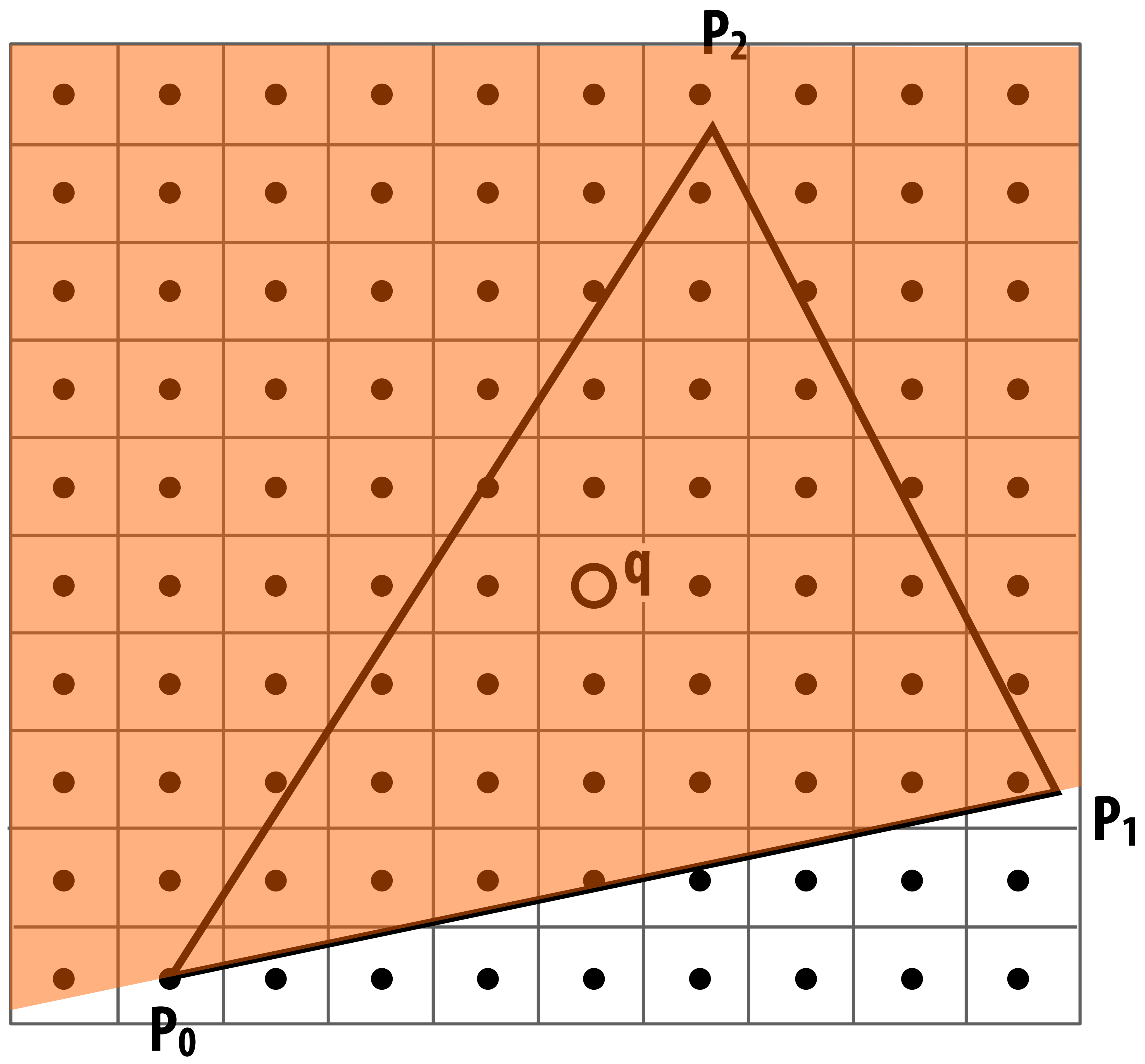

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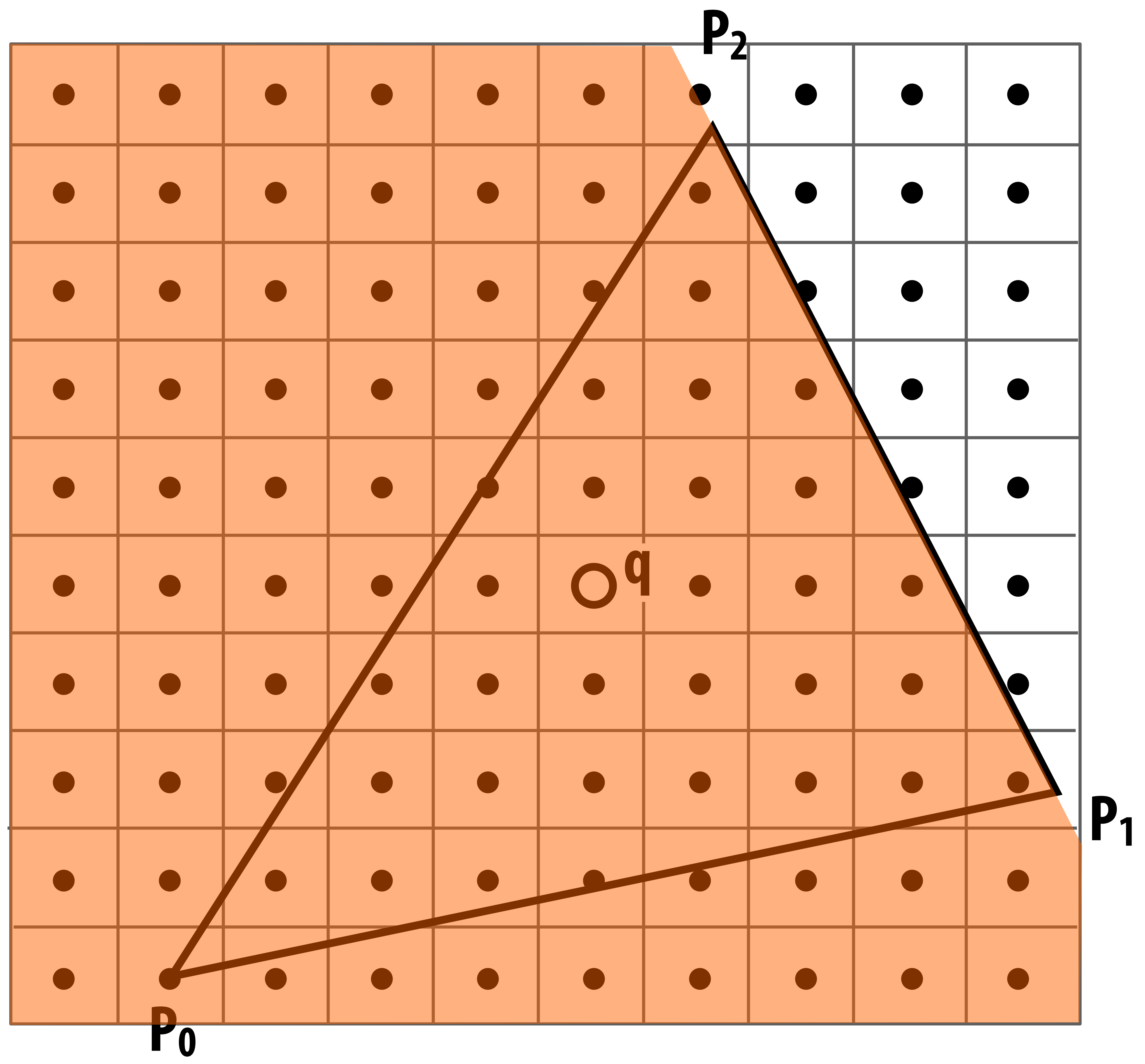

