Lecture 13:

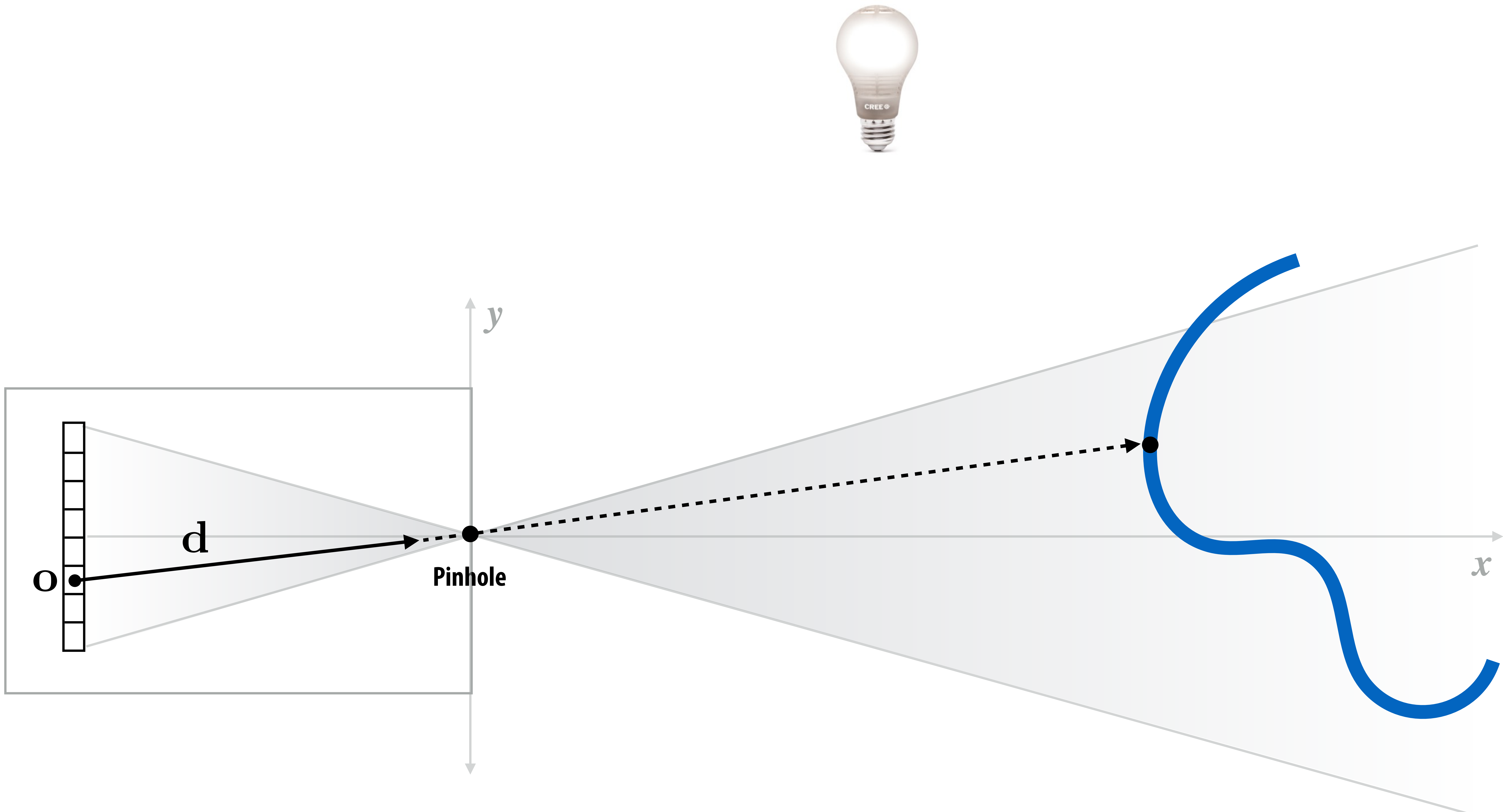
# Reflection

Computer Graphics

CMU 15-462/15-662, Fall 2015

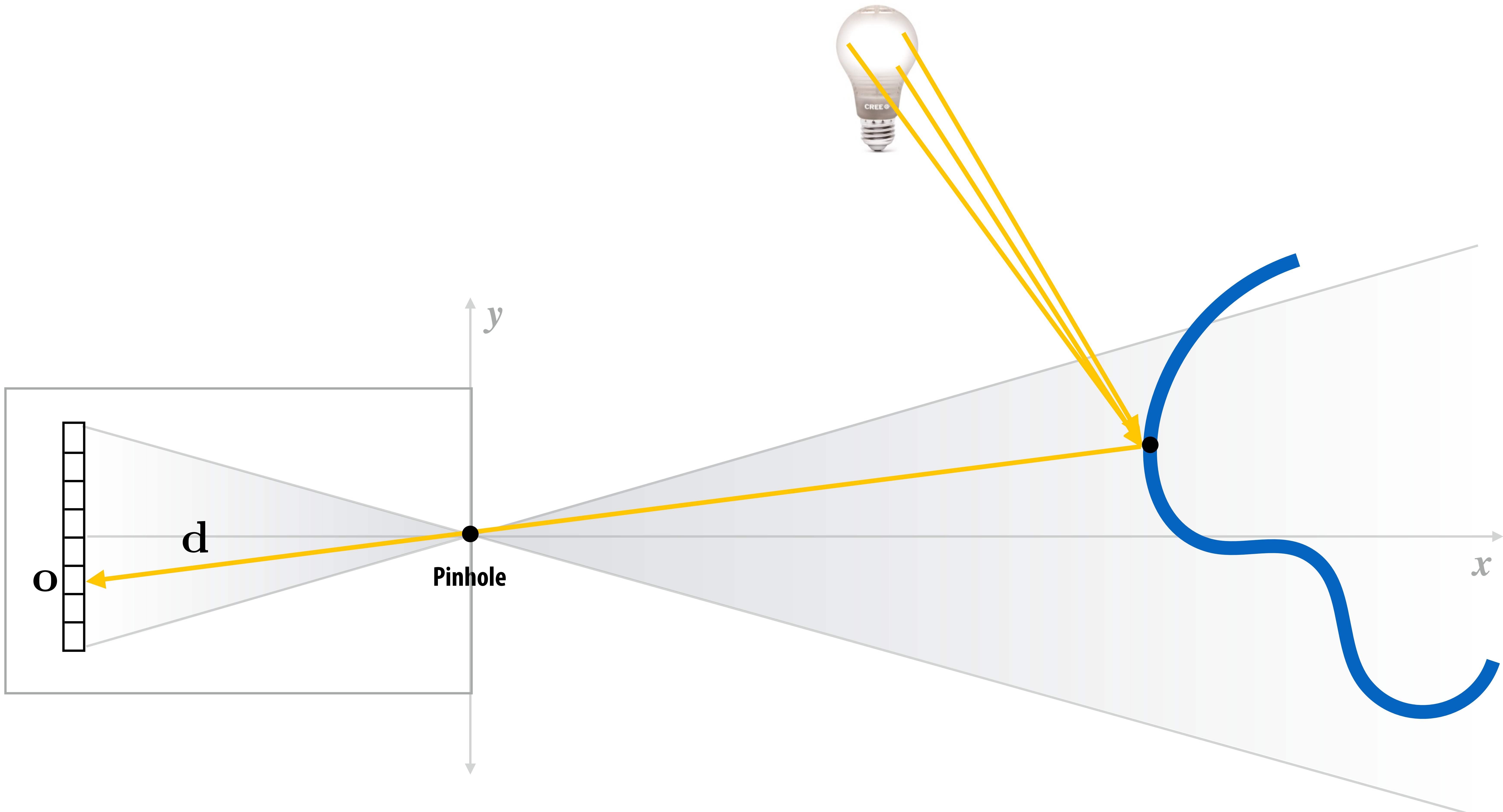
Slides credit: a majority of these slides were created by Matt Pharr and Pat Hanrahan

# Ray tracer measures radiance along a ray




In the ray tracing algorithms we've discussed so far: want to measure radiance traveling in the direction opposite the ray direction.

# Ray tracer measures radiance along a ray




**Radiance entering camera in direction *d* = light from scene light sources that is reflected off surface in direction *d*.**

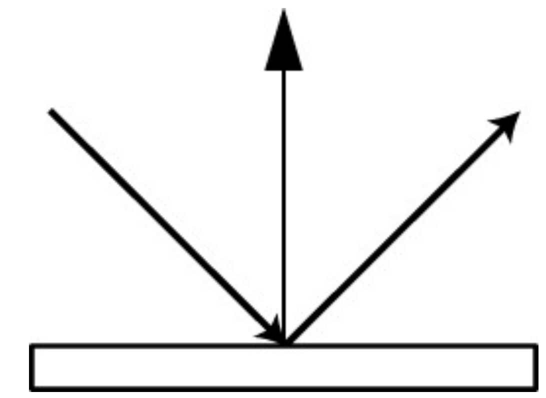
# Reflection models

- **Definition: reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident (same) side without change in frequency**

# Categories of reflection functions

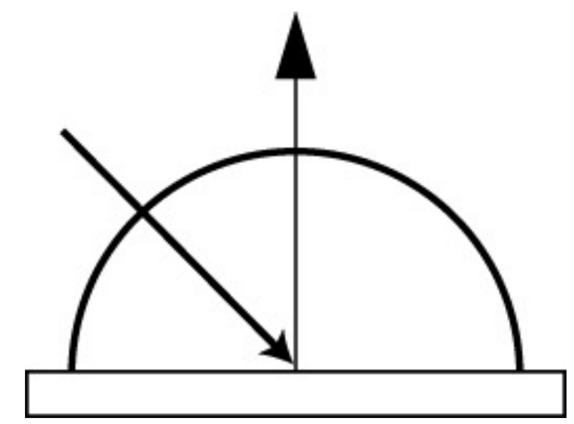
- **Ideal specular**

  Perfect mirror



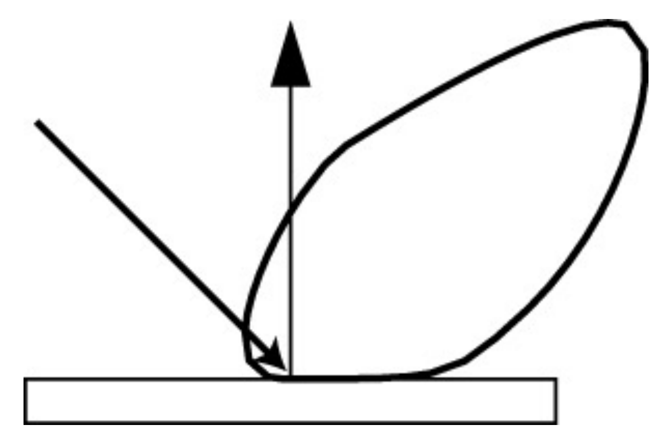
- **Ideal diffuse**

  Uniform reflection in all directions



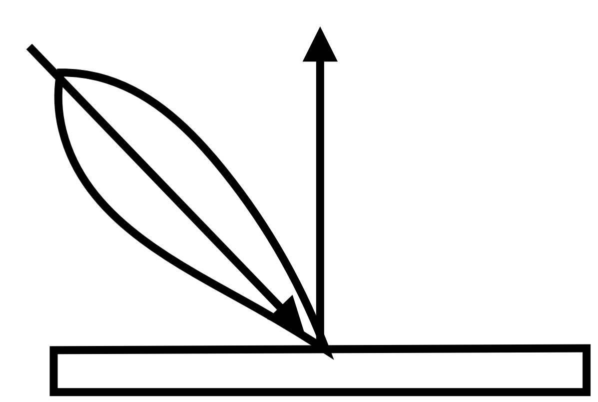

- **Glossy specular**

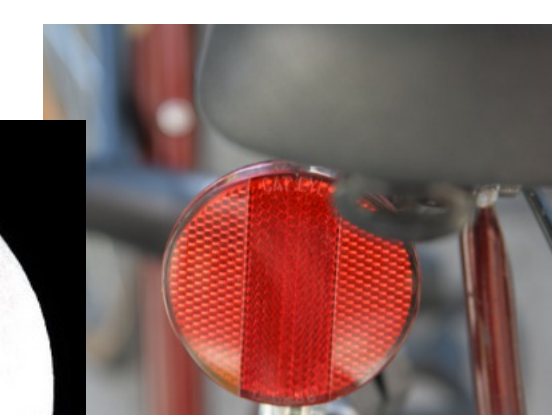
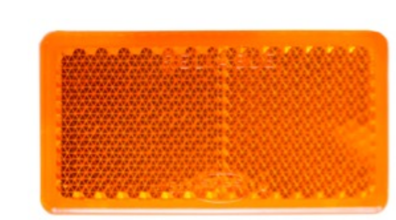
  Majority of light distributed in reflection direction



- **Retro-reflective**

  Reflects light back toward source



Diagrams illustrate how incoming light energy from given direction is reflected in various directions.

# Materials: diffuse

# Materials: plastic

# Materials: red semi-gloss paint

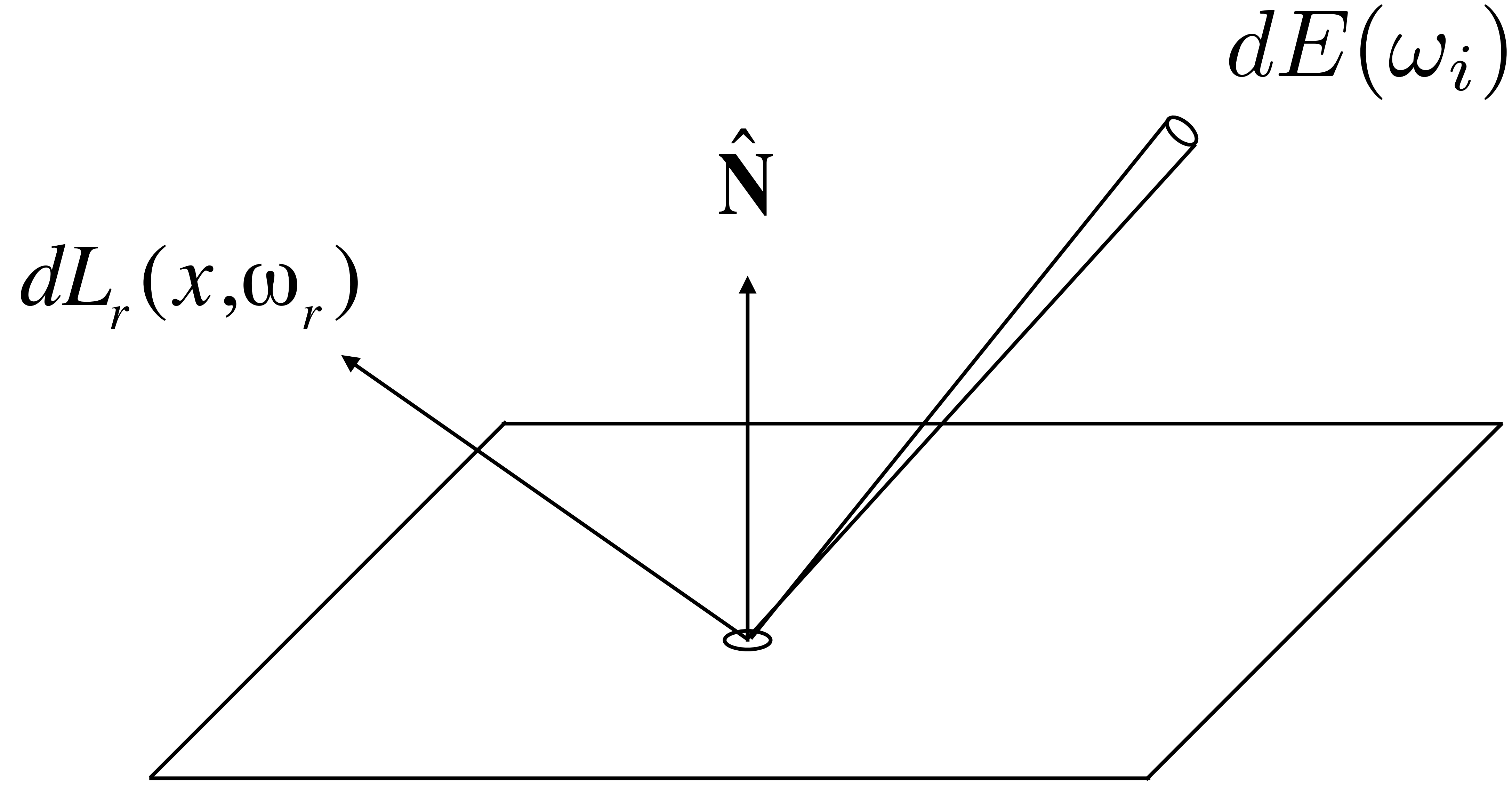
# Materials: Ford mystic lacquer paint

# Materials: mirror

# Materials: gold

# Materials

# Bidirectional reflectance distribution function (BRDF)




$$dL_r(\omega_r) \propto dE_i(\omega_i)$$

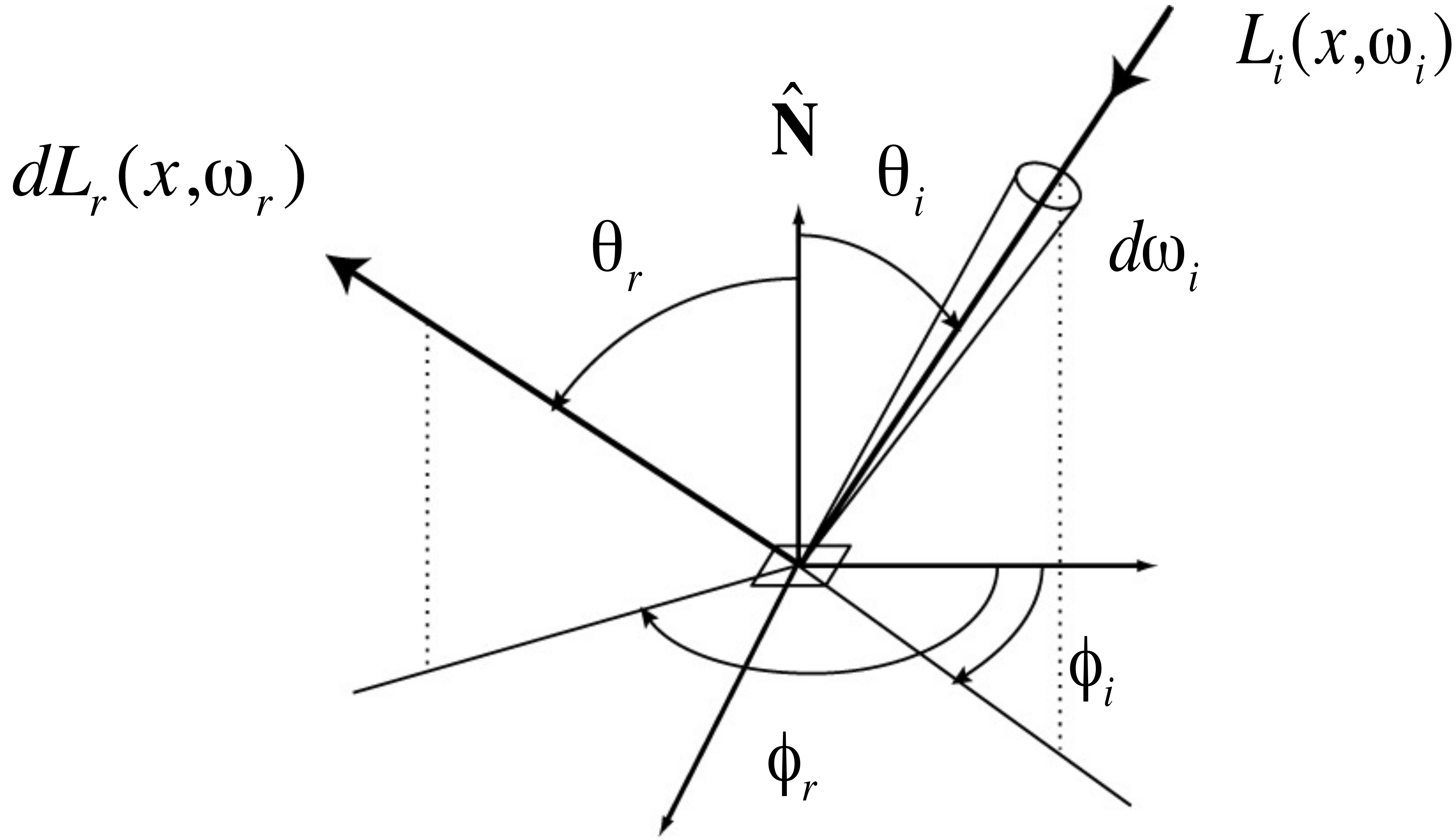
**Differential irradiance landing on surface from differential cone of directions** $\omega_i$

