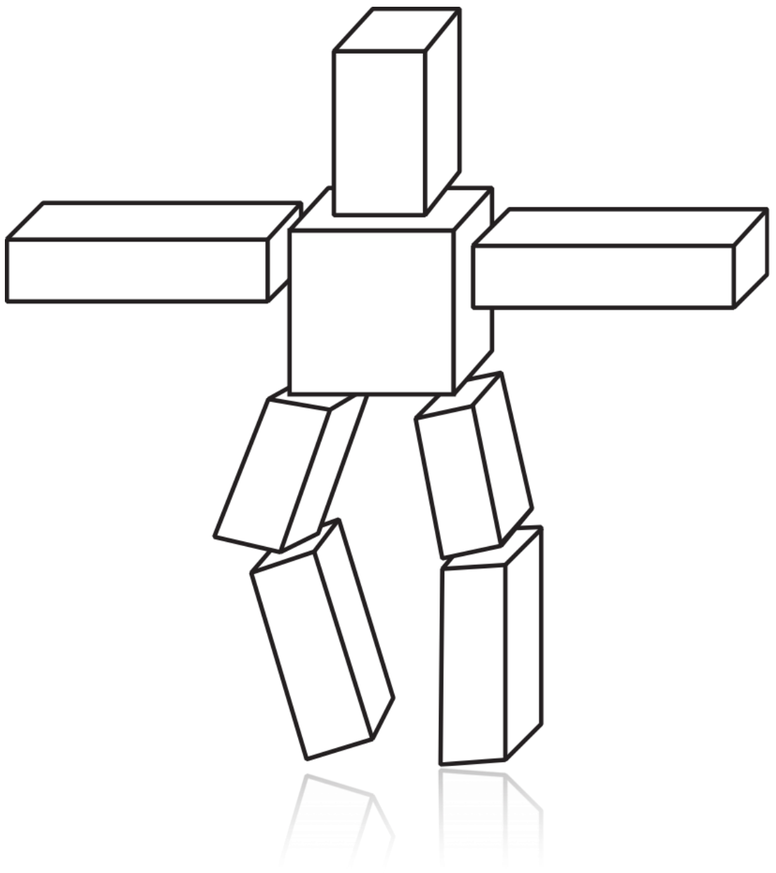
Introduction to Animation

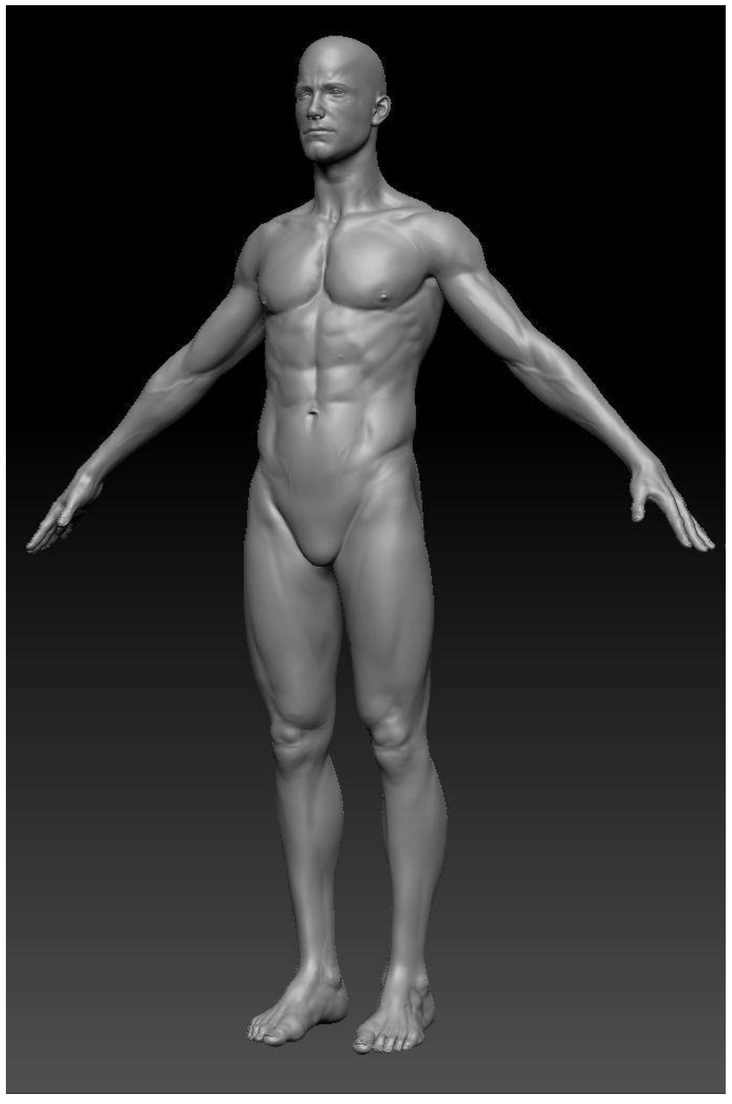
Computer Graphics
CMU 15-462/15-662, Fall 2016

Increasing the complexity of our models

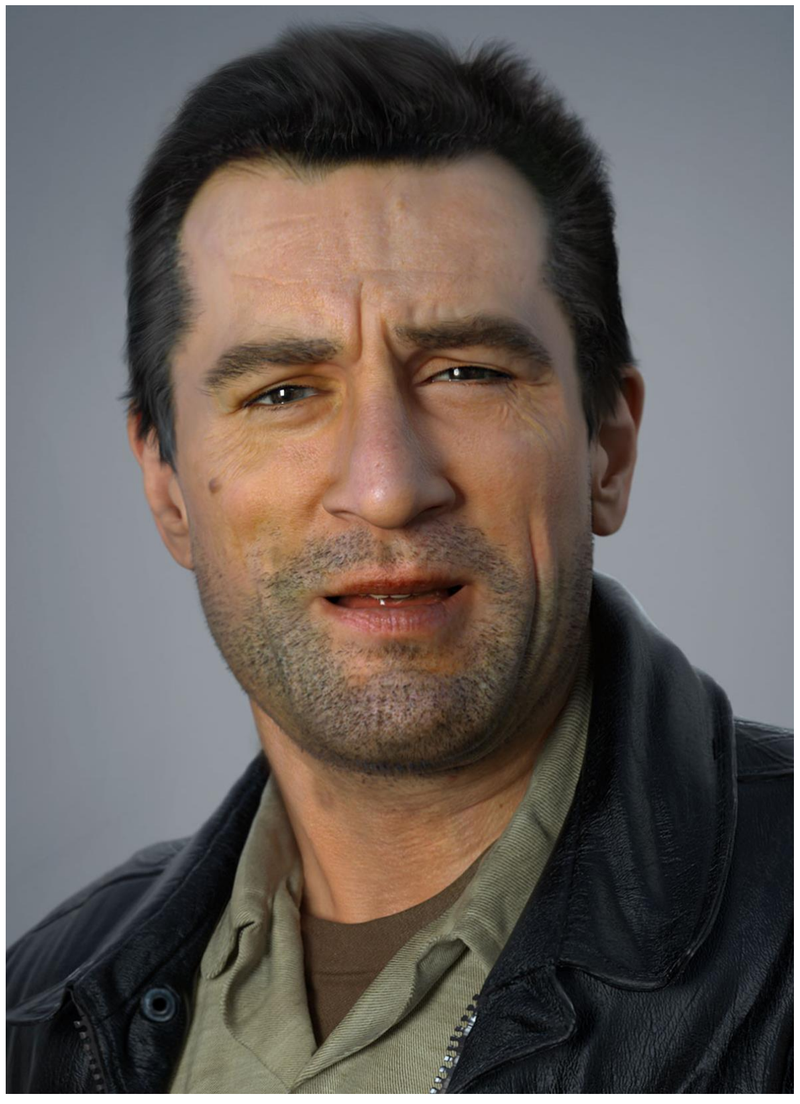
Transformations



Geometry



Materials, lighting

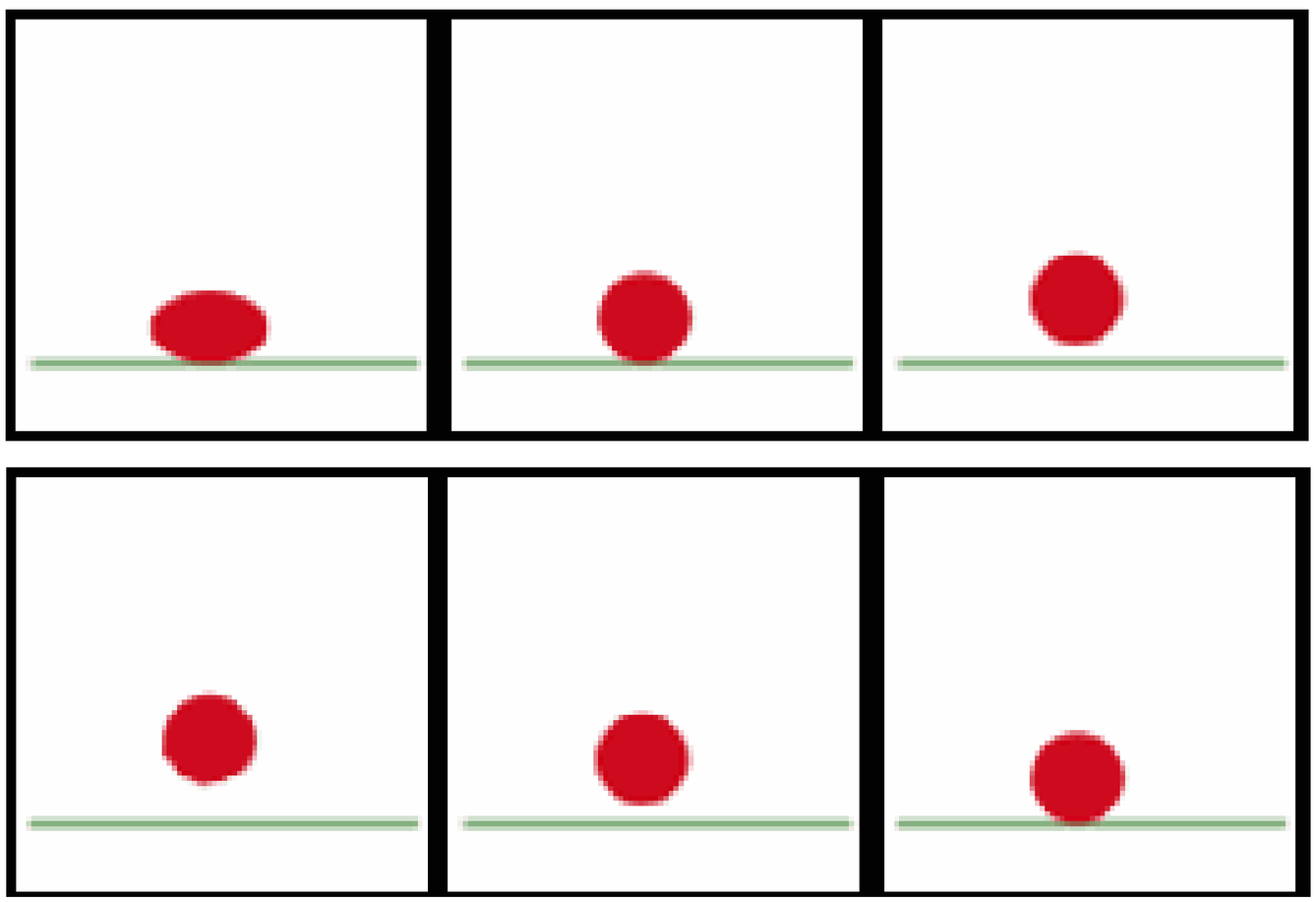


Increasing the complexity of our models

Motion: an essential ingredient in making virtual worlds come to life!

Animation

- **Creating the illusion of motion**
  - e.g. by rapidly displaying of a sequence of static images that minimally differ from each other.