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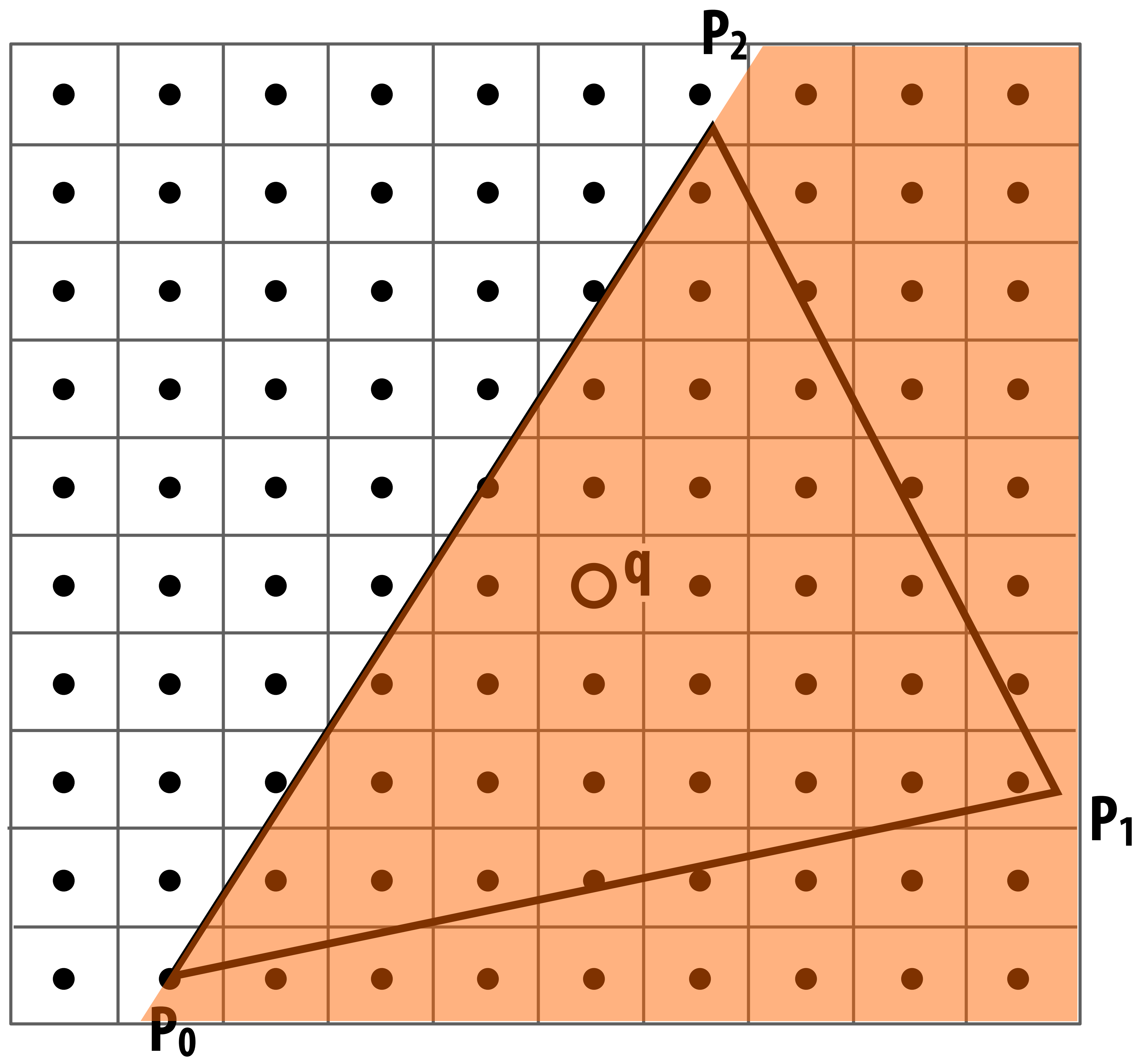
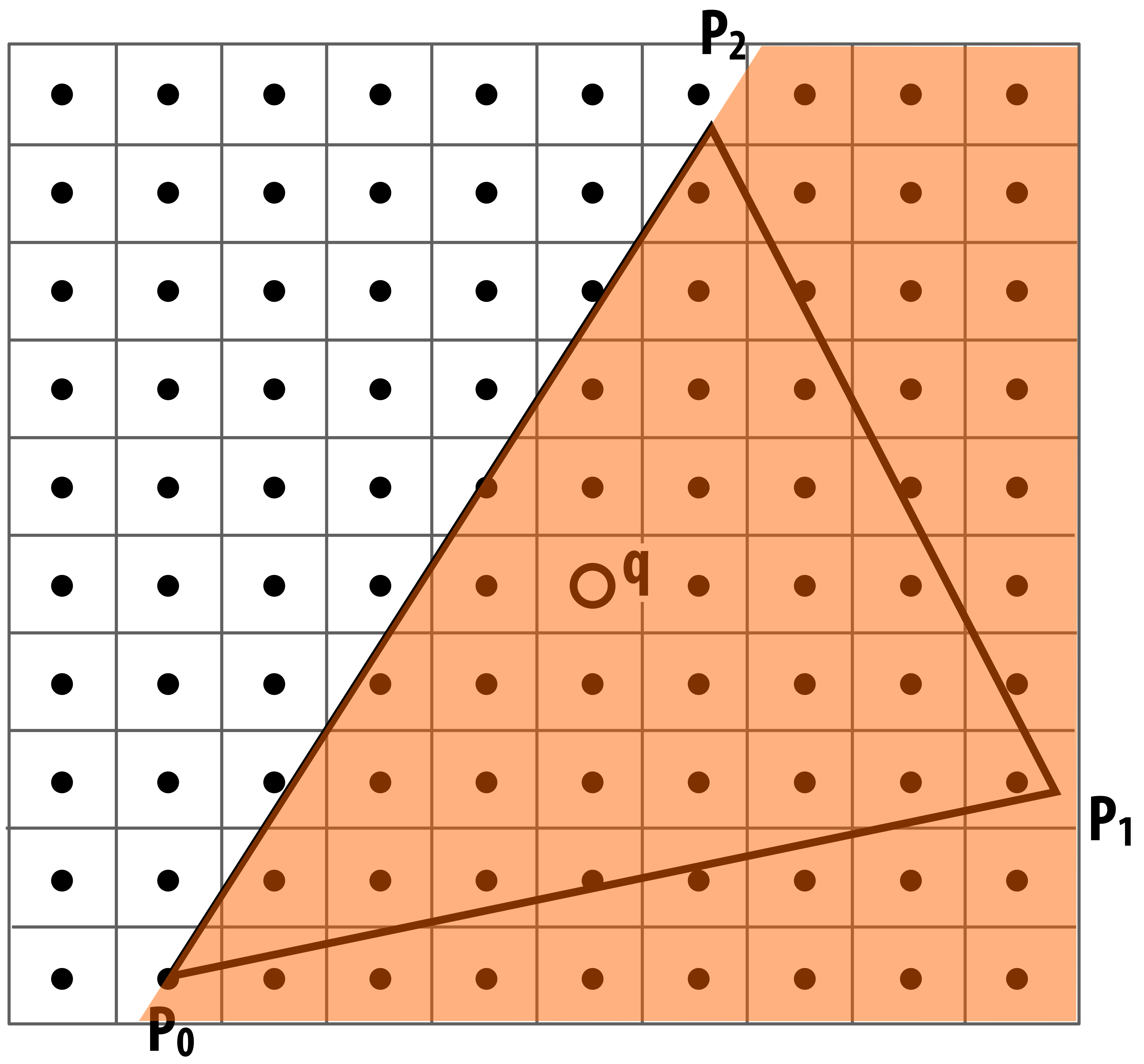


Point-in-triangle test

Q: How do we check if a given point q is inside a triangle?