$$dE(\omega_i) = dL(\omega_i) cos\,\theta_i$$

**Differential radiance reflected in direction** $\omega_r$ **(due to differential irradiance from** $\omega_i$**)**

$$dL_r(\omega_r)$$

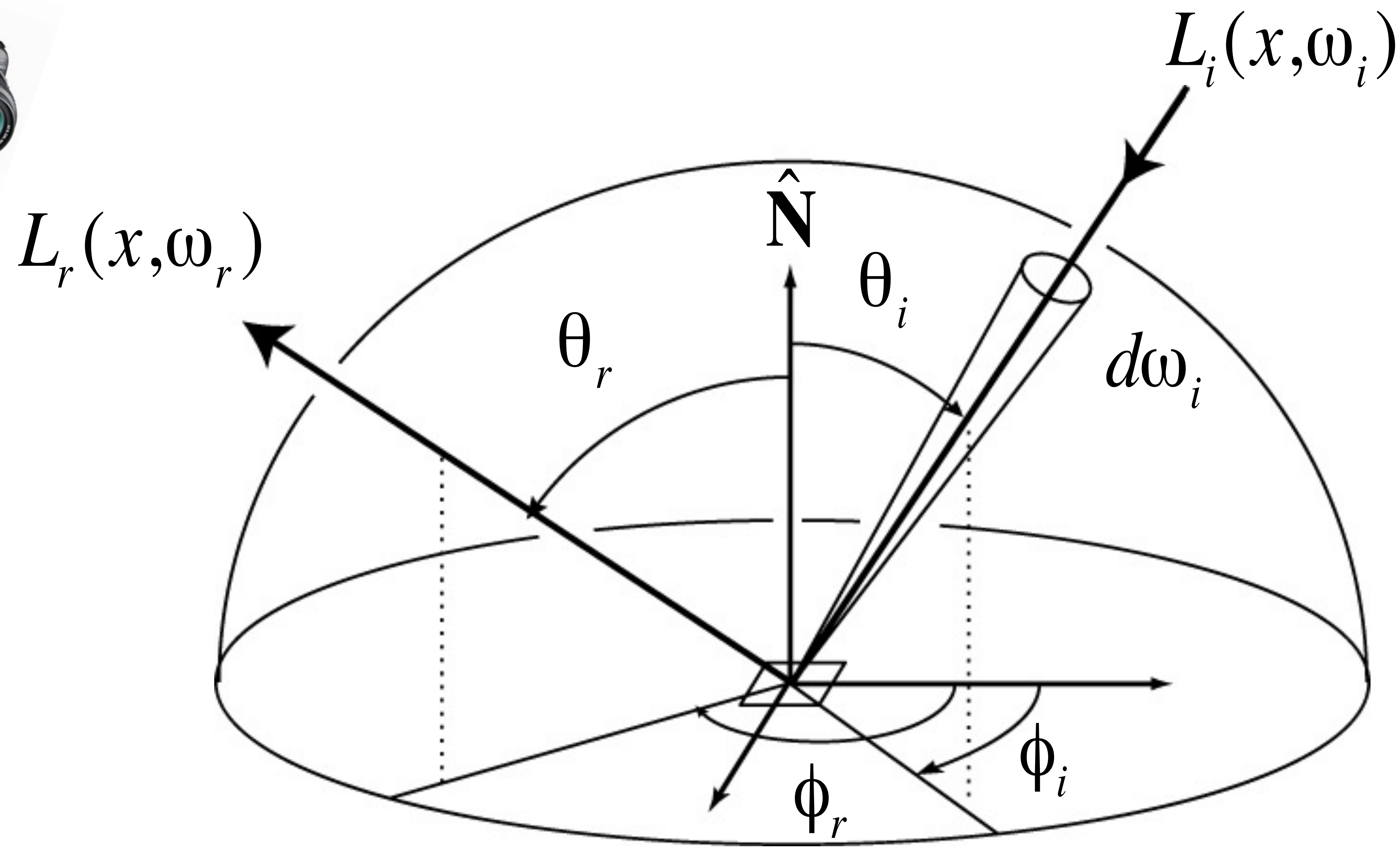
# Bidirectional reflectance distribution function (BRDF)




$$f_r(\omega_i \to \omega_r) = \frac{\mathrm{d}L_r(\omega_r)}{\mathrm{d}E_i(\omega_i)} = \frac{\mathrm{d}L_r(\omega_r)}{\mathrm{d}L_i(\omega_i)\ \cos\theta_i} \quad \left[\frac{1}{\mathrm{sr}}\right]$$

**BRDF defines the fraction of energy arriving from $\omega_i$ that is reflected in the direction $\omega_r$**

# The reflection equation




$$\mathrm{d}L_r(\omega_r) = f_r(\omega_i \to \omega_r)\,\mathrm{d}L_i(\omega_i)\,\cos\theta_i$$

$$L_r(\mathrm{p},\omega_r) = \int_{H^2} f_r(\mathrm{p},\omega_i \to \omega_r)\,L_i(\mathrm{p},\omega_i)\,\cos\theta_i\,\mathrm{d}\omega_i$$

# Solving the reflection equation

$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \, \mathrm{d}\omega_i$$

- **Basic Monte Carlo estimate:**
  - **Generate directions $\omega_j$ sampled from some distribution $p(\omega)$**
    - **To reduce variance $p(\omega)$ should match BRDF or incident radiance function**
  - **Compute the estimator**

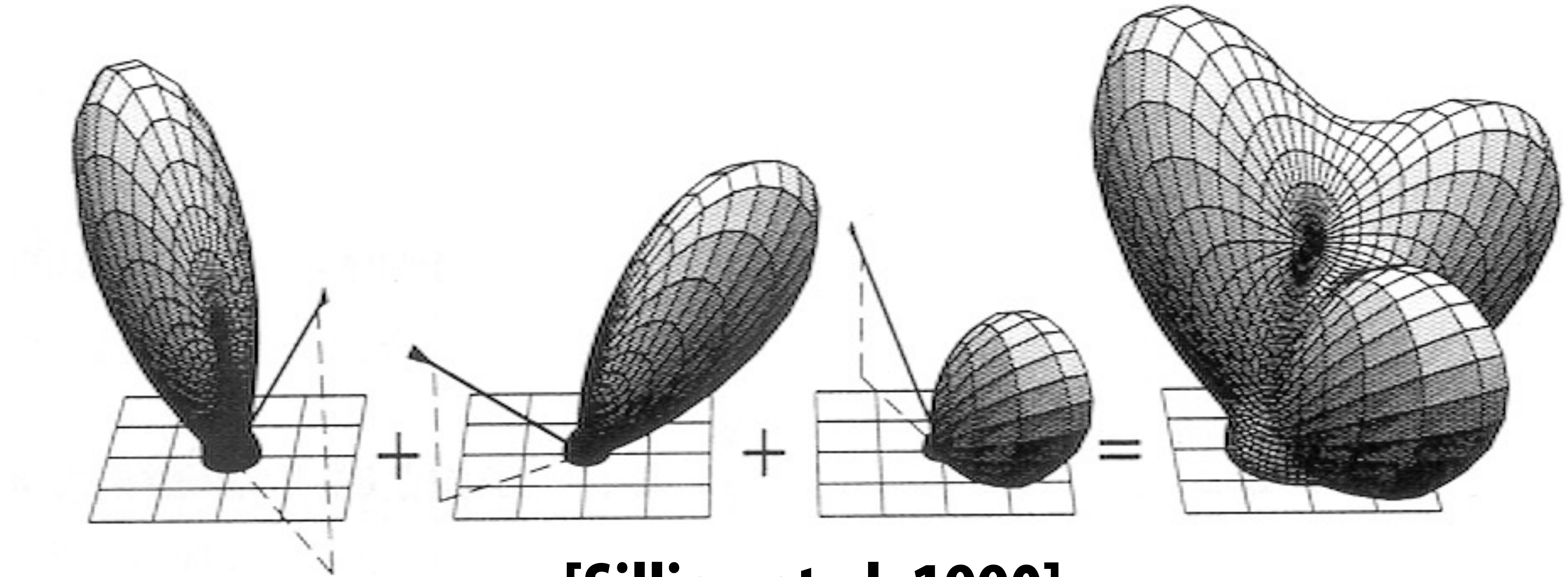
$$\frac{1}{N} \sum_{j=1}^{N} \frac{f_r(\mathrm{p}, \omega_j \to \omega_r)\, L_i(\mathrm{p}, \omega_j)\, \cos\theta_j}{p(\omega_j)}$$

# Estimating reflected light

```
// Assume:
//   Ray ray hits surface at point hit_p
//   Normal of surface at hit point is hit_n

Vector3D wr = -ray.d;   // outgoing direction
Spectrum Lr = 0.;
for (int i = 0; i < N; ++i) {
    Vector3D wi;        // sample incident light from this direction
    float pdf;          // p(wi)

    generate_sample(brdf, &wi, &pdf);     // generate sample according to brdf

    Spectrum f = brdf->f(wr, wi);
    Spectrum Li = trace_ray(Ray(hit_p, wi));  // compute incoming Li
    Lr += f * Li * fabs(dot(wi, hit_n)) / pdf;
}
return Lr / N;
```

# Properties of BRDFs

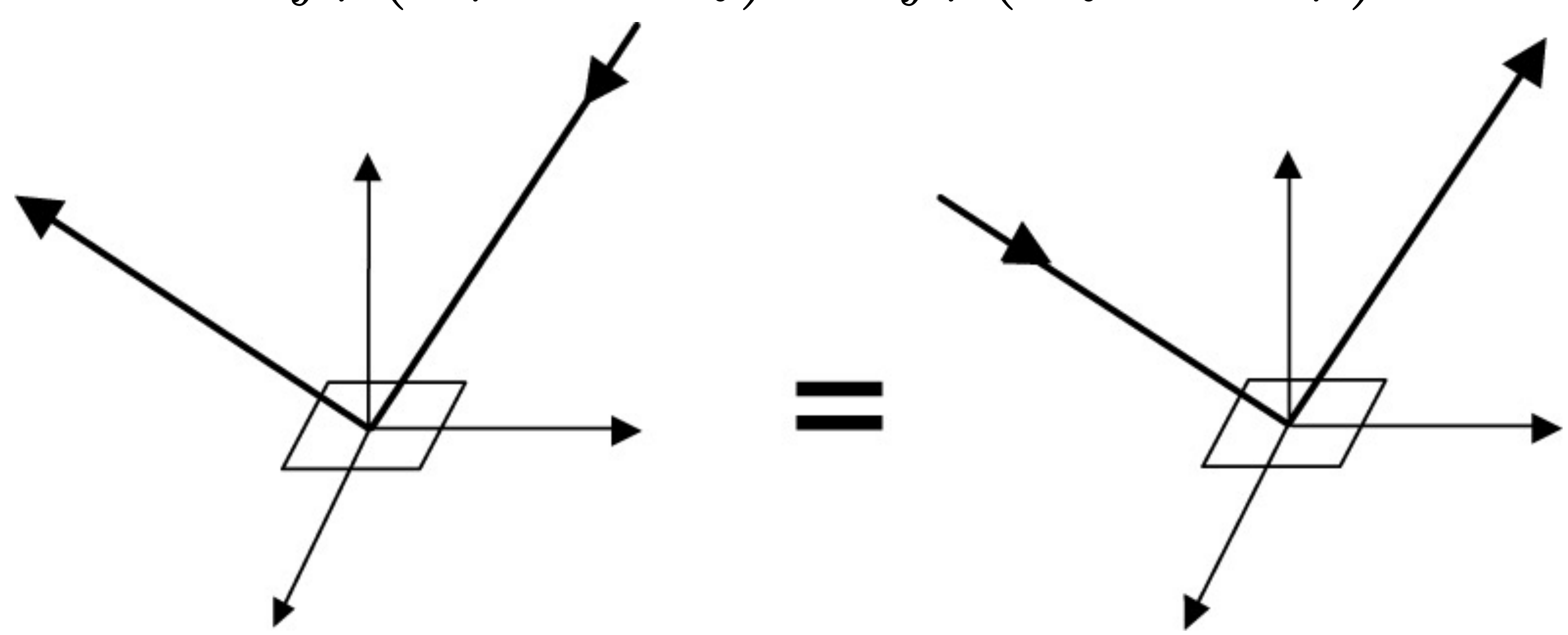
- **Linearity**



[Sillion et al. 1990]

- **Reciprocity principle**

$$f_r(\omega_r \to \omega_i) = f_r(\omega_i \to \omega_r)$$



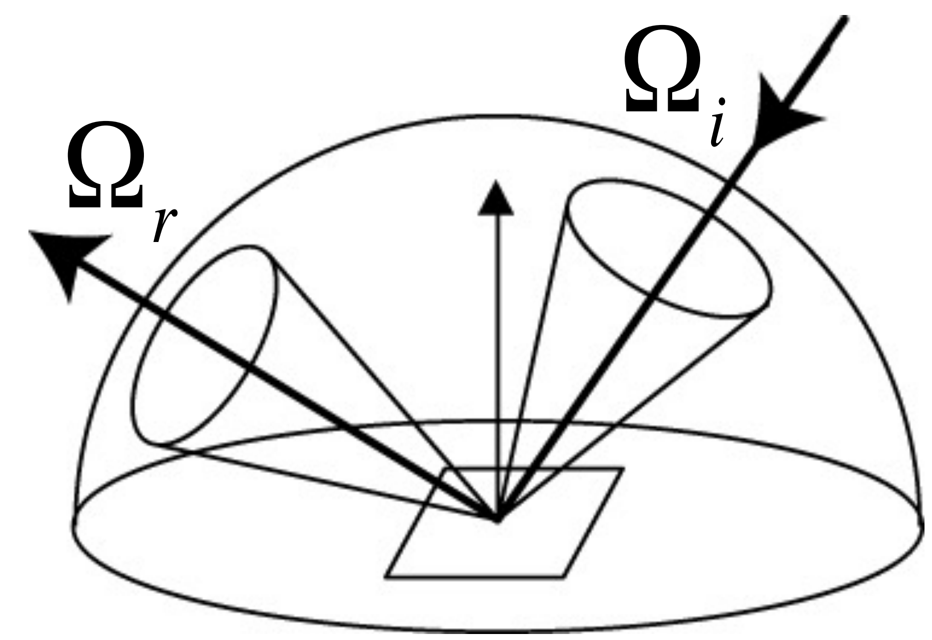
# Properties of BRDFs

- **Isotropic vs. anisotropic**
  - **If isotropic,** $f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_r(\theta_i, \theta_r, \phi_r - \phi_i)$
  - **Then, from reciprocity,**

$$f_r(\theta_i, \theta_r, \phi_r - \phi_i) = f_r(\theta_r, \theta_i, \phi_i - \phi_r) = f_r(\theta_i, \theta_r, |\phi_r - \phi_i|)$$

# Energy conservation

$$f_r(\omega_i \to \omega_r) = \frac{\mathrm{d}L_r(\omega_i \to \omega_r)}{\mathrm{d}E_i} \quad \left[\frac{1}{\mathrm{sr}}\right]$$




