Meshes and Manifolds

Computer Graphics
CMU 15-462/15-662

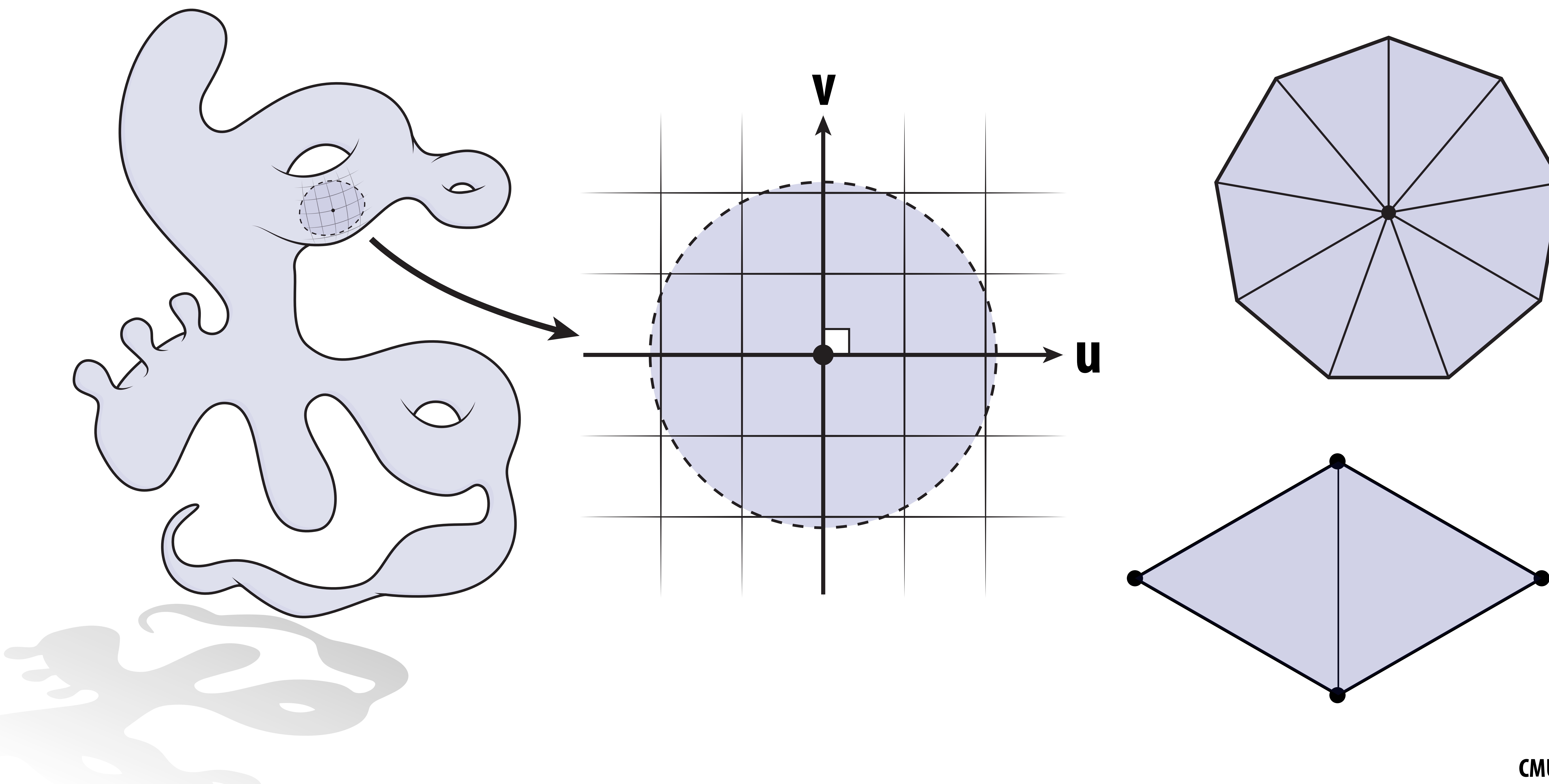
Last time: overview of geometry

- Many types of geometry in nature
- Demand sophisticated representations
- Two major categories:
  - IMPLICIT - “tests” if a point is in shape
  - EXPLICIT - directly “lists” points
- Lots of representations for both
- Today:
  - what is a surface, anyway?
  - nuts & bolts of polygon meshes
  - geometry processing / resampling

Geometry

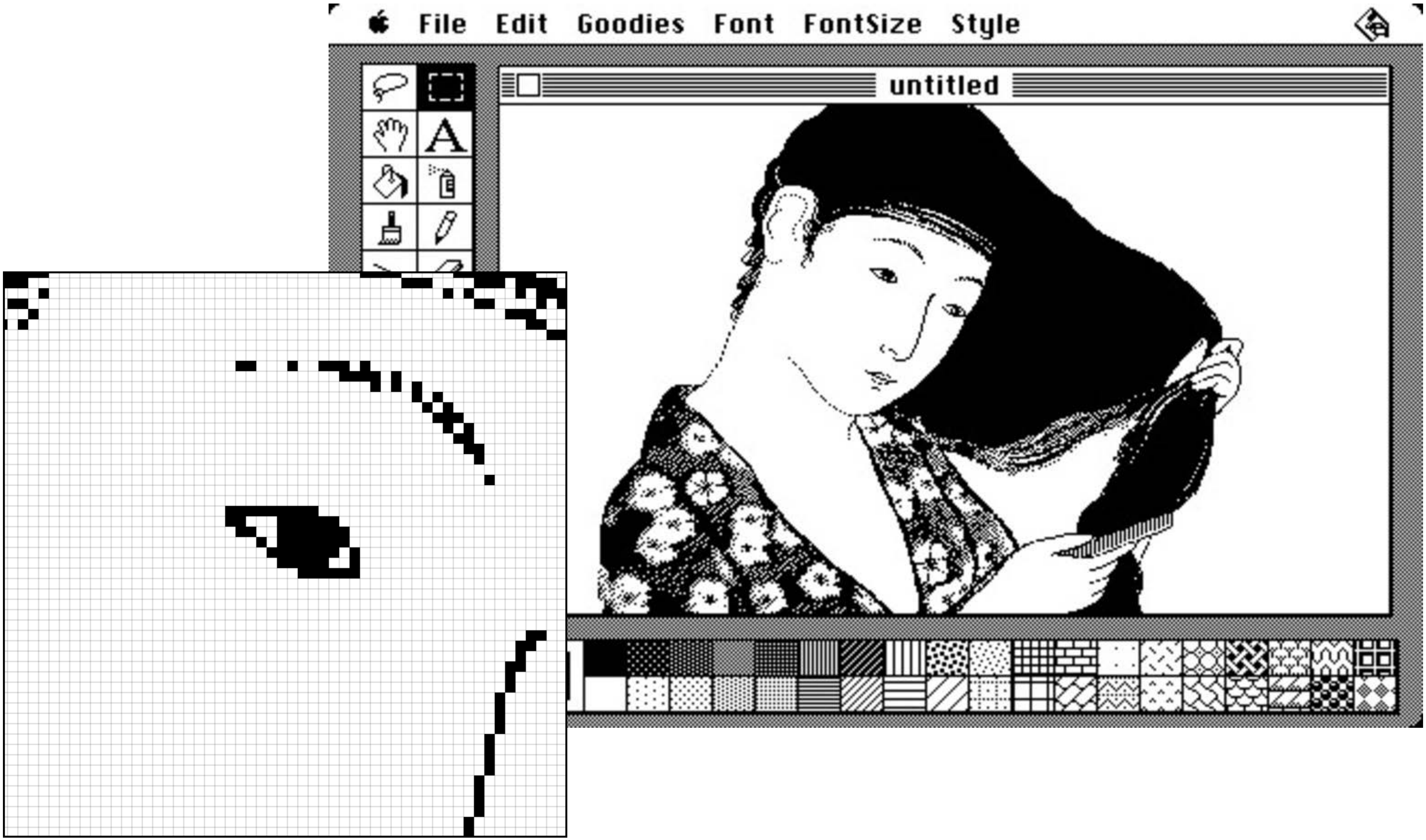
Manifold Assumption

- Today we're going to introduce the idea of *manifold* geometry
- Can be hard to understand motivation at first!
- So first, let's revisit a more familiar example...



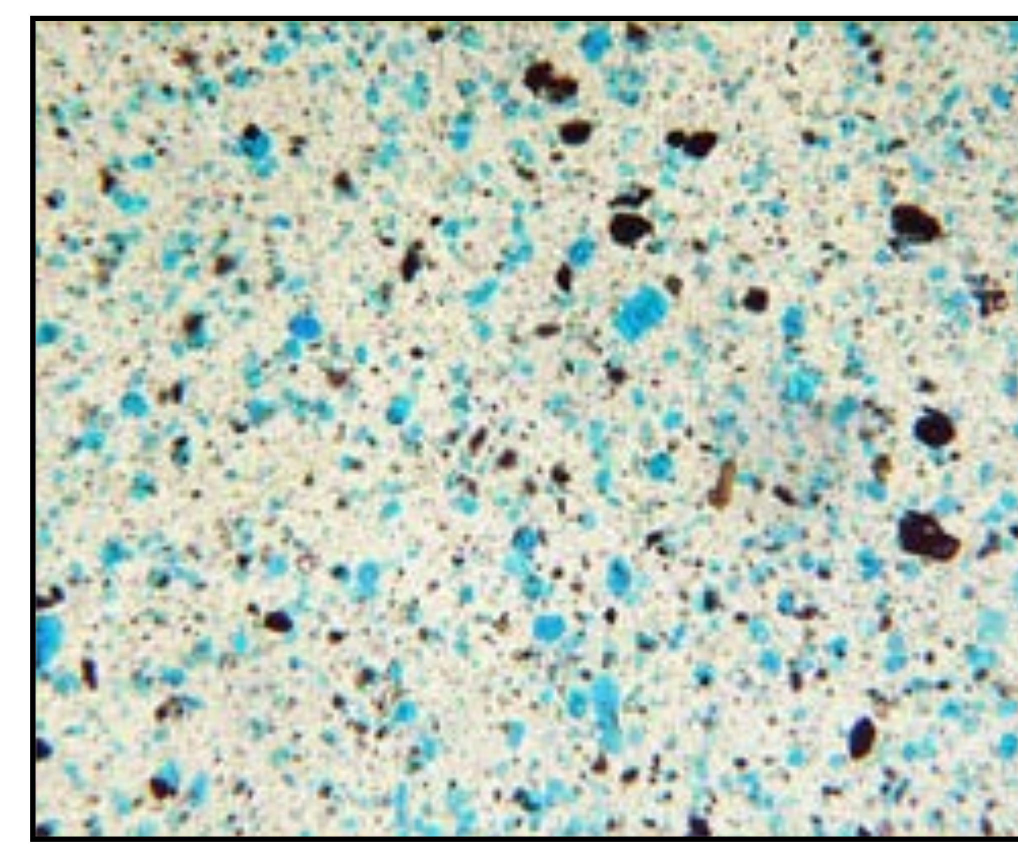
Bitmap Images, Revisited

To encode images, we used a *regular grid* of pixels:



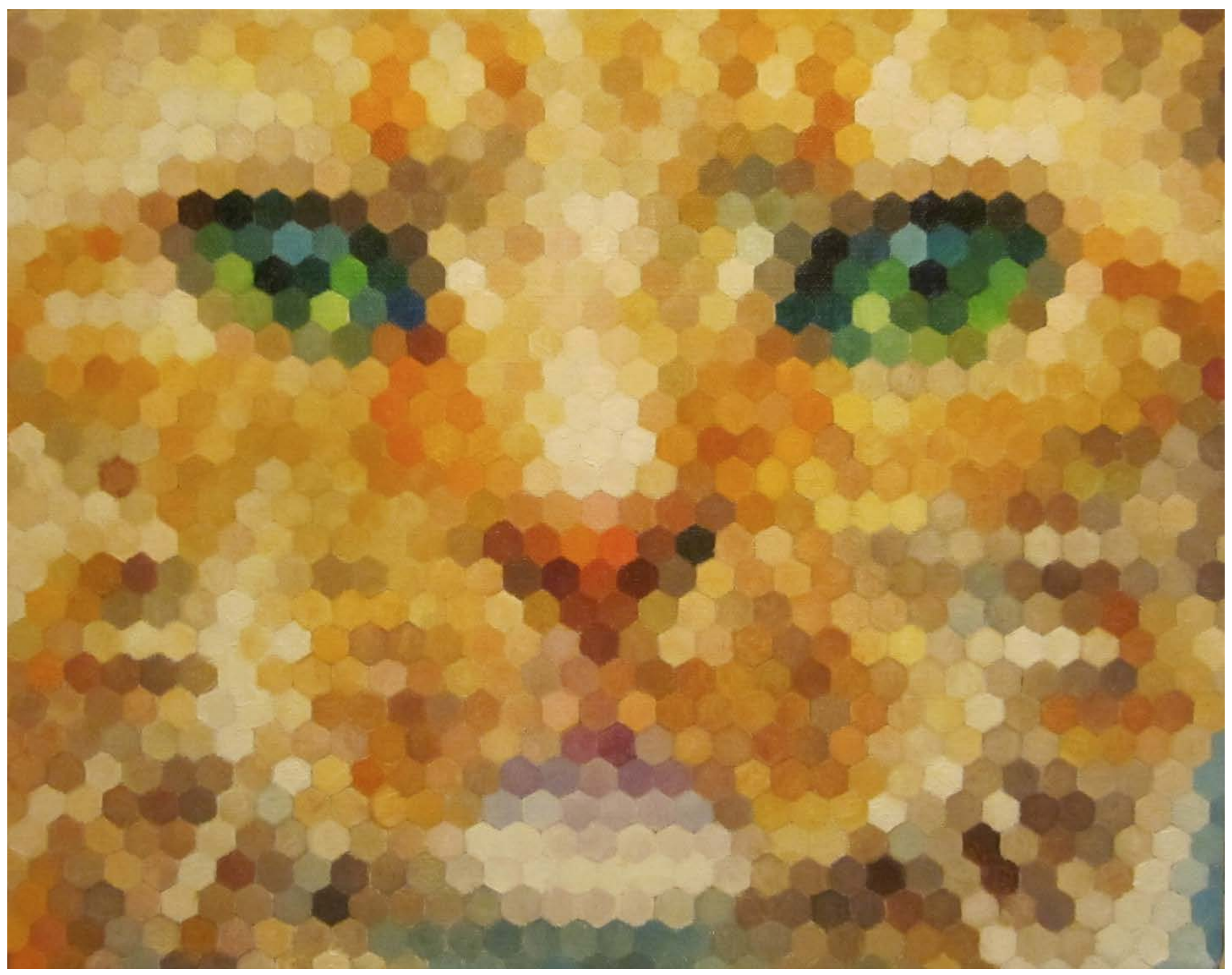
But images are not fundamentally made of little squares:

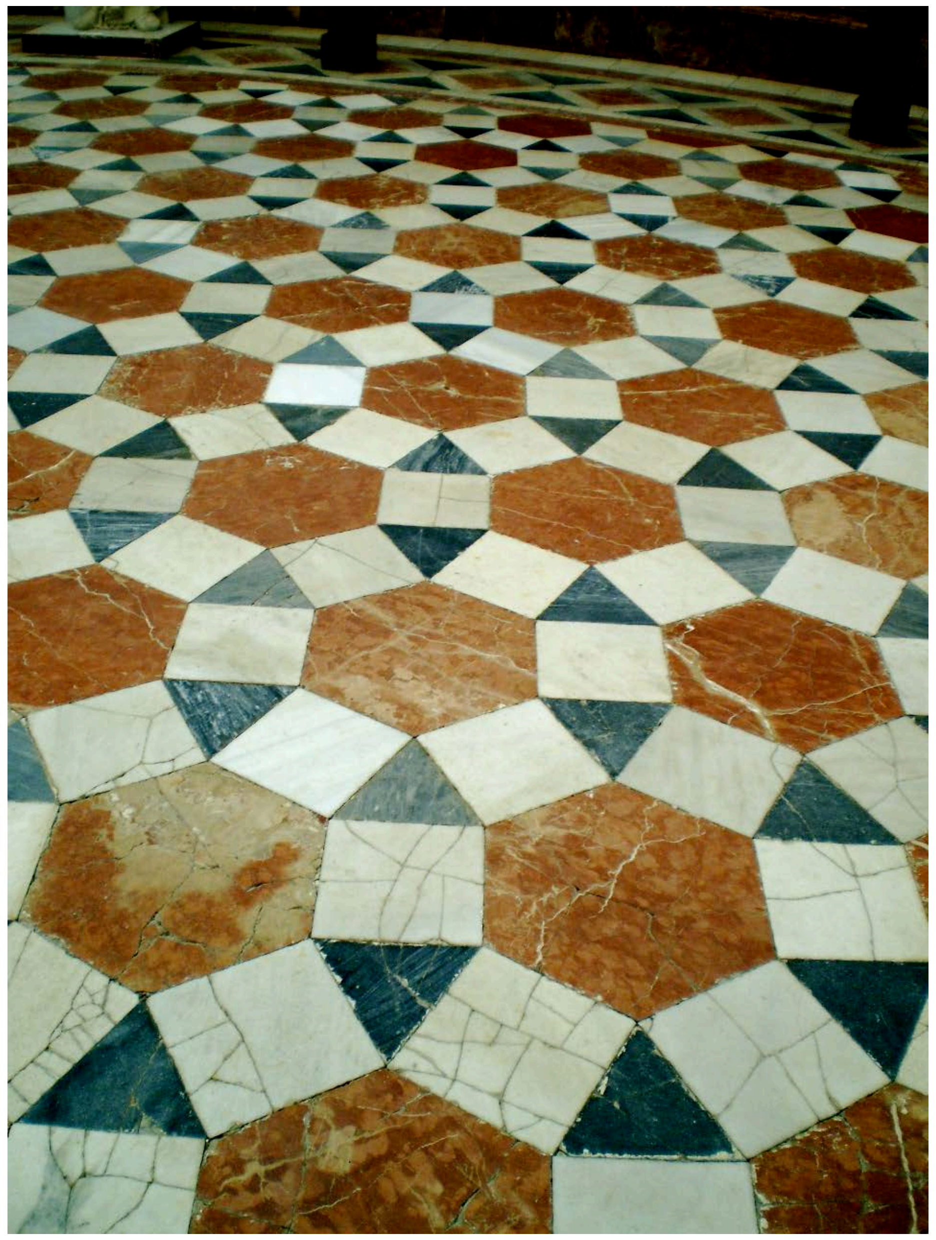
Goyō Hashiguchi, *Kamisuki* (ca 1920)



photomicrograph of paint

So why did we choose a square grid?





...rather than dozens of possible alternatives?

Regular grids make life easy

- One reason: SIMPLICITY / EFFICIENCY
  - E.g., always have four neighbors
  - Easy to index, easy to filter…
  - Storage is just a list of numbers
- Another reason: GENERALITY
  - Can encode basically any image
- Are regular grids *always* the best choice for bitmap images?
  - No! E.g., suffer from anisotropy, don't capture edges, ...
  - But *more often than not* are a pretty good choice
- Will see a similar story with geometry...

	`(i,j-1)`	
`(i-1,j)`	`(i,j)`	`(i+1,j)`
	`(i,j+1)`	

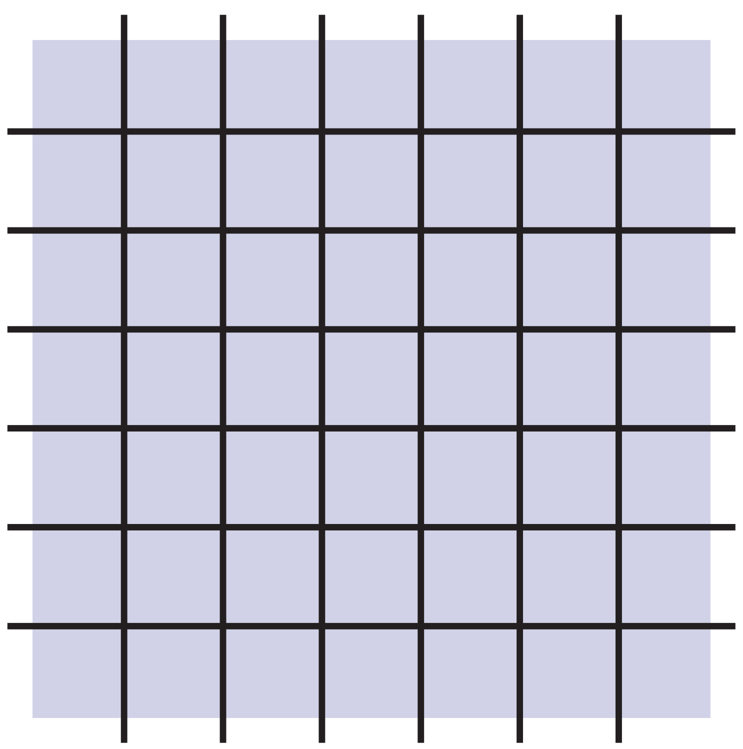
So, how should we encode surfaces?

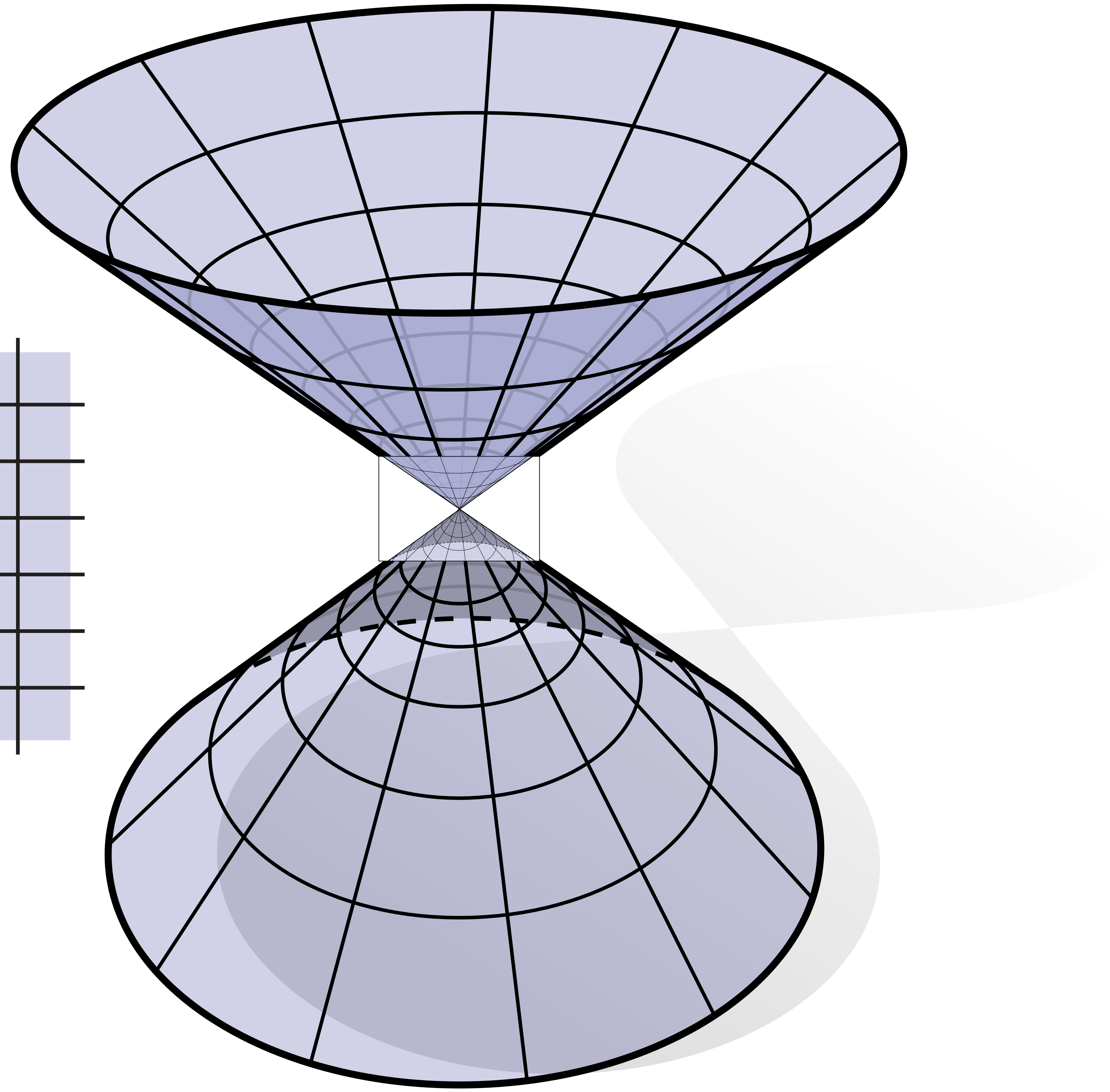
Smooth Surfaces

- Intuitively, a ***surface*** is the boundary or “shell” of an object
- (Think about the candy shell, not the chocolate.)
- Surfaces are *manifold*:
  - If you zoom in far enough, can draw a regular coordinate grid
  - E.g., the Earth from space vs. from the ground

Isn’t every shape manifold?

- No, for instance:

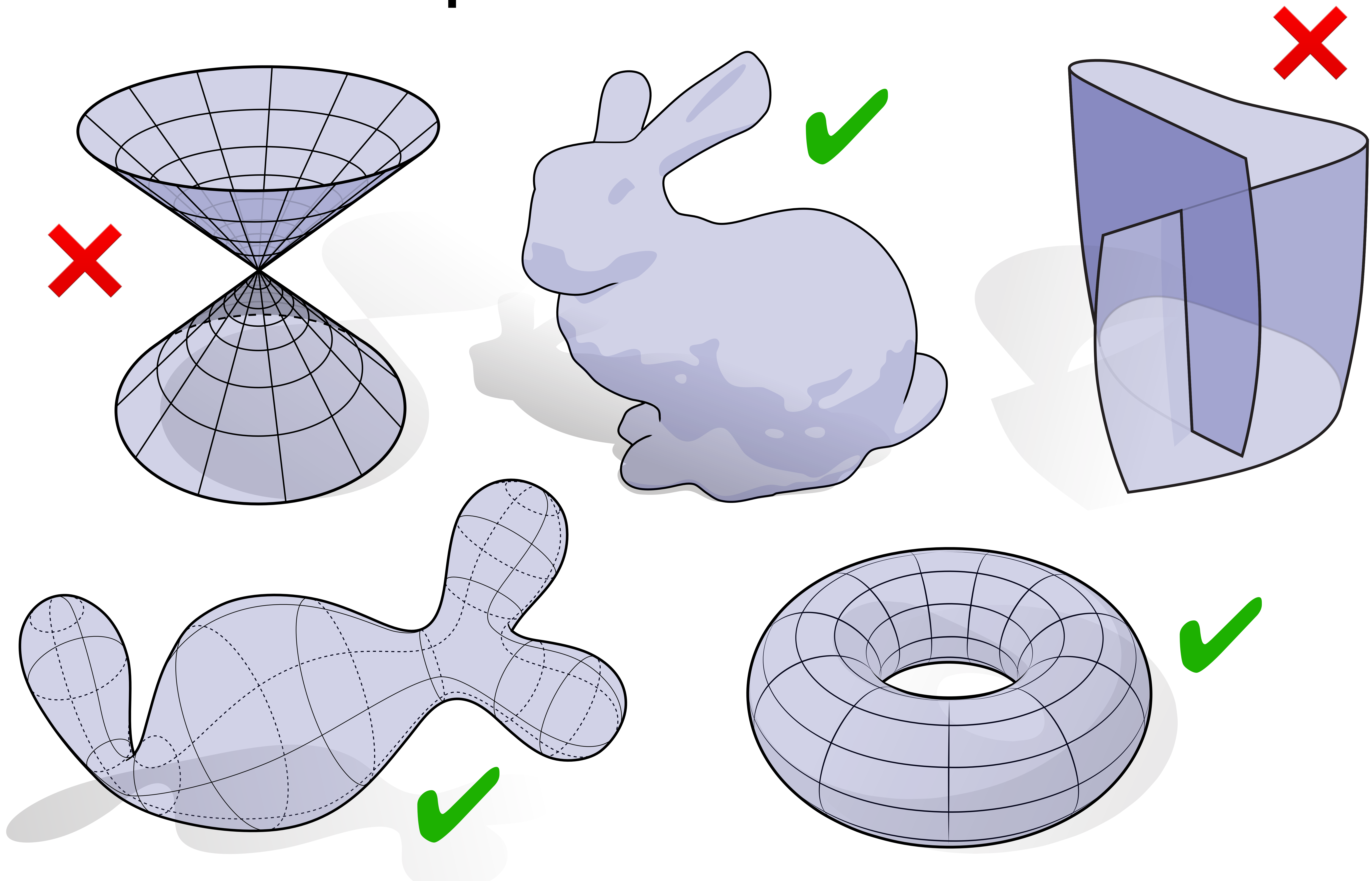




Can’t draw ordinary 2D grid at center, no matter how close we get.

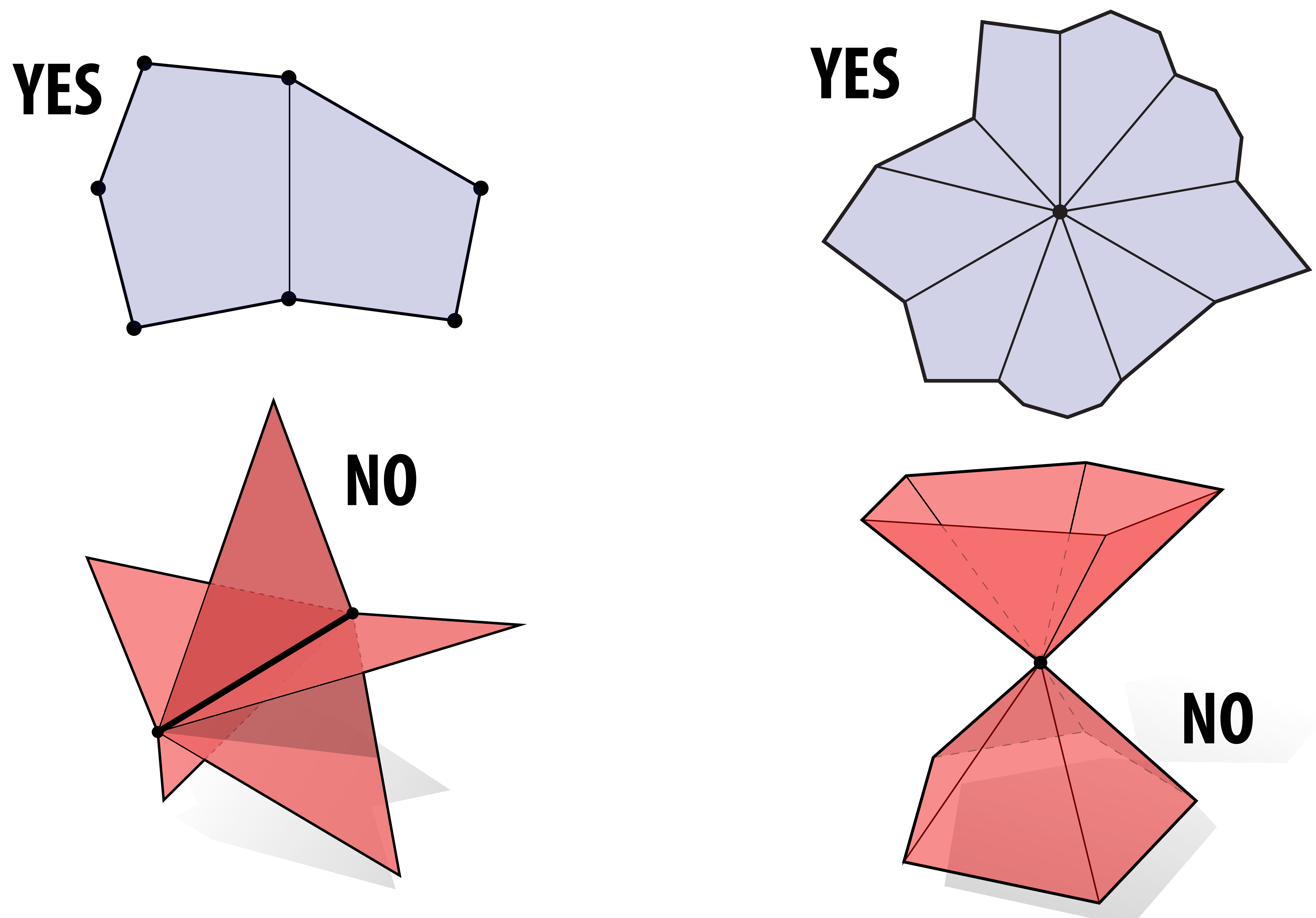
Examples—Manifold vs. Nonmanifold

- Which of these shapes are manifold?