A: Check if it's contained in three <u>half planes</u> associated with the edges.

Half plane test is then an exercise in linear algebra/ vector calculus:



GIVEN: points P_i, P_j along an edge, and a query point q

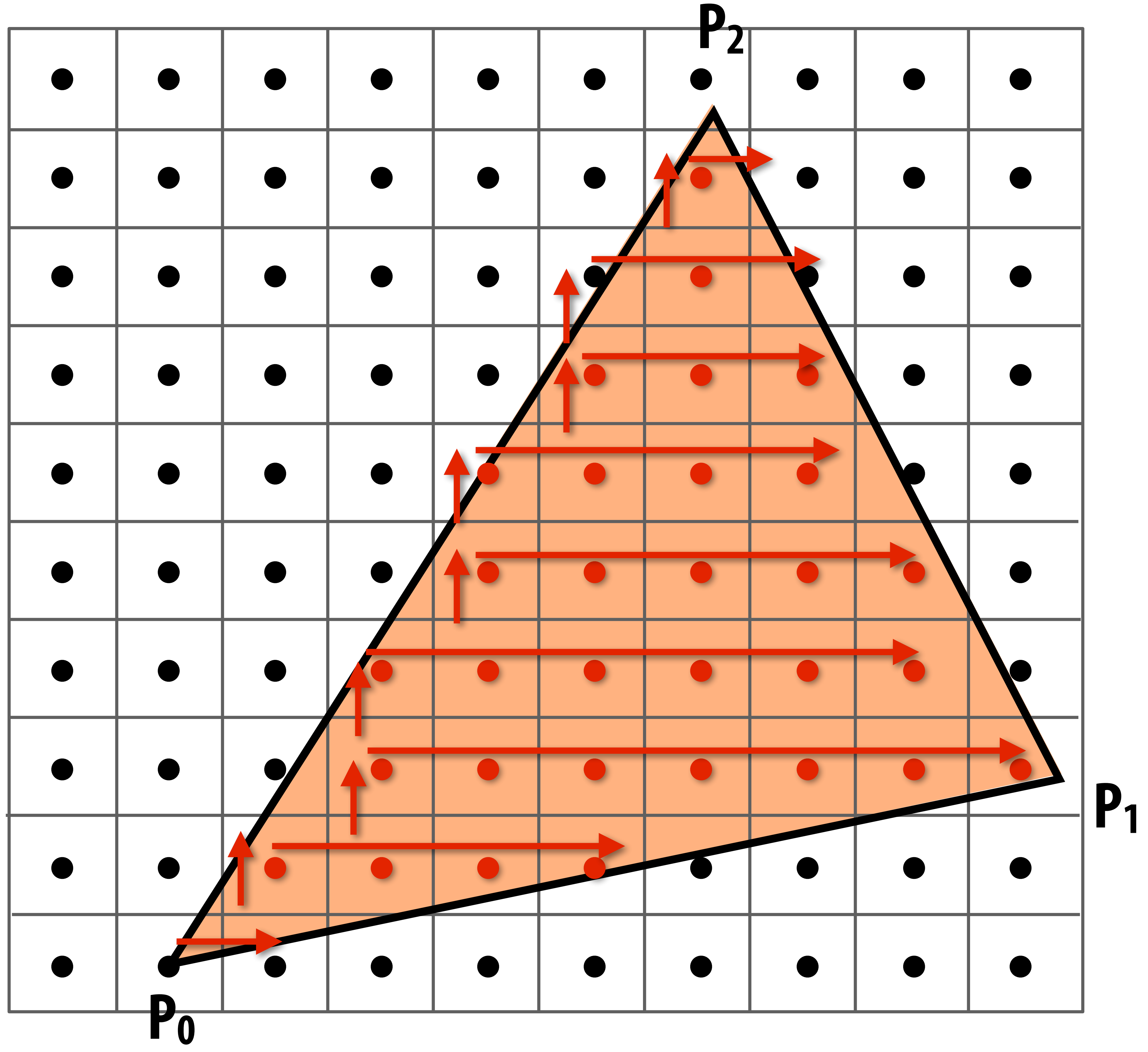
FIND: whether q is to the "left" or "right" of the line from P_i to P_j

(Careful to consider triangle coverage edge rules…)