**Outgoing energy cannot exceed incoming energy (reflection does not create energy)**

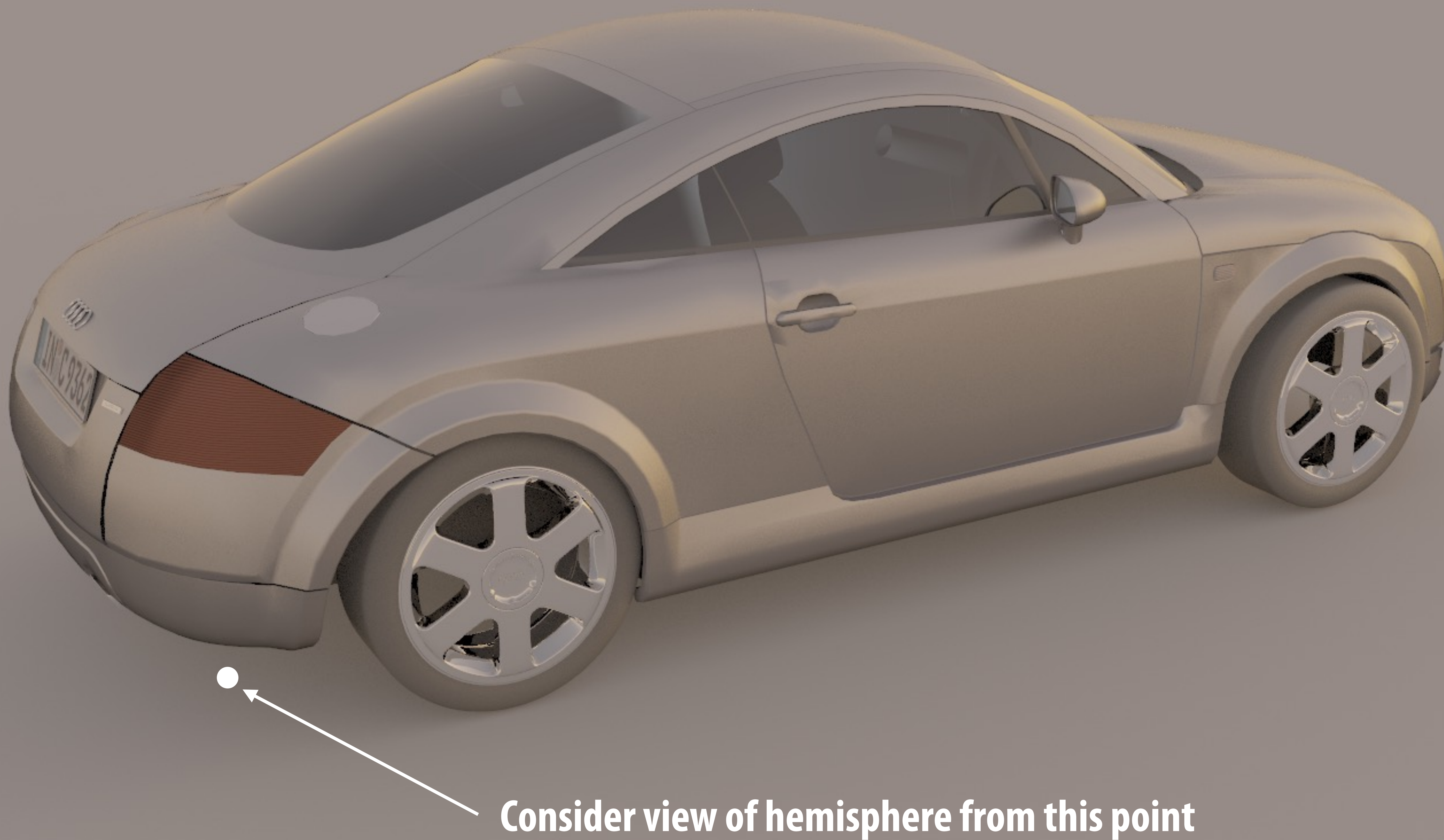
$$\begin{aligned}\frac{\mathrm{d}\Phi_r}{\mathrm{d}\Phi_i} &= \frac{\int_{\Omega_r} L_r(\omega_r)\cos\theta_r\,\mathrm{d}\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i\,\mathrm{d}\omega_i} \\ &= \int_{\Omega_r}\int_{\Omega_i} \frac{f_r(\omega_i \to \omega_r)\,L_i(\omega_i)\;\cos\theta_i \mathrm{d}\omega_i \cos\theta_r\,\mathrm{d}\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i\,\mathrm{d}\omega_i} \\ &\leq 1\end{aligned}$$

# Energy conservation

**Overall fraction of light reflected by surface**
**(assuming constant incident light from all directions)**

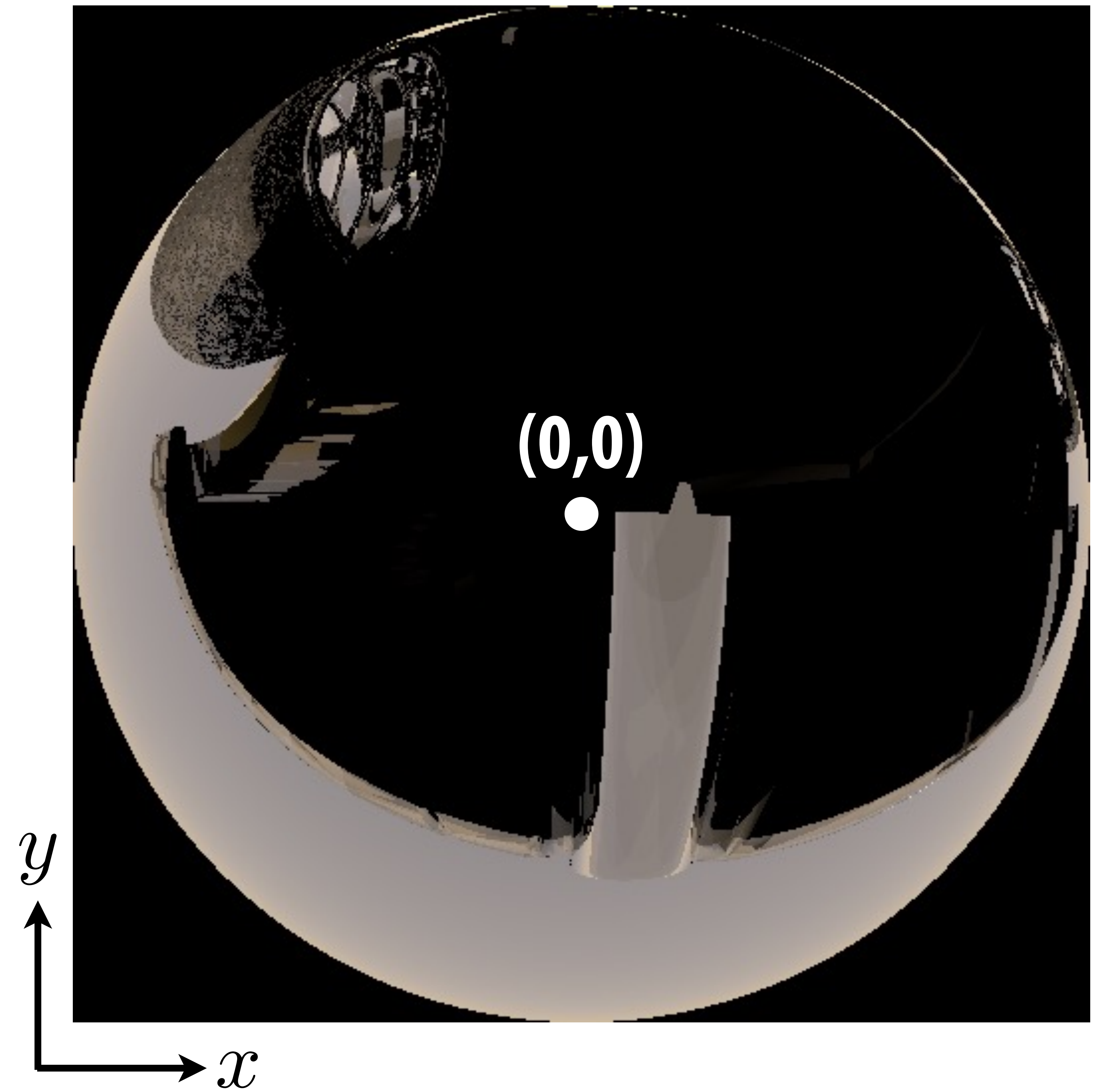
$$\begin{aligned}\rho &= \frac{\int_{H^2}\int_{H^2} f_r(\omega_i \to \omega_r)\, C\ \cos\theta_i \mathrm{d}\omega_i \cos\theta_r\, \mathrm{d}\omega_r}{\int_{H^2} C \cos\ \theta_i d\omega_i} \\ &= \frac{1}{\pi}\int_{H^2}\int_{H^2} f_r(\omega_i \to \omega_r)\, C\ \cos\theta_i \mathrm{d}\omega_i \cos\theta_r\, \mathrm{d}\omega_r \\ &\leq 1\end{aligned}$$

# Hemispherical incident radiance



# Hemispherical incident radiance

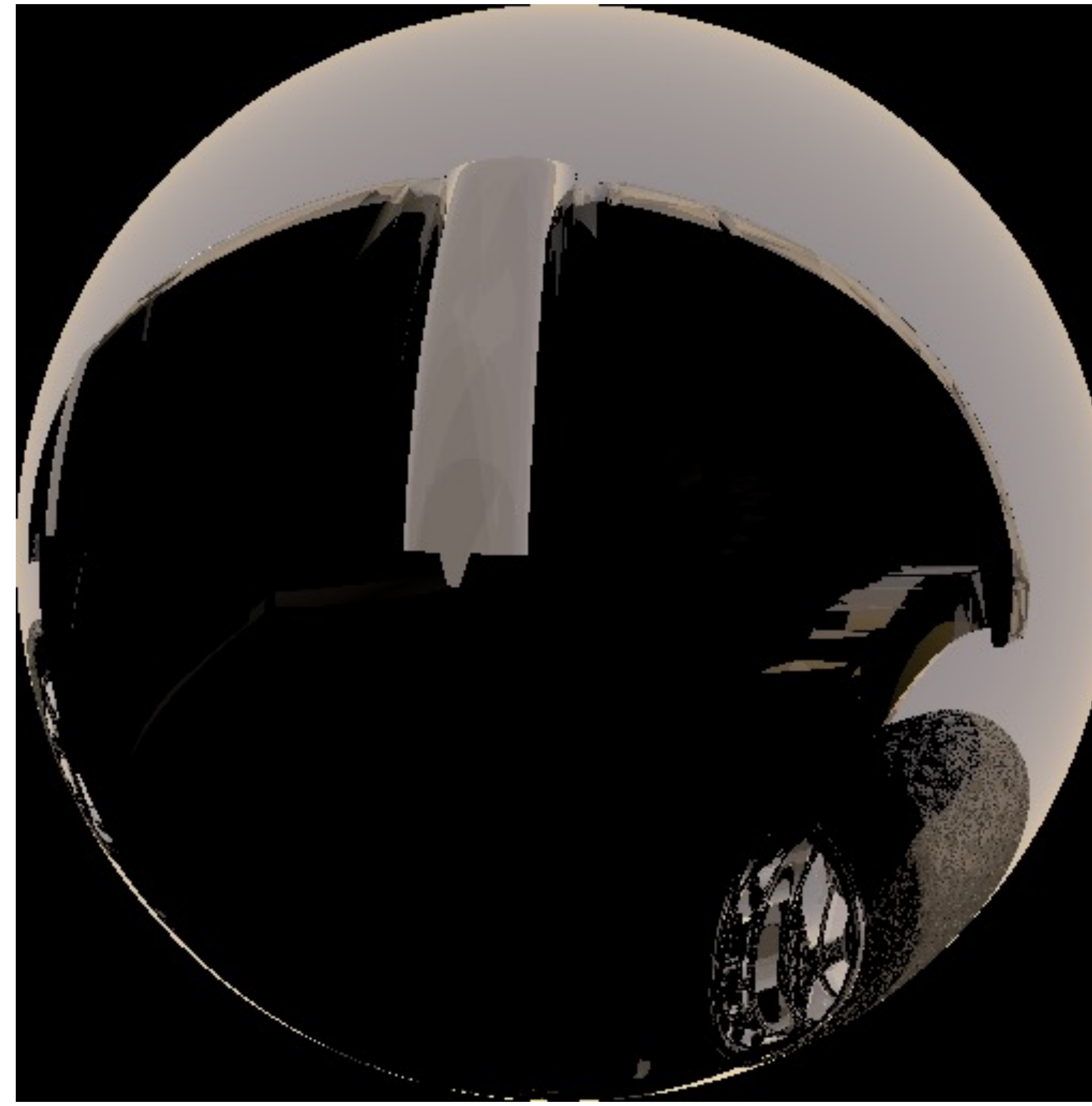
**At any point on any surface in the scene, there's an incident radiance field that gives the directional distribution of illumination at the point**