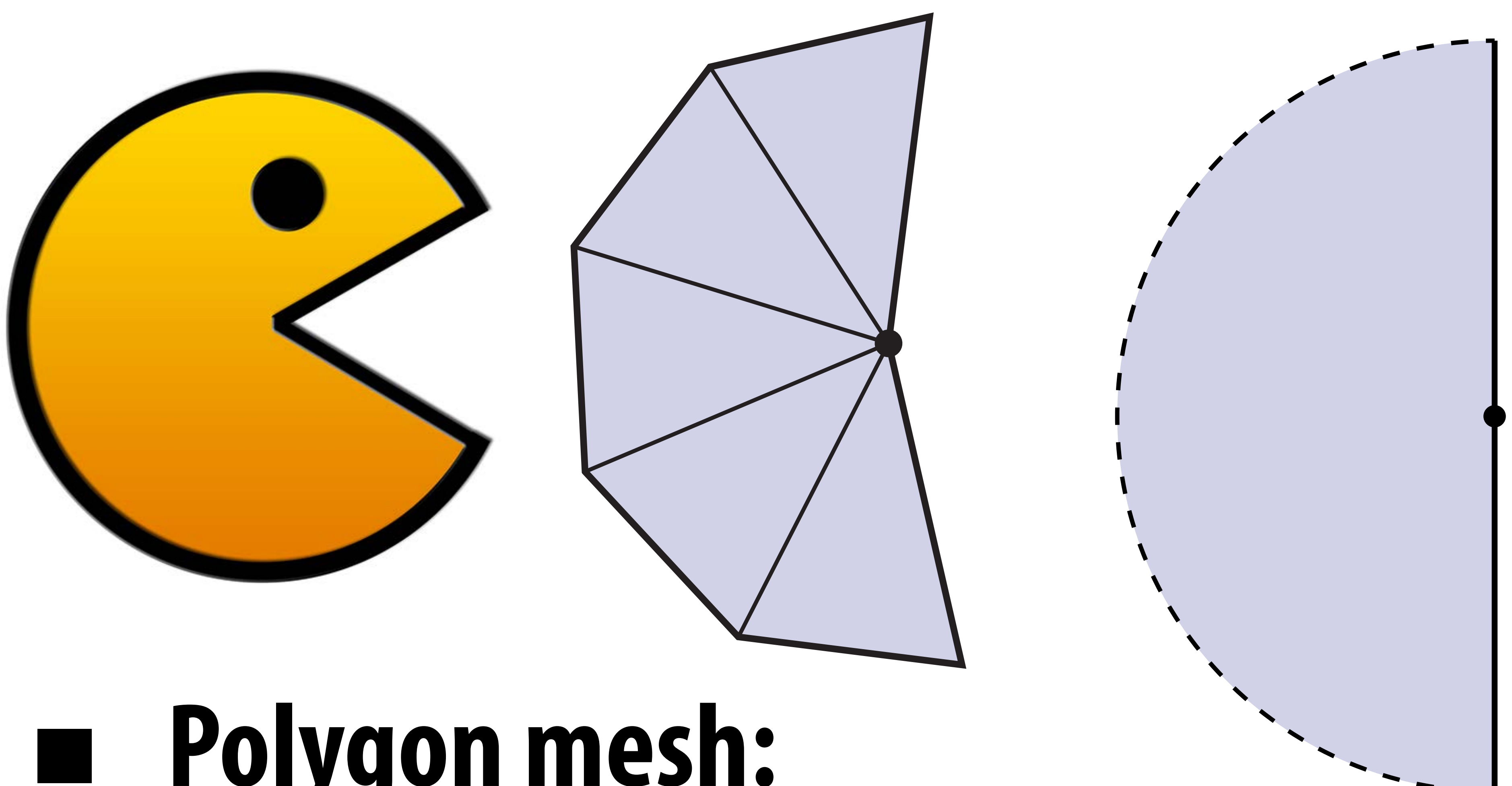
A manifold polygon mesh has fans, not fins

- For polygonal surfaces just two easy conditions to check:
  1. Every edge is contained in only two polygons (no “fins”)
  2. The polygons containing each vertex make a single “fan”



What about boundary?

- The boundary is where the surface "ends."
- E.g., waist & ankles on a pair of pants.
- Locally, looks like a *half* disk
- Globally, each boundary forms a loop

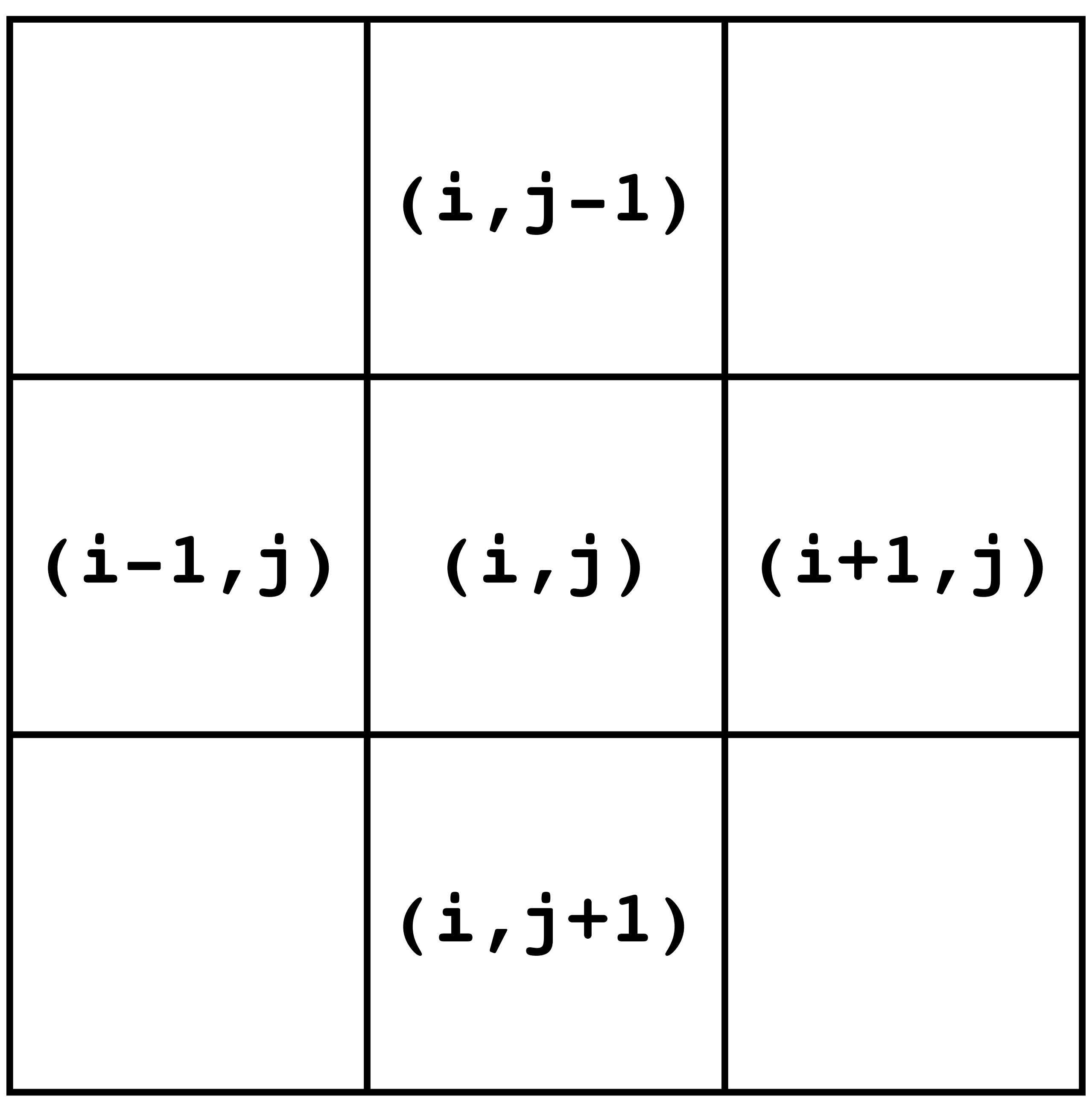


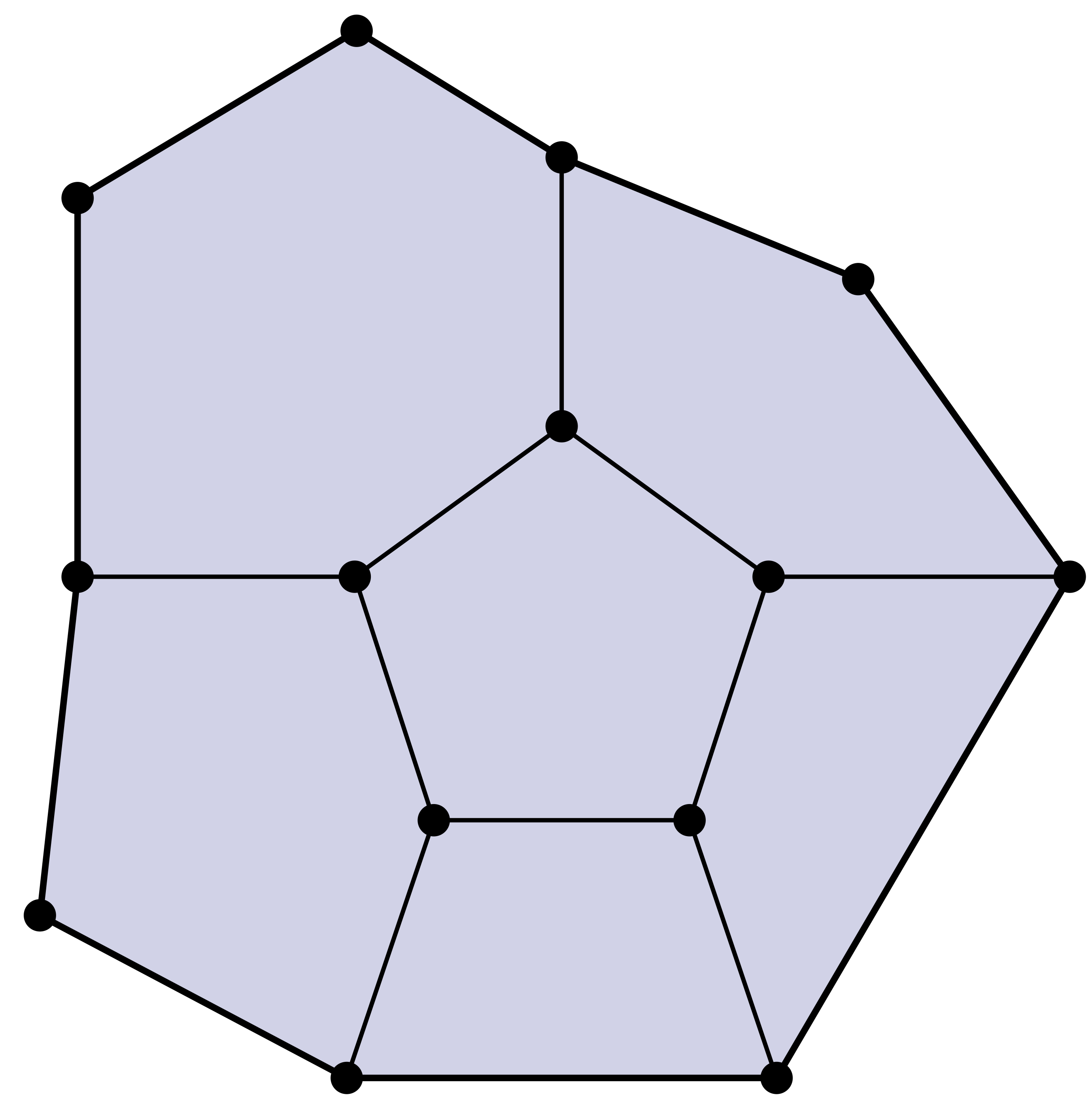
- Polygon mesh:
  - one polygon per boundary edge
  - boundary vertex looks like "pacman"

Ok, but why is the manifold assumption *useful?*

Keep it Simple!

- Same motivation as for images:
  - make some assumptions about our geometry to keep data structures/algorithms simple and efficient
  - in *many common cases*, doesn't fundamentally limit what we can do with geometry





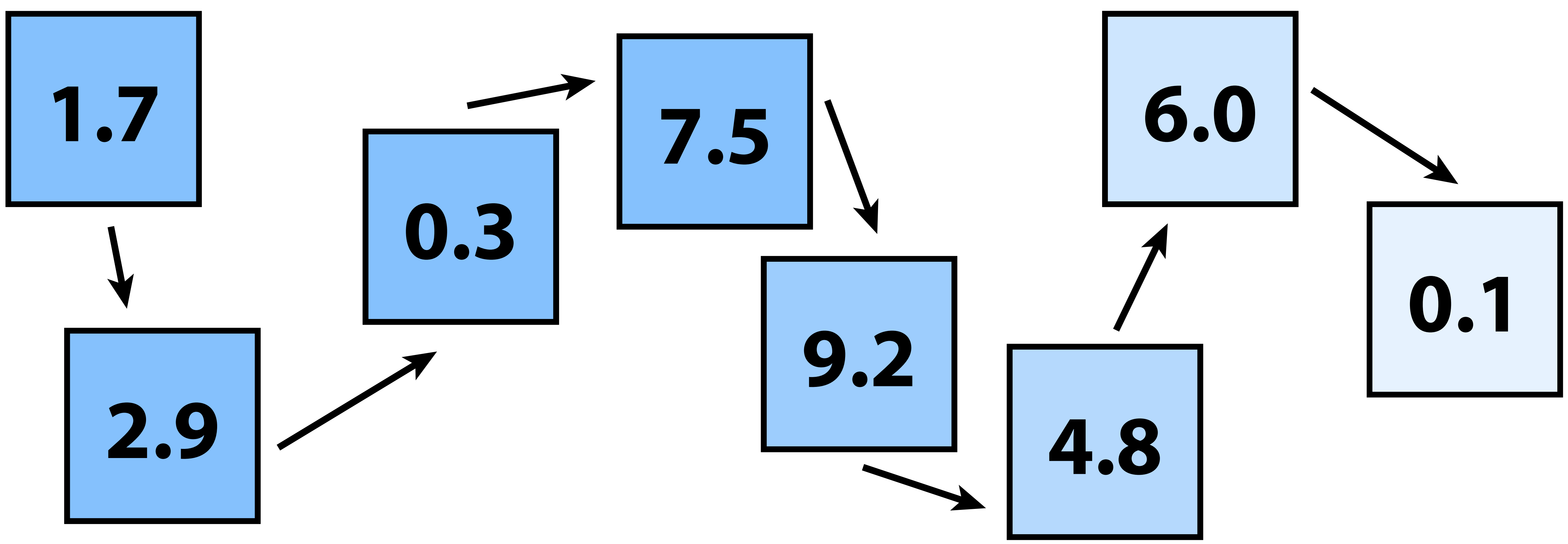
How do we actually encode all this data?