Traditional approach: incremental traversal

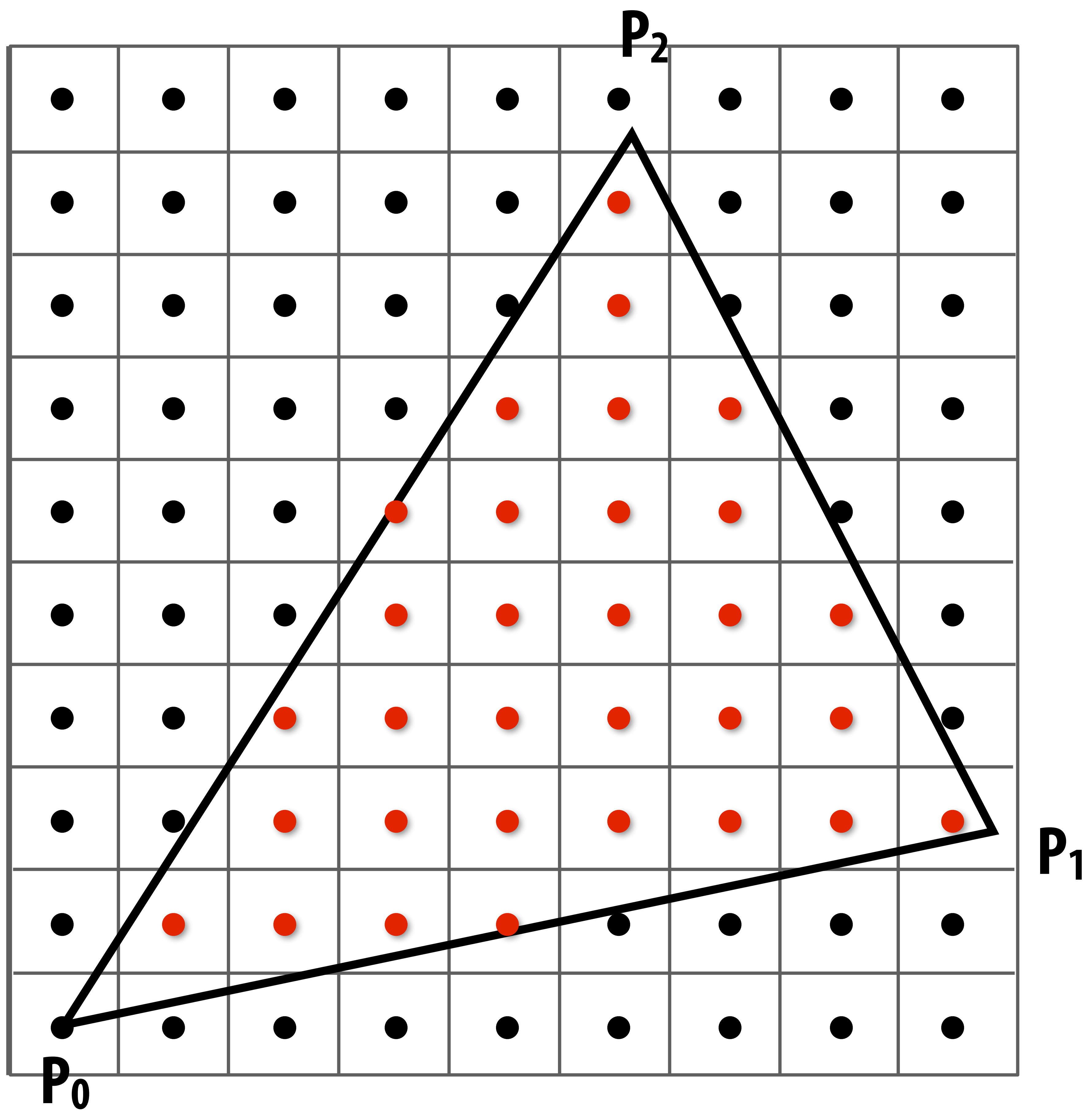
Since half-plane check looks *very* similar for different points, can save arithmetic by clever "incremental" schemes.

Incremental approach also visits pixels in an order that improves memory coherence: backtrack, zig-zag, Hilbert/Morton curves, …