$$L_i(\omega) = L_i(x, y, \sqrt{1 - x^2 - y^2})$$

# Ideal specular reflection

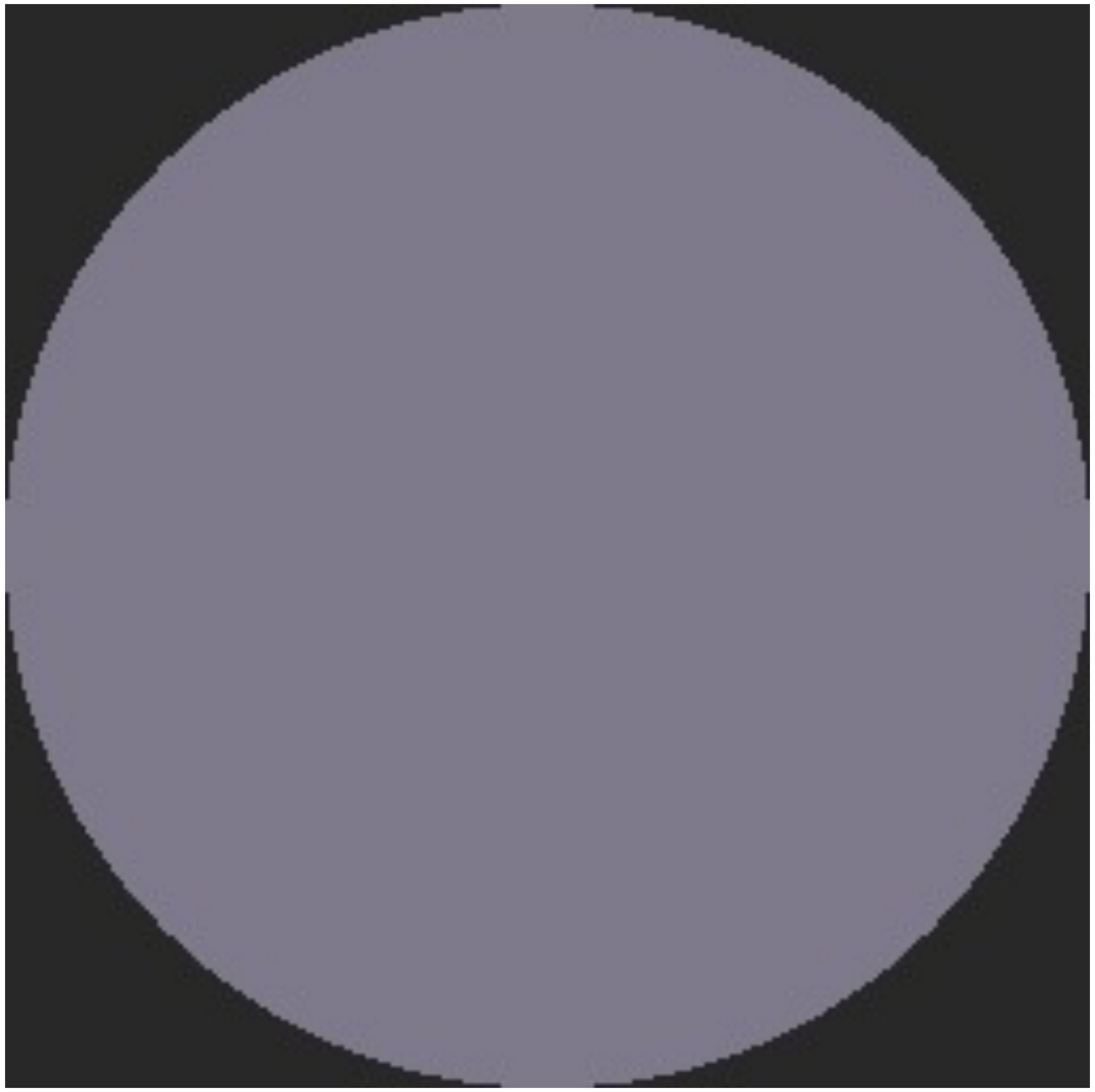
Incident radiance



Exitant radiance

# Diffuse reflection

Incident radiance



Exitant radiance

**Exitant radiance is the same in all directions**

# Plastic

Incident radiance

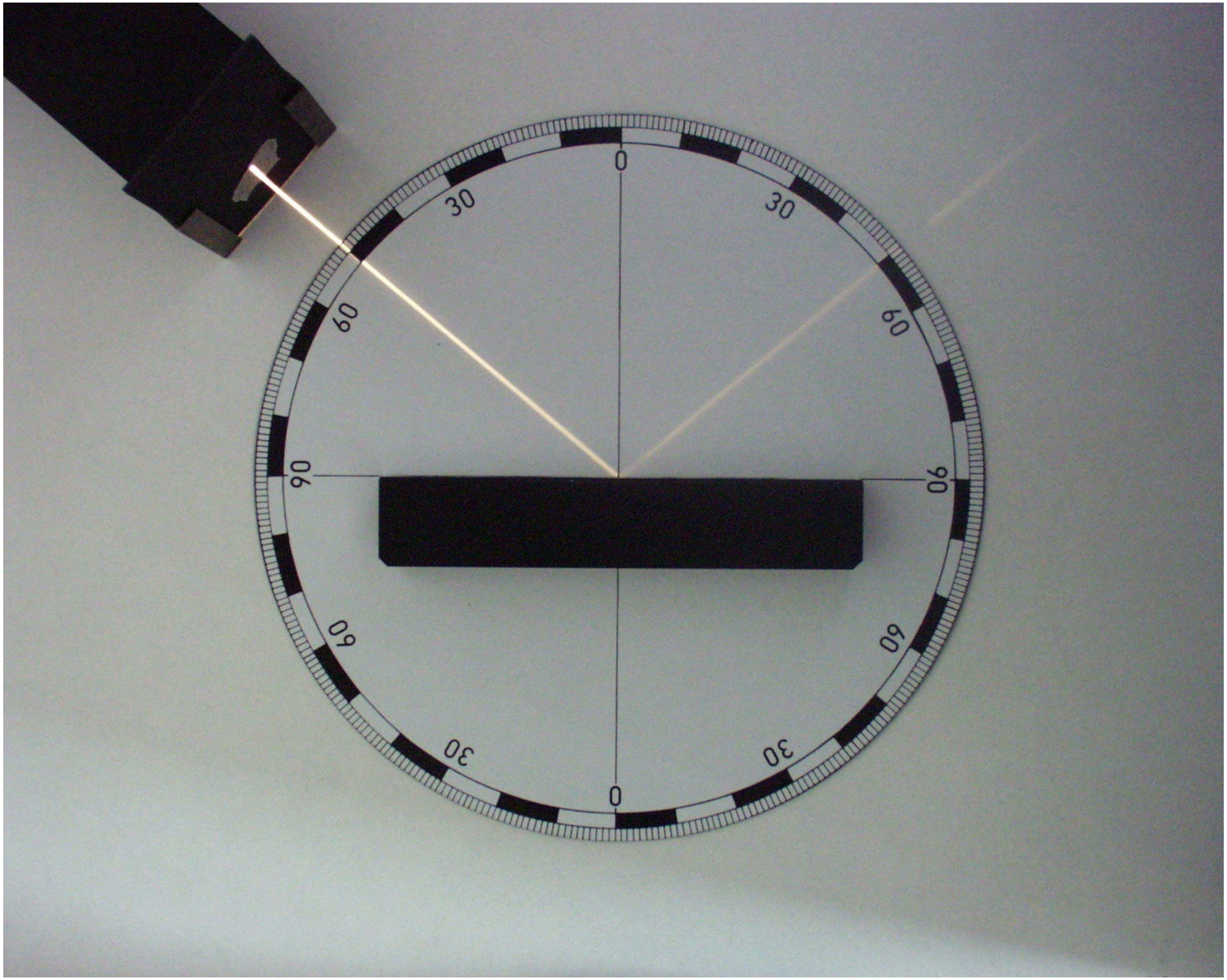
Exitant radiance

# Copper

Incident radiance

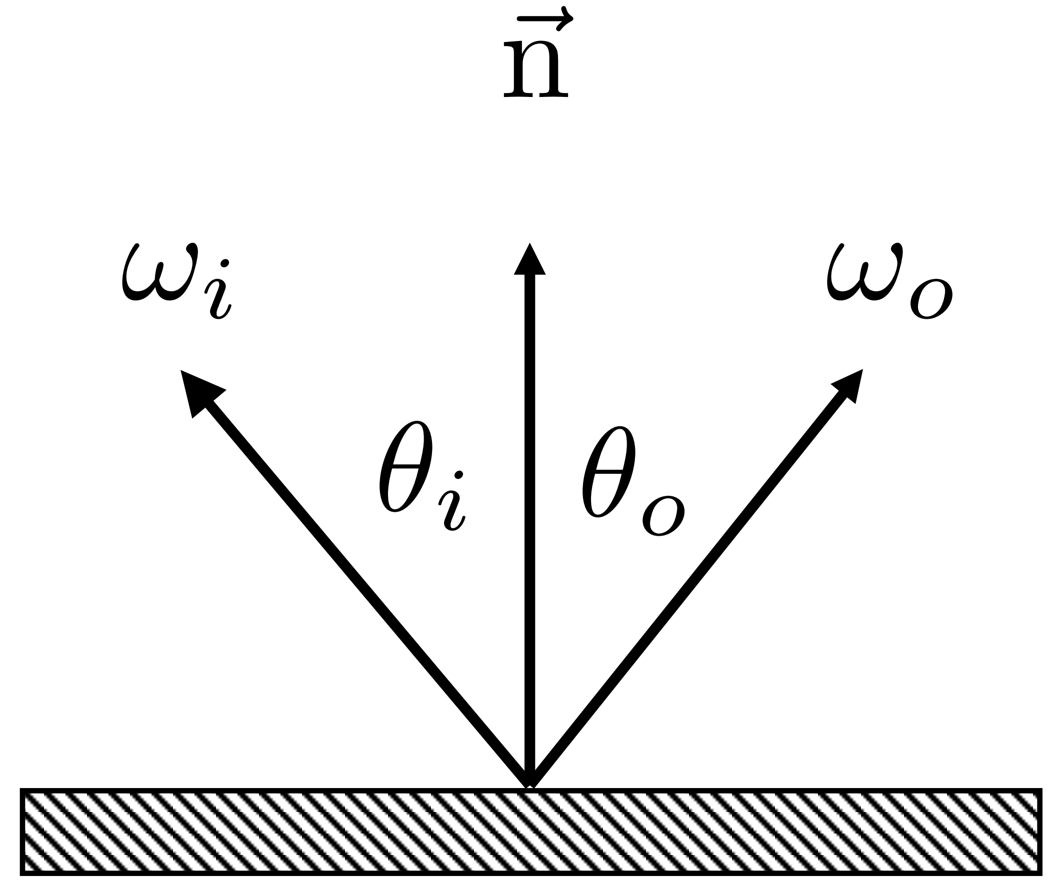
Exitant radiance

# Perfect specular reflection



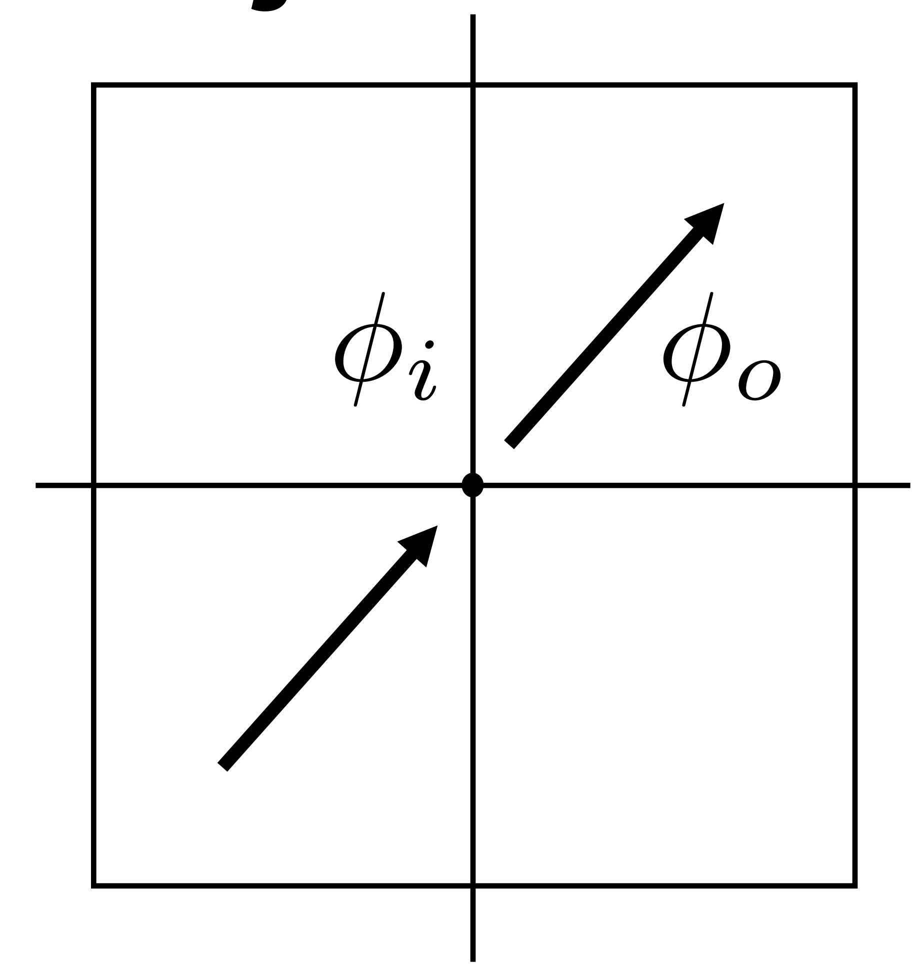

[Zátonyi Sándor]

# Perfect specular reflection




$$\theta = \theta_o = \theta_i$$

**Top-down view**
**(looking down on surface)**

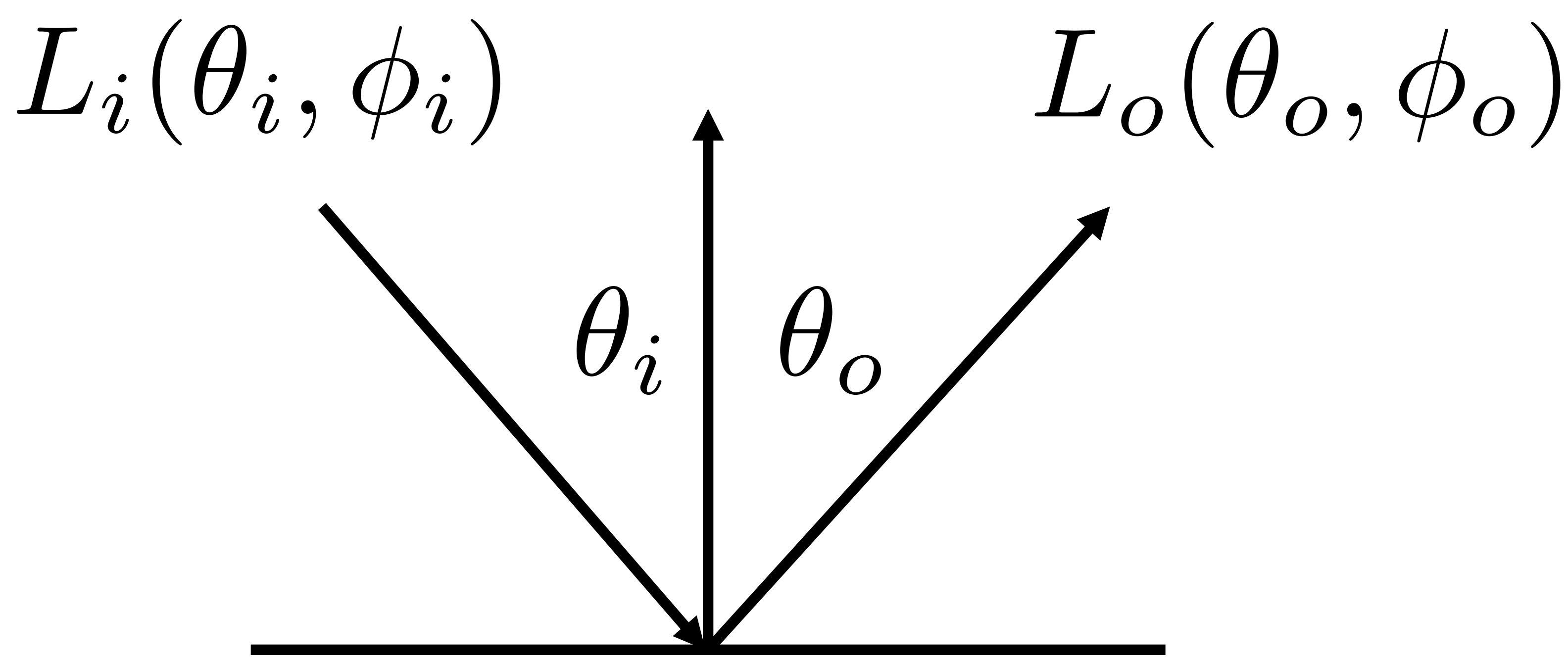



$$\phi_o = (\phi_i + \pi) \bmod 2\pi$$

$$\omega_o + \omega_i = 2\cos\theta\,\vec{n} = 2(\omega_i \cdot \vec{n})\vec{n}$$

$$\omega_o = -\omega_i + 2(\omega_i \cdot \vec{n})\vec{n}$$

# Specular reflection BRDF




$$L_o(\theta_o, \phi_o) = L_i(\theta_o, \phi_o \pm \pi)$$

$$f_r(\theta_i, \phi_i; \theta_o, \phi_o) = \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i}\delta(\phi_i - \phi_o \pm \pi)$$

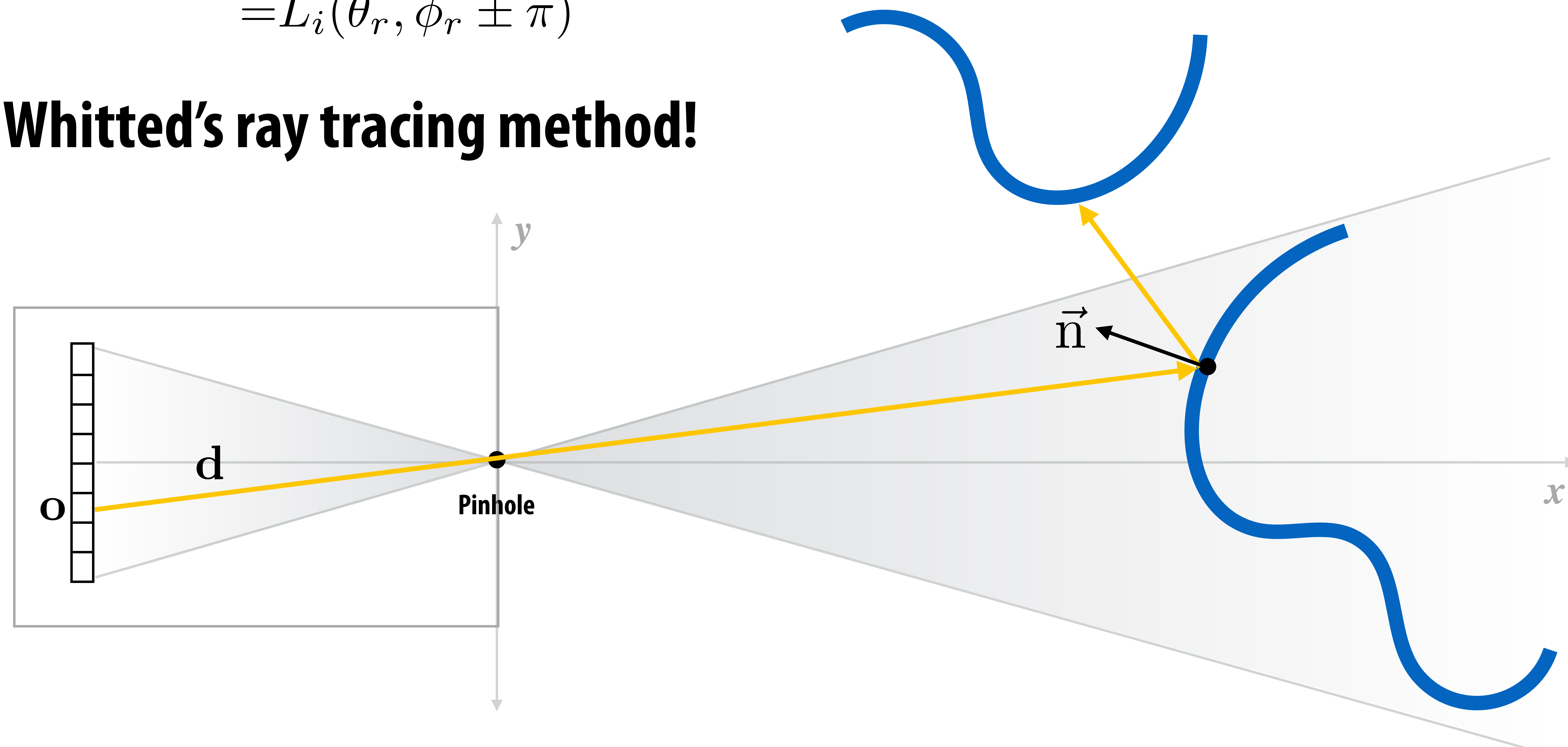
# Specular reflection and the reflection equation

$$L_o(\theta_o, \phi_o) = \int f_r(\theta_i, \phi_i; \theta_o, \phi_o)\, L_i(\theta_i, \phi_i)\; \cos\theta_i \, \mathrm{d}\cos\theta_i \, \mathrm{d}\phi_i$$

$$= \int \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i} \delta(\phi_i - \phi_o \pm \pi)\, L_i(\theta_i, \phi_i)\; \cos\theta_i \, \mathrm{d}\cos\theta_i \, \mathrm{d}\phi_i$$

$$= L_i(\theta_r, \phi_r \pm \pi)$$

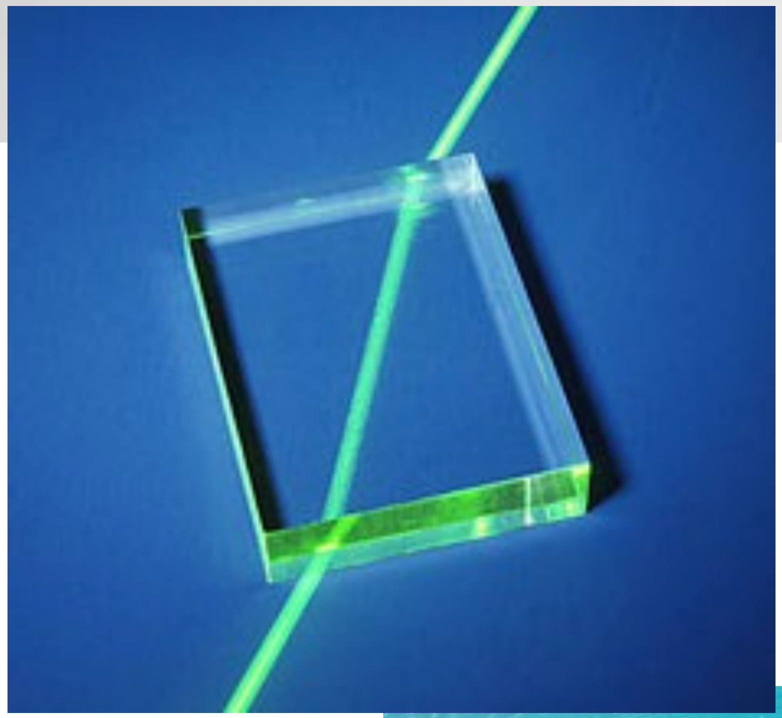
## Whitted's ray tracing method!