Warm up: storing numbers

- Q: What data structures can we use to store a list of numbers?
- One idea: use an *array* (constant time lookup, coherent access)

1.7	2.9	0.3	7.5	9.2	4.8	6.0	0.1

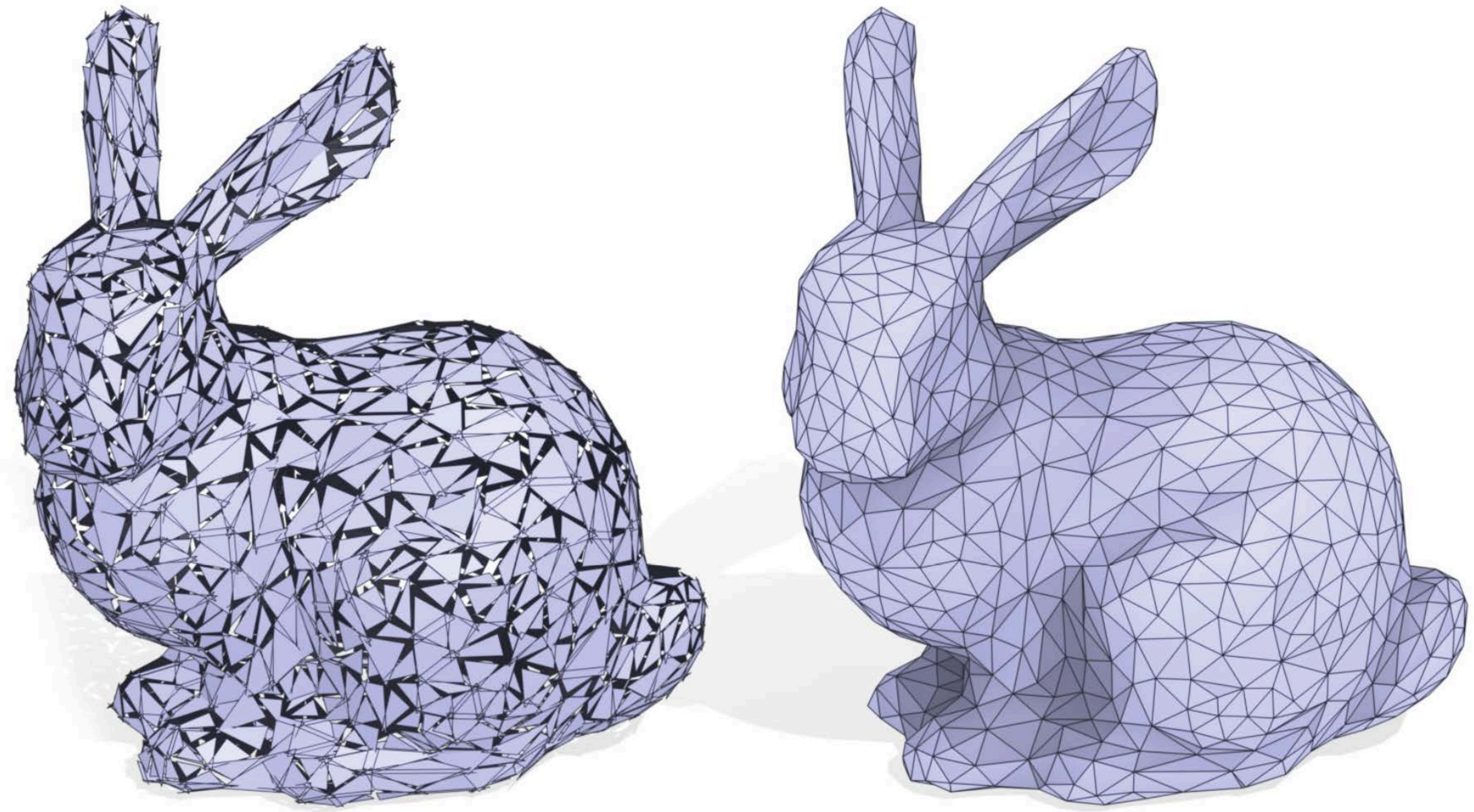
- Alternative: use a linked list (linear lookup, incoherent access)

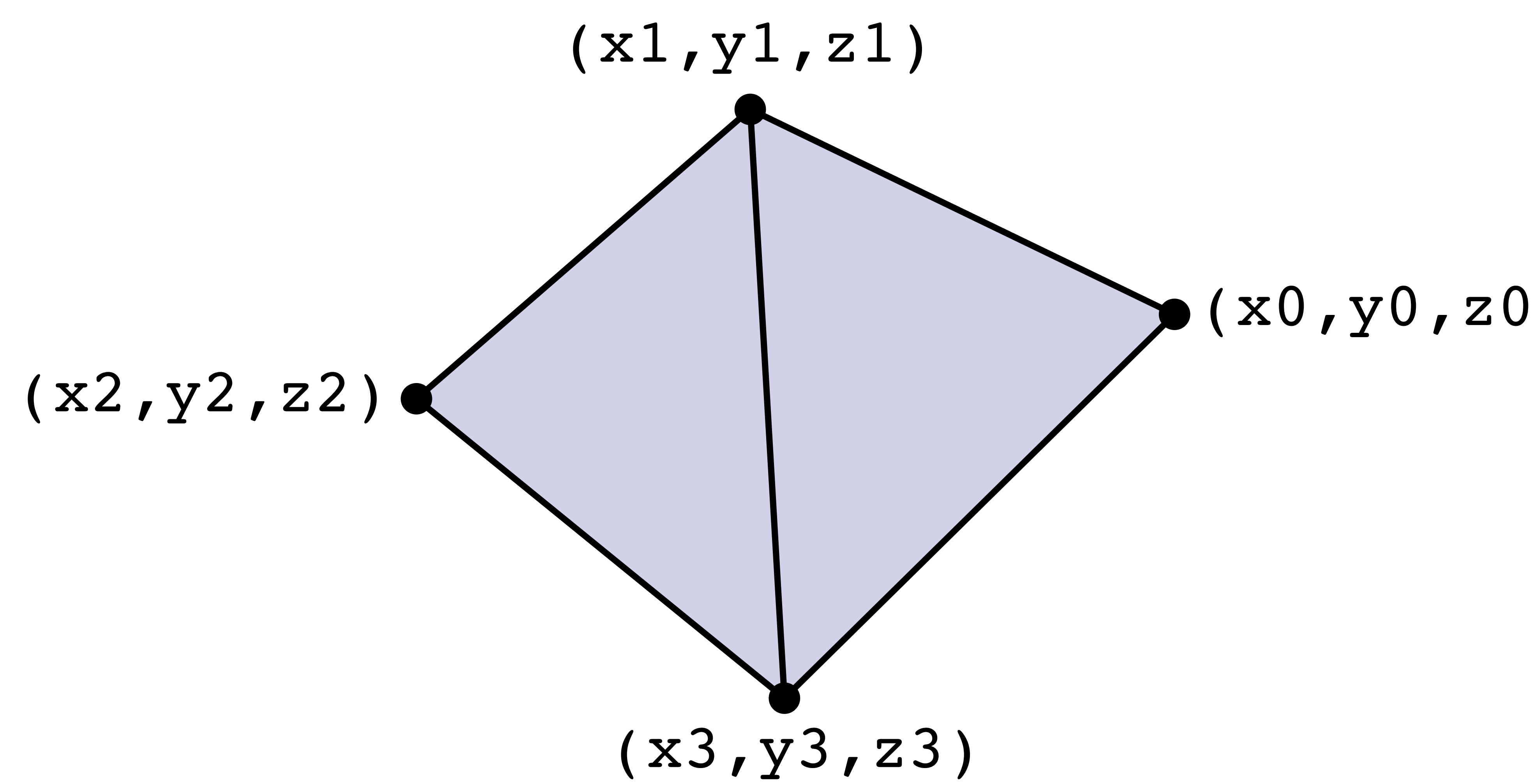


- Q: Why bother with the linked list?
- A: For one, we can easily insert numbers wherever we like...

Polygon Soup

- Most basic idea:
  - For each triangle, just store three coordinates
  - No other information about connectivity
  - Not much different from point cloud! ("Triangle cloud?")
- Pros:
  - Really stupidly simple
- Cons:
  - Redundant storage
  - Hard to do much beyond simply drawing the mesh on screen
  - Need spatial data structures (later) to find neighbors





x0,y0,z0	x1,y1,z1	x3,y3,z3
x1,y1,z1	x2,y2,z2	x3,y3,z3

Adjacency List (Array-like)

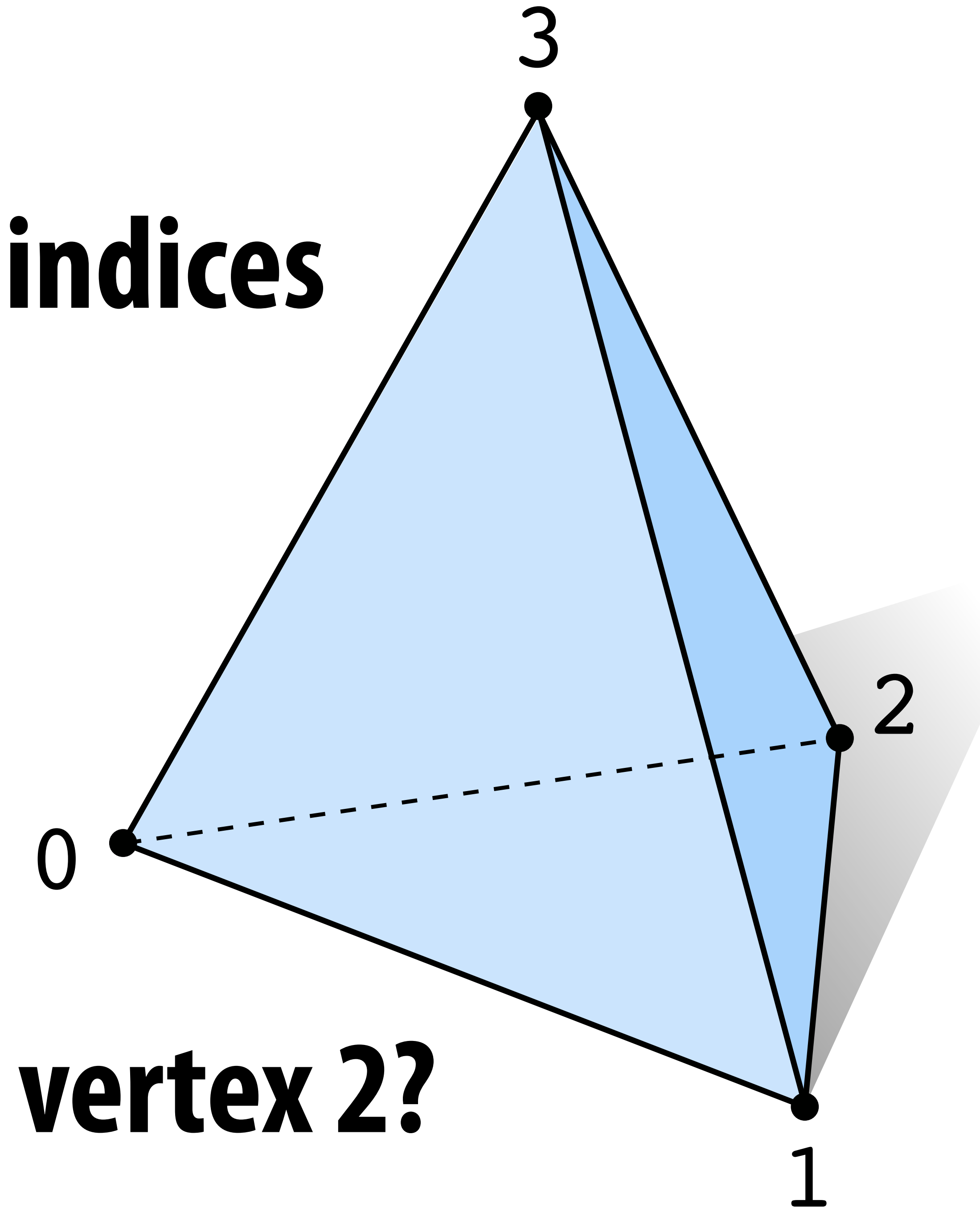
- Store triples of coordinates (x,y,z), tuples of indices
- E.g., tetrahedron:

VERTICES

	x	y	z
0:	−1	−1	−1
1:	1	−1	1
2:	1	1	−1
3:	−1	1	1

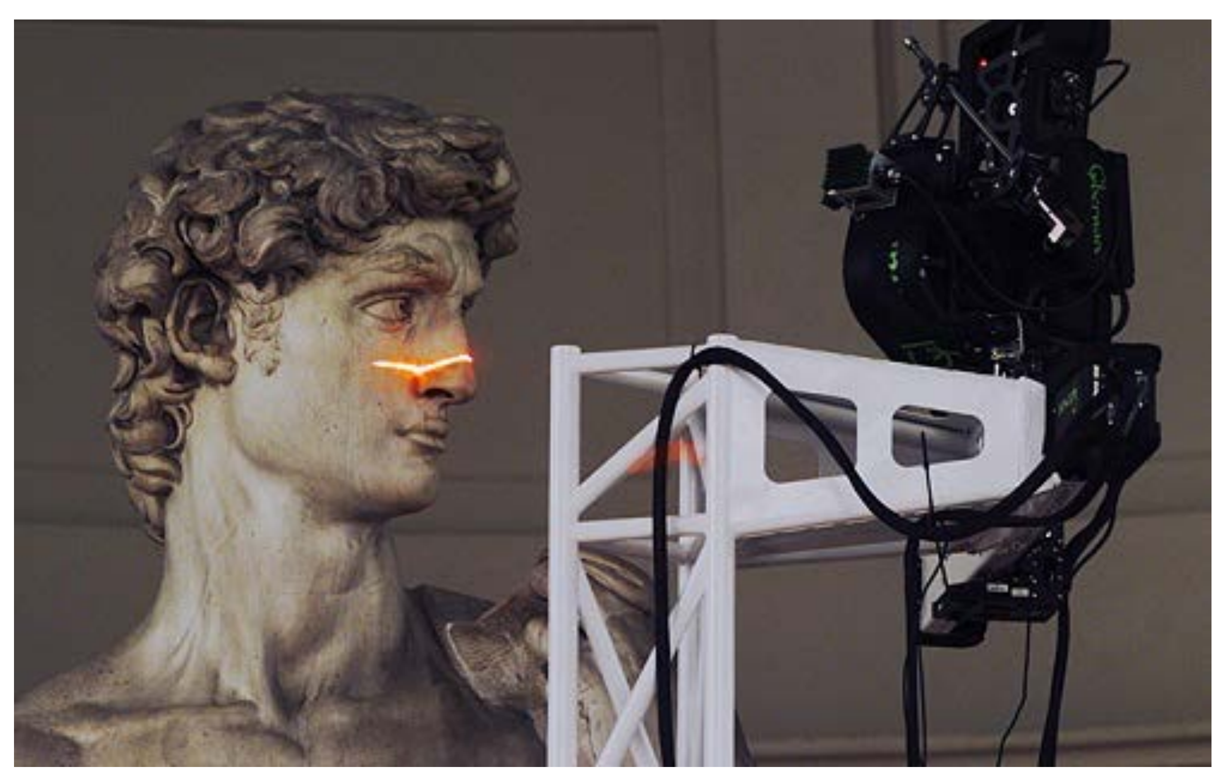
POLYGONS

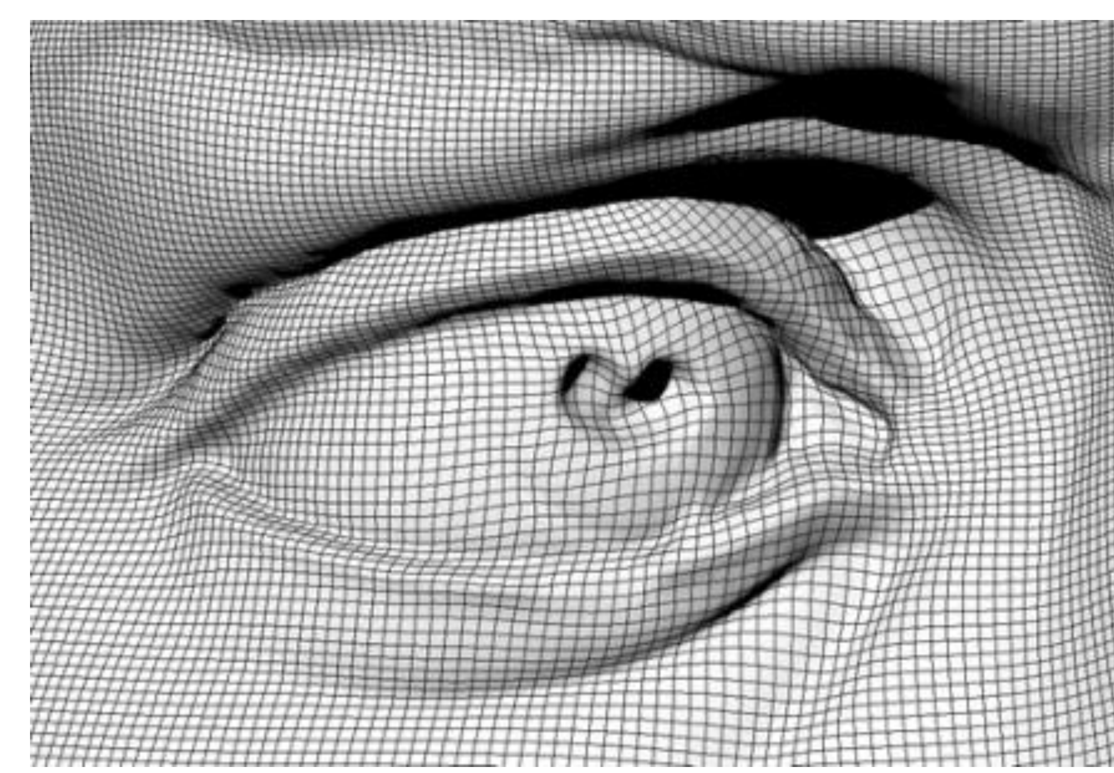
i	j	k
0	2	1
0	3	2
3	0	1
3	1	2



- Q: How do we find all the polygons touching vertex 2?
- Ok, now consider a more complicated mesh:

~1 *billion* polygons





Very **expensive to find the neighboring polygons! (What's the cost?)**

Incidence Matrices

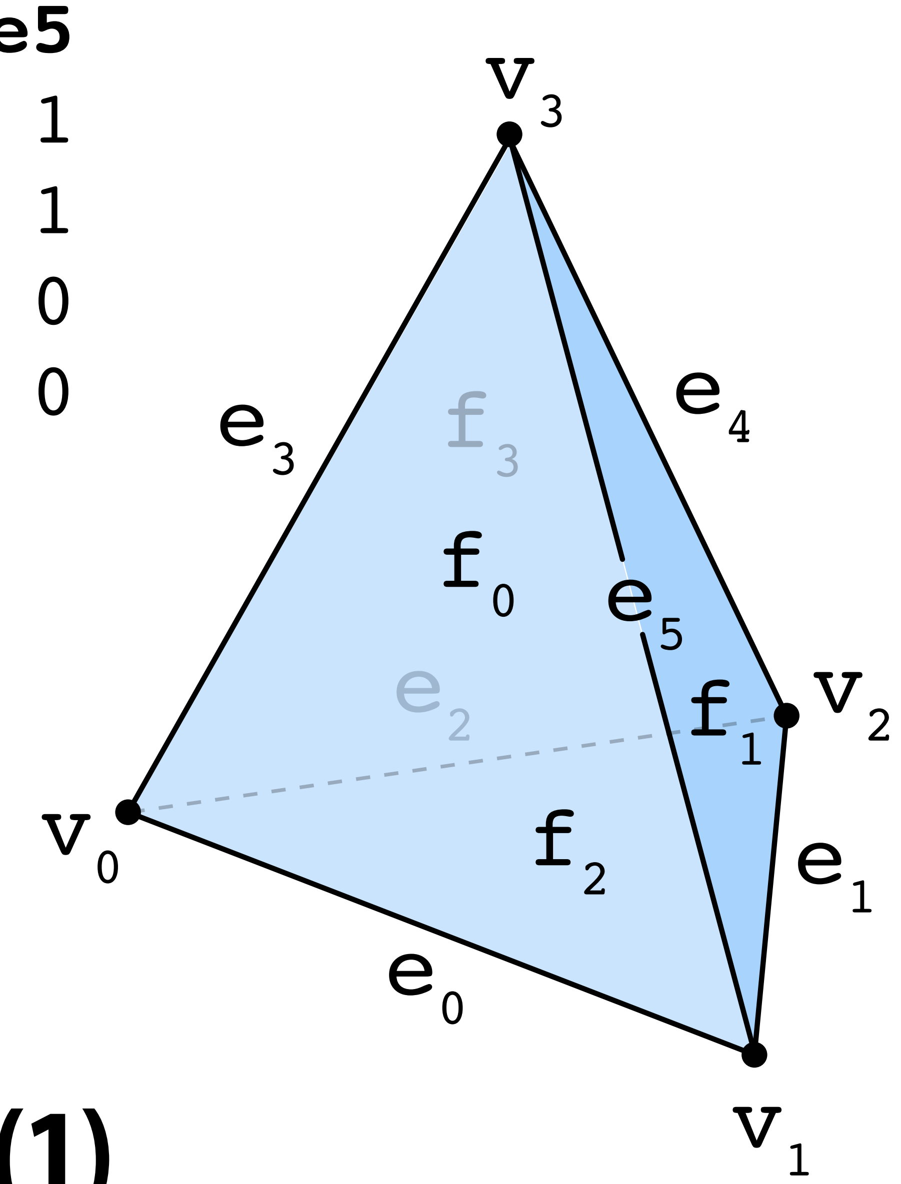
- If we want to know who our neighbors are, why not just store a list of neighbors?
- Can encode all neighbor information via *incidence matrices*
- E.g., tetrahedron:

VERTEX↔EDGE

	v0	v1	v2	v3
e0	1	1	0	0
e1	0	1	1	0
e2	1	0	1	0
e3	1	0	0	1
e4	0	0	1	1
e5	0	1	0	1

EDGE↔FACE

	e0	e1	e2	e3	e4	e5
f0	1	0	0	1	0	1
f1	0	1	0	0	1	1
f2	1	1	1	0	0	0
f3	0	0	1	1	1	0



- 1 means "touches"; 0 means "does not touch"
- Instead of storing lots of 0's, use *sparse matrices*
- Still large storage cost, but finding neighbors is now O(1)
- Hard to change connectivity, since we used fixed indices
- Bonus feature: mesh does not have to be manifold

Aside: Sparse Matrix Data Structures

$$\begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 4 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 7 & 0 \end{bmatrix} \end{array}$$

- **Ok, but how do we actually store a "sparse matrix"?**
- **Lots of possible data structures:**
  - **<u>Associative array</u> from `(row, column)` to `value`**
    - **easy to lookup/set entries, fast (e.g., hash table)**
    - **harder to do matrix operations (e.g., multiplication)**
  - **<u>Array of linked lists</u> (one per row)**
    - **conceptually simple**
    - **slow access time, incoherent memory access**
  - **<u>Compressed column format</u>—pack entries in list**
    - **hard to add/modify entries**
    - **fast for actual matrix operations**
- **In practice: often build up entries using an "easier" data structure, convert to compressed format for computation**

```
(row,col)  val
(0,0) -> 4
(0,1) -> 2
(1,2) -> 3
(2,1) -> 7
```

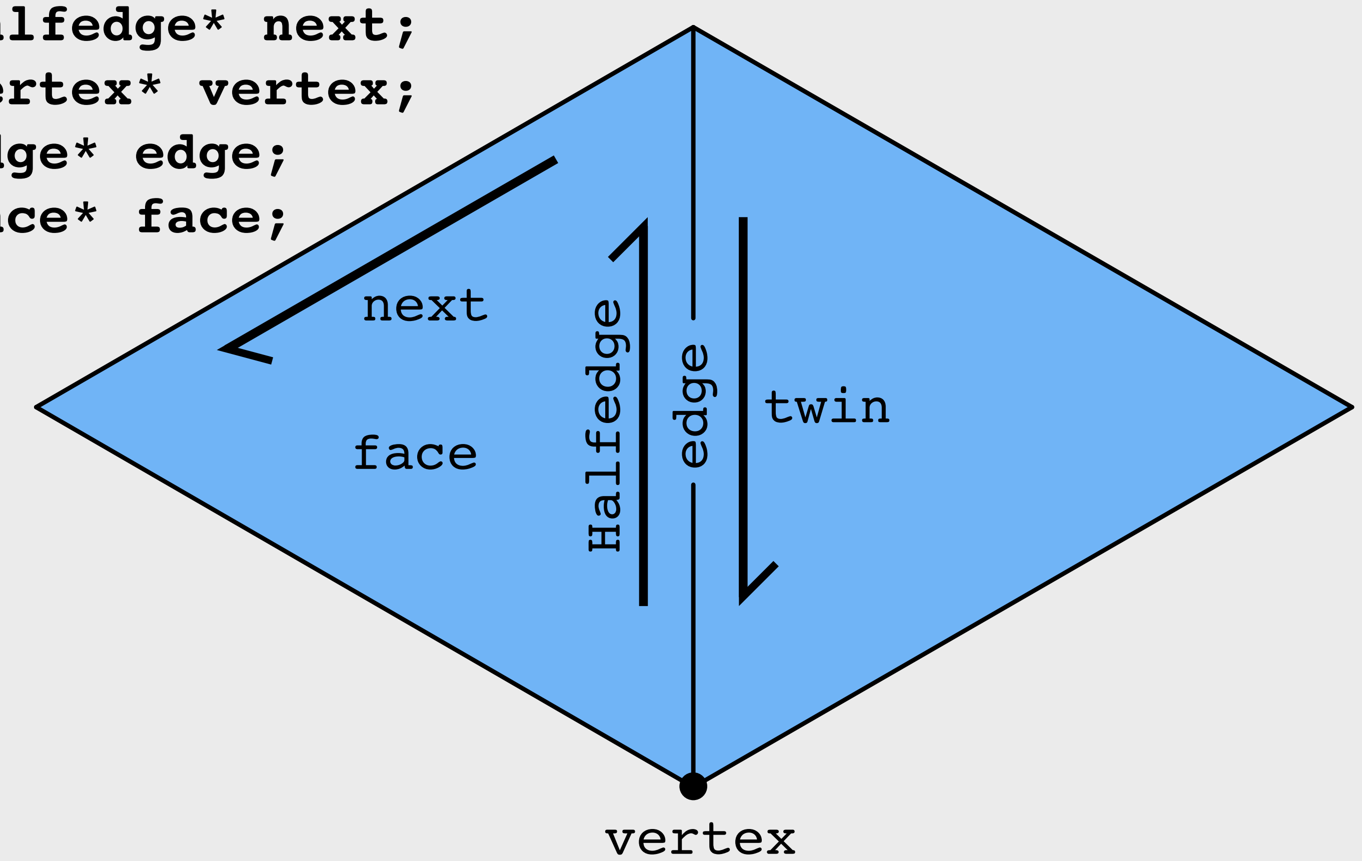
```
          (col,val)    (col,val)
row 0:    (0,4)   -->  (1,2)
    1:    (2,3)
    2:    (1,7)
```

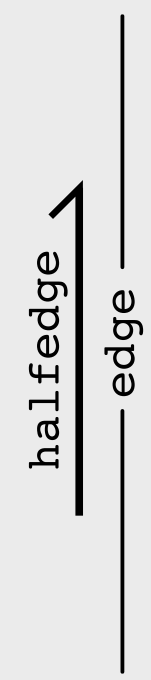
```
values            4,2,7,3
row indices       0,0,2,1
cumulative        1,3,4
# entries
by column
```

Halfedge Data Structure (Linked-list-like)

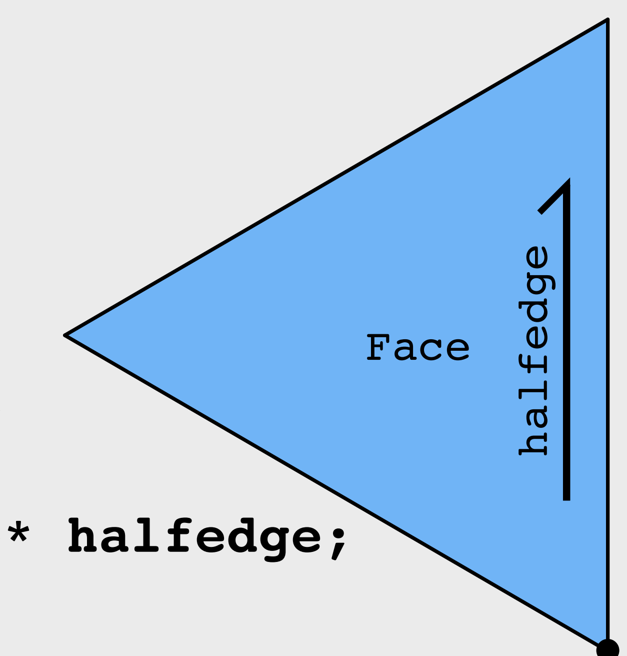
- Store *some* information about neighbors
- Don't need an exhaustive list; just a few key pointers
- Key idea: two *halfedges* act as "glue" between mesh elements:

```
struct Halfedge
{
   Halfedge* twin;
   Halfedge* next;
   Vertex* vertex;
   Edge* edge;
   Face* face;
};
```

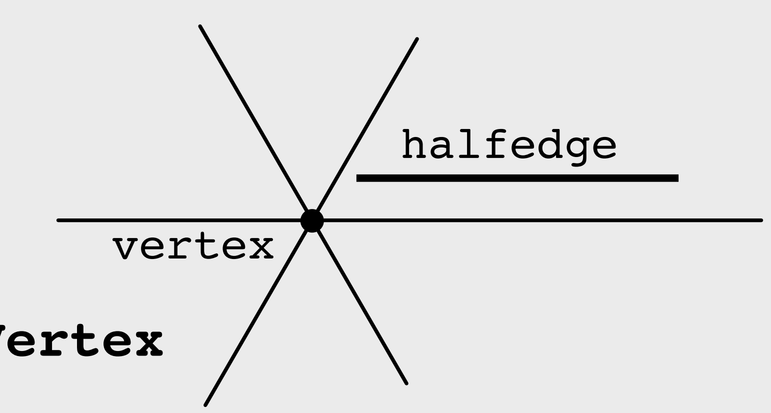





```
struct Edge
{
   Halfedge* halfedge;
};
```




```
struct Face
{
   Halfedge* halfedge;
};
```



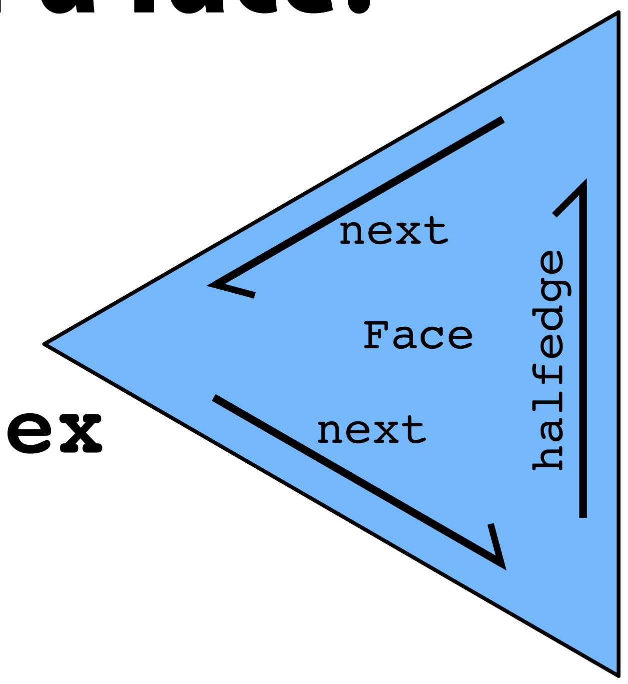

```
struct Vertex
{
   Halfedge* halfedge;
};
```

- Each vertex, edge face points to just *one* of its halfedges.