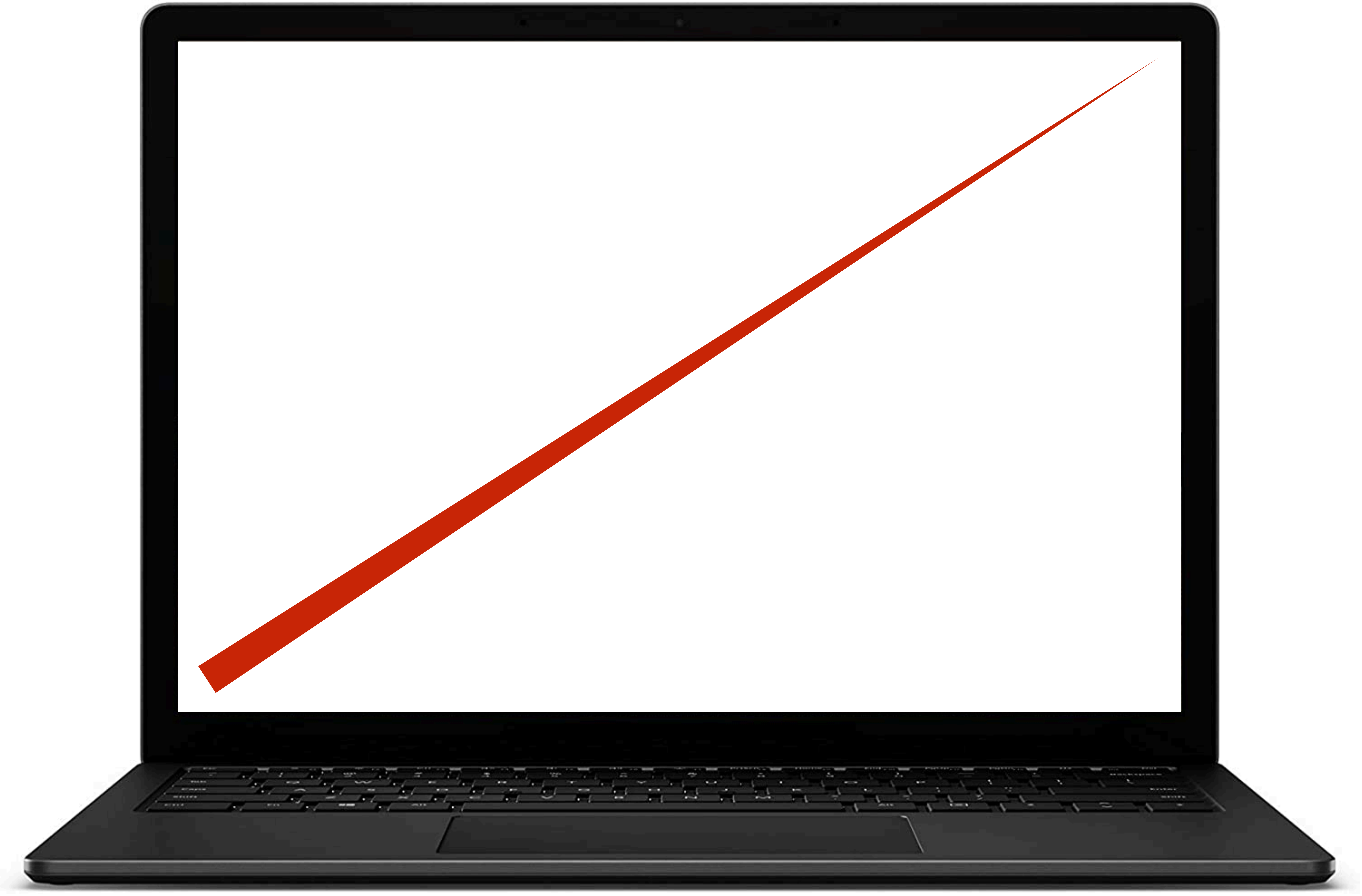
Modern approach: parallel coverage tests

- Incremental traversal is very serial; modern hardware is highly parallel
- Alternative: test all samples in triangle "bounding box" in parallel
- Wide parallel execution overcomes cost of extra tests (most triangles cover many samples, especially when super-sampling)
- All tests share some "setup" calculations
- Modern graphics processing unit (GPU) has special-purpose hardware for efficiently performing point-in-triangle tests



Q: What's a case where the naïve parallel approach is still very inefficient?