# Transmission

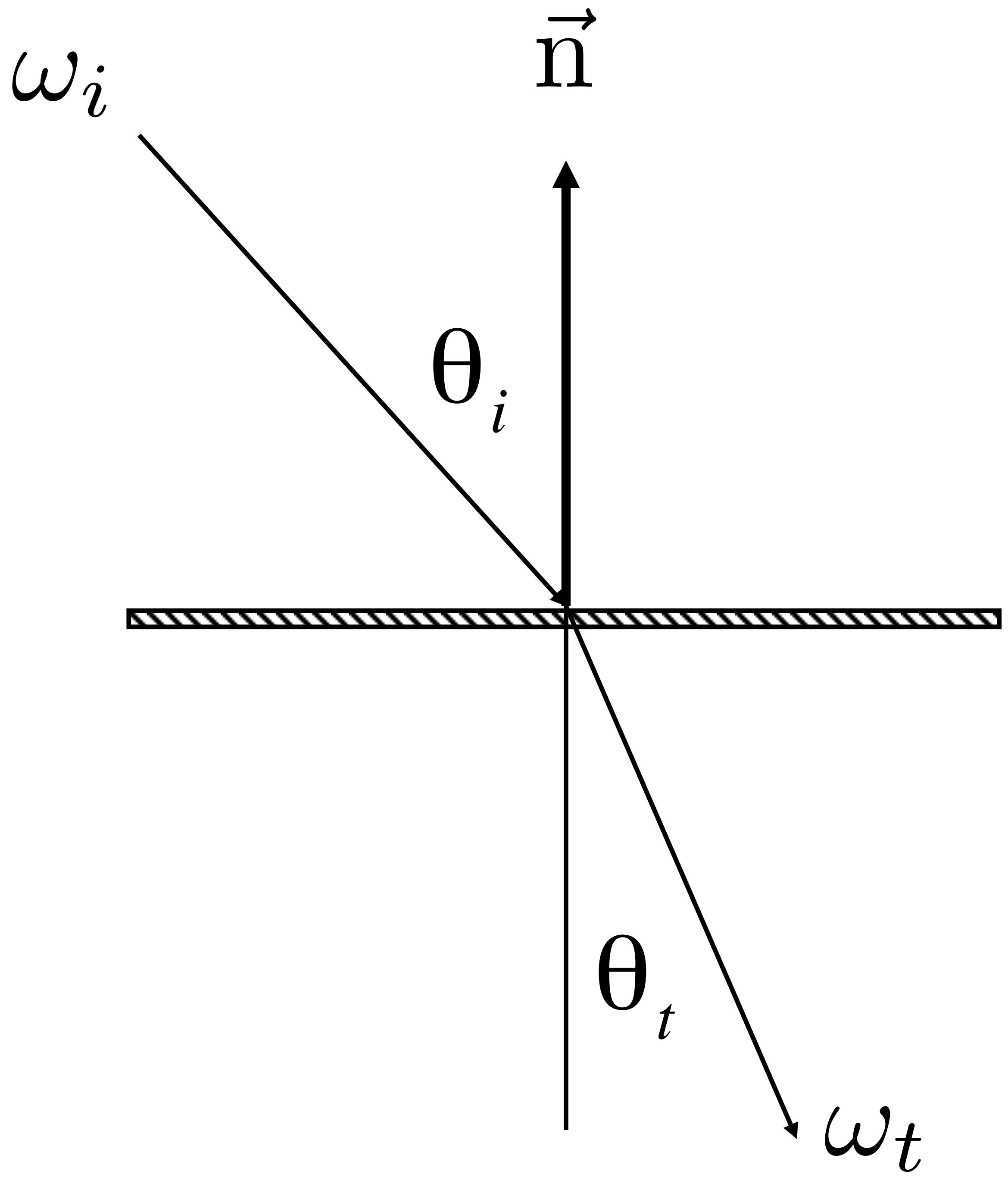
In addition to reflecting off surface, light may be transmitted through surface.

Light refracts when it enters a new medium.

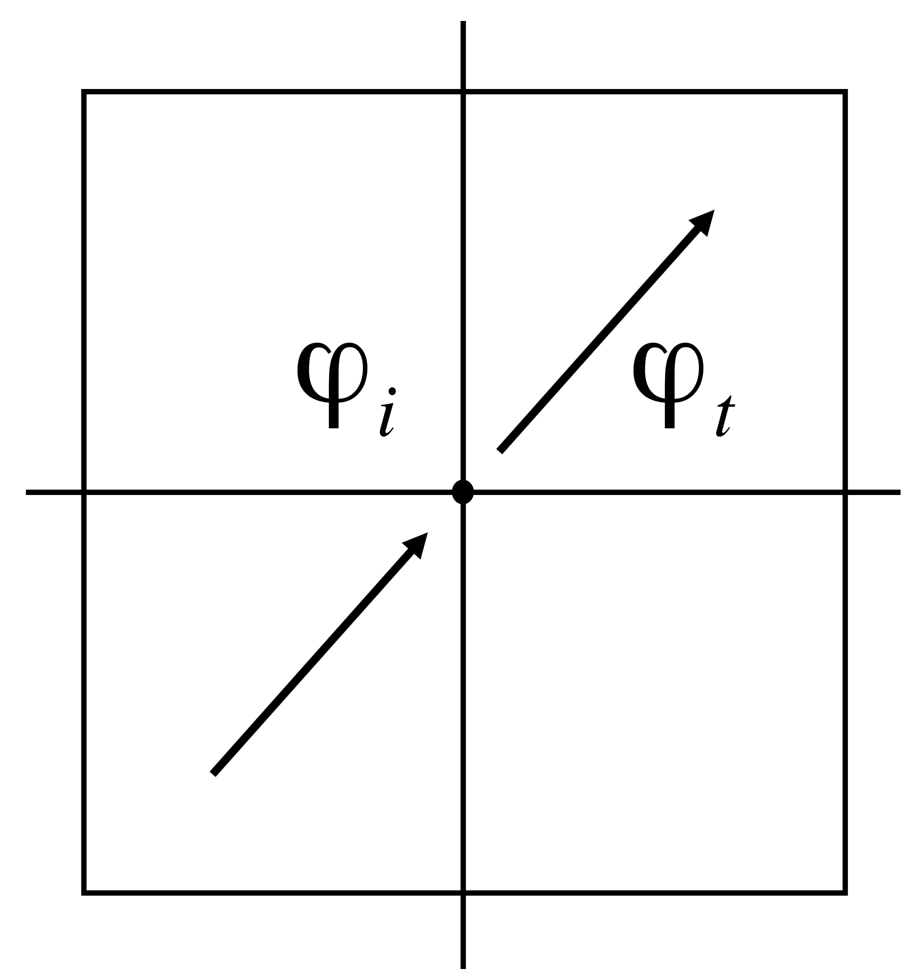


# Snell's Law

**Transmitted angle depends on index of refraction of medium incident ray is in and index of refraction of medium light is entering.**




$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$

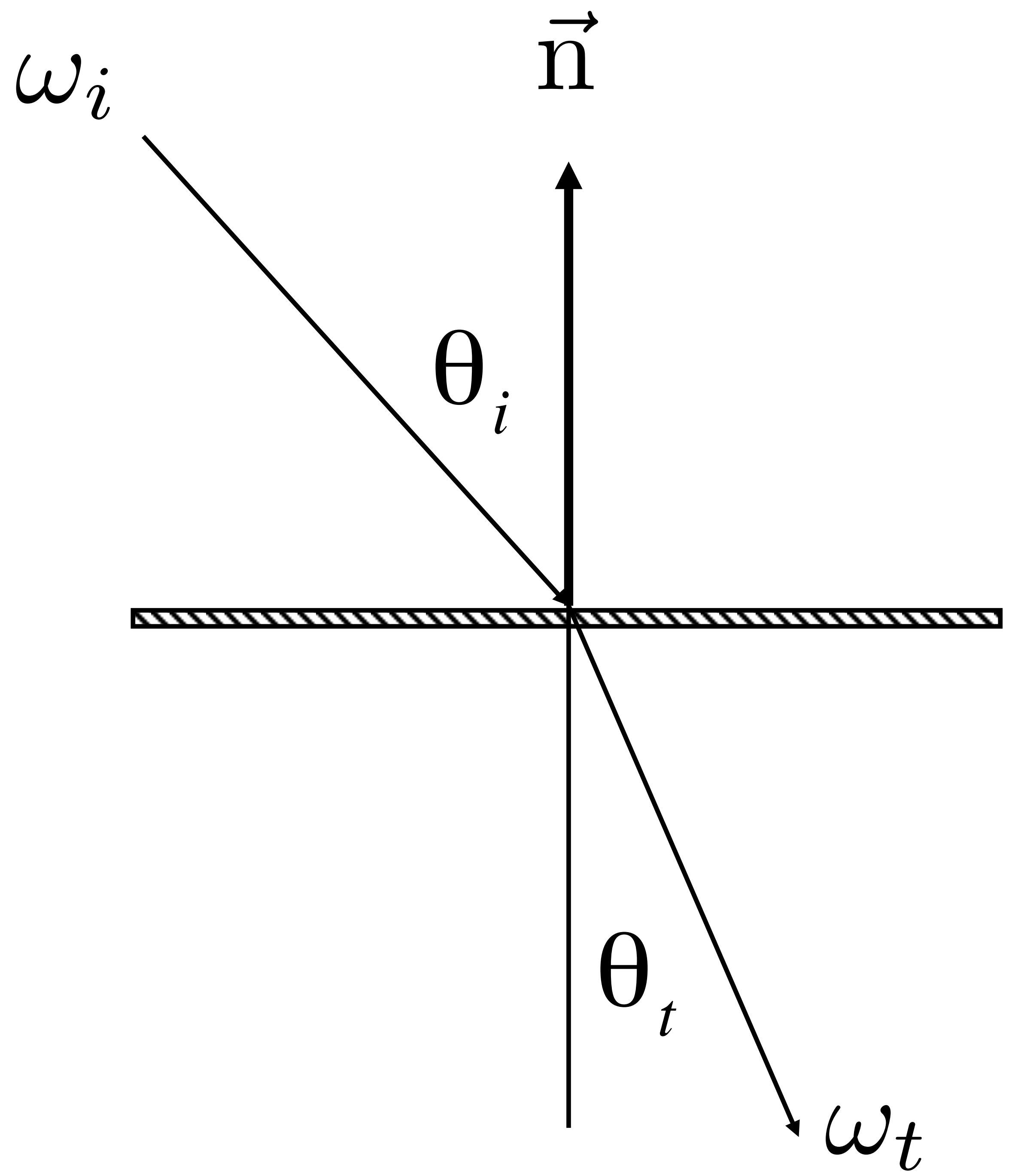



$$\varphi_t = \varphi_i \pm \pi$$

| Medium | $\eta$* |
|---|---|
| Vacuum | 1.0 |
| Air (sea level) | 1.00029 |
| Water (20°C) | 1.333 |
| Glass | 1.5-1.6 |
| Diamond | 2.42 |

* index of refraction is wavelength dependent (these are averages)

# Law of refraction




$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$

$$\begin{aligned}\cos\theta_t &= \sqrt{1-\sin^2\theta_t} \\ &= \sqrt{1-\left(\frac{\eta_i}{\eta_t}\right)^2 \sin^2\theta_i} \\ &= \sqrt{1-\left(\frac{\eta_i}{\eta_t}\right)^2 (1-\cos^2\ \theta_i)}\end{aligned}$$

## Total internal reflection:

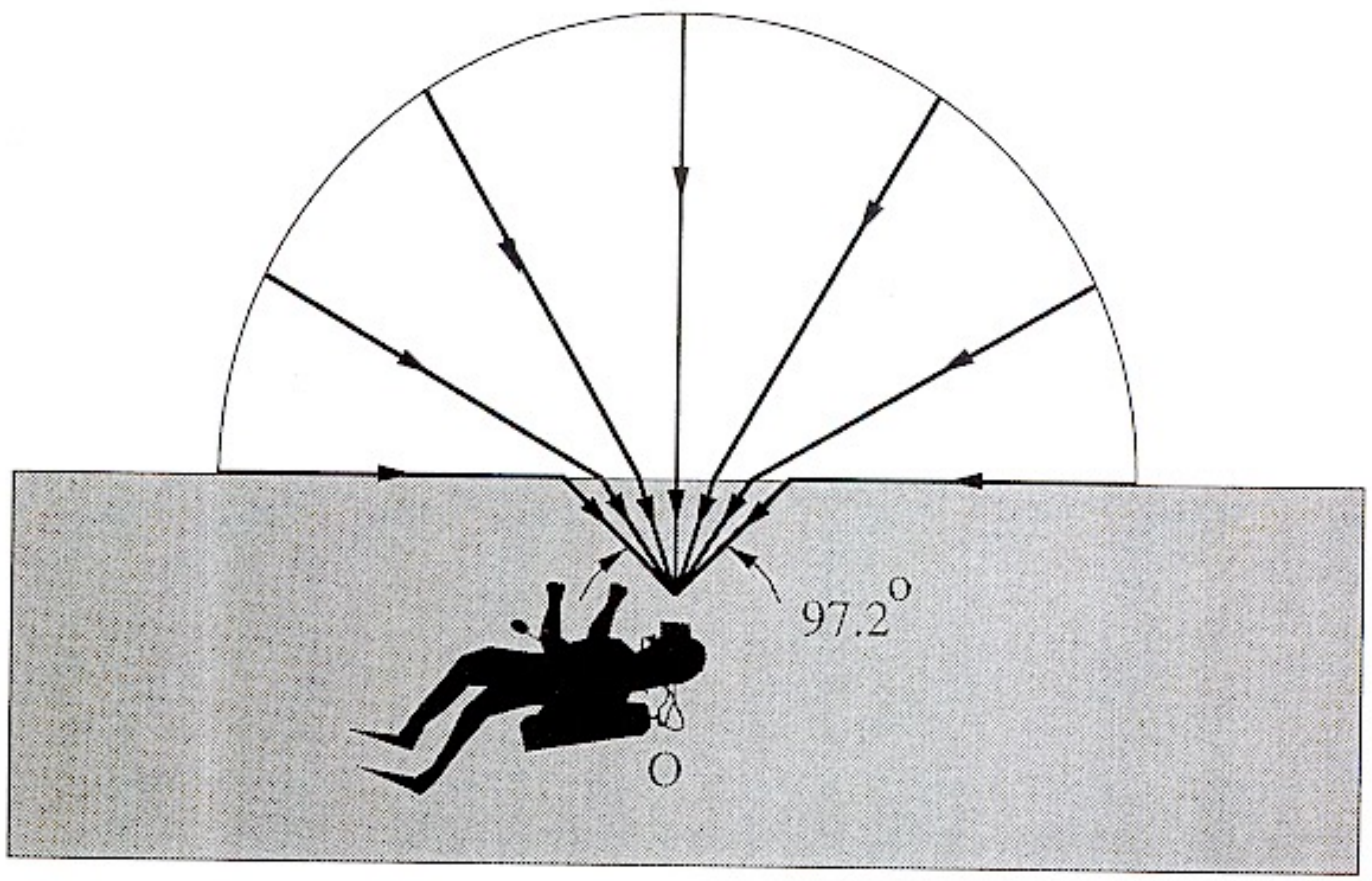
$$1-\left(\frac{\eta_i}{\eta_t}\right)^2 (1-\cos^2\theta_i) < 0$$

**When light is moving from a more optically dense medium to a less optically dense medium:** $\frac{\eta_i}{\eta_t} > 1$

**Light incident on boundary from large enough angle will not exit medium.**

# Optical manhole

## Total internal reflection




[Livingston and Lynch]