Halfedge makes mesh traversal easy

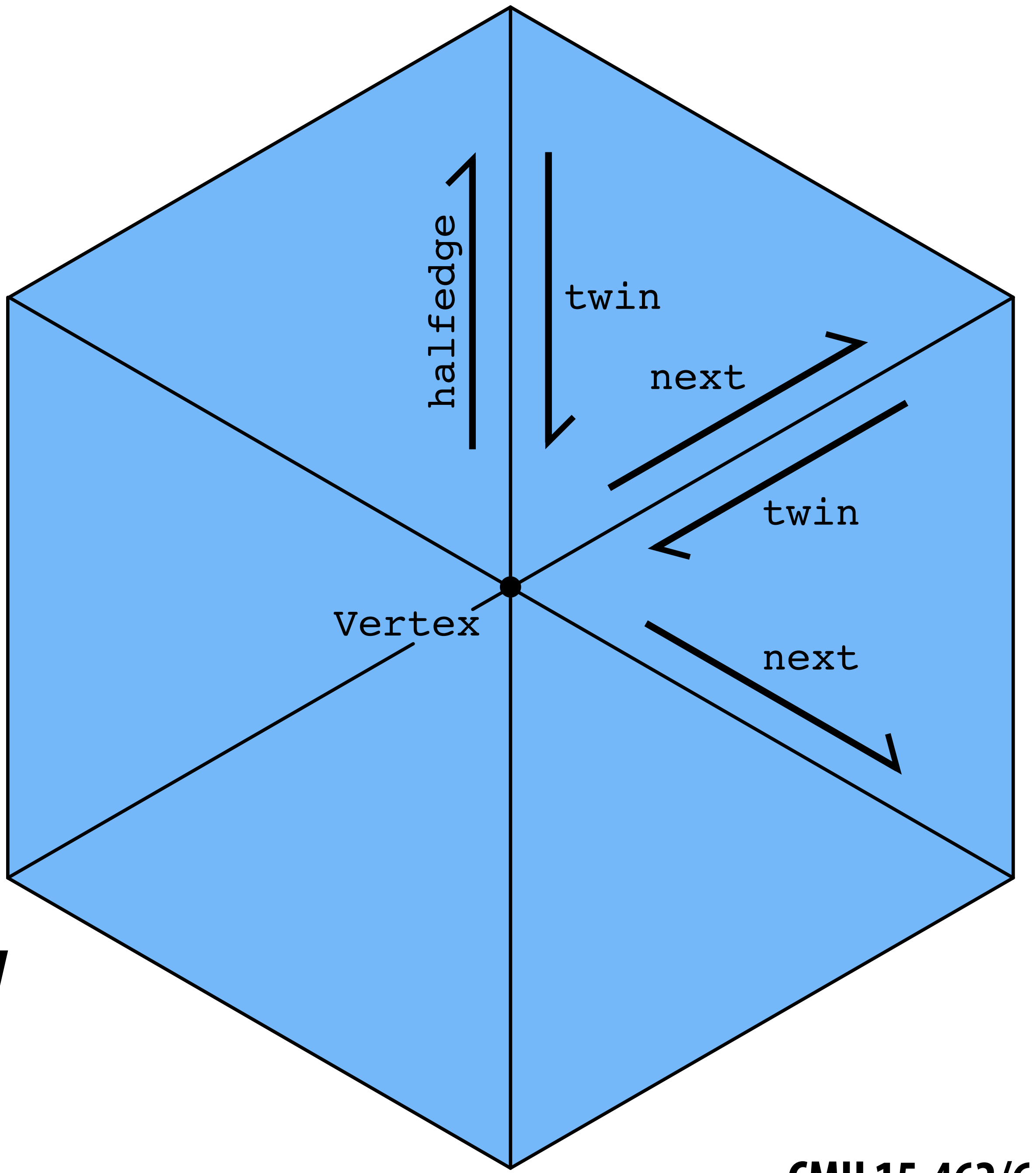
- Use “twin” and “next” pointers to move around mesh
- Use “vertex”, “edge”, and “face” pointers to grab element
- Example: visit all vertices of a face:

```
Halfedge* h = f->halfedge;
do {
   h = h->next;
   // do something w/ h->vertex
}
while( h != f->halfedge );
```



- Example: visit all neighbors of a vertex:

```
Halfedge* h = v->halfedge;
do {
   h = h->twin->next;
}
while( h != v->halfedge );
```



- Note: only makes sense if mesh is *manifold!*

Halfedge connectivity is *always* manifold

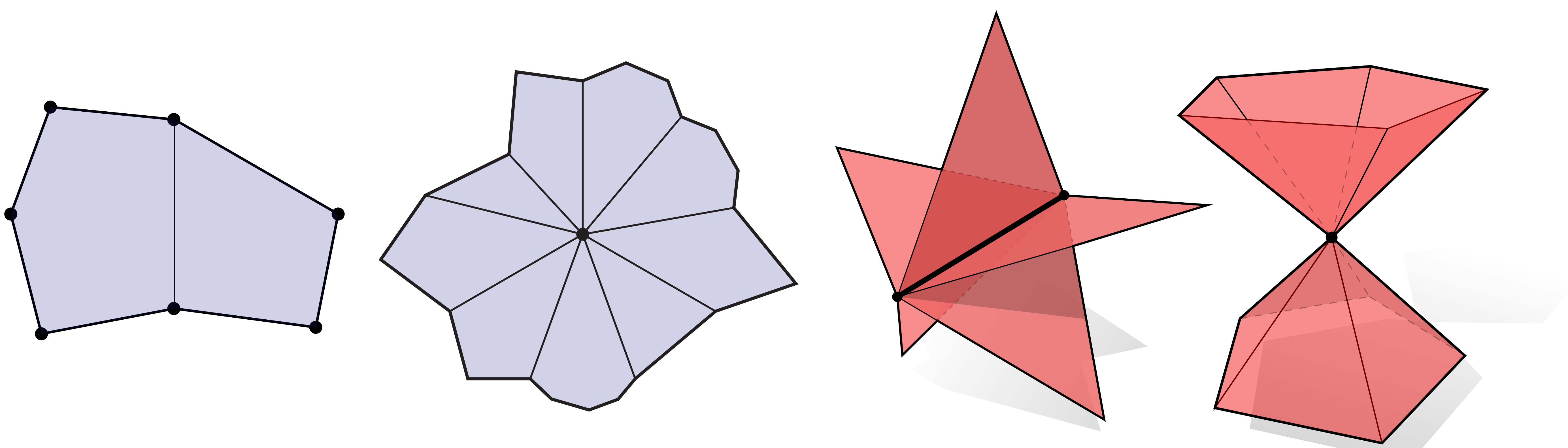
- Consider simplified halfedge data structure
- Require only "common-sense" conditions

```
struct Halfedge {
   Halfedge *next, *twin;
};
```

(pointer to yourself!)

```
twin->twin == this
twin != this
every he is someone's "next"
```

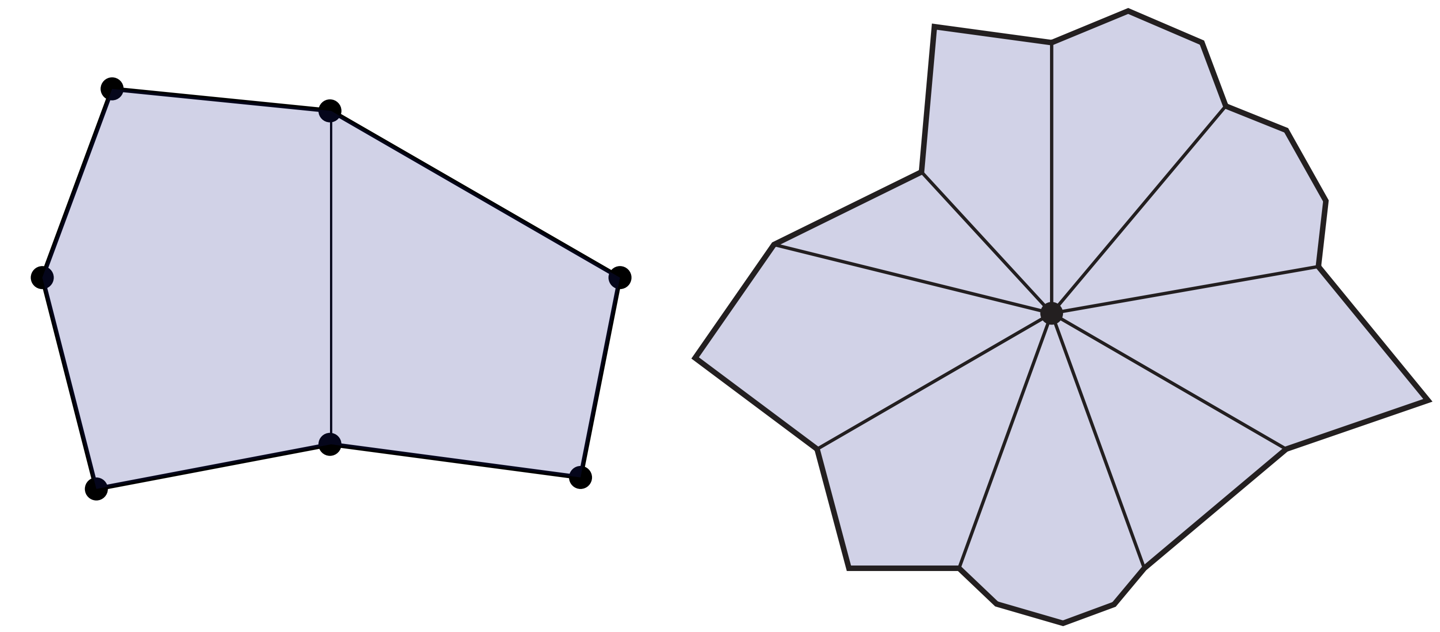
- Keep following `next`, and you'll get faces.
- Keep following `twin` and you'll get edges.
- Keep following `next->twin` and you'll get vertices.

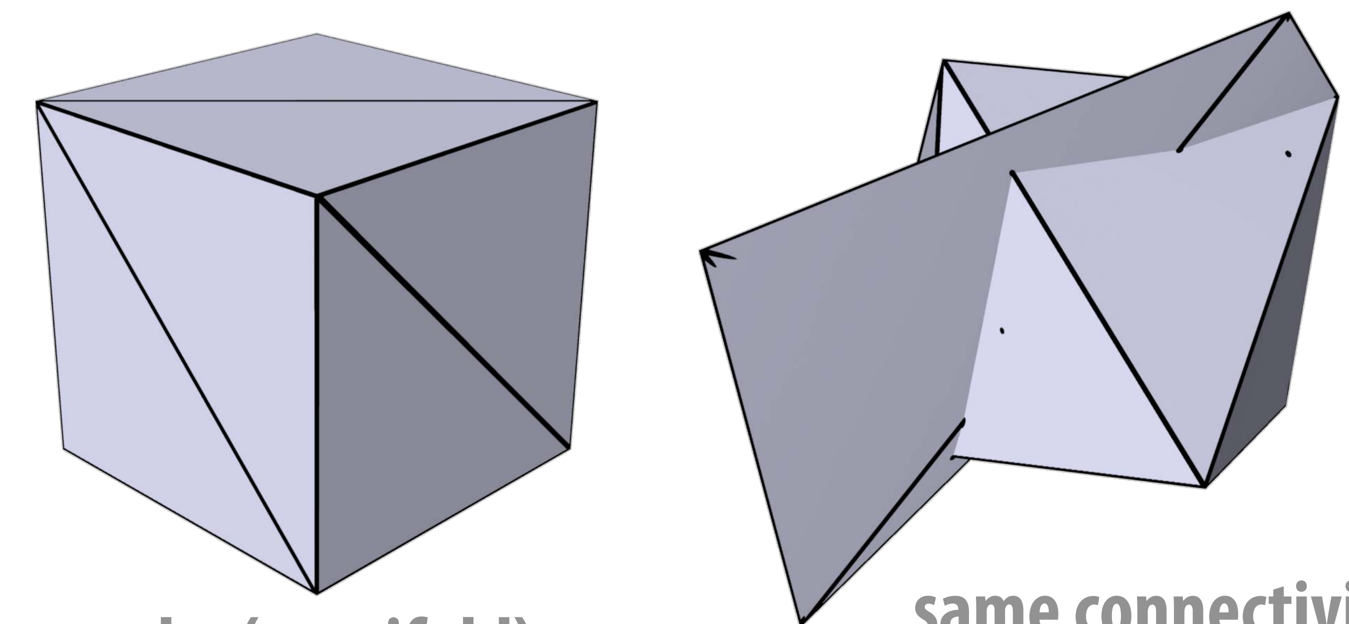


Q: Why, therefore, is it impossible to encode the red figures?

Connectivity vs. Geometry

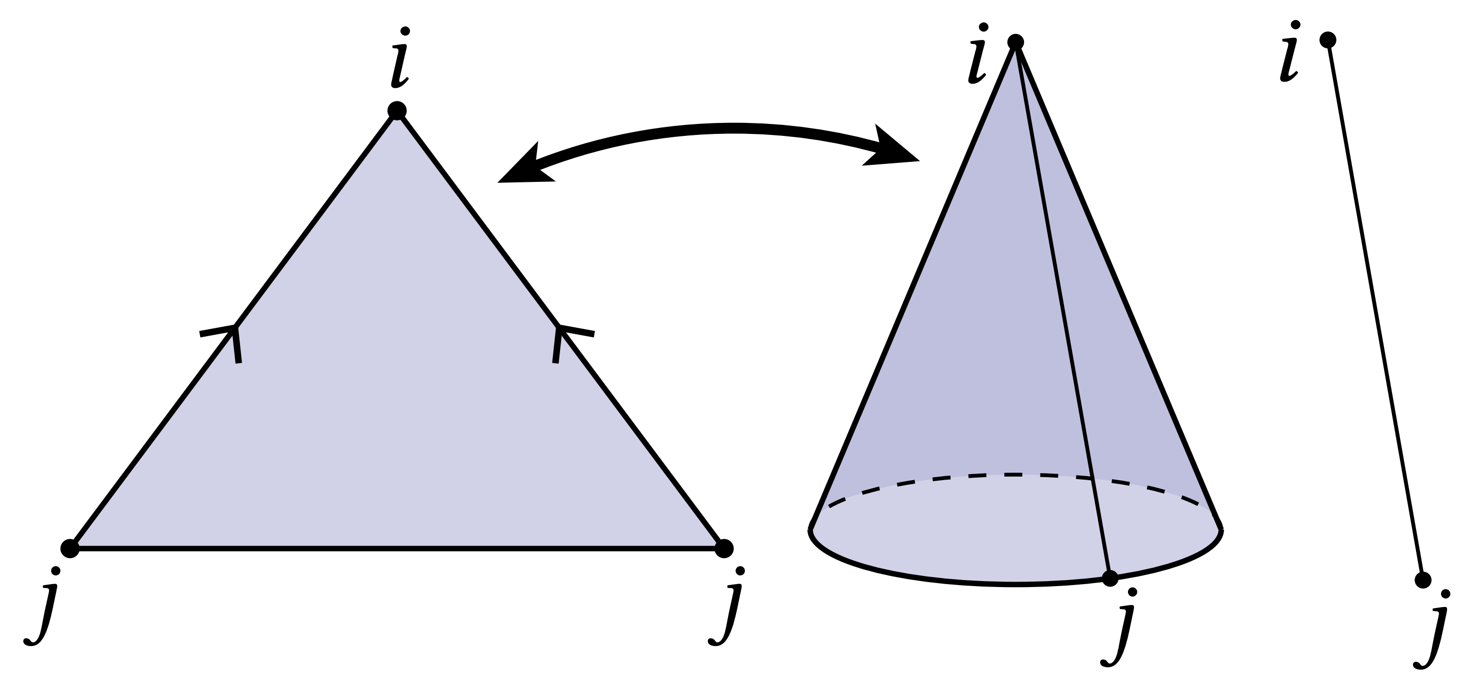
- **Recall manifold conditions (fans not fins):**
  - **every edge contained in two faces**
  - **every vertex contained in one fan**
- **These conditions say <u>nothing</u> about vertex positions! Just connectivity**
- **Hence, can have perfectly good (manifold) connectivity, even if geometry is awful**
- **In fact, sometimes you can have perfectly good manifold connectivity for which <u>any</u> vertex positions give "bad" geometry!**
- **Can lead to confusion when debugging: mesh looks "bad", even though connectivity is fine**