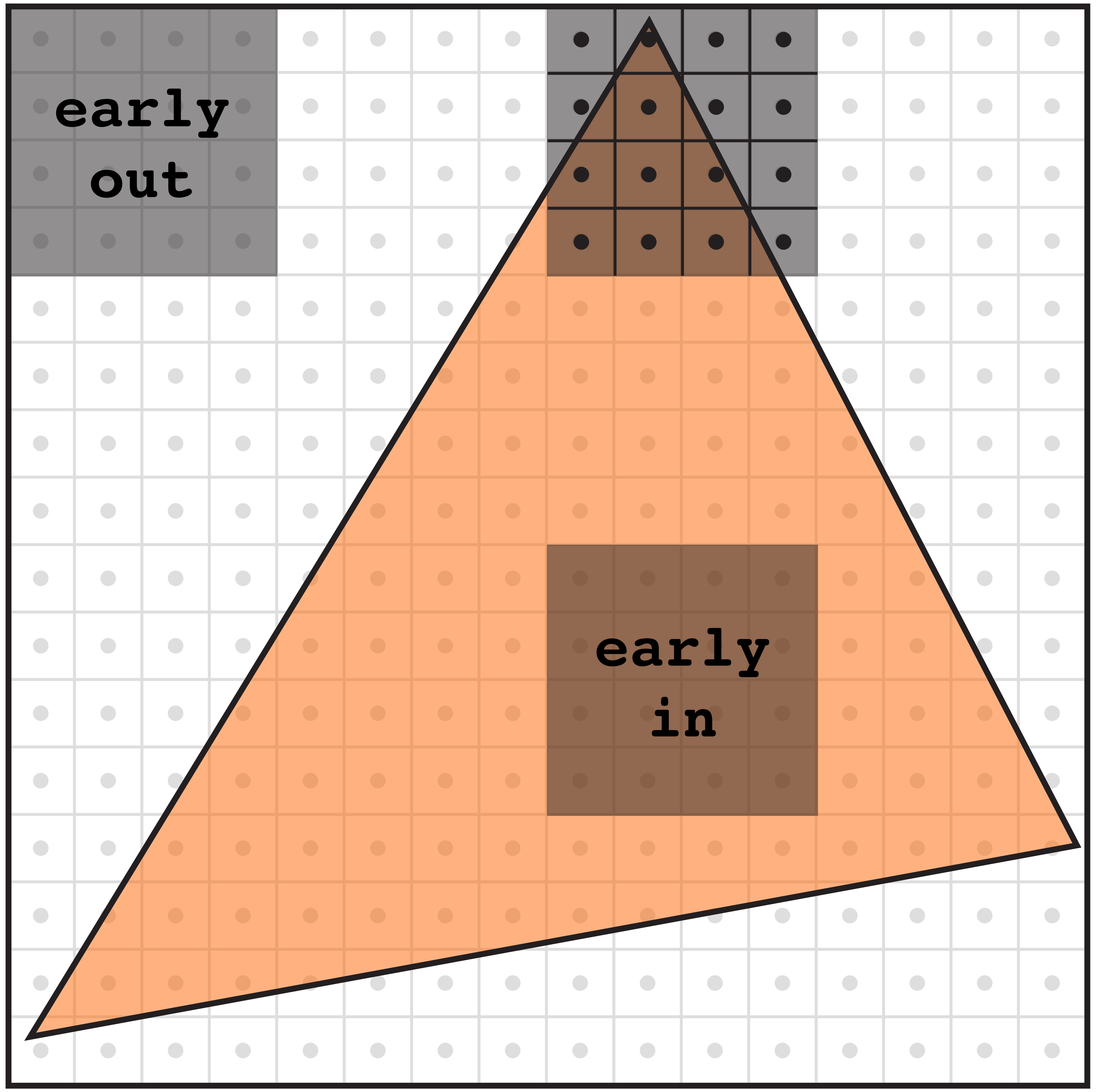
Naïve approach can be (very) wasteful…



Hybrid approach: tiled triangle traversal

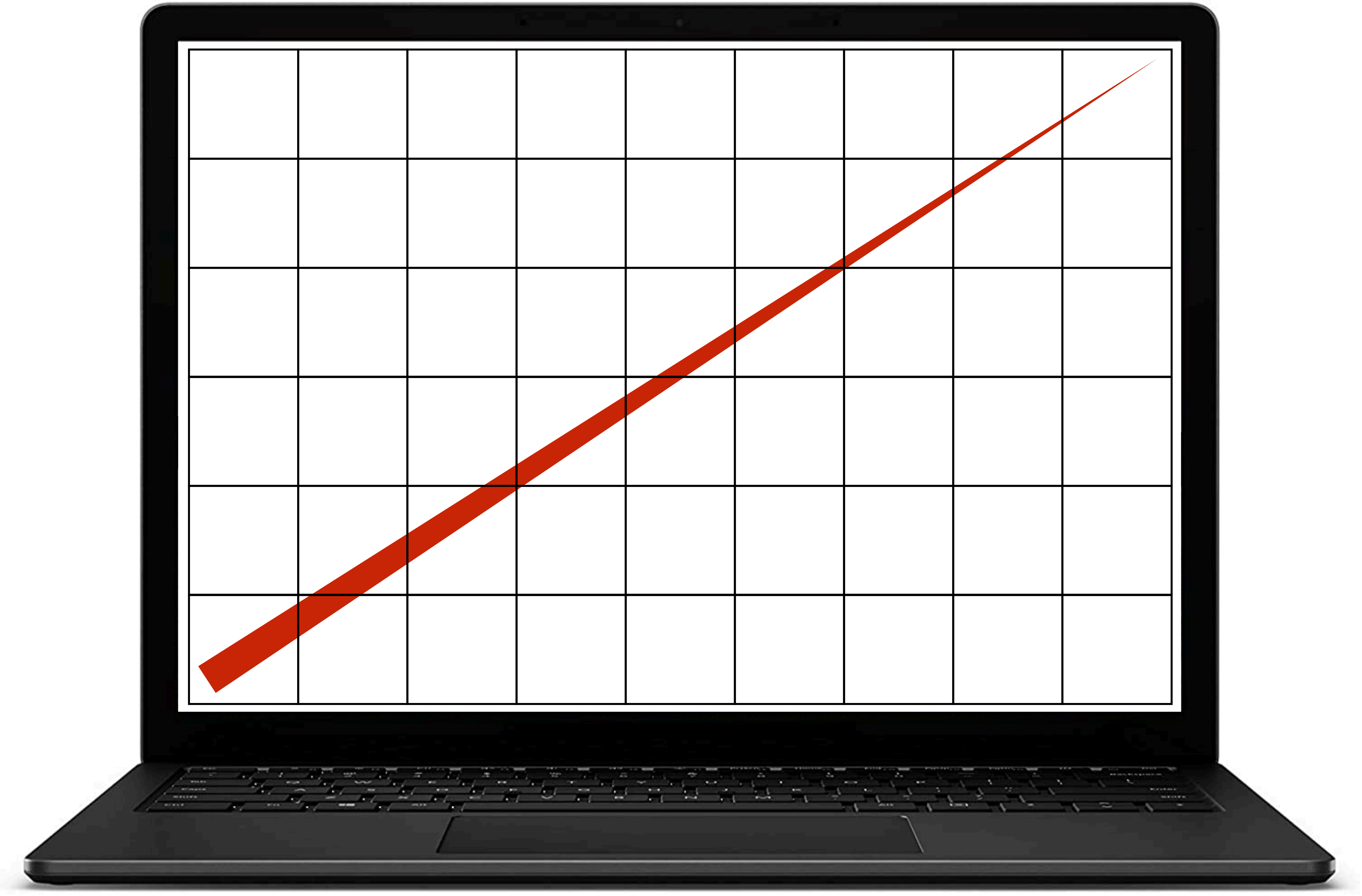
Idea: work "coarse to fine":