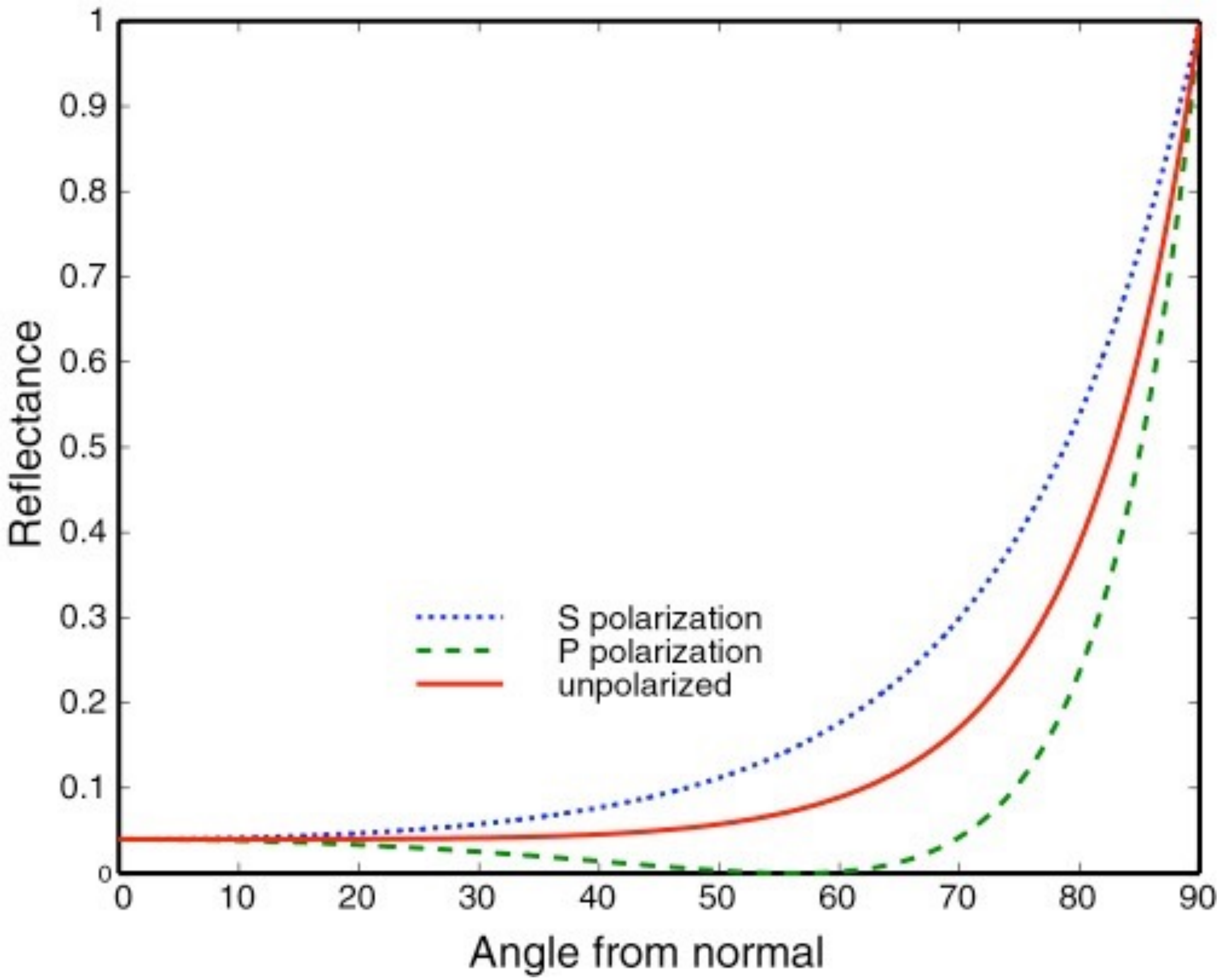
# Fresnel reflection

**Reflectance depends on angle of incidence and polarization of light**

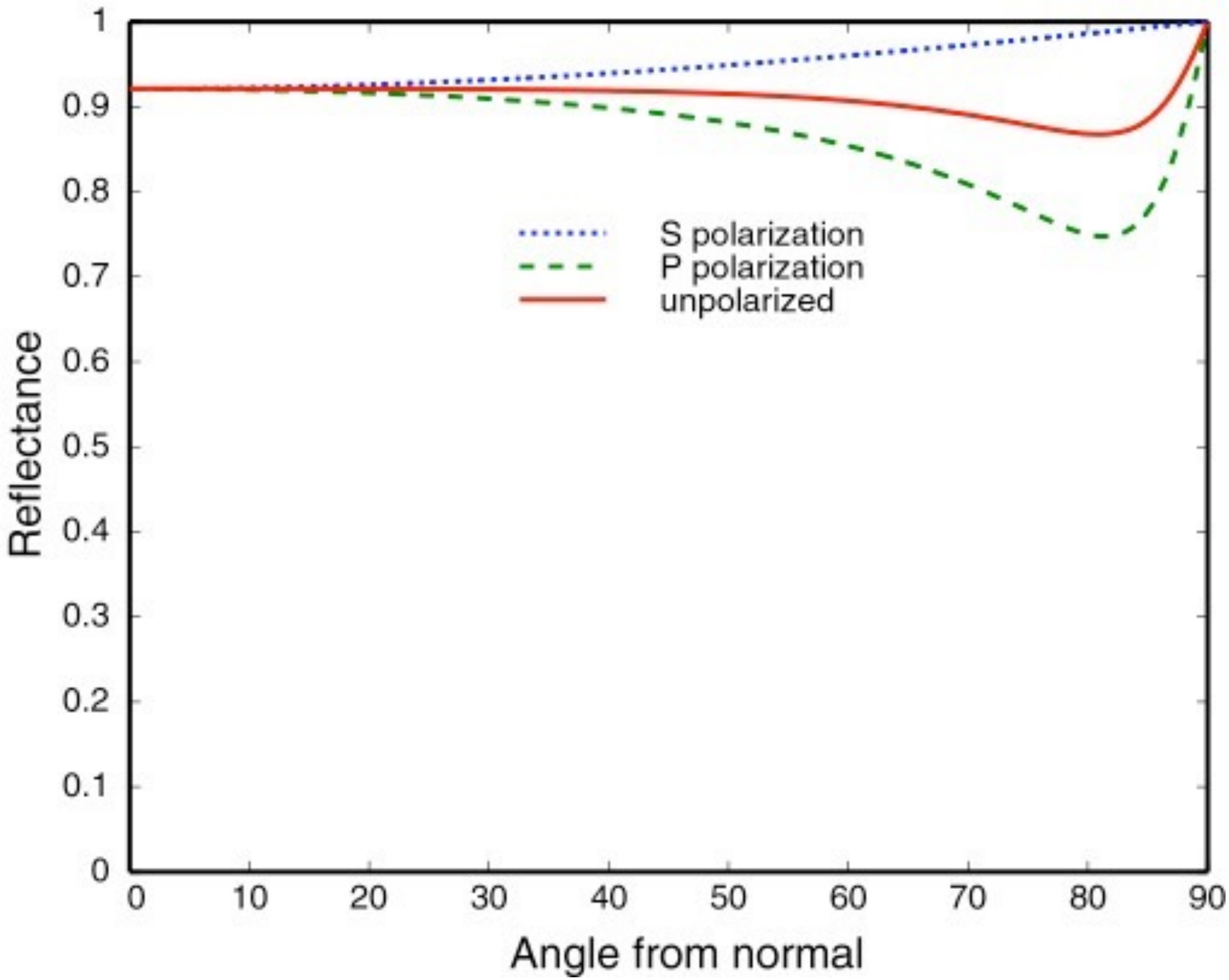

**This example: reflectance increases with grazing angle**

[Lafortune et al. 1997]

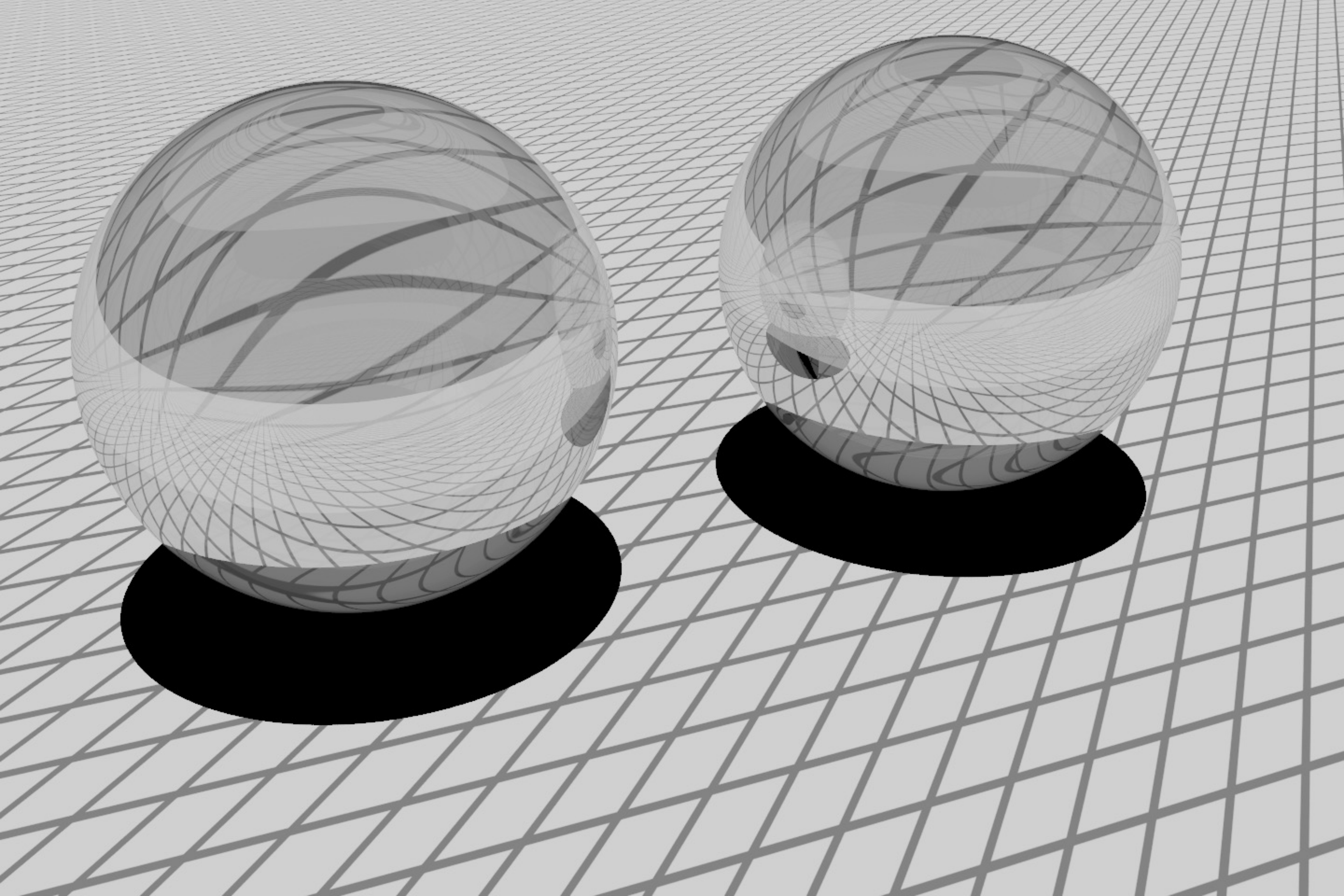
# Fresnel reflection (dielectric, $\eta$=1.5)



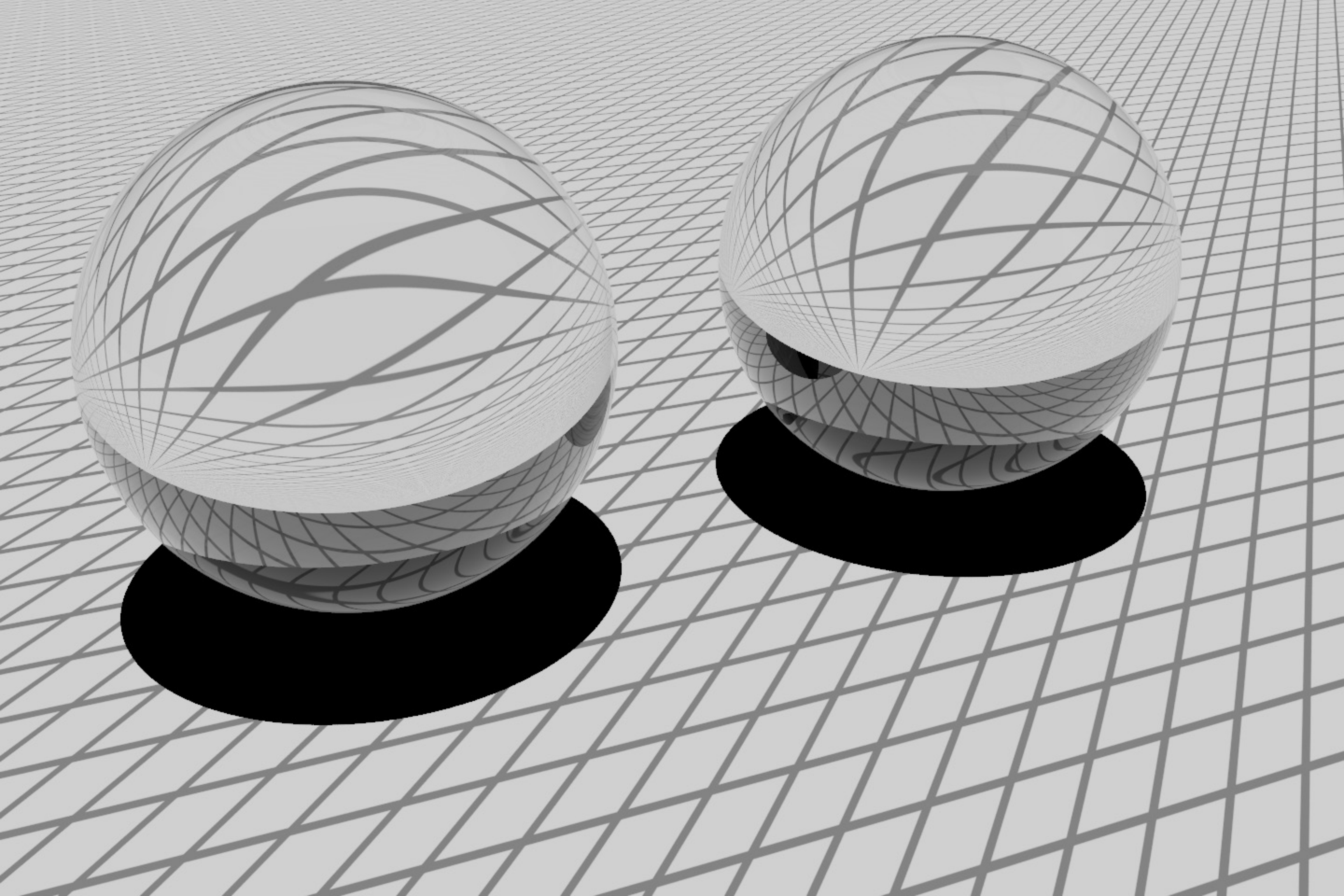
# Fresnel reflectance (conductor)



# Without Fresnel (fixed reflectance/transmission)

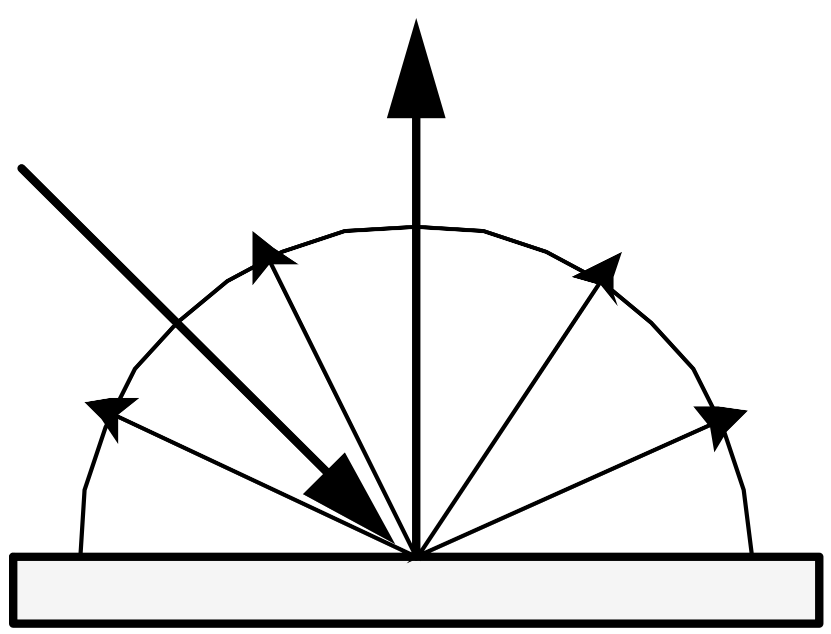


# Glass with Fresnel reflection/transmission



# Lambertian reflection

## Assume light is equally likely to be reflected in each output direction



$$f_r = c$$

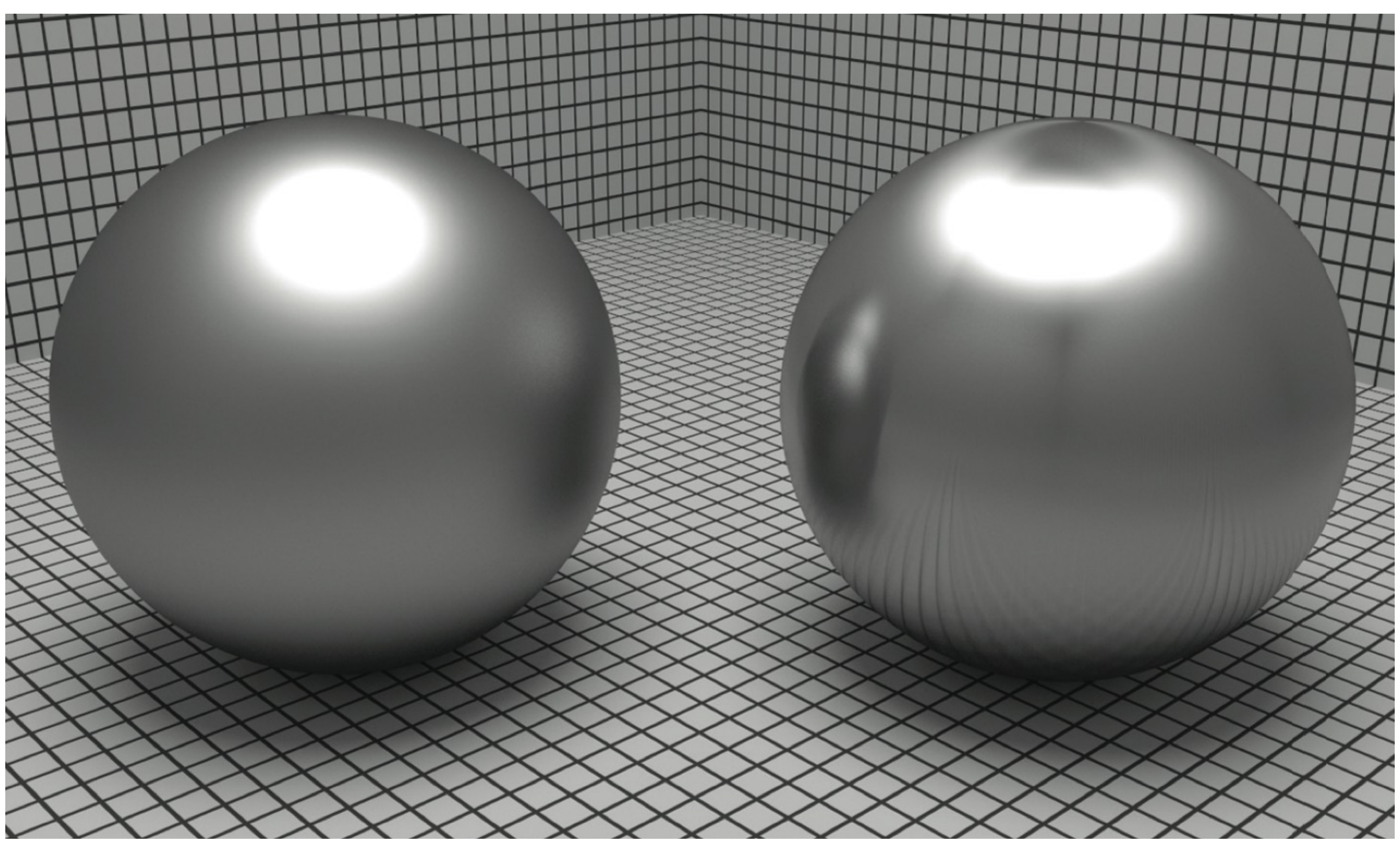
$$\begin{aligned} L_o(\omega_o) &= \int_{H^2} f_r \, L_i(\omega_i) \, \cos\theta_i \, \mathrm{d}\omega_i \\ &= f_r \int_{H^2} L_i(\omega_i) \, \cos\theta_i \, \mathrm{d}\omega_i \\ &= f_r E \end{aligned}$$

$$f_r = \frac{\rho}{\pi}$$

# Anisotropic reflection

**Reflection depends on azimuthal angle $\phi$**

**Results from oriented microstructure of surface**
**e.g., brushed metal**

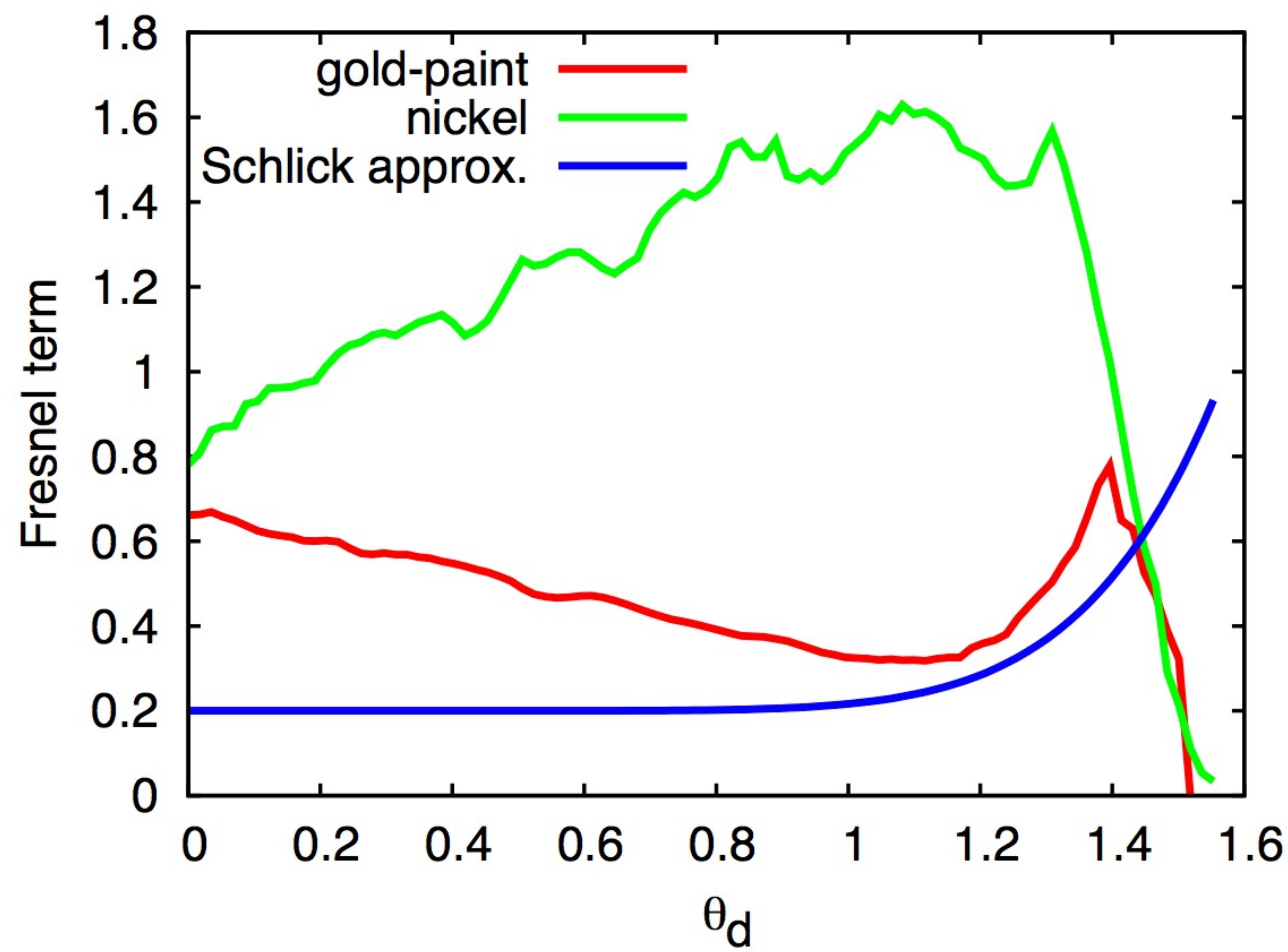


# Measuring BRDFs

# Measuring BRDFs: motivation

- Avoid need to develop / derive models
  - Automatically includes all of the scattering effects present
- Can accurately render with real-world materials
  - Useful for product design, special effects, ...
- Theory vs. practice:

[Bagher et al. 2012]



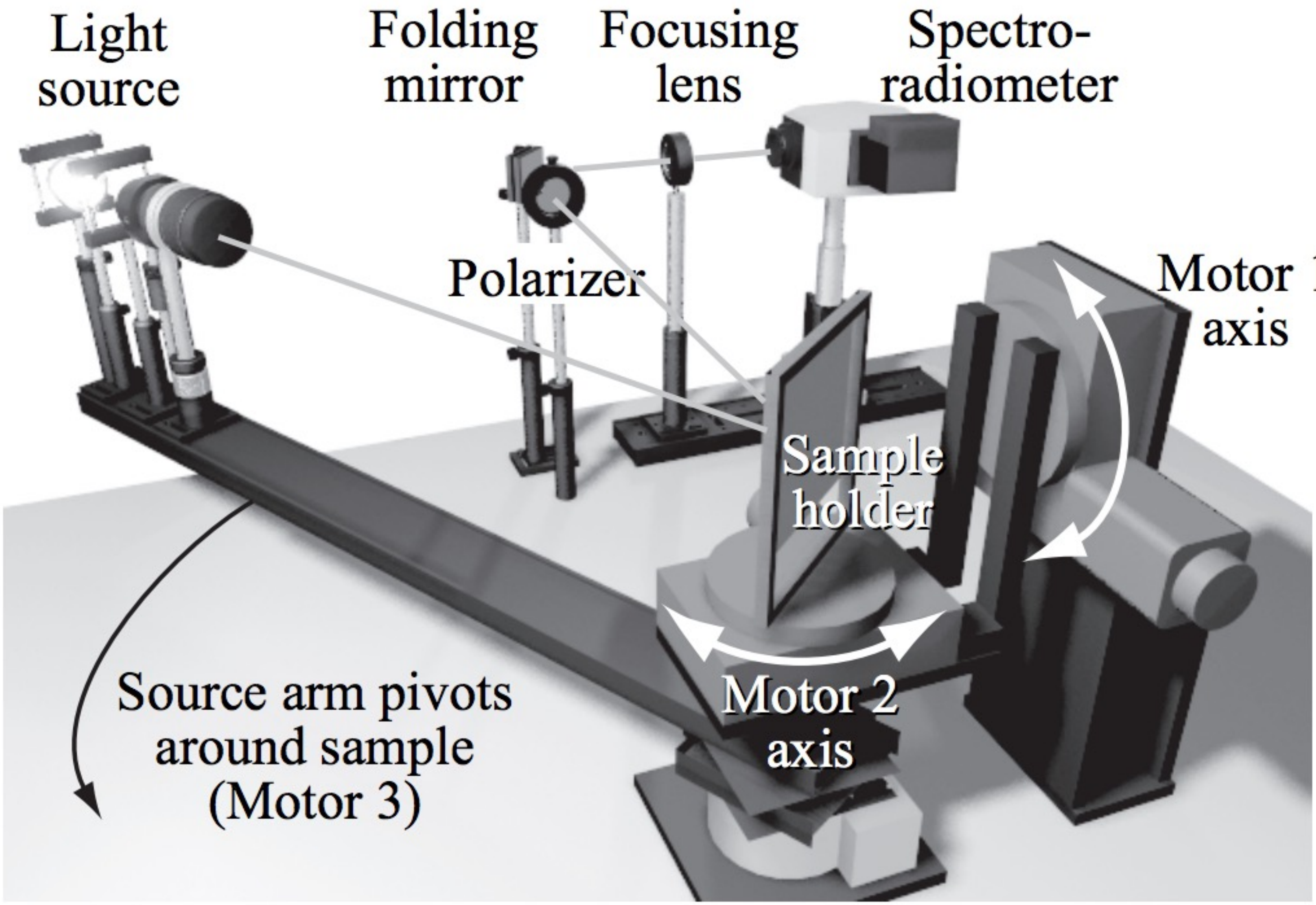
# Measuring BRDFs

- **General approach:**

```
foreach outgoing direction wo
    move light to illuminate surface with a thin beam from wo
    for each incoming direction wi
        move sensor to be at direction wi from surface
        measure incident radiance
```

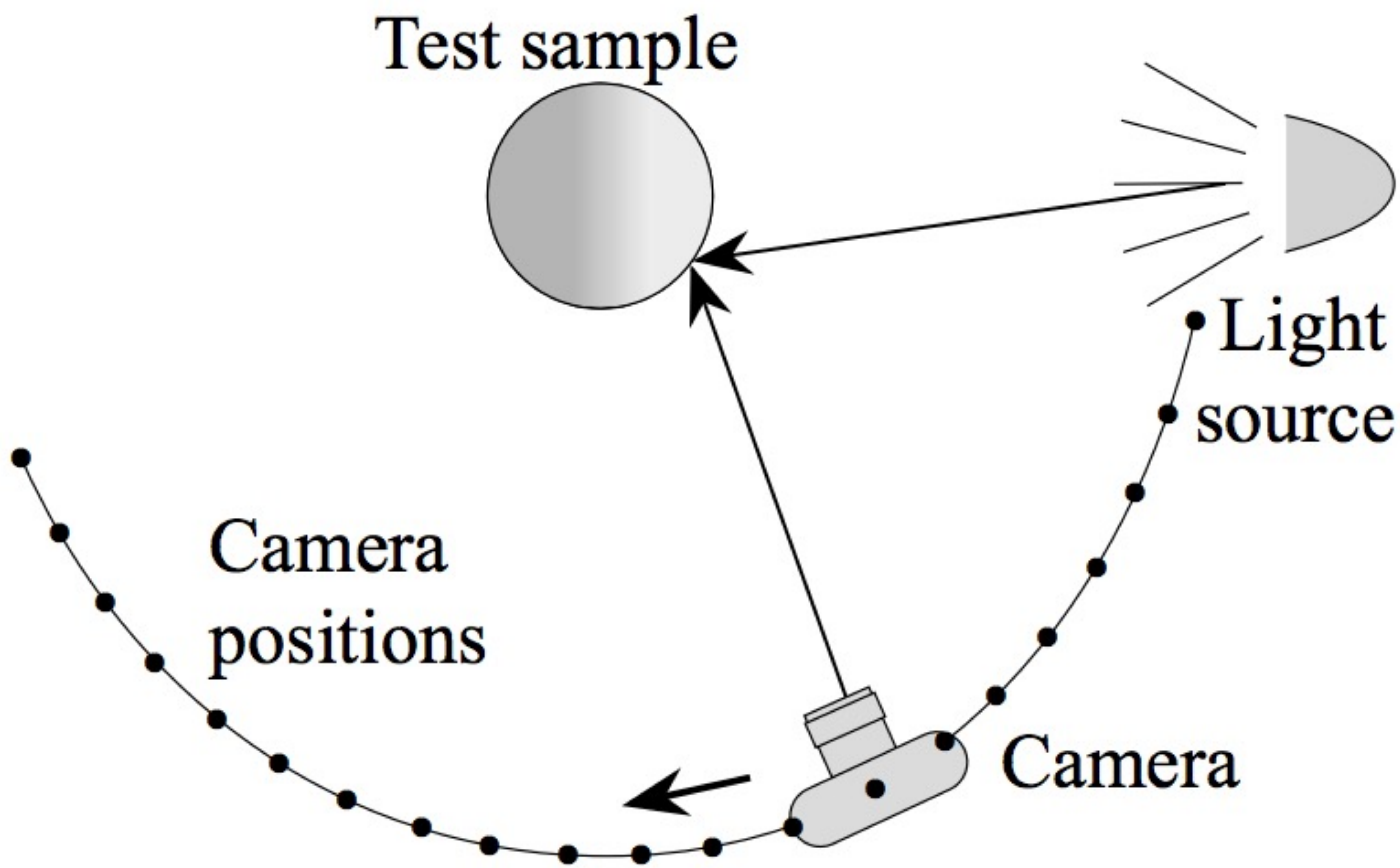
- **Improving efficiency:**
  - **Isotropic surfaces reduce dimensionality from 4D to 3D**
  - **Reciprocity reduces # of measurements by half**
  - **Clever optical systems…**

# Measuring BRDFs: gonioreflectometer




[Li et al. 2005]

# Image-based BRDF measurement




[Marschner et al. 1999]

# Challenges in measuring BRDFs

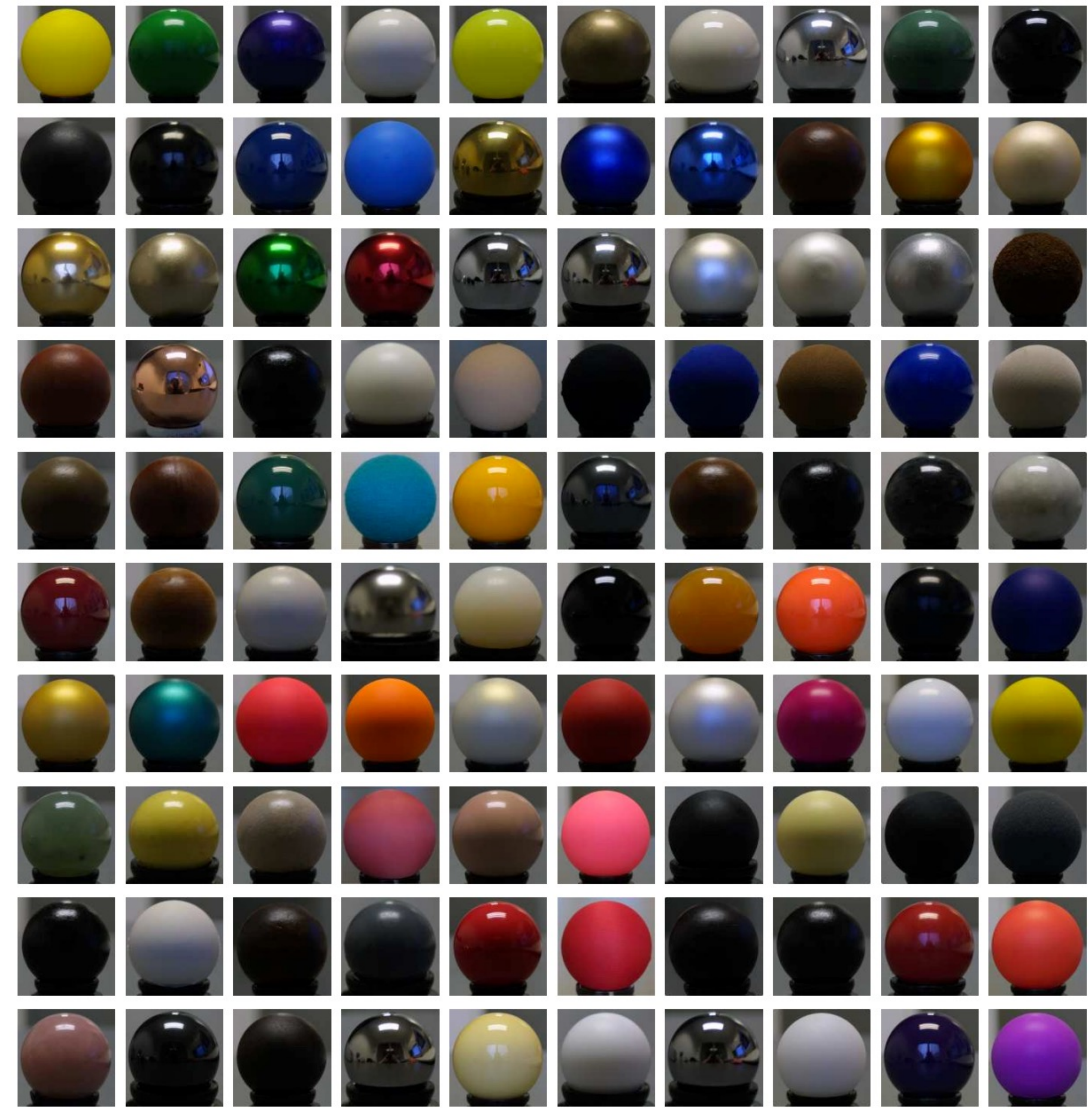
- Accurate measurements at grazing angles
  - Important due to Fresnel effects
- Measuring with dense enough sampling to capture high frequency specularities
- Retro-reflection
- Spatially-varying reflectance, ...

# Representing measured BRDFs

- **Desirable qualities**
  - **Compact representation**
  - **Accurate representation of measured data**
  - **Efficient evaluation for arbitrary pairs of directions**
  - **Good distributions available for importance sampling**

# Tabular representation

- Store regularly-spaced samples in $(\theta_i, \theta_o, |\phi_i - \phi_o|)$
  - Better: reparameterize angles to better match specularities
- Generally need to resample measured values to table
- Very high storage requirements



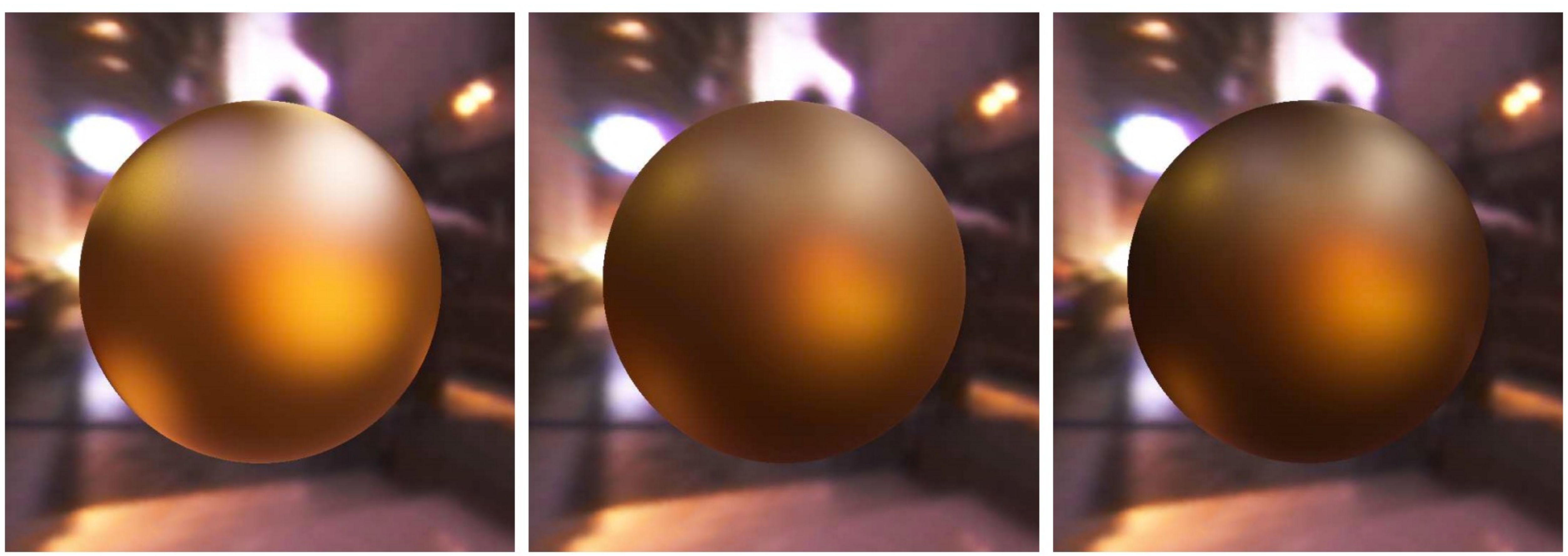
MERL BRDF Database
[Matusik et al. 2004]
90*90*180 measurements

# Basis functions

- **Can fit existing models, e.g. Cook-Torrance, 3 parameters per wavelength,** $k_d, k_s, k_e$