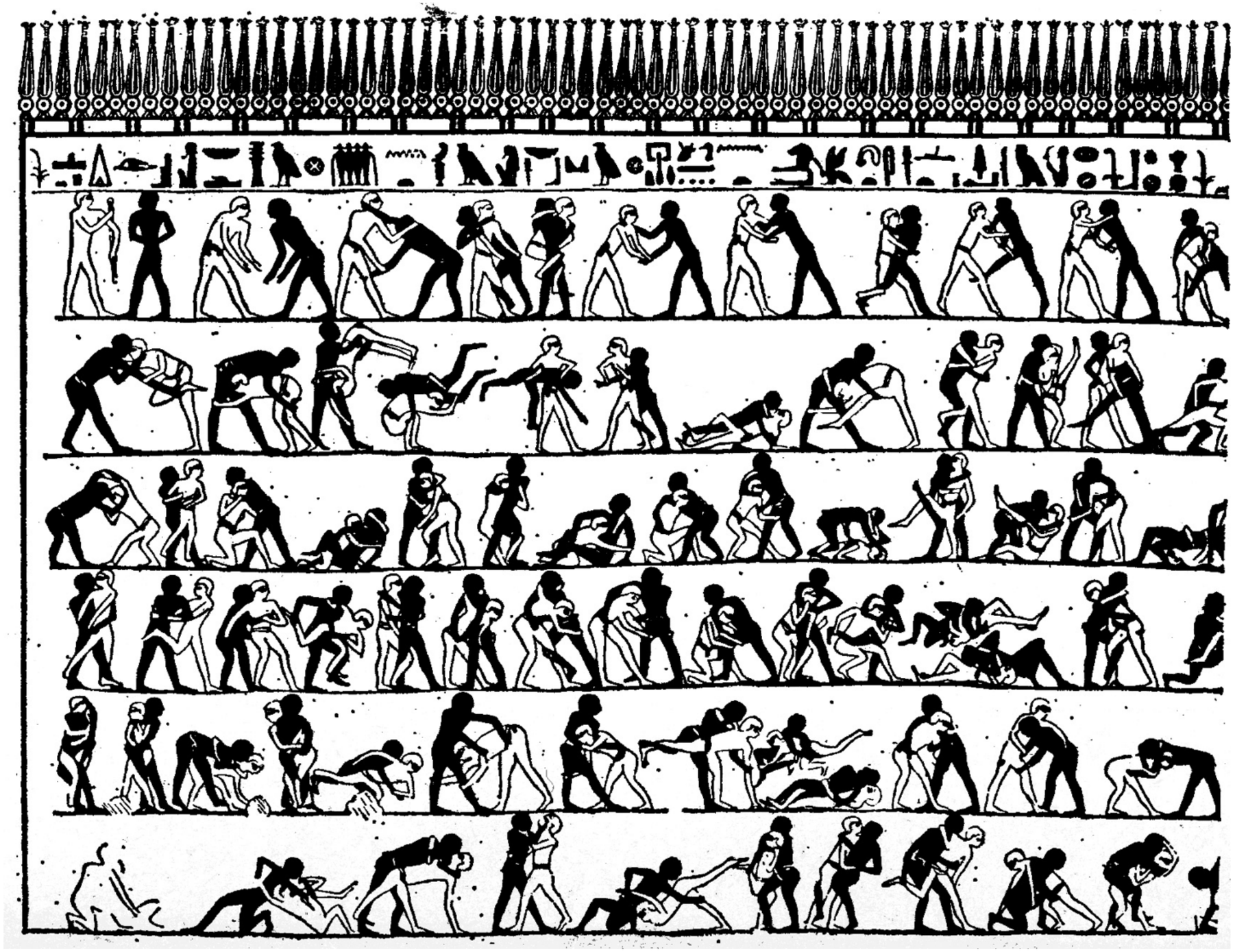


Early Animation

Early Animation

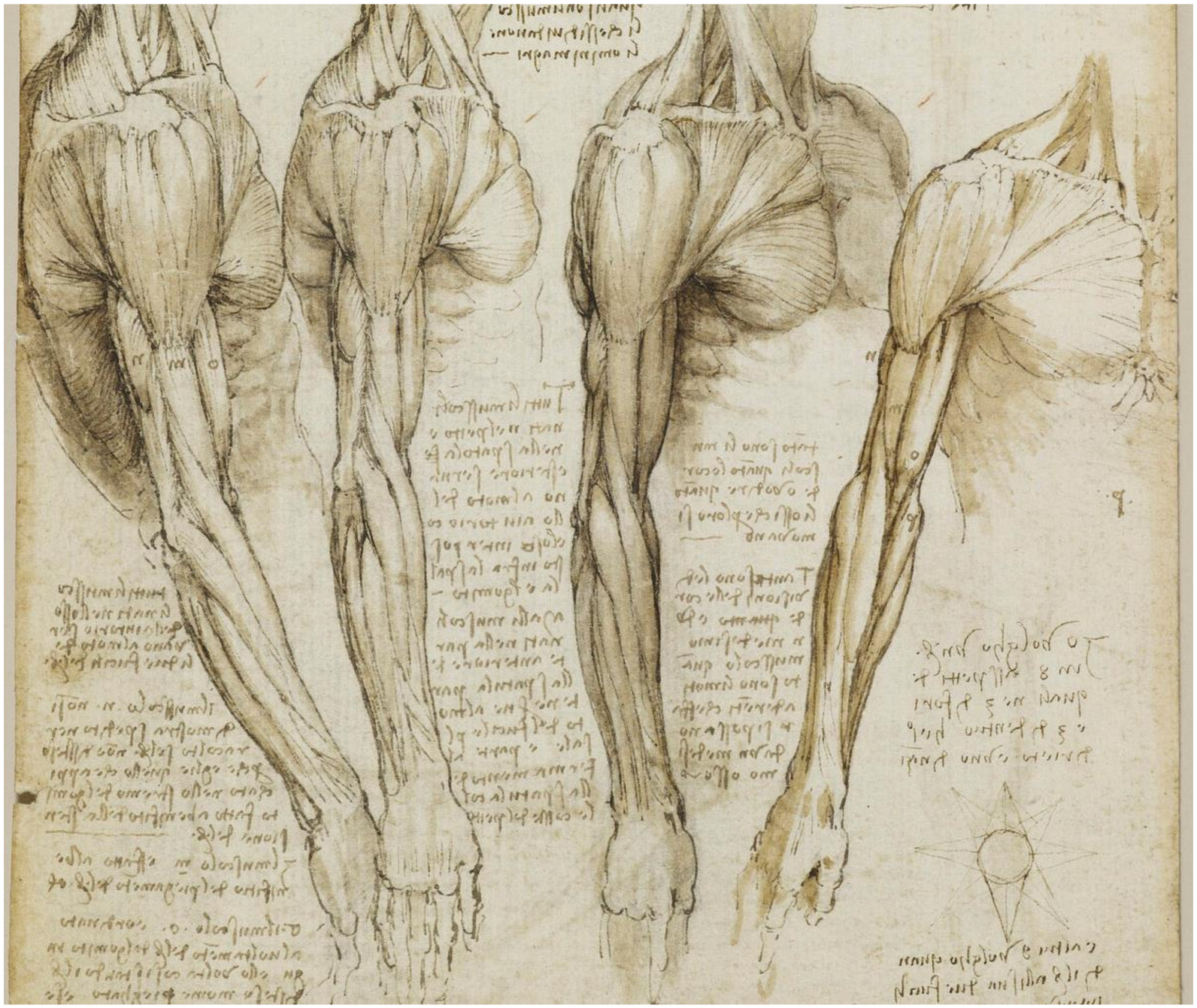
(Shahr-e Sukhteh, Iran 3200 BCE)

Early Animation



(tomb of Khnumhotep, Egypt 2400 BCE)

Early Animation



Leonardo da Vinci (1510)

Early Animation

- **16th century Mechanical Monk**

https://www.youtube.com/watch?v=kie96iRTq5M

Modern day automatons

https://www.youtube.com/watch?v=B4J9TBlFxAg

Early Animation

(Phenakistoscope, 1831)

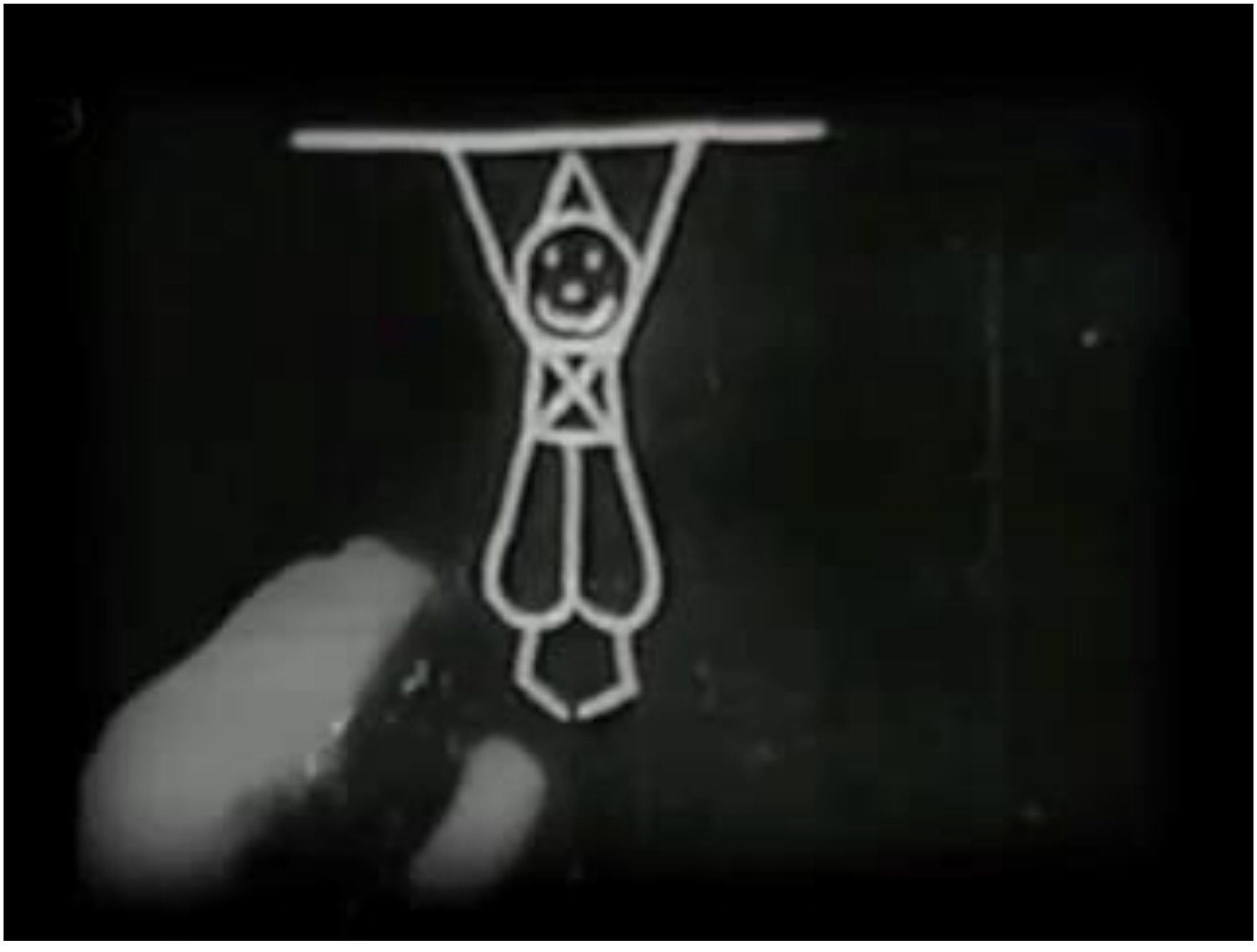
Zoetrope - 3D Printed Animation

First Film

- **Originally used as scientific tool rather than for entertainment**
- **Critical *technology* that accelerated development of animation**

Eadweard Muybridge, *"Sallie Gardner"* (1878)

First Animation on Film



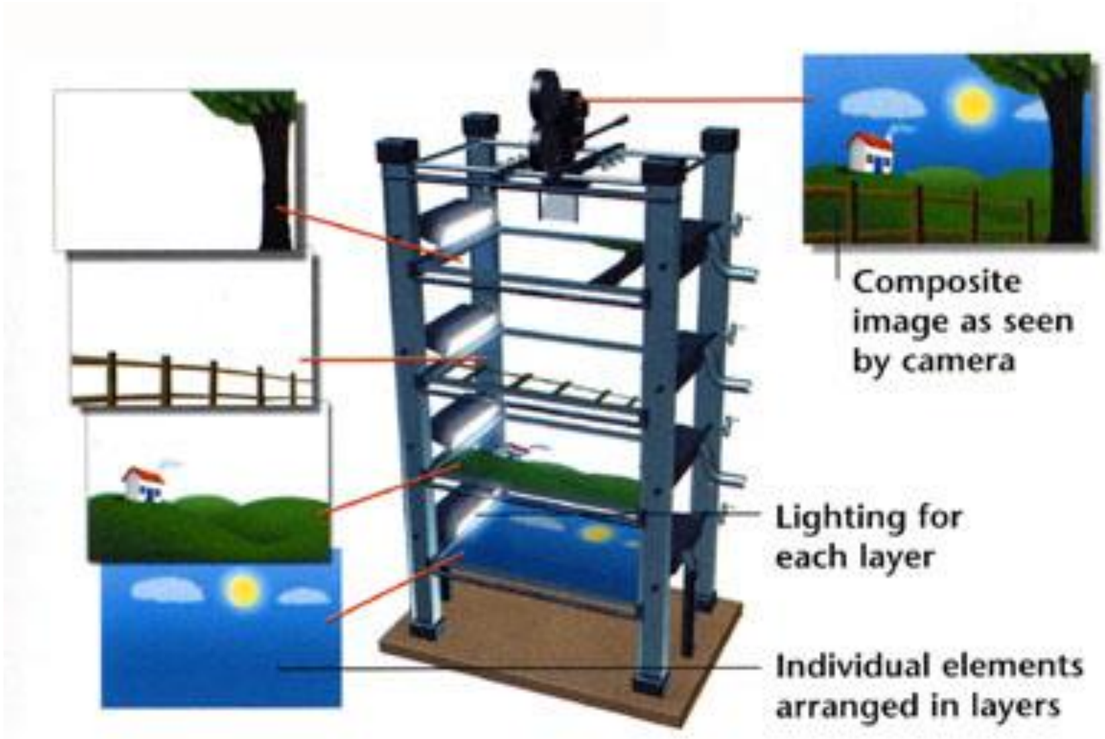
Emile Cohl, “Fantasmagorie” (1908)

First Feature-Length Animation

Lotte Reiniger, “Die Abenteuer des Prinzen Achmed” (1926)

First Hand-Drawn Feature-Length Animation

Disney, “Snow White and the Seven Dwarves” (1937)



https://www.youtube.com/watch?v=YdHTlUGN1zw

12 Principles of animation

1) Squash and Stretch - defining the rigidity & mass of an object by distorting its shape during an action.

2) Timing - spacing actions to define the weight & size of objects & the personality of characters.

3) Anticipation - The preparation for an action.

4) Staging - Presenting an idea so that it is unmistakably clear.

5) Follow Through & Overlapping Action - The termination of an action & establishing its relationship to the next action.

6) Straight Ahead Action & Pose-To-Pose Action - The two contrasting approaches to the creation of movement.

7) Slow In and Out - The spacing of in-between frames to achieve subtlety of timing & movements.

8) Arcs - The visual path of action for natural movement.

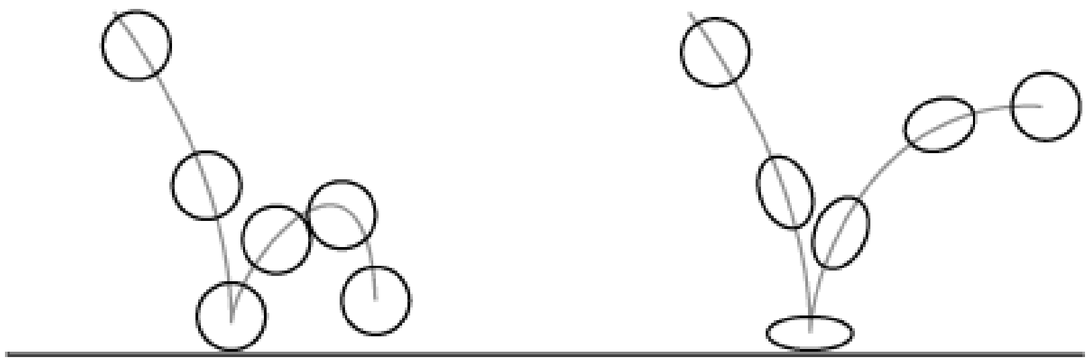
9) Exaggeration - Accentuating the essence of an idea via the design & the action.