- First, check if large blocks intersect the triangle
- If not, skip this block entirely ("early out")
- If the block is contained inside the triangle, know <u>all</u> samples are covered ("early in")
- Otherwise, test individual sample points in the block, in parallel

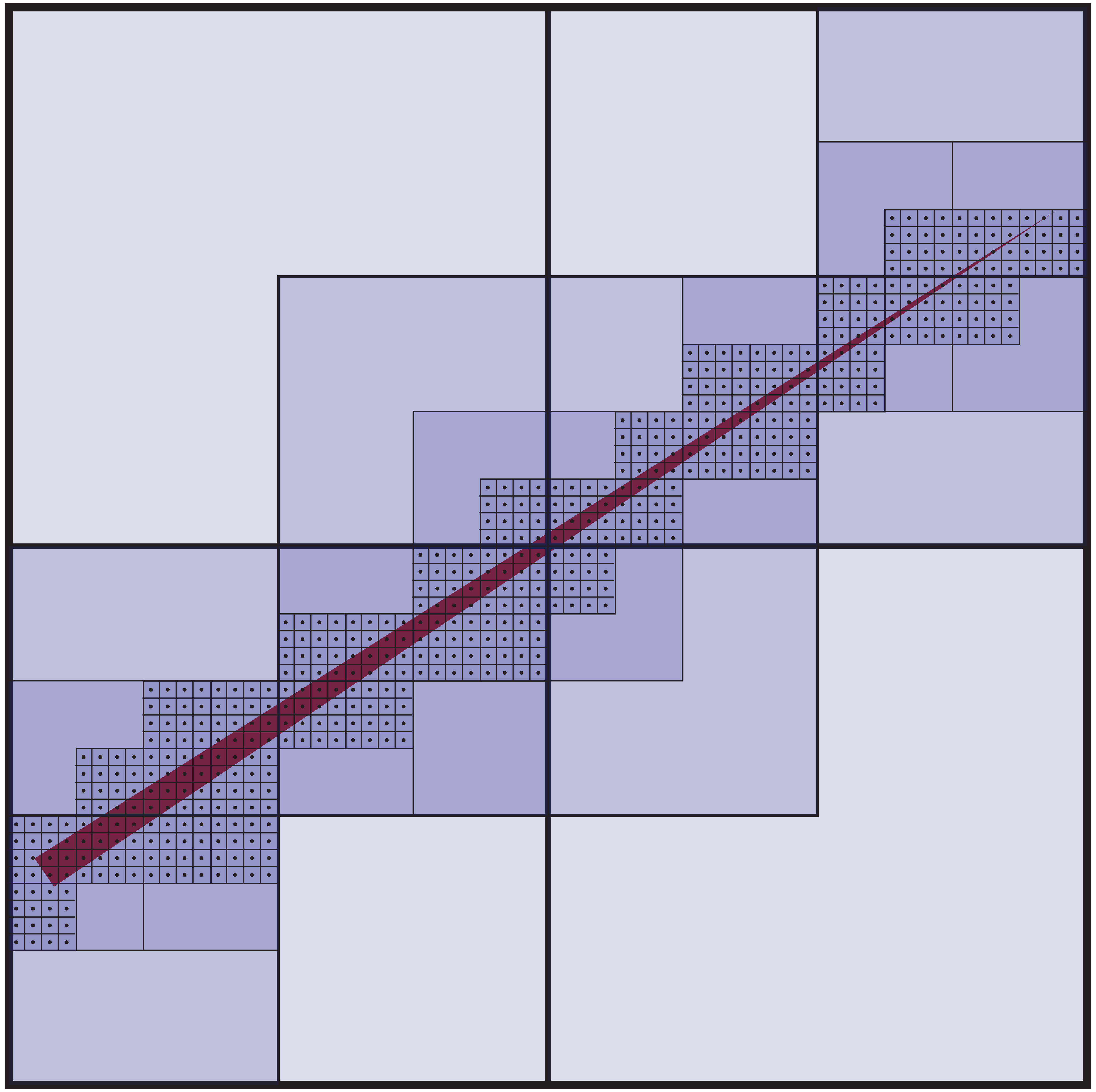


This how real graphics hardware works!

Can we do even better for this example?



Hierarchical strategies in computer graphics



Q: Better way to find finest blocks?   A: Maybe: incremental traversal!

Summary

- Can frame many graphics problems in terms of <u>sampling</u> and <u>reconstruction</u>
  - sampling: turn a continuous signal into `digital` information
  - reconstruction: turn `digital` information into a continuous signal
  - <u>aliasing</u> occurs when the reconstructed signal presents a false sense of what the original signal looked like
- Can frame rasterization as sampling problem
  - sample coverage function into pixel grid
  - reconstruct by emitting a "little square" of light for each pixel
  - aliasing manifests as jagged edges, shimmering artifacts, …
  - reduce aliasing via *supersampling*
- Triangle rasterization is basic building block for graphics pipeline
  - amounts to three half-plane tests
  - atomic operation—make it fast!
  - several strategies: incremental, parallel, blockwise, hierarchical…

Next time: 3D Transformations