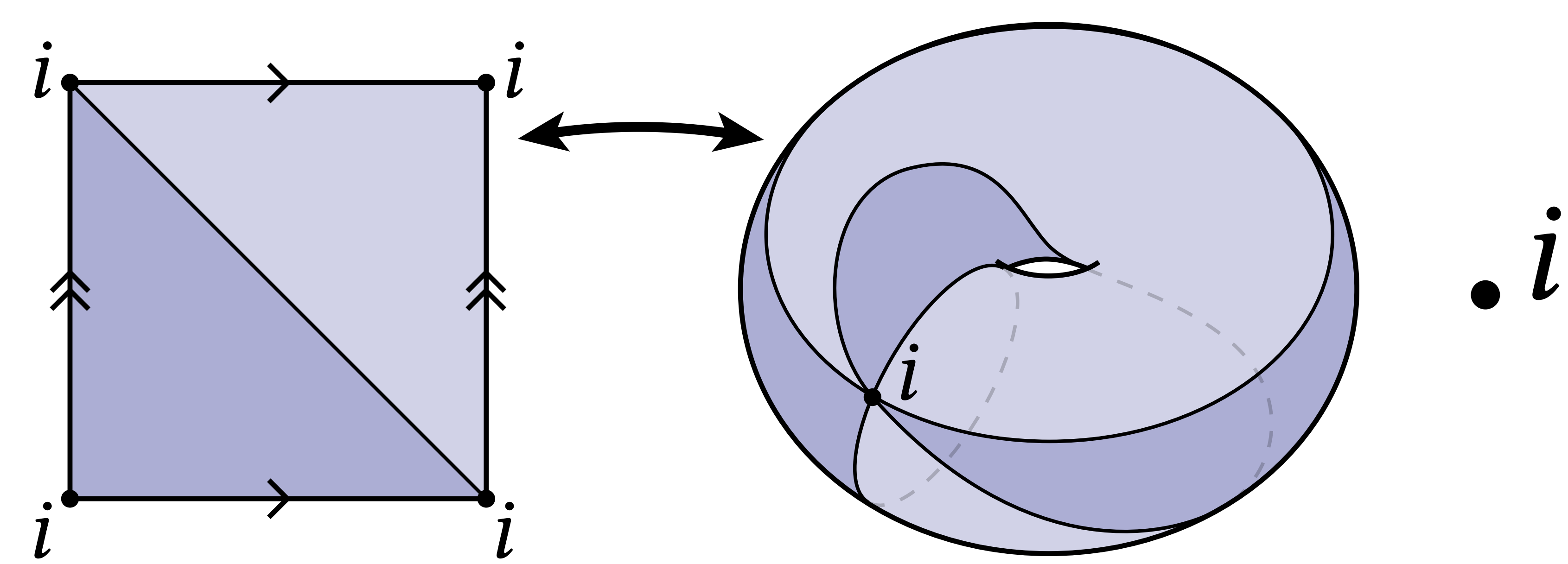


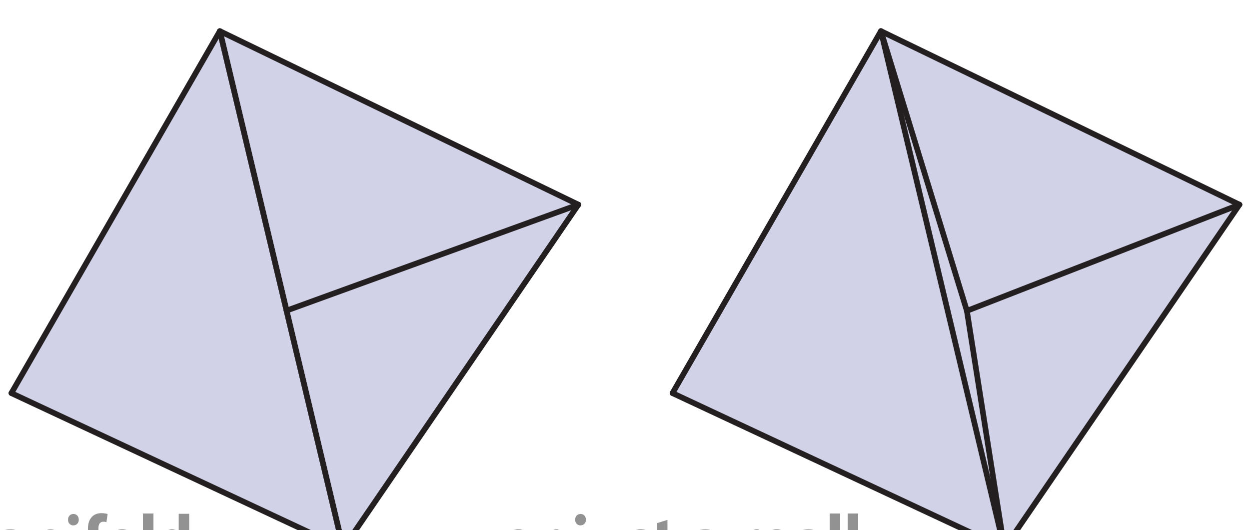


cube (manifold)

same connectivity, random vertex positions





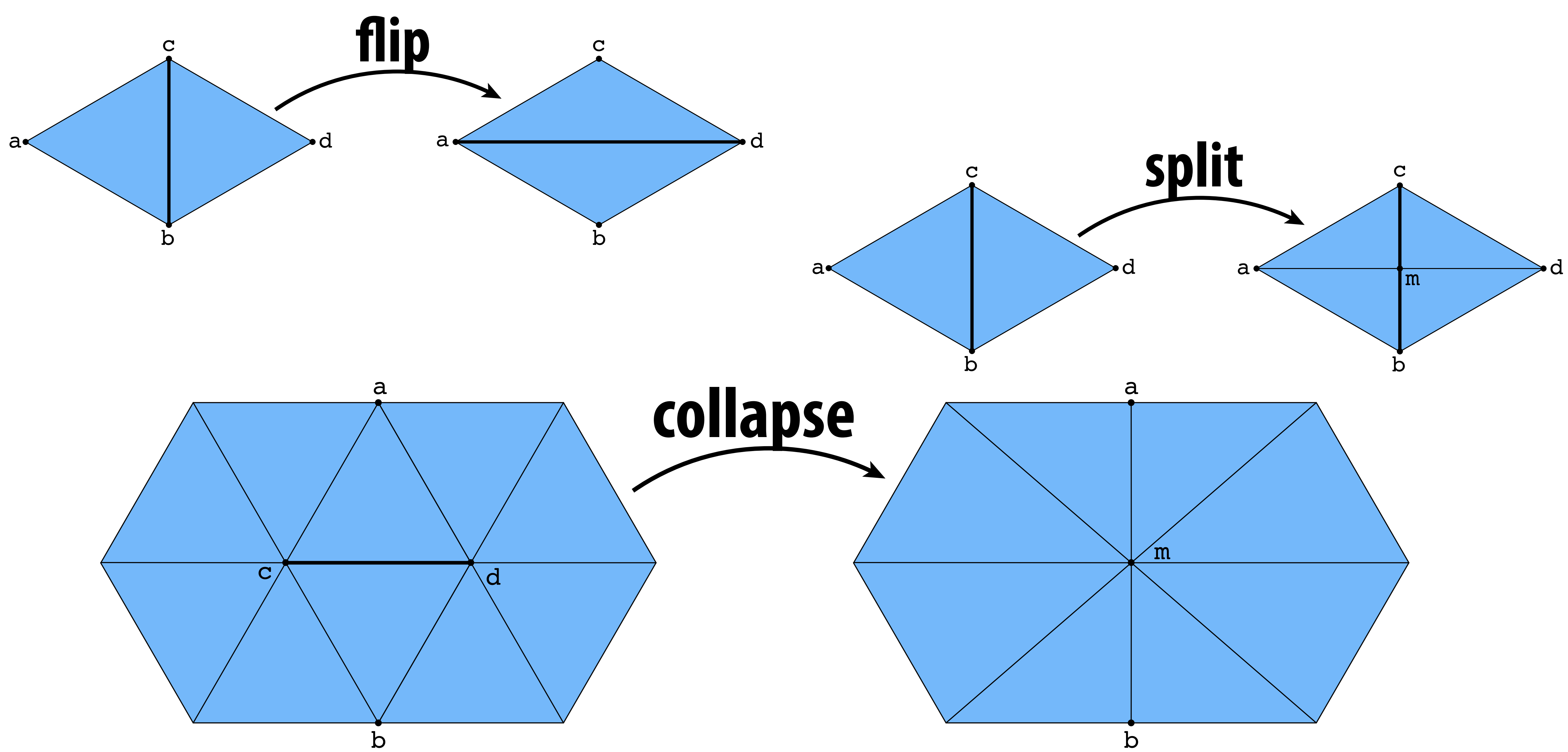


non manifold connectivity?

…or just a really skinny triangle?

Halfedge meshes are easy to edit

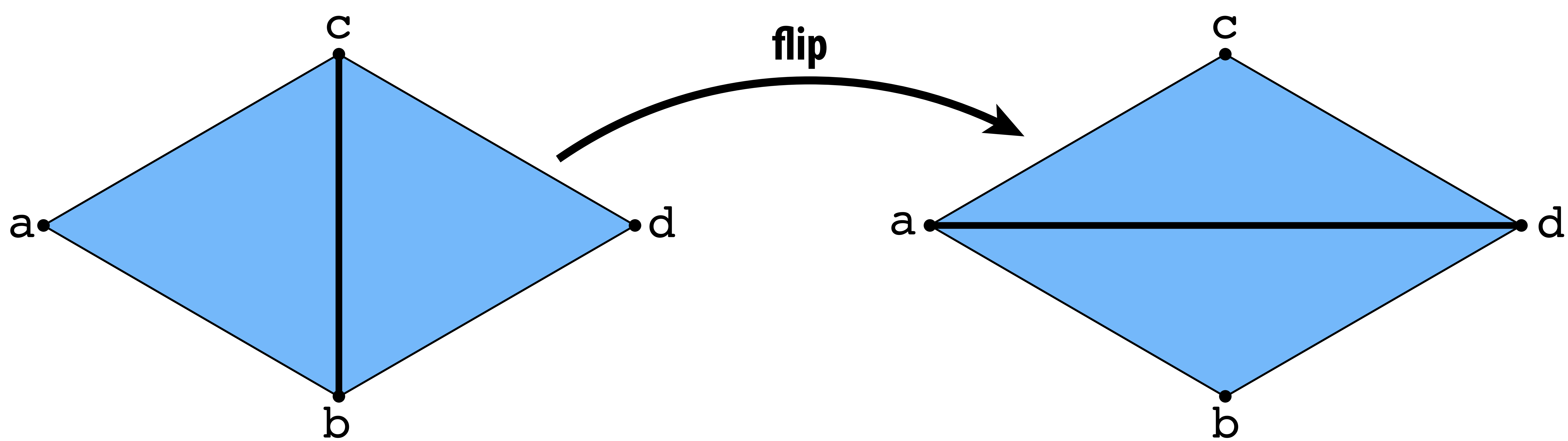
- Remember key feature of linked list: insert/delete elements
- Same story with halfedge mesh ("linked list on steroids")
- E.g., for triangle meshes, several atomic operations:



- How? Allocate/delete elements; reassigning pointers.
- Must be careful to preserve manifoldness!

Edge Flip (Triangles)

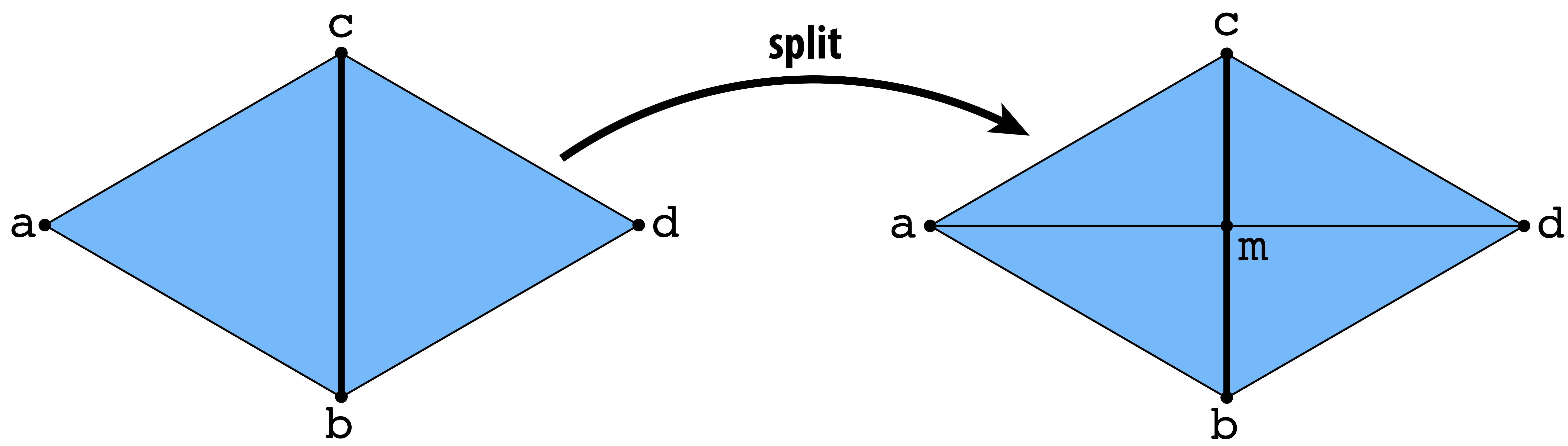
- **Triangles (a,b,c), (b,d,c) become (a,d,c), (a,b,d):**



- **Long list of pointer reassignments (`edge->halfedge = ...`)**
- **However, no elements created/destroyed.**
- **Q: What happens if we flip twice?**
- **Challenge: can you implement edge flip such that pointers are *unchanged* after two flips?**

Edge Split (Triangles)

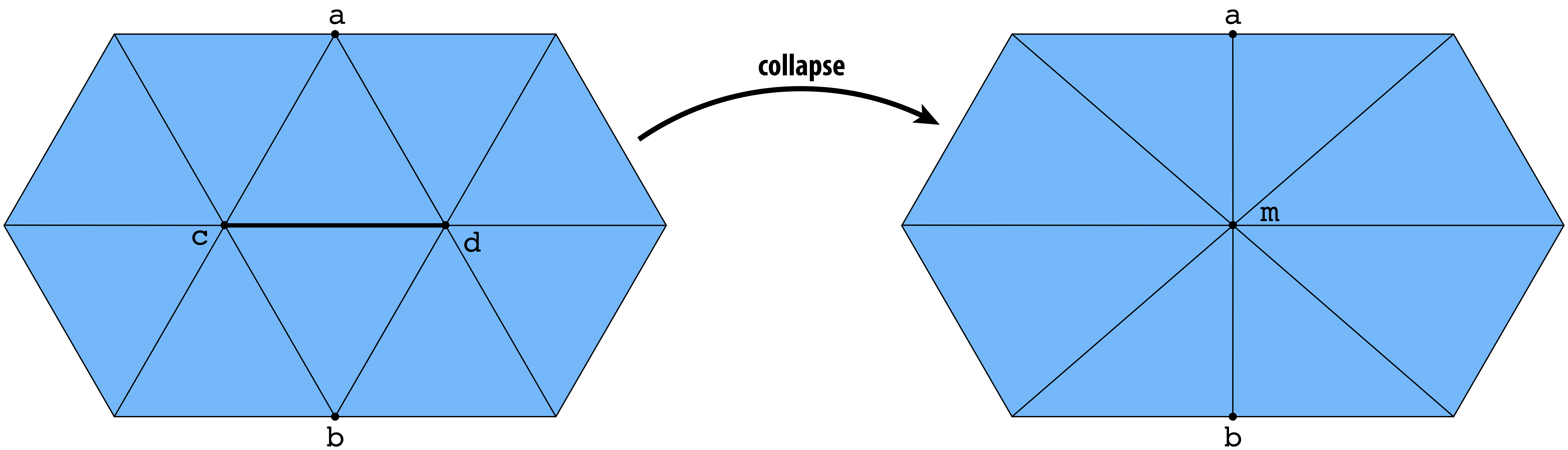
- Insert midpoint m of edge (c,b), connect to get four triangles:



- This time, have to *add* new elements.
- Lots of pointer reassignments.
- Q: Can we “reverse” this operation?