10) Secondary Action - The Action of an object resulting from another action

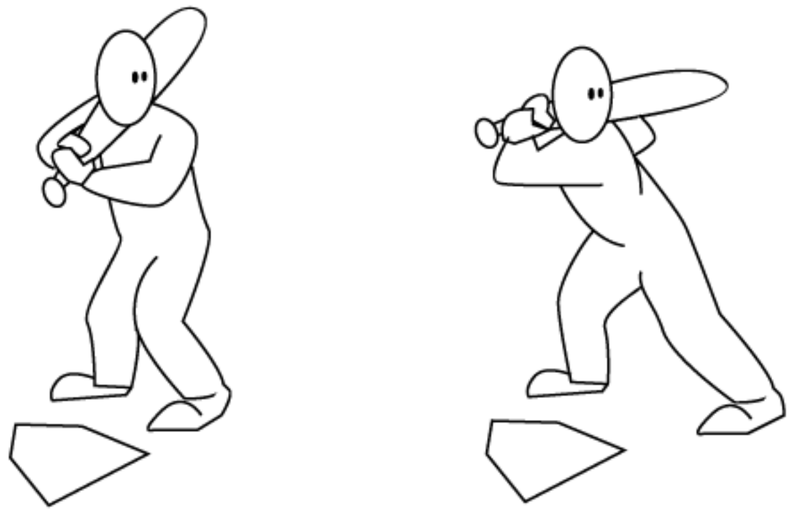
11) Appeal - Creating a design or an action that the audience enjoys watching.

12) Solid Drawing - Knowing them can dramatically improve one's ability to create good, strong poses and compose them with well crafted environments.

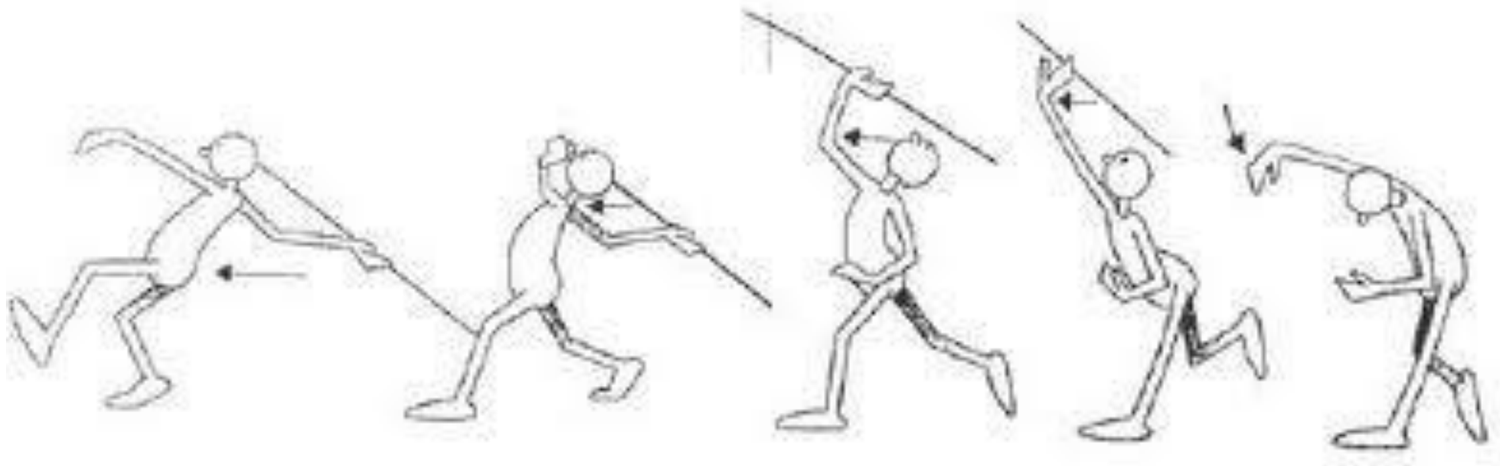
12 Principles of animation



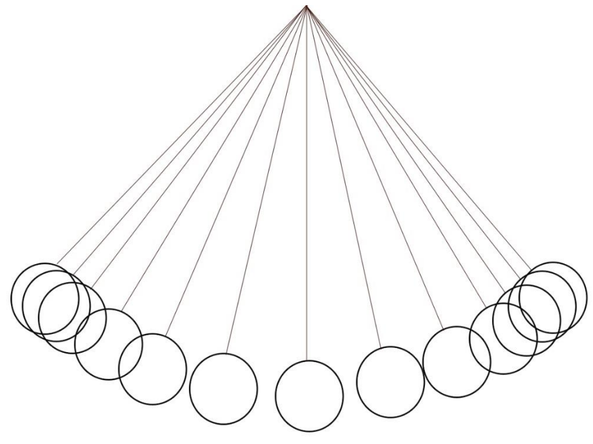
Squash and Stretch



Anticipation



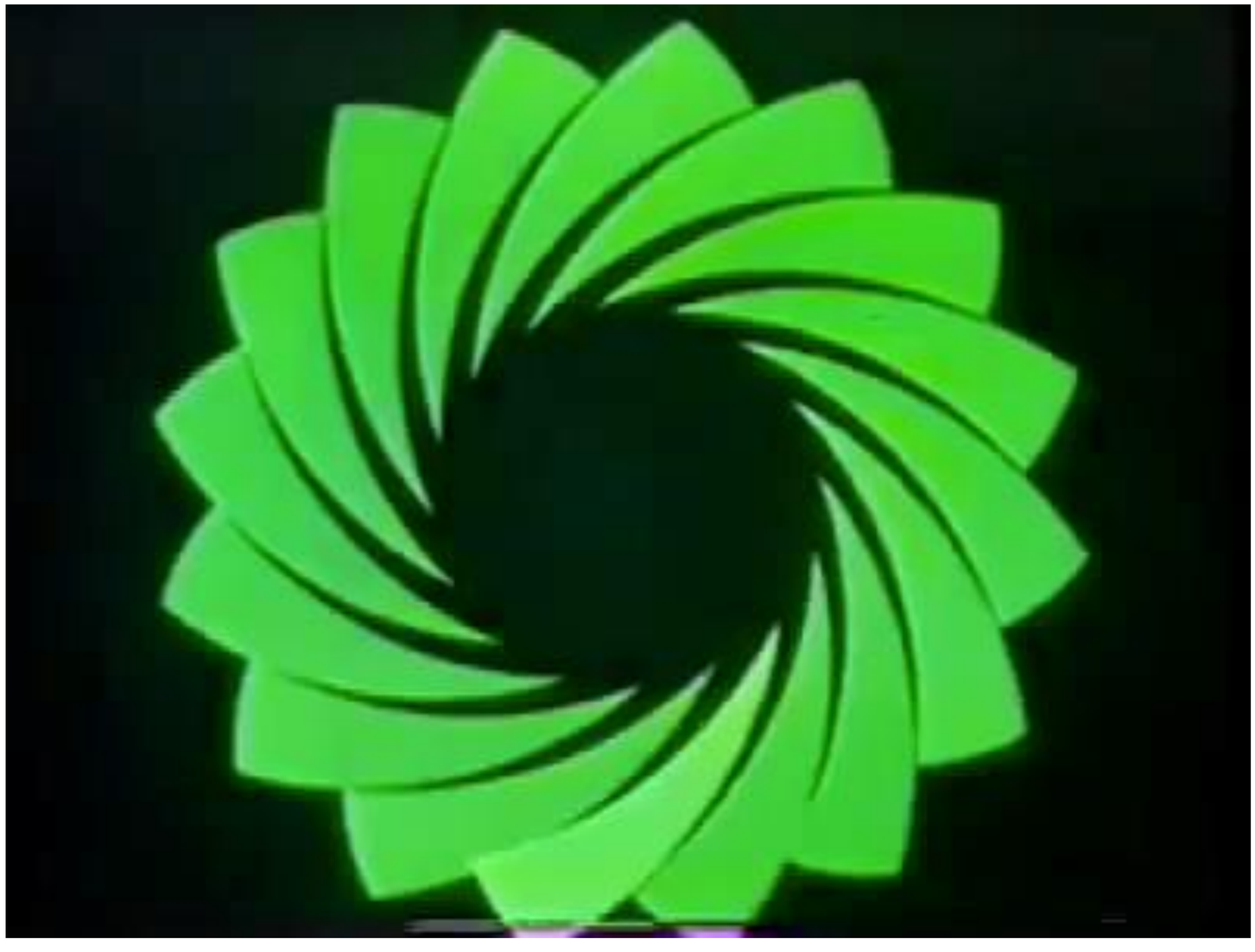
Follow through



Slow In, Slow Out

First Computer-Generated Animation

- New *technology*, also developed as a scientific tool
- Again turbo-charged the development of animation

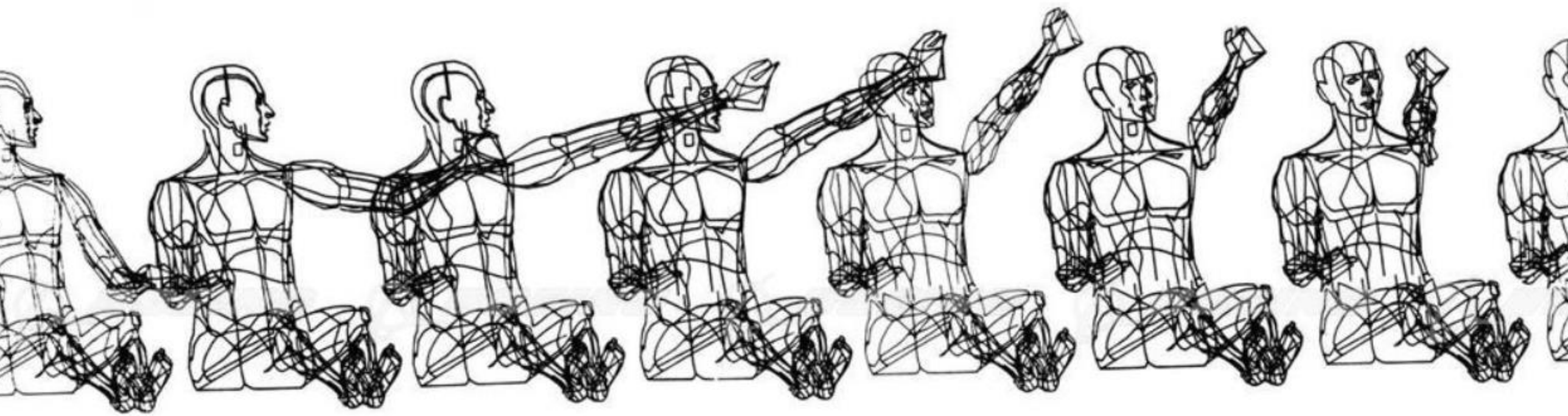


John Whitney, “Catalog” (1961)

First Digital-Computer-Generated Animation

Ivan Sutherland, “Sketchpad” (1963)

First 3D Computer Animation

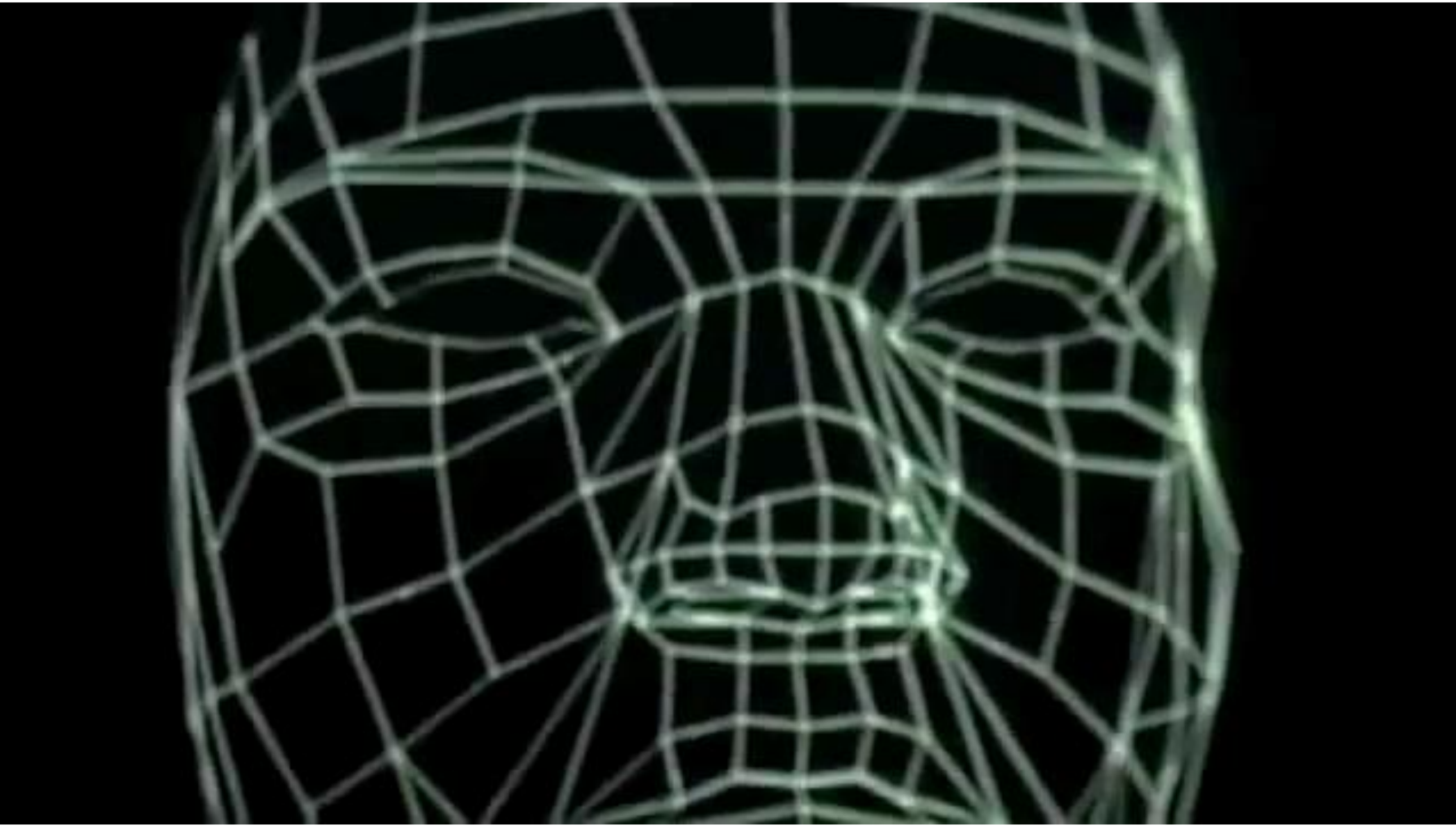


William Fetter, “Boeing Man” (1964)

Early Computer Animation

Nikolay Konstantinov, “Kitty” (1968)

Early Computer Animation



Ed Catmull & Fred Park, “Computer Animated Faces” (1972)

First major feature film to use CGI

Ed Catmull & Fred Park worked on "Futureworld" (1976)

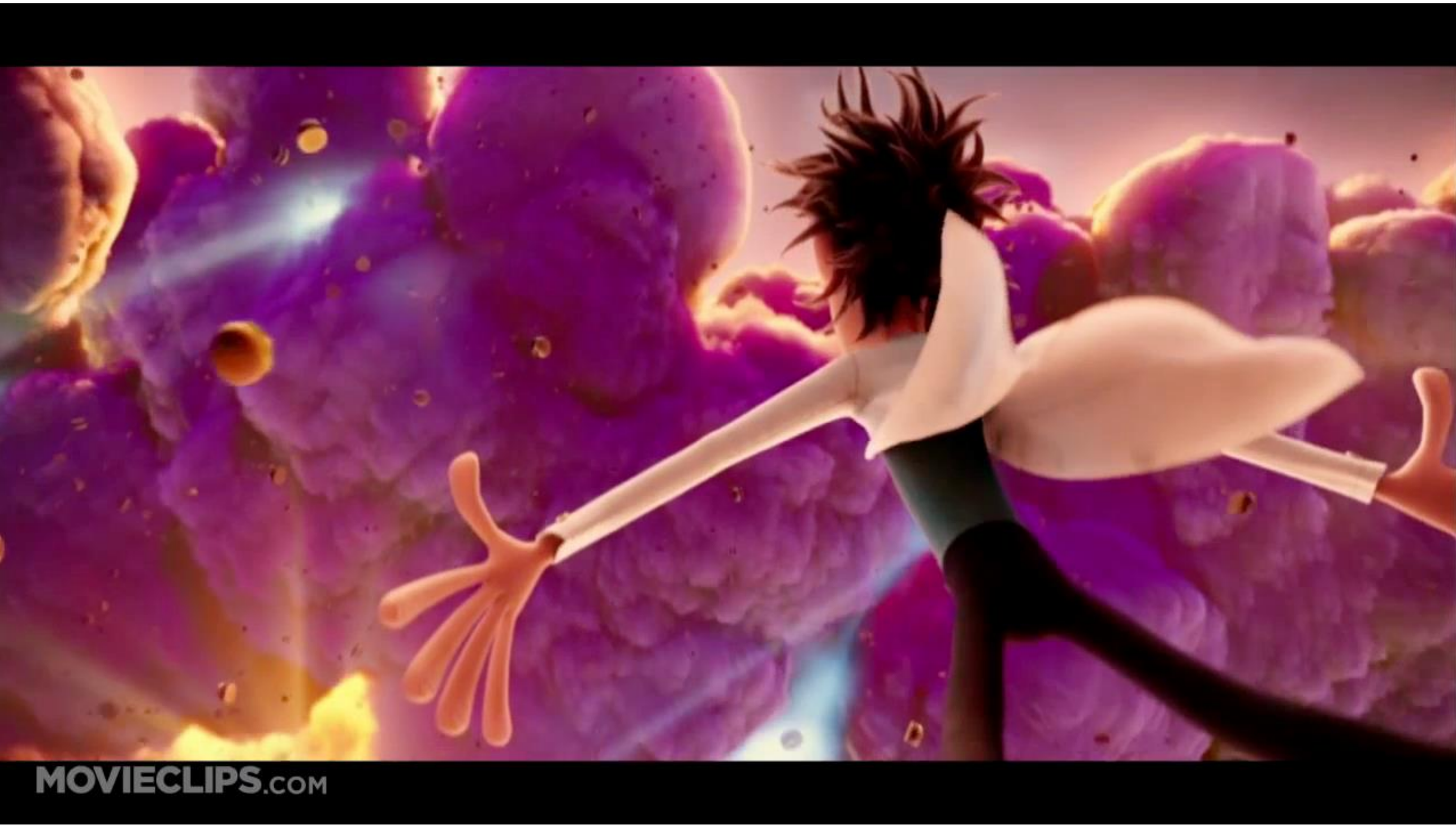
First *Attempted* CG Feature Film

NYIT [Williams, Heckbert, Catmull, …], “The Works” (1984)

First CG Feature Film

Pixar, “Toy Story” (1995)

Computer Animation - Present Day



Sony Pictures Animation, “Cloudy With a Chance of Meatballs” (2009)

Computer Animation - Present Day

Generating Motion (Hand-Drawn)

- Senior artist draws *keyframes*
- Assistant draws *inbetweens*
- Tedious / labor intensive (opportunity for technology!)

keyframe

keyframe

keyframe

inbetweens ("tweening")

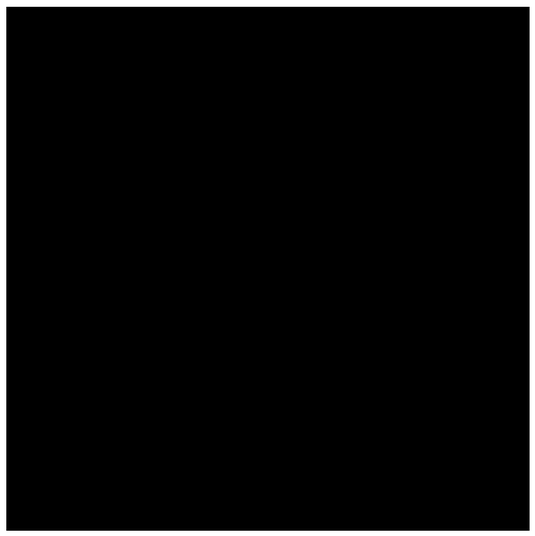
Generating Motion (Stop-motion animation)

- **Same idea, but with physical props**

  - armatures, clay models, etc…

Perception of Motion

- **Original (but debunked) theory: *persistence of vision* ("streaking")**
- **The eye is not a camera! More modern explanation:**
  - ***beta/phi phenomenon*: visual memory/anticipation built in brain—not eyeball**

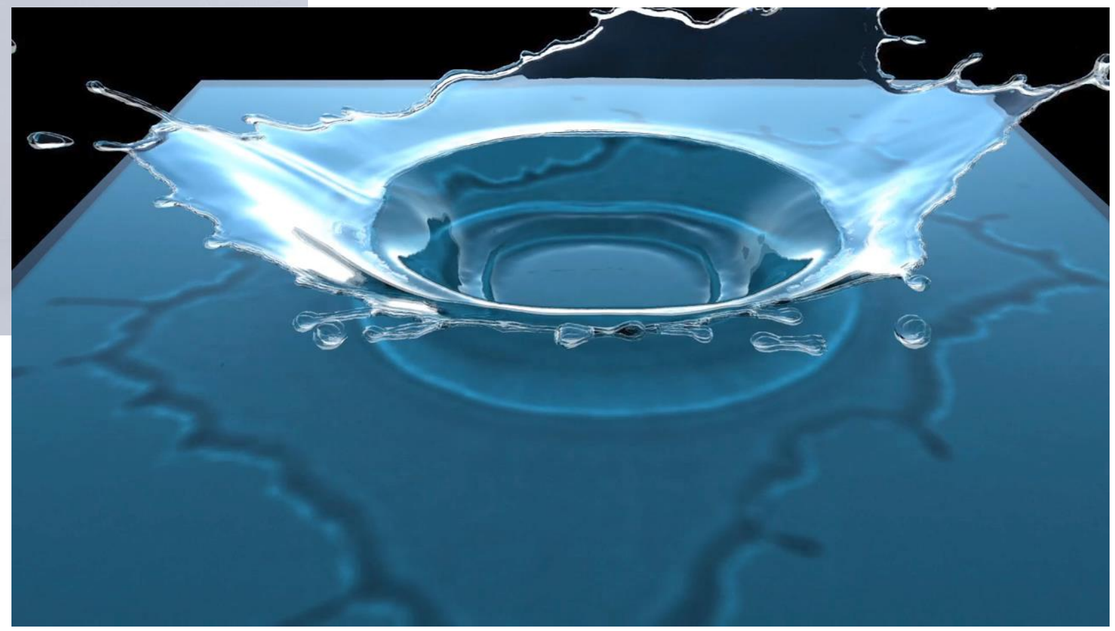


beta

How do we describe motion on a computer?

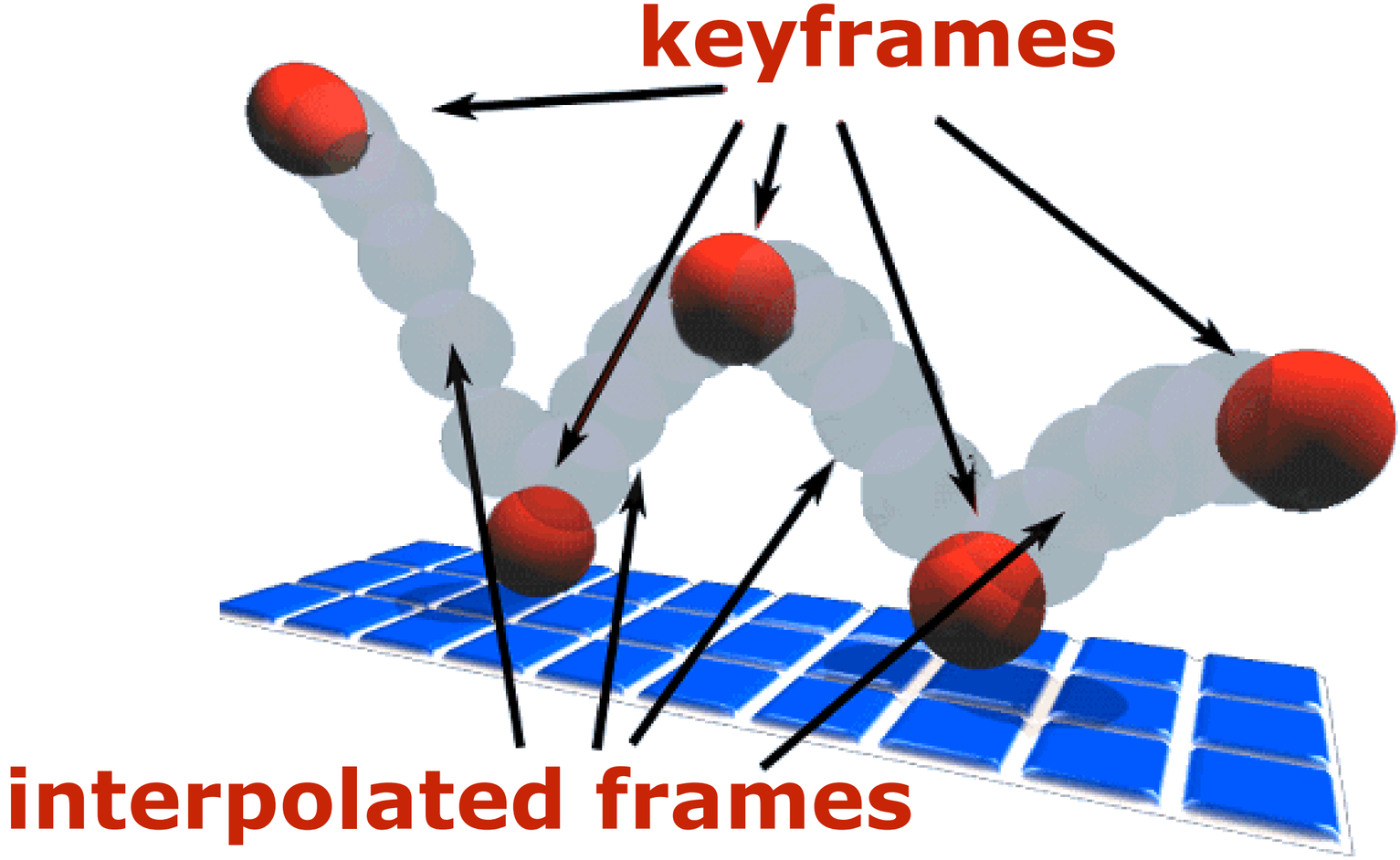
Basic Techniques in Computer Animation

- Artist-directed (e.g., keyframing)
- Data-driven (e.g., motion capture)
- Procedural (e.g., simulation)