Edge Collapse (Triangles)

- **Replace edge (b,c) with a single vertex m:**

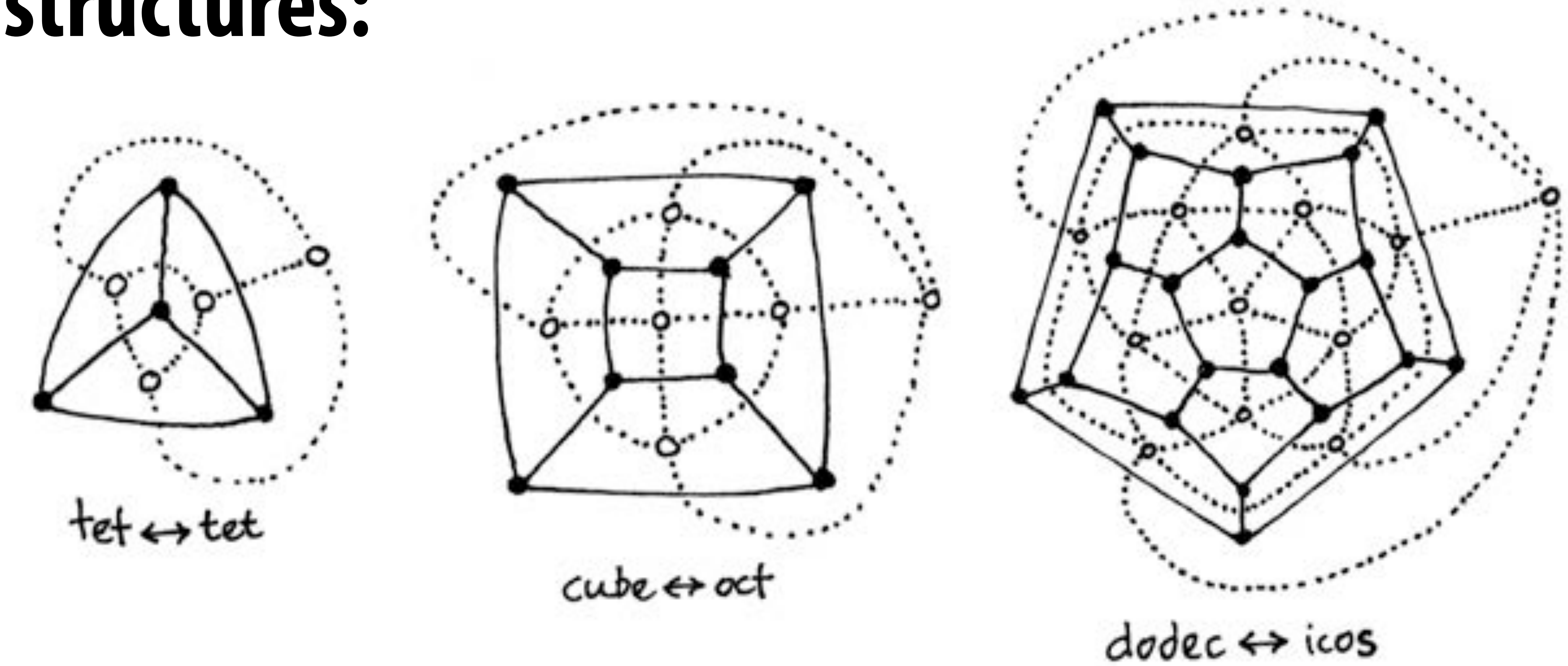


- **Now have to *delete* elements.**
- **Still lots of pointer assignments!**
- **Q: How would we implement this with an adjacency list?**
- **Any other good way to do it? (E.g., different data structure?)**

Alternatives to Halfedge

Paul Heckbert (former CMU prof.)
quadedge code - http://bit.ly/1QZLHos

- Many very similar data structures:
  - winged edge
  - corner table
  - quadedge
  - ...



- Each stores local neighborhood information
- Similar tradeoffs relative to simple polygon list:
  - CONS: additional storage, incoherent memory access
  - PROS: better access time for individual elements, intuitive traversal of local neighborhoods
- With some thought*, can design halfedge-type data structures with coherent data storage, support for non manifold connectivity, etc.

*see for instance http://geometry-central.net/

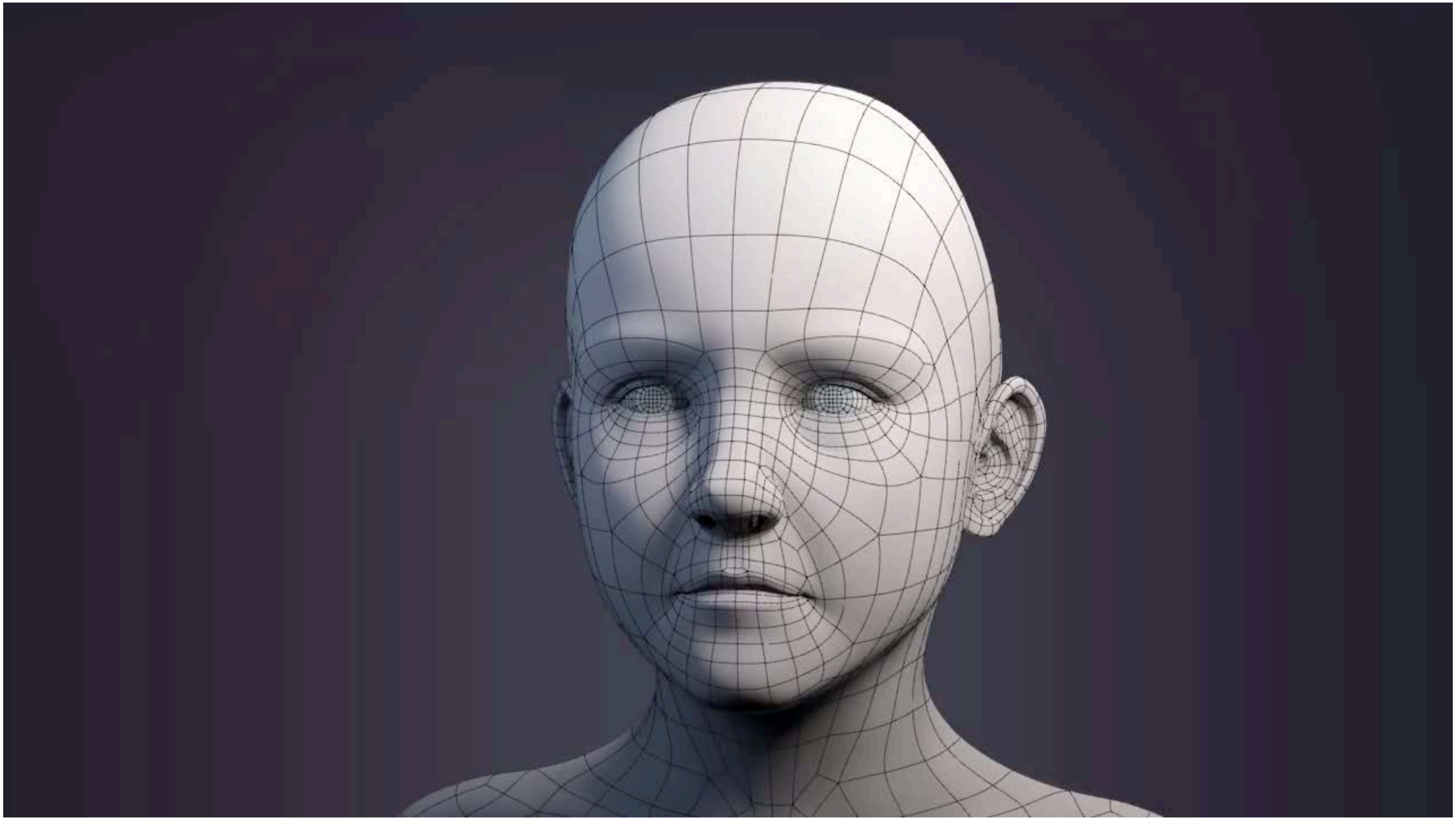
Comparison of Polygon Mesh Data Strucutres

	Adjacency List	Incidence Matrices	Halfedge Mesh
constant-time neighborhood access?	NO	YES	YES
easy to add/remove mesh elements?	NO	NO	YES
nonmanifold geometry?	YES	YES	NO

Conclusion: pick the right data structure for the job!

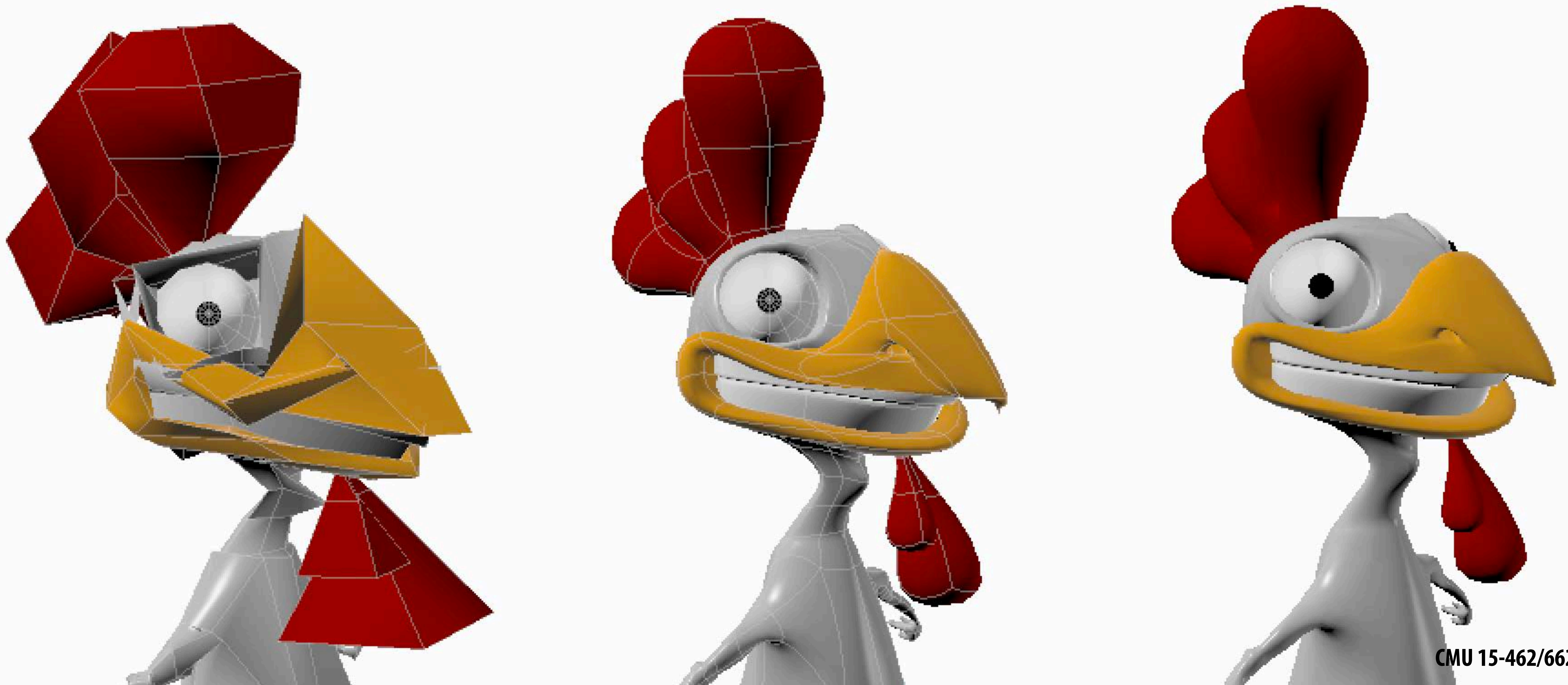
Ok, but what can we actually *do* with our fancy new data structures?

Subdivision Modeling



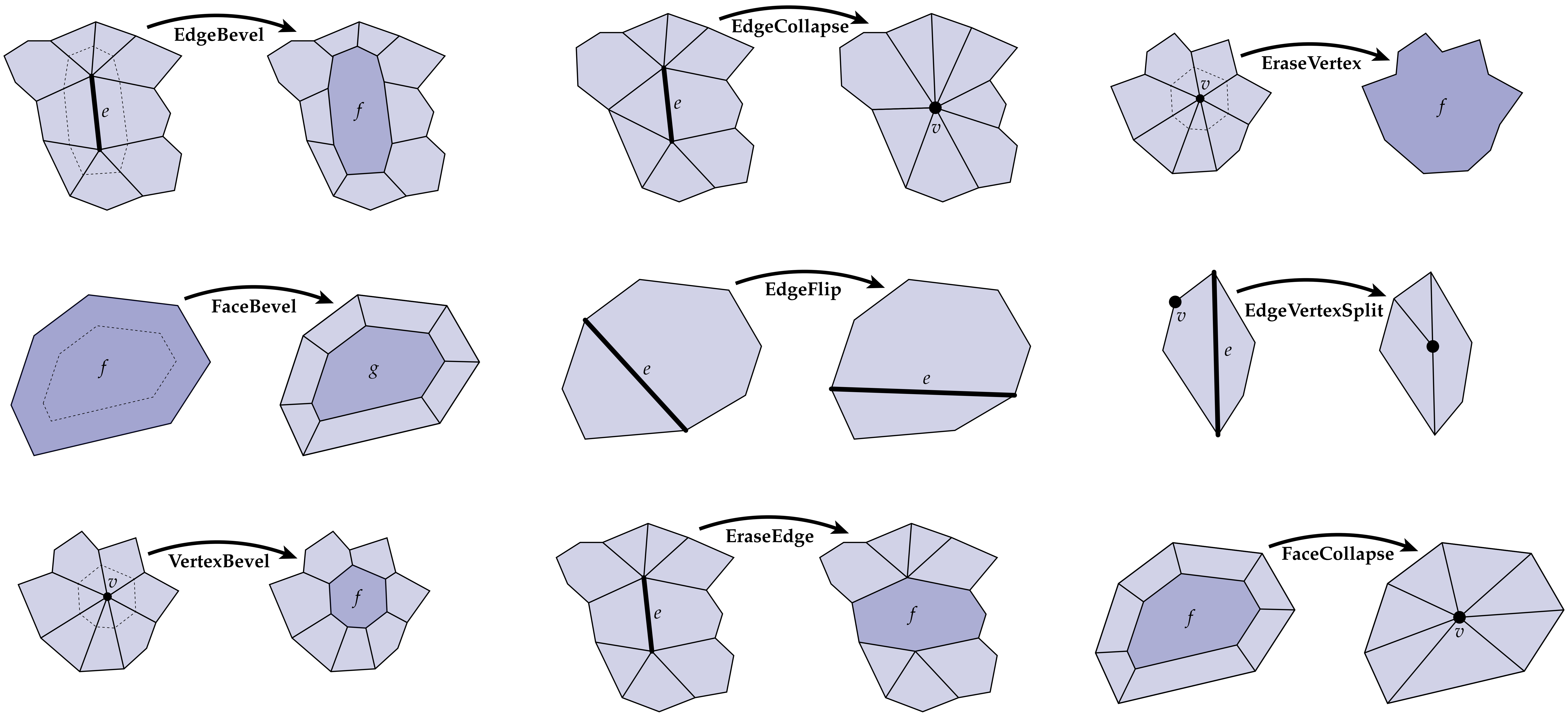
Subdivision Modeling

- Common modeling paradigm in modern 3D tools:
  - Coarse "control cage"
  - Perform local operations to control/edit shape
  - Global subdivision process determines final surface



Subdivision Modeling—Local Operations

- **For general polygon meshes, we can dream up lots of local mesh operations that might be useful for modeling:**



. . . and many, many more!

Next Time: Digital Geometry Processing

- Extend traditional digital signal processing (audio, video, etc.) to deal with *geometric* signals:
  - upsampling / downsampling / resampling / filtering ...
  - aliasing (reconstructed surface gives “false impression”)