Keyframing

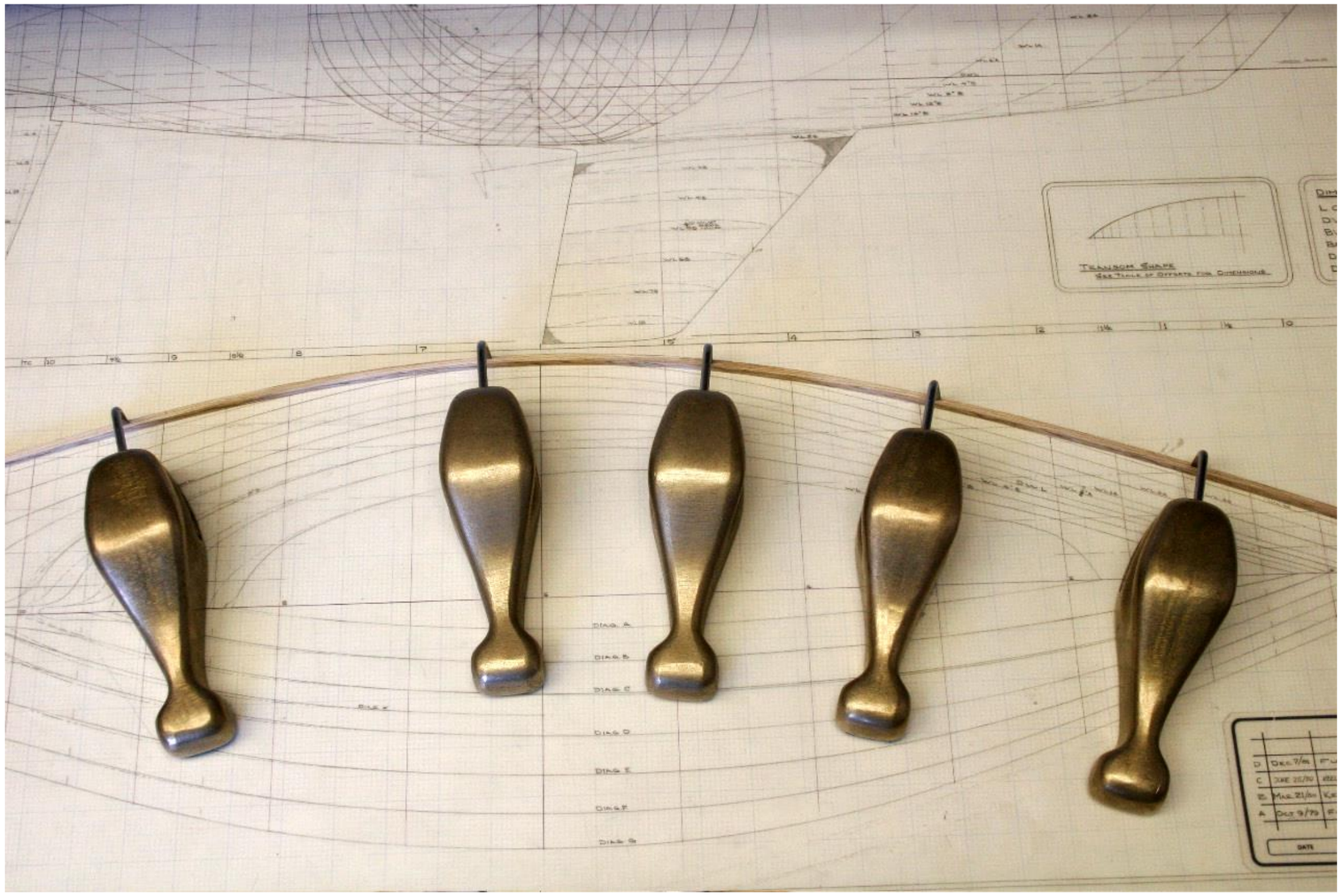
- Basic idea:
  - specify important events only, computer fills in the rest via interpolation/approximation
- “Events” don’t have to be position
- Could be color, light intensity, camera configuration, ...

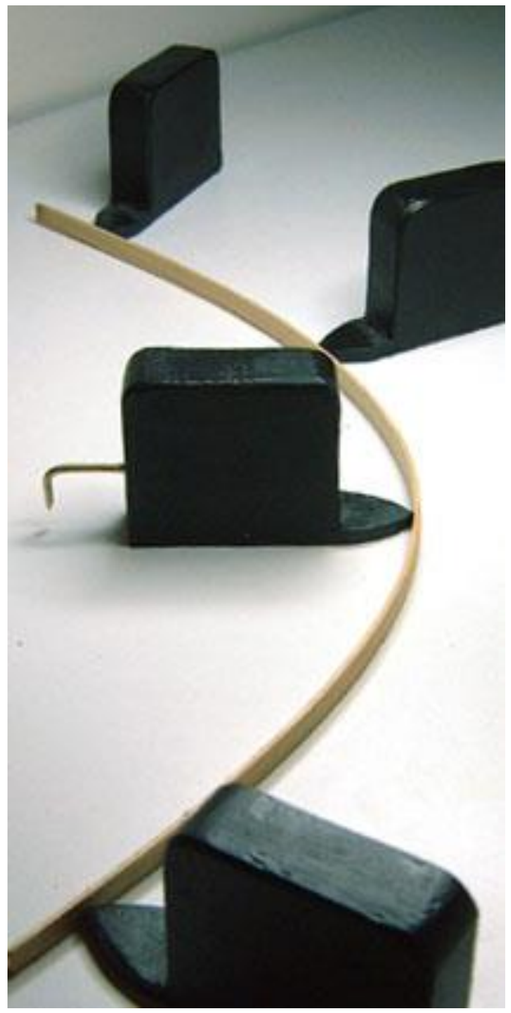


How do you interpolate data?

Spline Interpolation

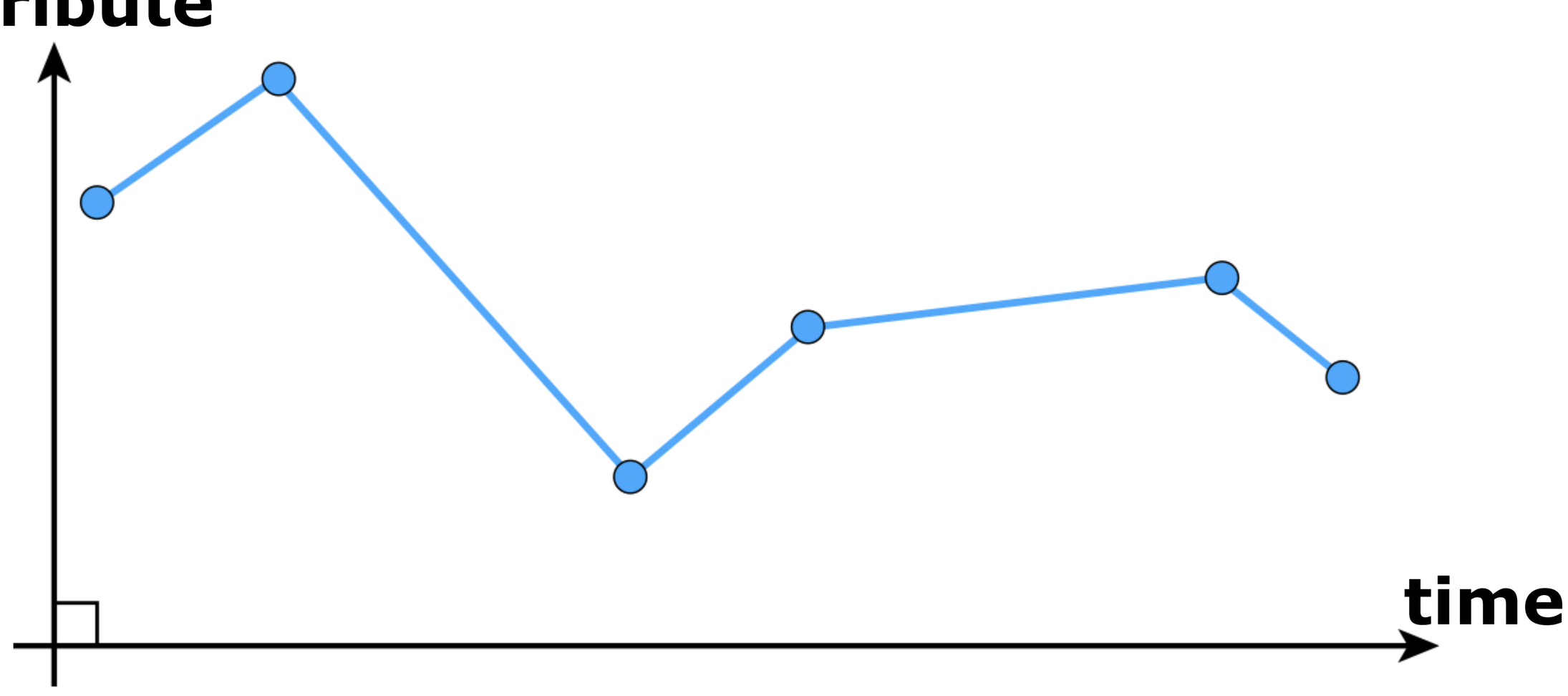
- Mathematical theory of interpolation arose from study of thin strips of wood or metal ("splines") under various forces
- Good summary in Levin, "The Elastica: A Mathematical History"





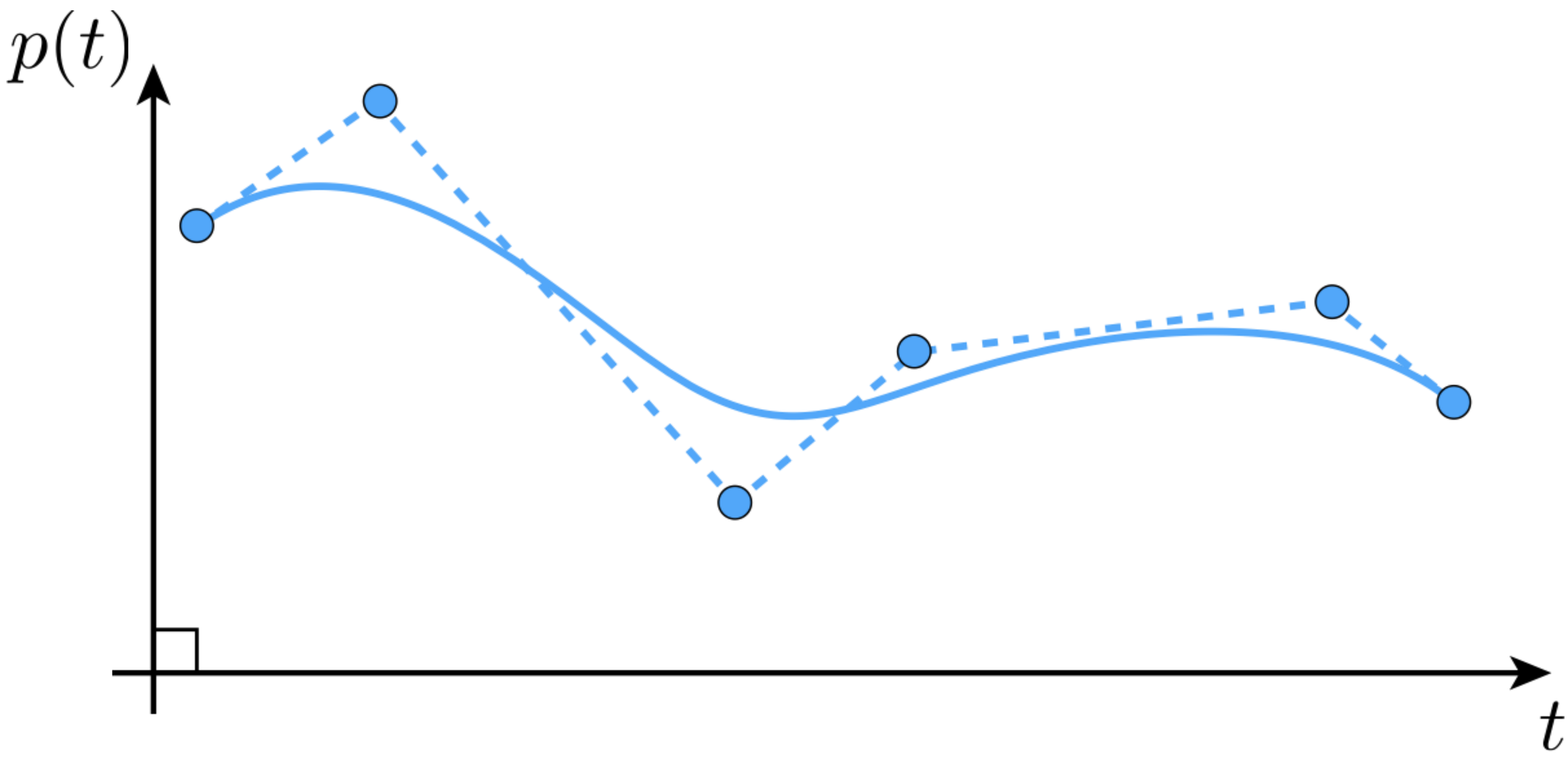
Interpolation

- Basic idea: “connect the dots”
- E.g., *piecewise linear interpolation*
- Simple, but yields rather rough motion
  - infinite acceleration – why is this far from ideal?



Piecewise Polynomial Interpolation

- **Common interpolant: piecewise polynomial "spline"**



Basic motivation: get better continuity than piecewise linear!

Splines

- In general, a *spline* is any piecewise polynomial function
- In 1D, spline interpolates data over the real line:

$$(t_i, f_i), \quad i = 0, \ldots, n$$

"knots" values

$t_i < t_{i+1}$

- "Interpolates" just means that the function *exactly* passes through those values:

$$f(t_i) = f_i \quad \forall i$$

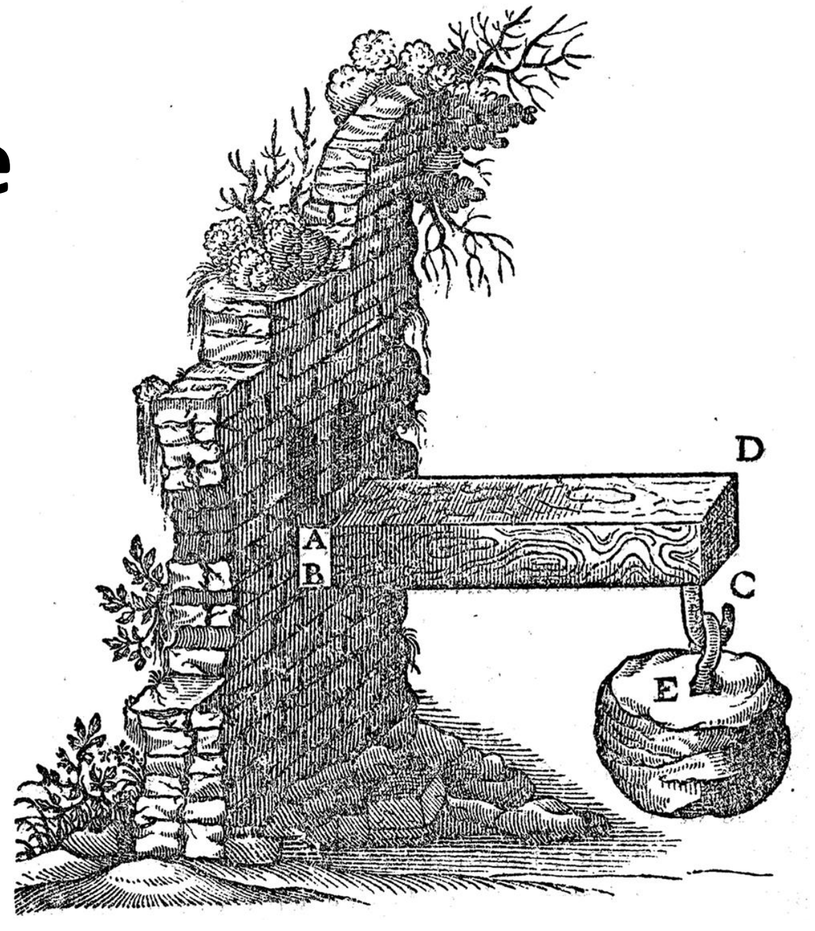
- The only other condition is that the function is a *polynomial* when restricted to any interval between knots:

$$\text{for } t_i \leq t \leq t_{i+1}, f(t) = \sum_{j=1}^{d} c_i t^j =: p_i(t)$$

degree polynomial coefficients

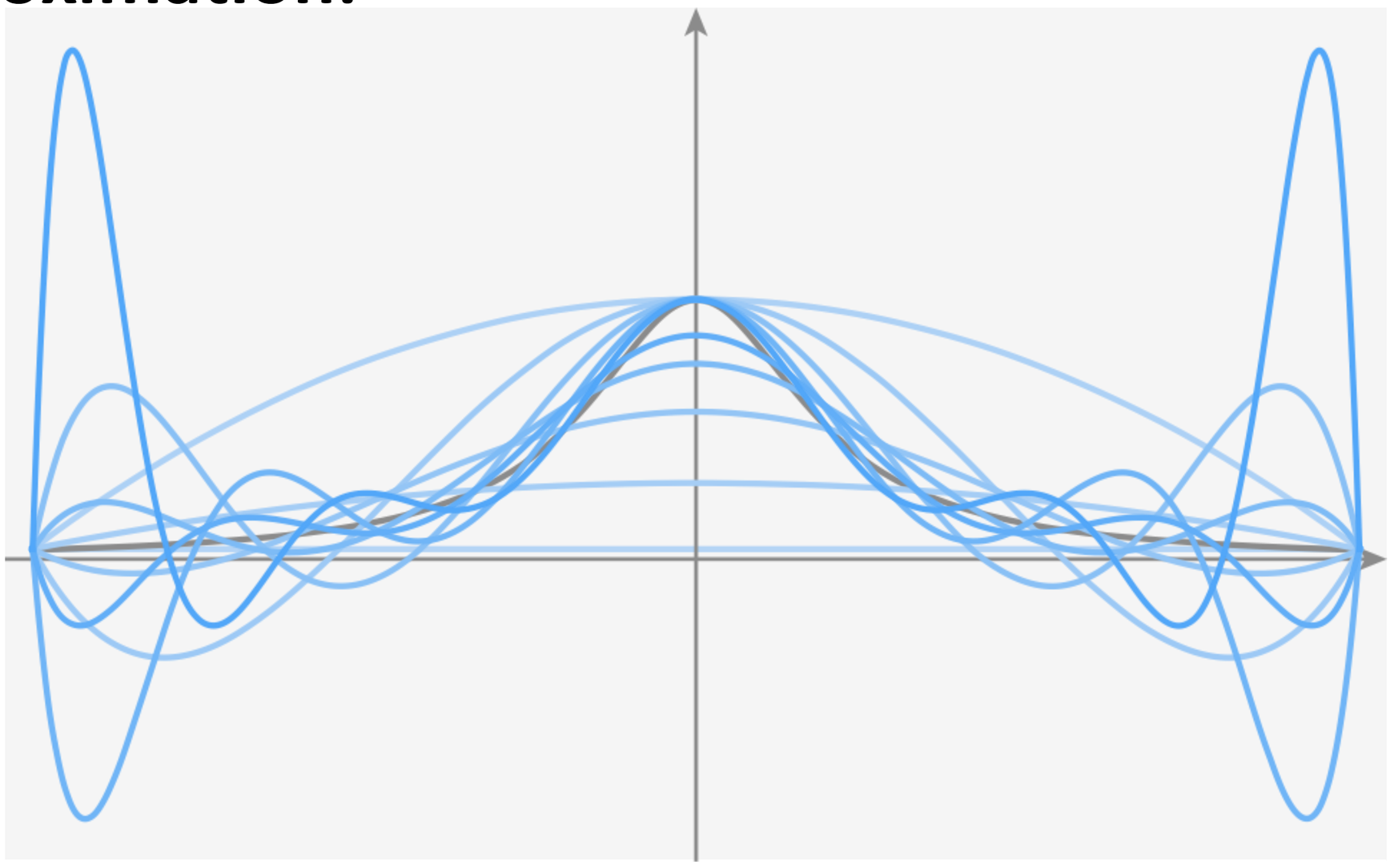
Splines

- Splines most commonly used for interpolation are *cubic* (d=3)
- Schoenberg: piecewise cubics give exact solution to elastic spline problem under assumption of small displacements
- More precisely: among all curves interpolating set of data points, minimizes norm of second derivative (*not* curvature)
- Q: Why use model based on physical splines?



Runge‘s Phenomenon

- Tempting to use higher-degree polynomials, in order to get higher-order continuity
- Can lead to oscillation, ultimately *worse* approximation:

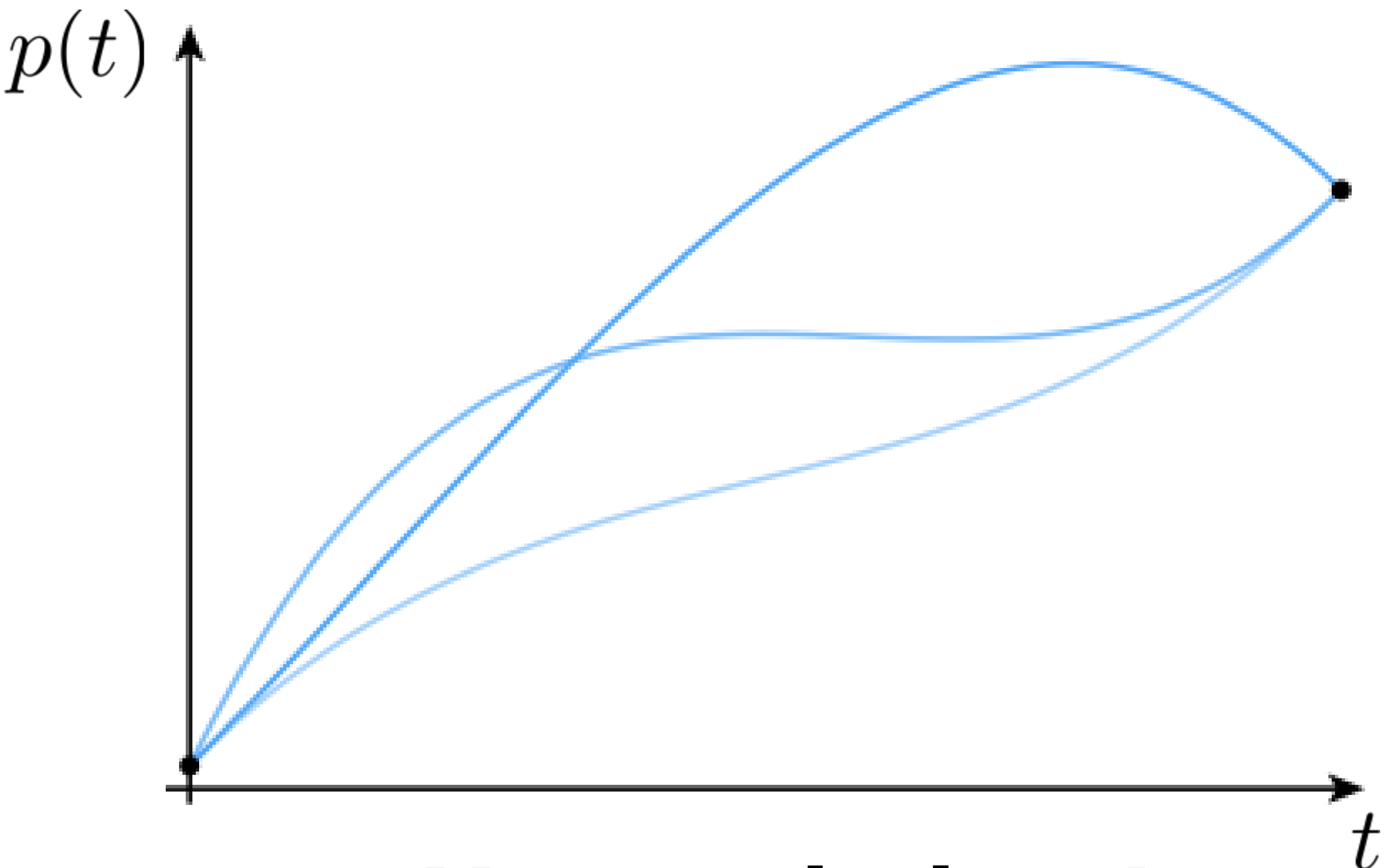


Fitting a Cubic Polynomial to Endpoints

- **Consider a *single* cubic polynomial**

$$p(t) = at^3 + bt^2 + ct + d$$

- **Suppose we want it to match given endpoints:**



Many solutions!

Cubic Polynomial - Degrees of Freedom

- **Why are there so many different solutions?**
- **Cubic polynomial has four *degrees of freedom (DOFs)*, namely four coefficients (a,b,c,d) that we can manipulate/control**
- **Only need *two* degrees of freedom to specify endpoints:**

$$p(t) = at^3 + bt^2 + ct + d$$

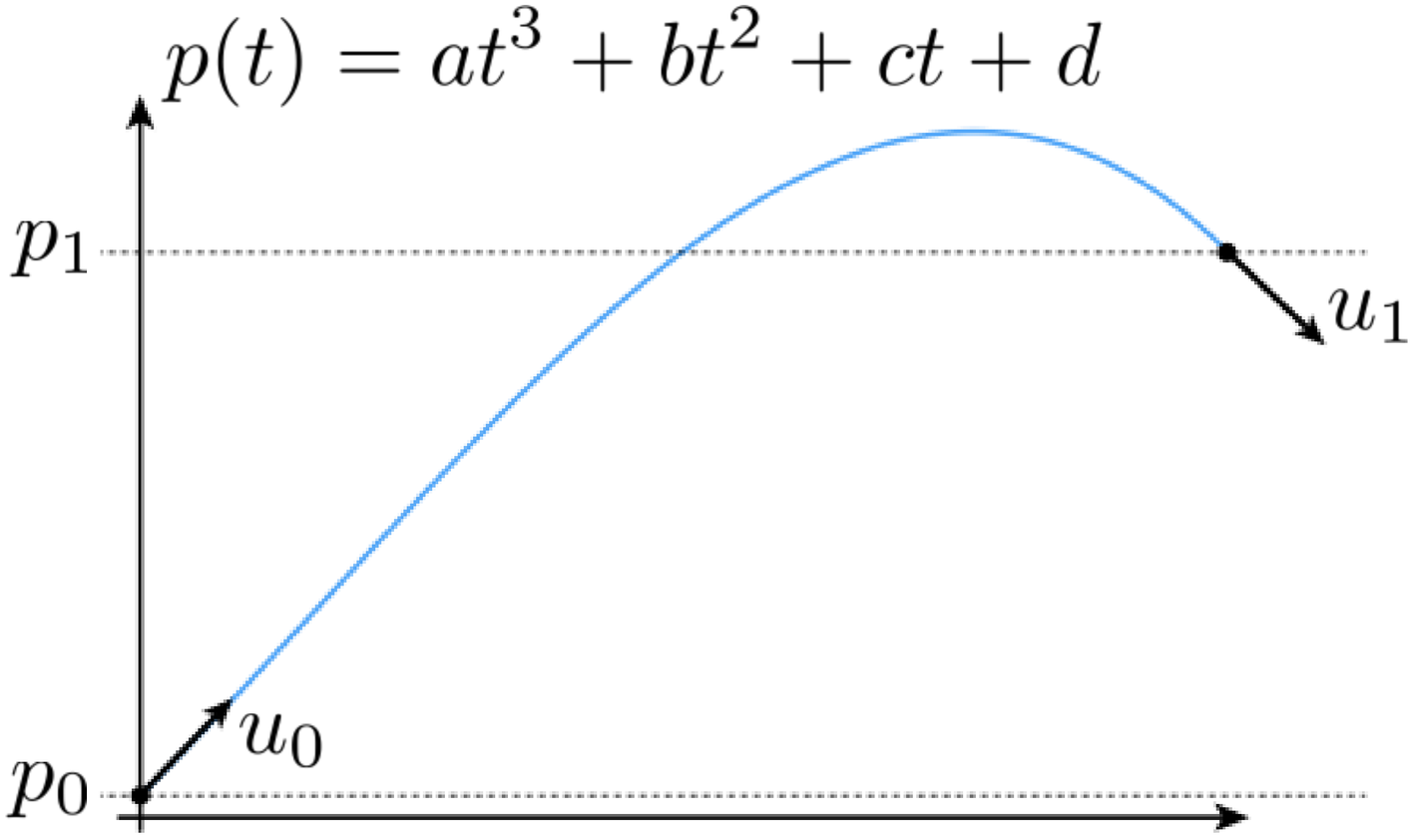
$$p(0) = p_0 \qquad \Rightarrow d = p_0$$

$$p(1) = p_1 \qquad \Rightarrow a + b + c + d = p_1$$

- **Overall, four unknowns but only *two* equations**
- **Not enough to uniquely determine the curve!**

Fitting Cubic to Endpoints and Derivatives

- **What if we also match *derivatives* at endpoints?**



$$p(0) = p_0 \qquad \Rightarrow d = p_0$$

$$p(1) = p_1 \qquad \Rightarrow a + b + c + d = p_1$$

$$p'(0) = u_0 \qquad \Rightarrow c = u_0$$

$$p'(1) = u_1 \qquad \Rightarrow 3a + 2b + c = u_1$$

Splines as Linear Systems

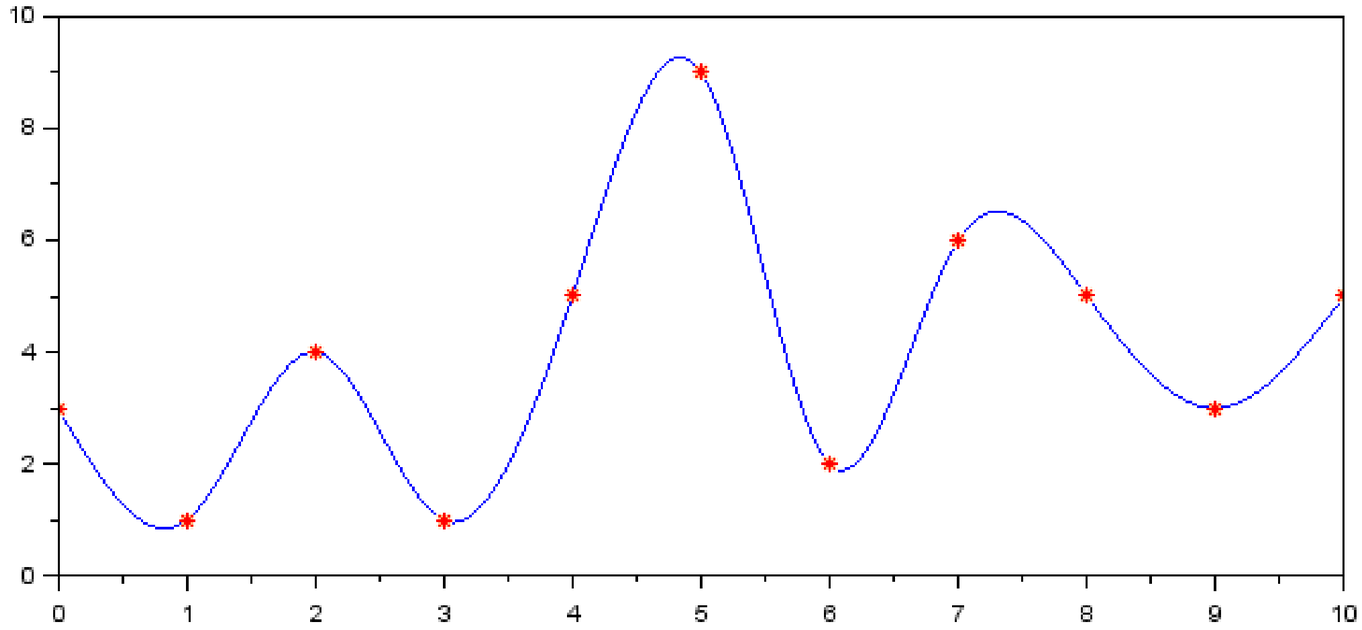
- This time, we have four equations in four unknowns
- Could also express as a matrix equation:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} p_0 \\ p_1 \\ u_0 \\ u_1 \end{bmatrix}$$

- Often, this is the game we will play:
  - each condition on values or derivatives leads to a linear equality
  - if we have m degrees of freedom, we need m conditions to determine spline

Natural Splines

- ***Piecewise* spline made of cubic polynomials p_i**



- **Want: interpolation at both endpoints:**

$$p_i(t_i) = f_i,\ p_i(t_{i+1}) = f_{i+1},\ i = 0, \ldots, n-1$$

- **Want tangents to agree at endpoints ("C^1 continuity"):**

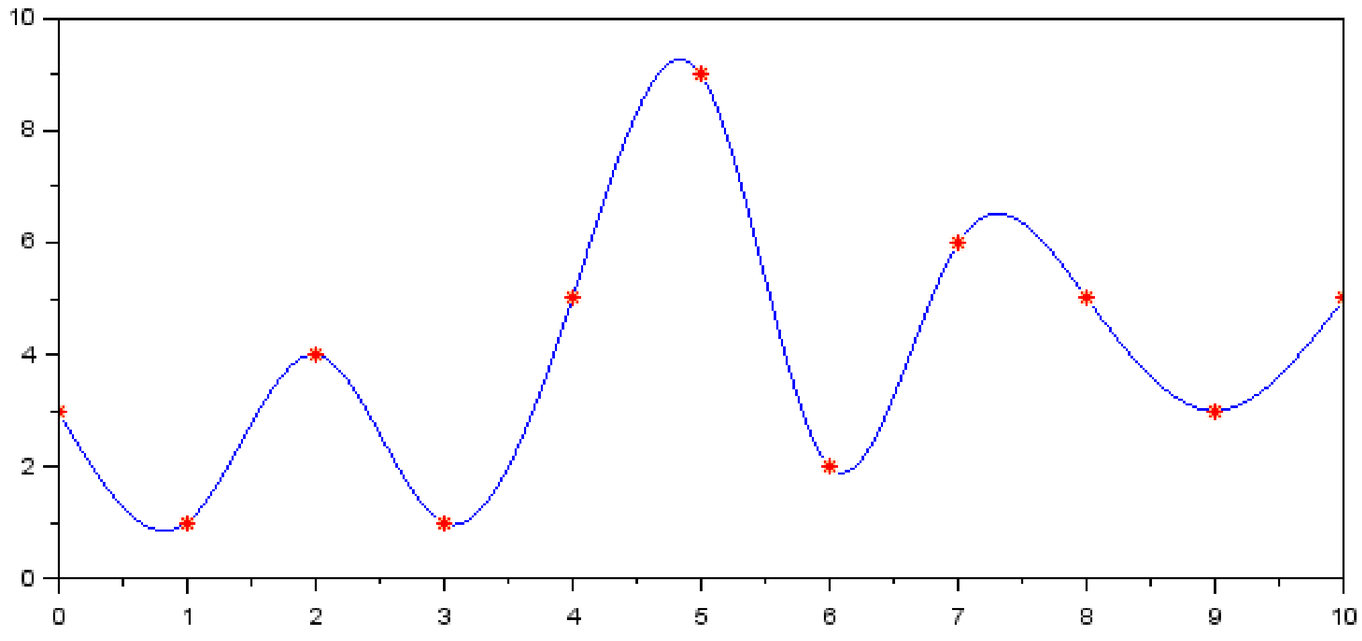
$$p'(t_{i+1}) = p'_{i+1}(t_{i+1}),\ i = 0, \ldots, n-2$$

- **Want curvature to agree at endpoints ("C^2 continuity"):**

$$p''(t_{i+1}) = p''_{i+1}(t_{i+1}),\ i = 0, \ldots, n-2$$

Natural Splines

- How many degrees of freedom? How many equations?
  - 4n DOFs, 2n+(n-1)+(n-1) = 4n-2 equations
- Pin down remaining DOFs by setting curvature to zero at endpoints (this is what makes the curve “natural”)

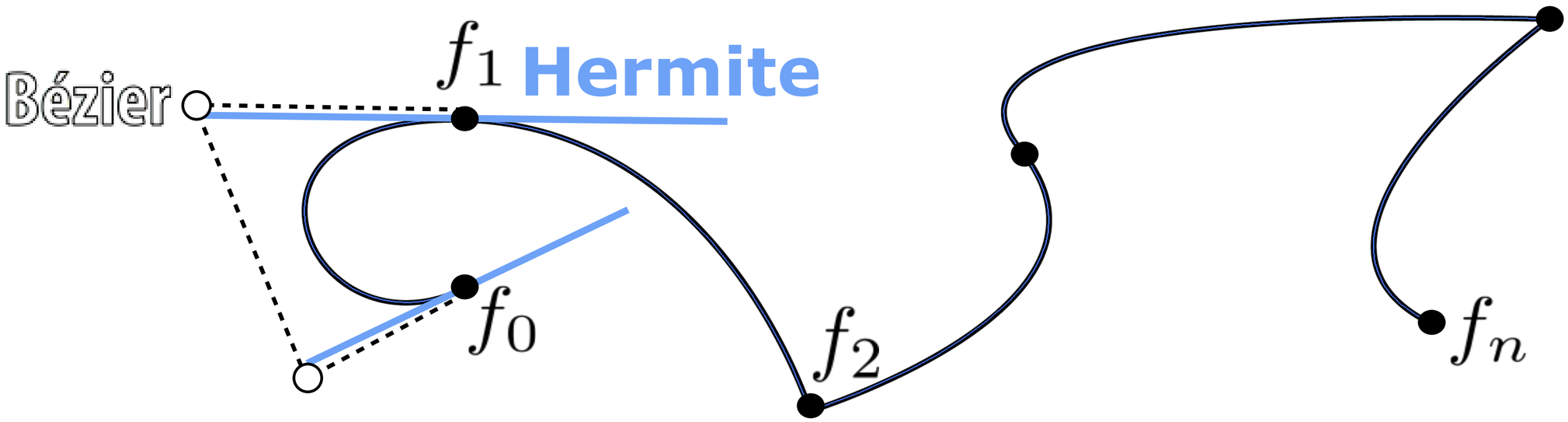


Spline Desiderata

- In general, what are some properties of a “good” spline?
  - INTERPOLATION: spline passes *exactly* through data points
  - CONTINUITY: at least *twice* differentiable everywhere
  - LOCALITY: moving one control point doesn’t affect whole curve
- How does our natural spline do?
  - INTERPOLATION: yes, by construction
  - CONTINUITY: C^2 everywhere
  - LOCALITY: no, coefficients depend on global linear system
- Many other types of splines we can consider
- Spoiler: there is “no free lunch” with cubic splines (can’t simultaneously get all three properties)

Review: Hermite/Bézier Splines

- Discussed briefly in introduction to geometry
- Each cubic "piece" specified by endpoints and tangents:



- Equivalently: by four points (Bézier form); just take difference!
- Commonly used for 2D vector art (Illustrator, SVG, ...)
- Can we get tangent continuity?
- Sure: set tangents to same value on both sides of knot!
  - E.g., f_1 above, but not f_2

Properties of Hermite/Bézier Spline

- More precisely, want endpoints to interpolate data:

$$p_i(t_i) = f_i,\ p_i(t_{i+1}) = f_{i+1},\ i = 0, \ldots, n-1$$

- Also want tangents to interpolate some given data:

$$p_i'(t_i) = u_i,\ p'(i)_{t_{i+1}} = u_{i+1},\ i = 0, \ldots, n-1$$

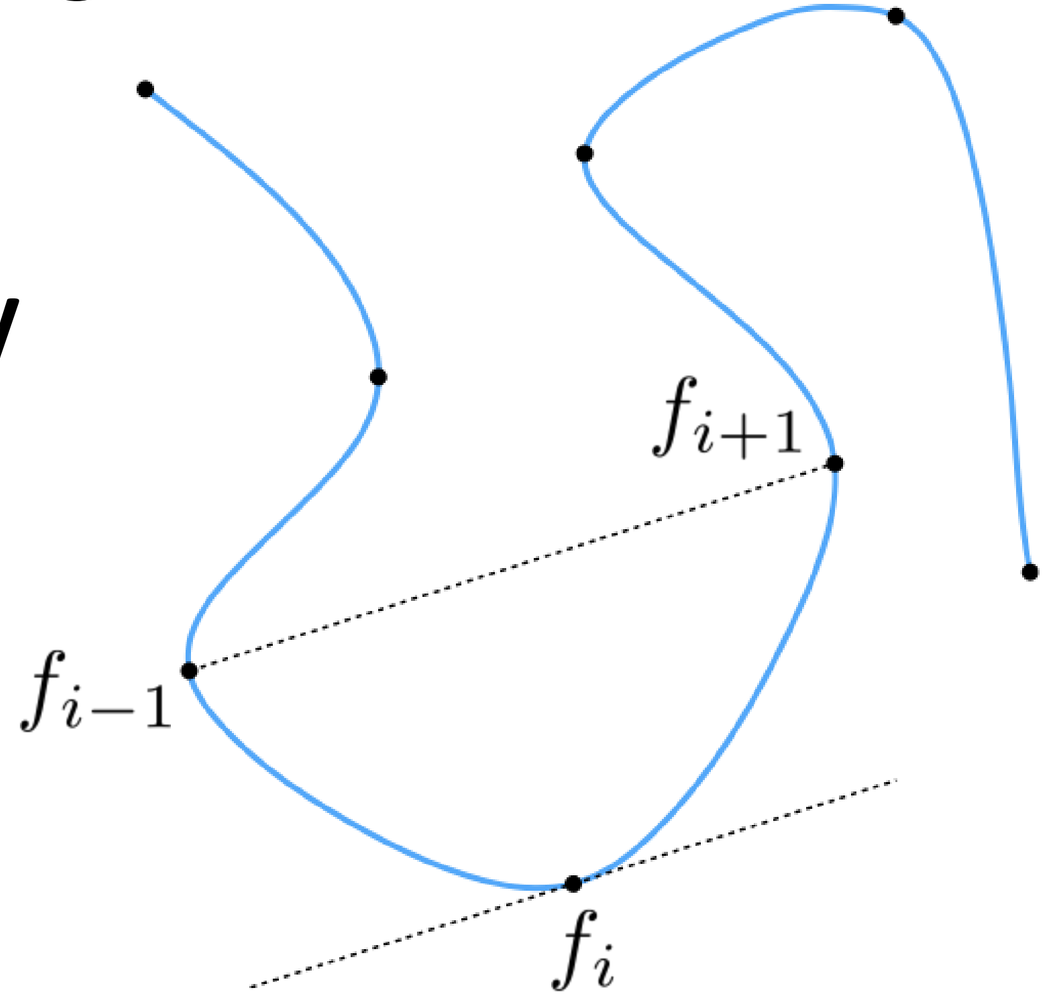
- How is this *different* from our natural spline's tangent condition?
- There, tangents didn't have to match any prescribed value—they merely had to be the same. Here, they are given.
- How many conditions overall?
  - 2n + 2n = 4n
- What properties does this curve have?
  - INTERPOLATION and LOCALITY, but not C^2 CONTINUITY

Catmull-Rom Splines

- Sometimes makes sense to specify *tangents,* but often more convenient to just specify *values*
- Catmull-Rom: specialization of Hermite spline, determined by values alone
- Basic idea: use difference of neighbors to define tangent

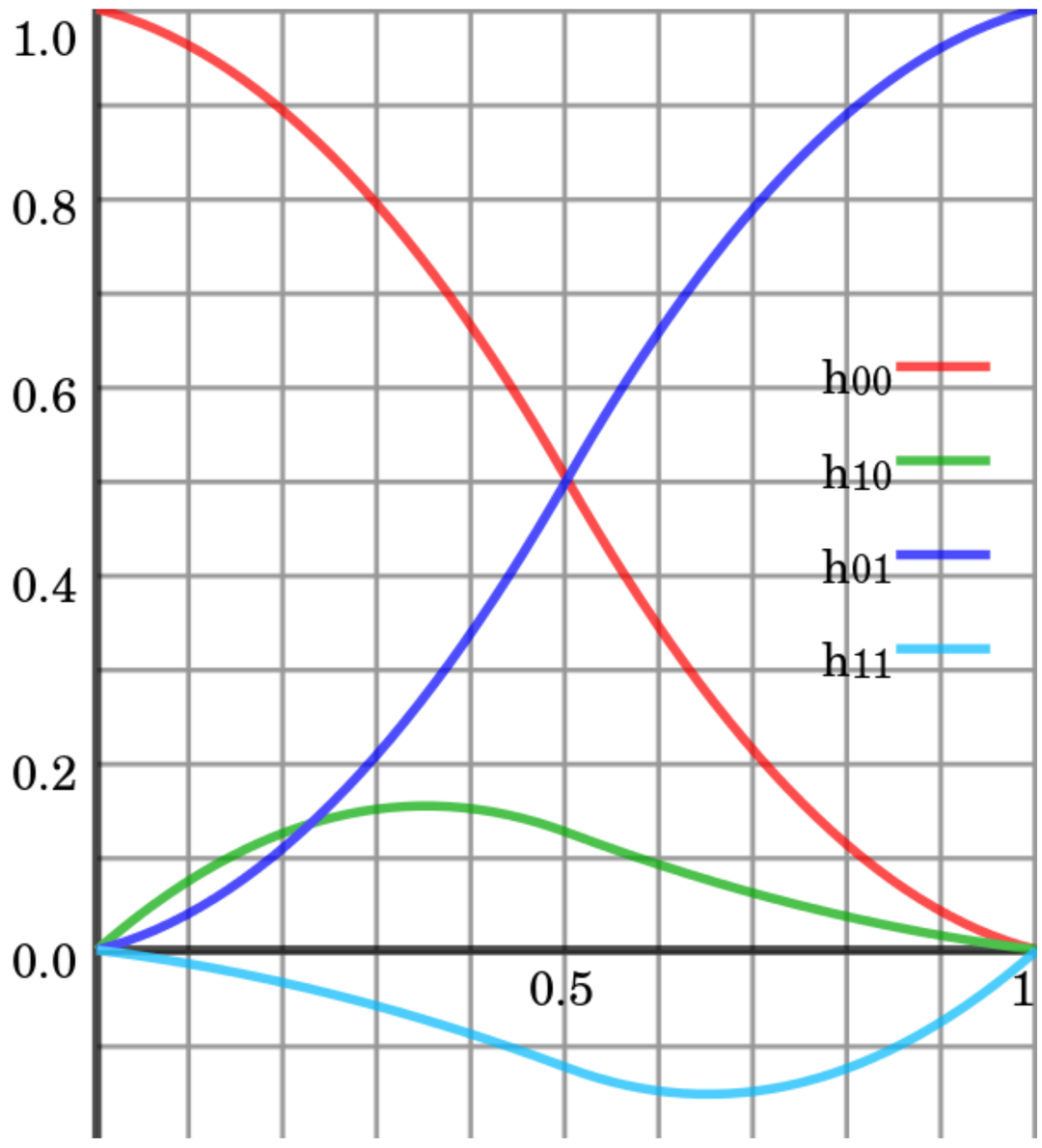
$$u_i := \frac{f_{i+1} - f_{i-1}}{t_{i+1} - t_{i-1}}$$

- All the same properties as any other Hermite spline
- Commonly used to interpolate motion in computer animation.
- Many, many variants, but Catmull-Rom is usually good starting point



Hermite/Bézier/Catmull-Rom Splines as sums of basis functions

$$\boldsymbol{p}(t) = (2t^3 - 3t^2 + 1)\boldsymbol{p}_0 + (t^3 - 2t^2 + t)\boldsymbol{m}_0 + (-2t^3 + 3t^2)\boldsymbol{p}_1 + (t^3 - t^2)\boldsymbol{m}_1$$



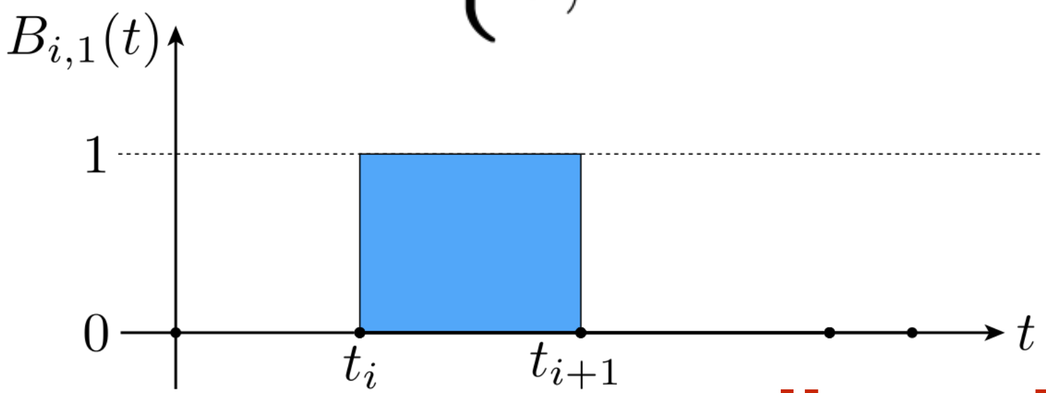
Spline Desiderata, Revisited

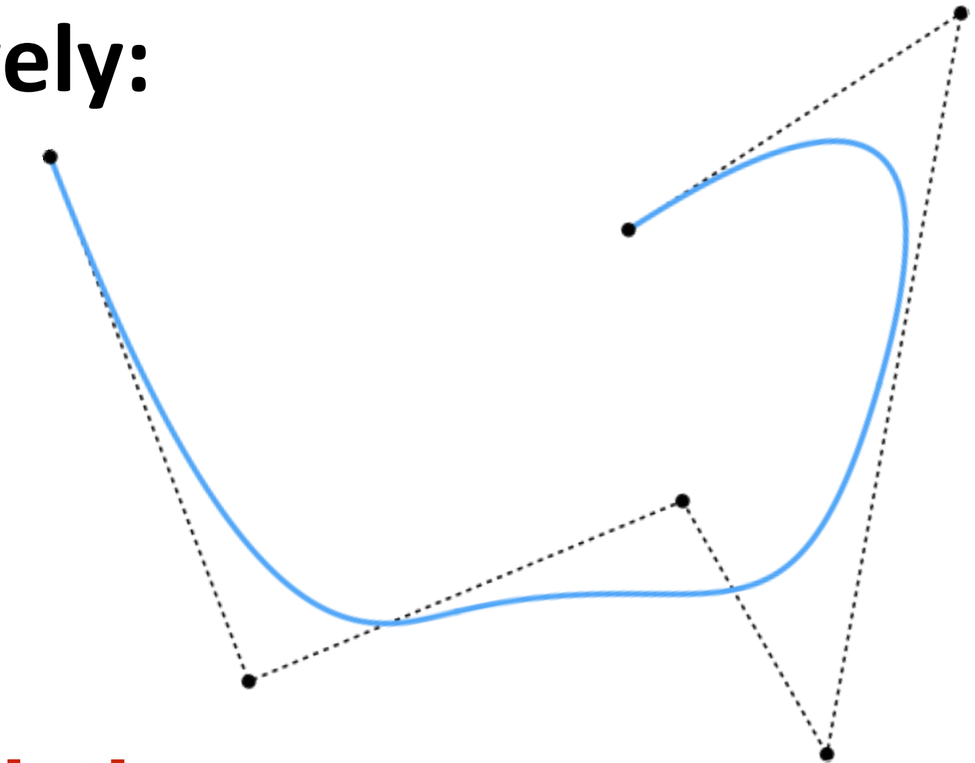
	INTERPOLATION	CONTINUITY	LOCALITY
natural	YES	YES	NO
Hermite	YES	NO	YES
???	NO	YES	YES

B-Splines

- **Get better continuity *and* local control! Sacrifice interpolation ☹**
- **B-spline *basis* defined recursively:**

$$B_{i,1}(t) := \begin{cases} 1, & \text{if } t_i \le t < t_{i+1} \\ 0, & \text{otherwise} \end{cases}$$





linear interpolation

$$B_{i,k}(t) := \frac{t-t_i}{t_{i+k-1}-t_i} B_{i,k-1}(t) + \frac{t_{i+k}-t}{t_{i+k}-t_{i+1}} B_{i+1,k-1}(t)$$

- **B-spline itself is then a linear combination of bases:**

$$f(t) := \sum_i a_i B_{i,d}$$

degree

Spline Desiderata, Revisited

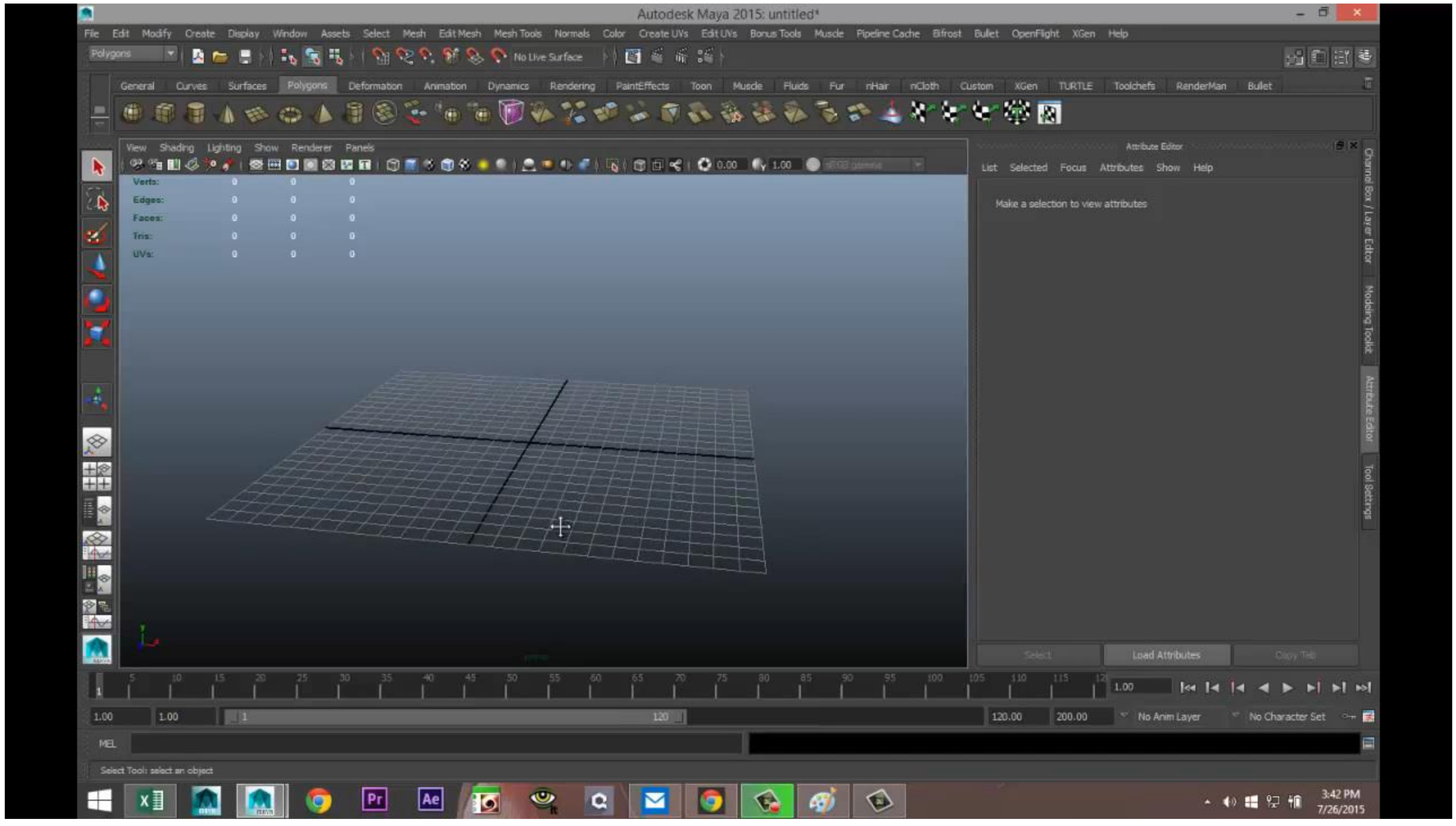
	INTERPOLATION	CONTINUITY	LOCALITY
natural	YES	YES	NO
Hermite	YES	NO	YES
B-Splines	NO	YES	YES

Ok, I get it: splines are great.

But what exactly are we interpolating?

Blend Shapes

- Keyframes represent surfaces (in correspondence)



- Simplest scheme: take linear combination of vertex positions
- Spline used to control choice of weights over time

Blend Shapes

- In the hands of a skilled animator…

courtesy Félix Ferrand

Another example: camera path

- Animate position, view direction, “up” direction
  - each path is a function f(t) = (x(t), y(t), z(t))
  - each component (x,y,z) is a spline

Zaha Hadid Architects—City of Dreams Hotel Tower