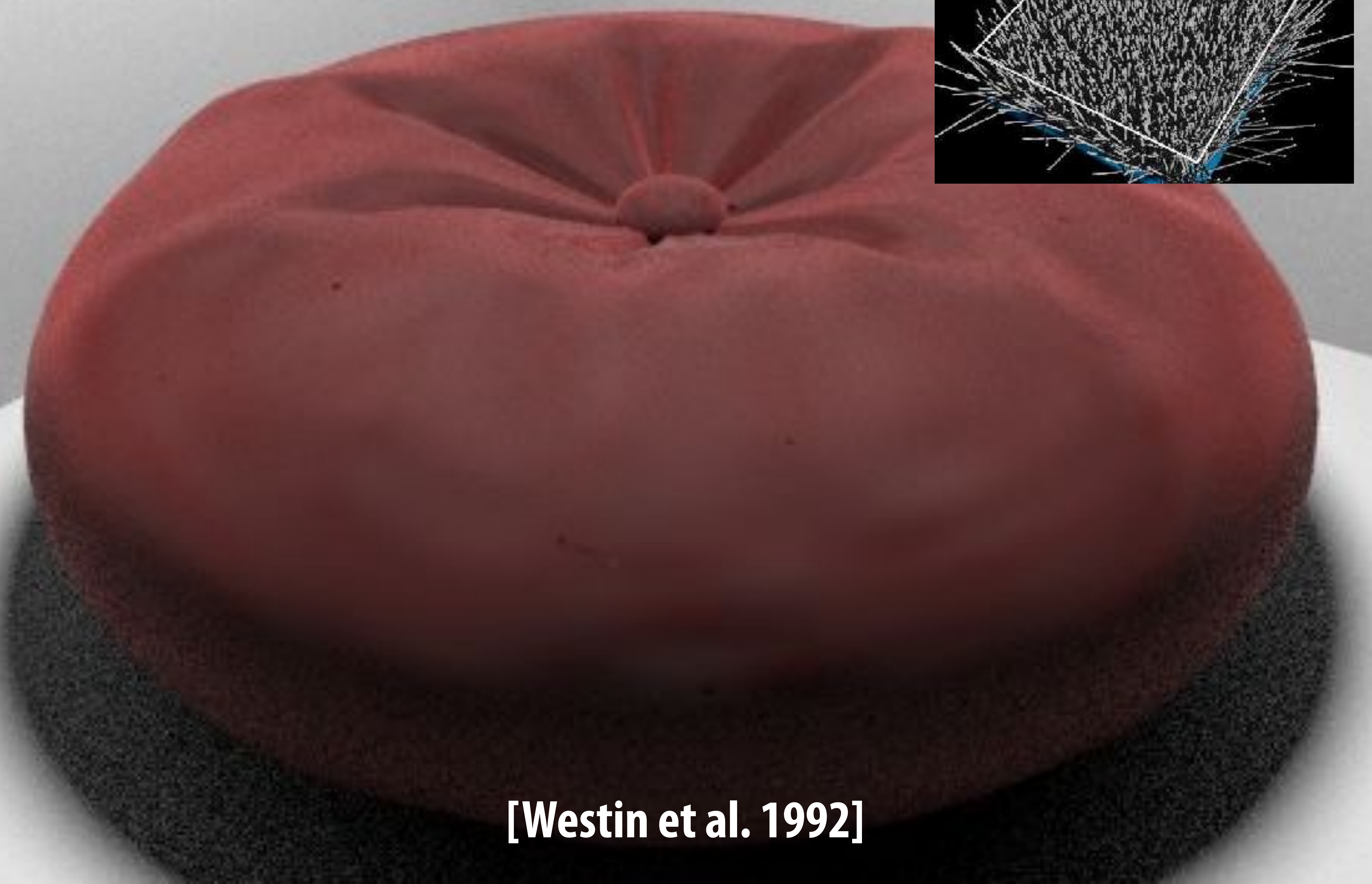
$$f_r(\omega_i \to \omega_o) = k_d + k_s(\vec{\mathrm{h}} \cdot \vec{\mathrm{n}})^{k_e} \qquad \vec{\mathrm{h}} = \widehat{\omega_i + \omega_o}$$

- **More sophisticated, e.g. [Bagher et al. 2012] 11 parameter model:**



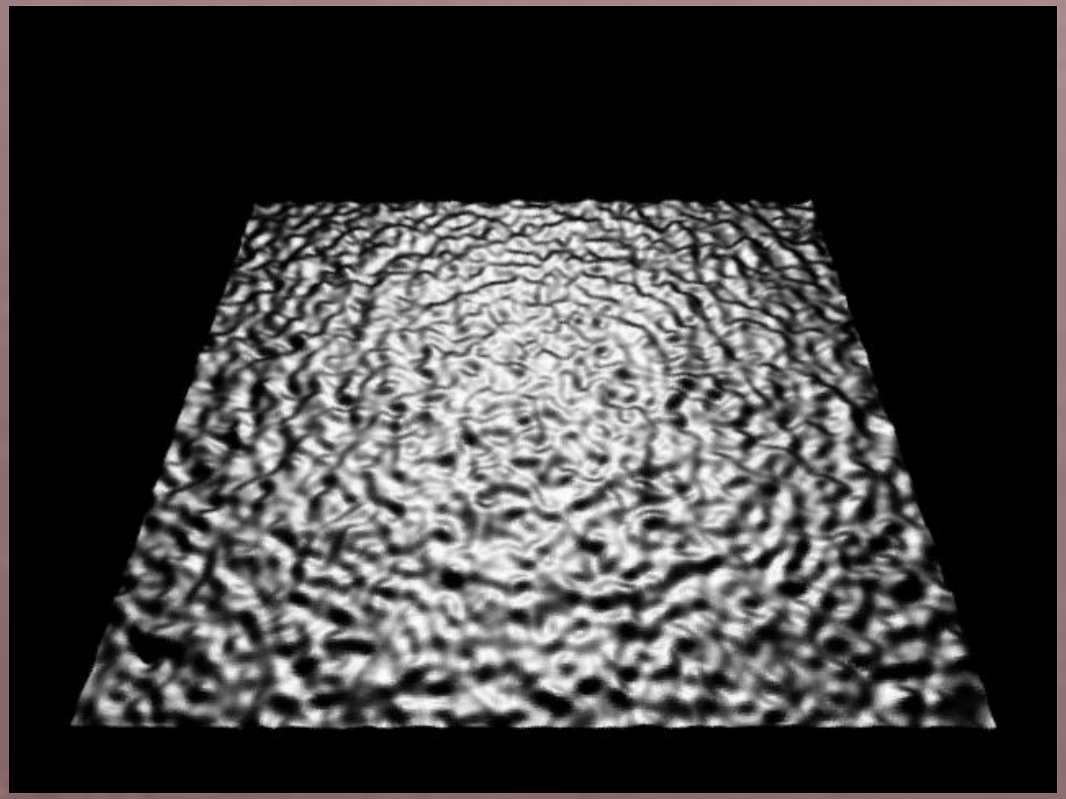
(b) Beckmann distribution (c) Ground truth (d) SGD distribution (ours)

# Simulation: velvet



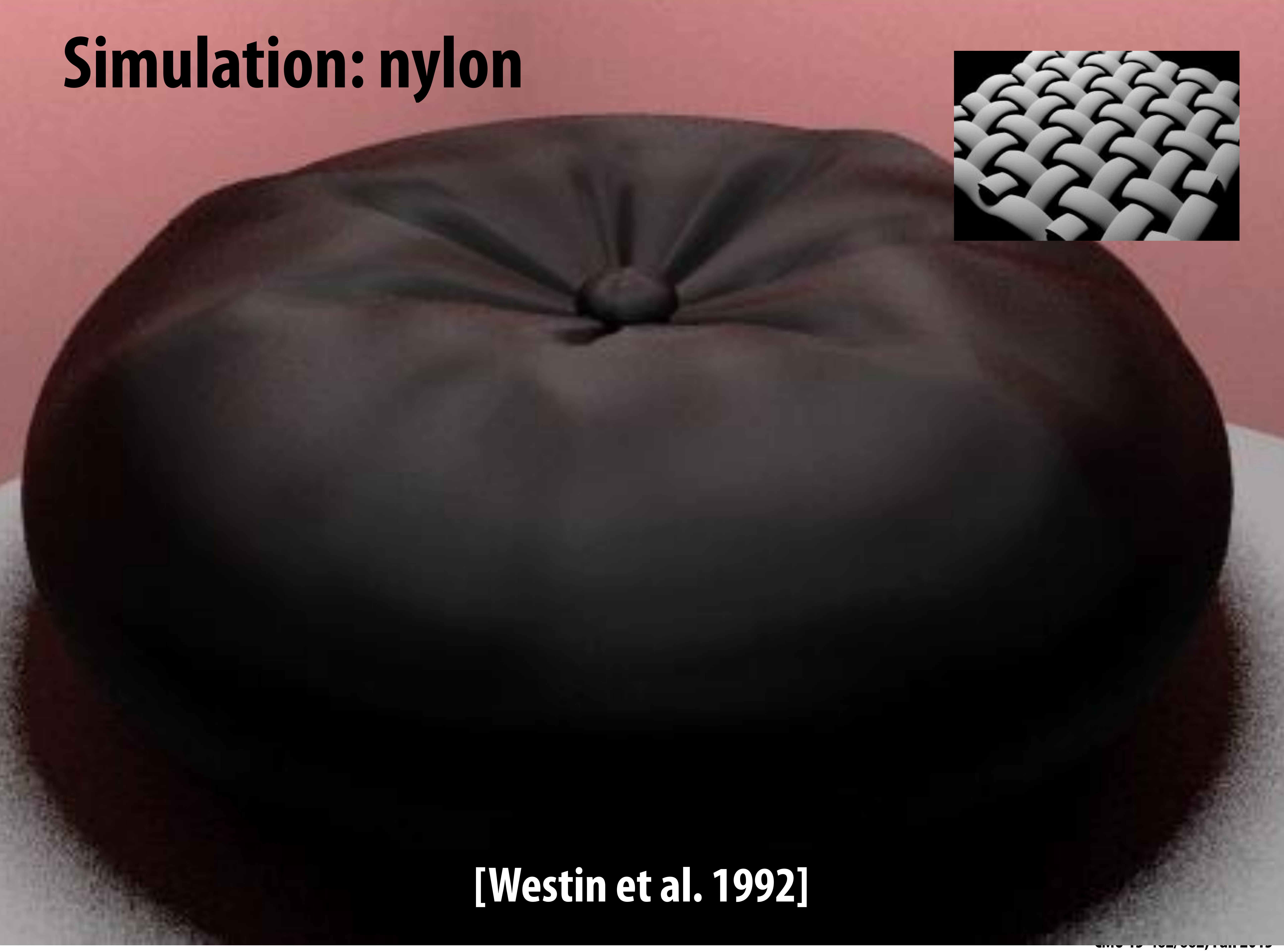
[Westin et al. 1992]

# Simulation: brushed aluminum



[Westin et al. 1992]

# Simulation: nylon



[Westin et al. 1992]

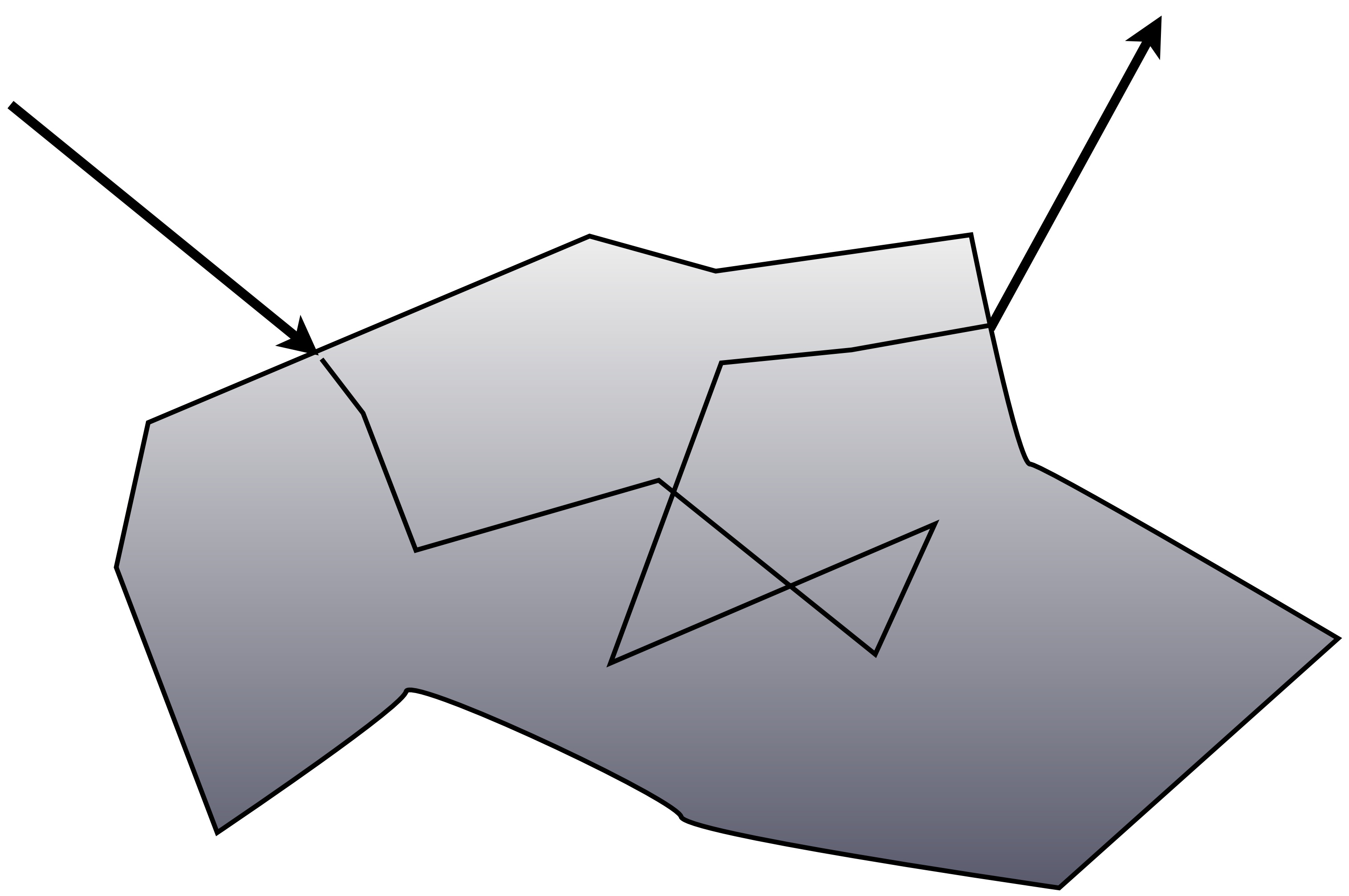
# Translucent materials: Jade

# Translucent materials: skin

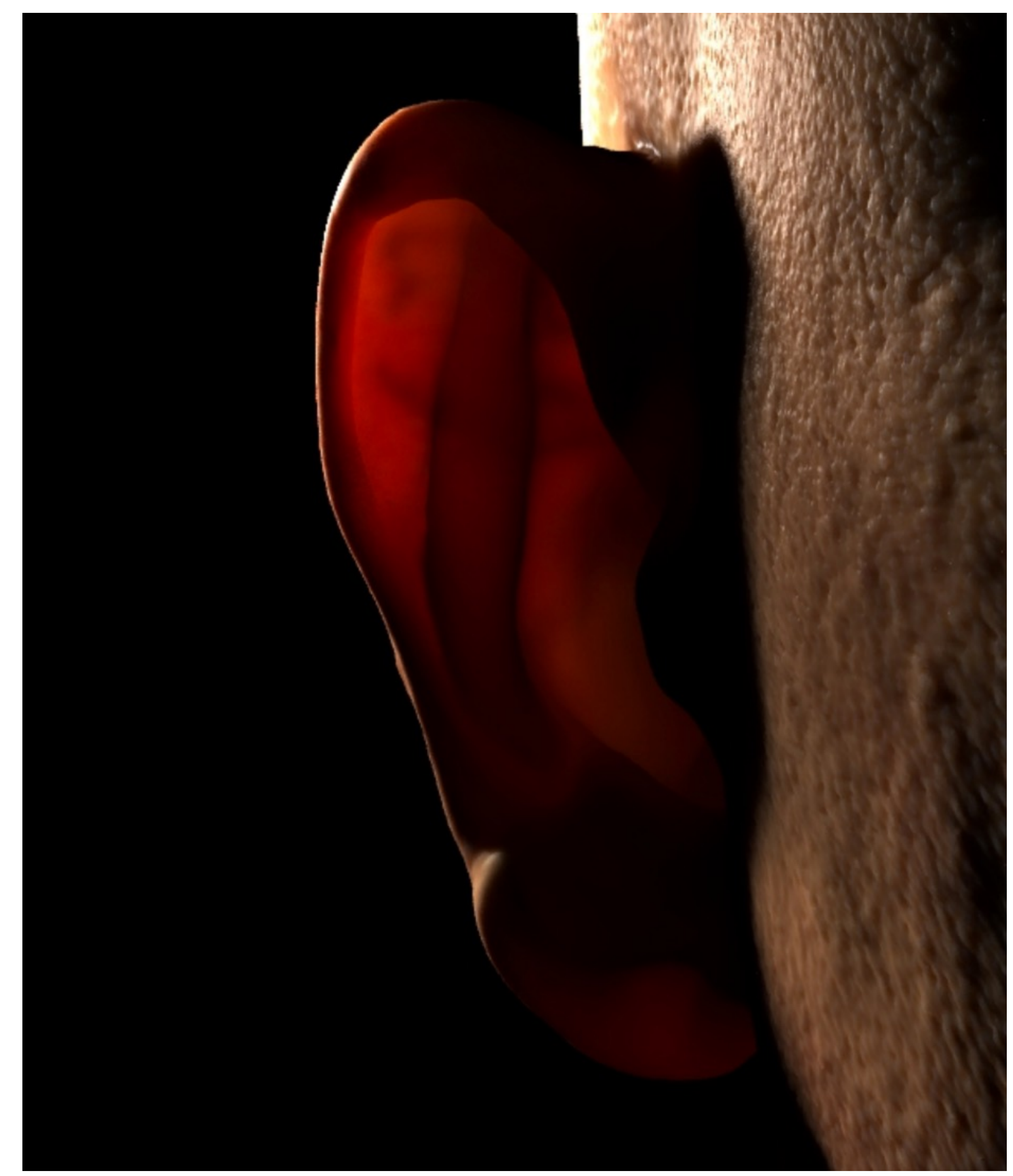
# Translucent materials: leaves

# Subsurface scattering

- Visual characteristics of many surfaces caused by light entering at different points than it exits
  - Violates a fundamental assumption of the BRDF



[Jensen et al 2001]



[Donner et al 2008]

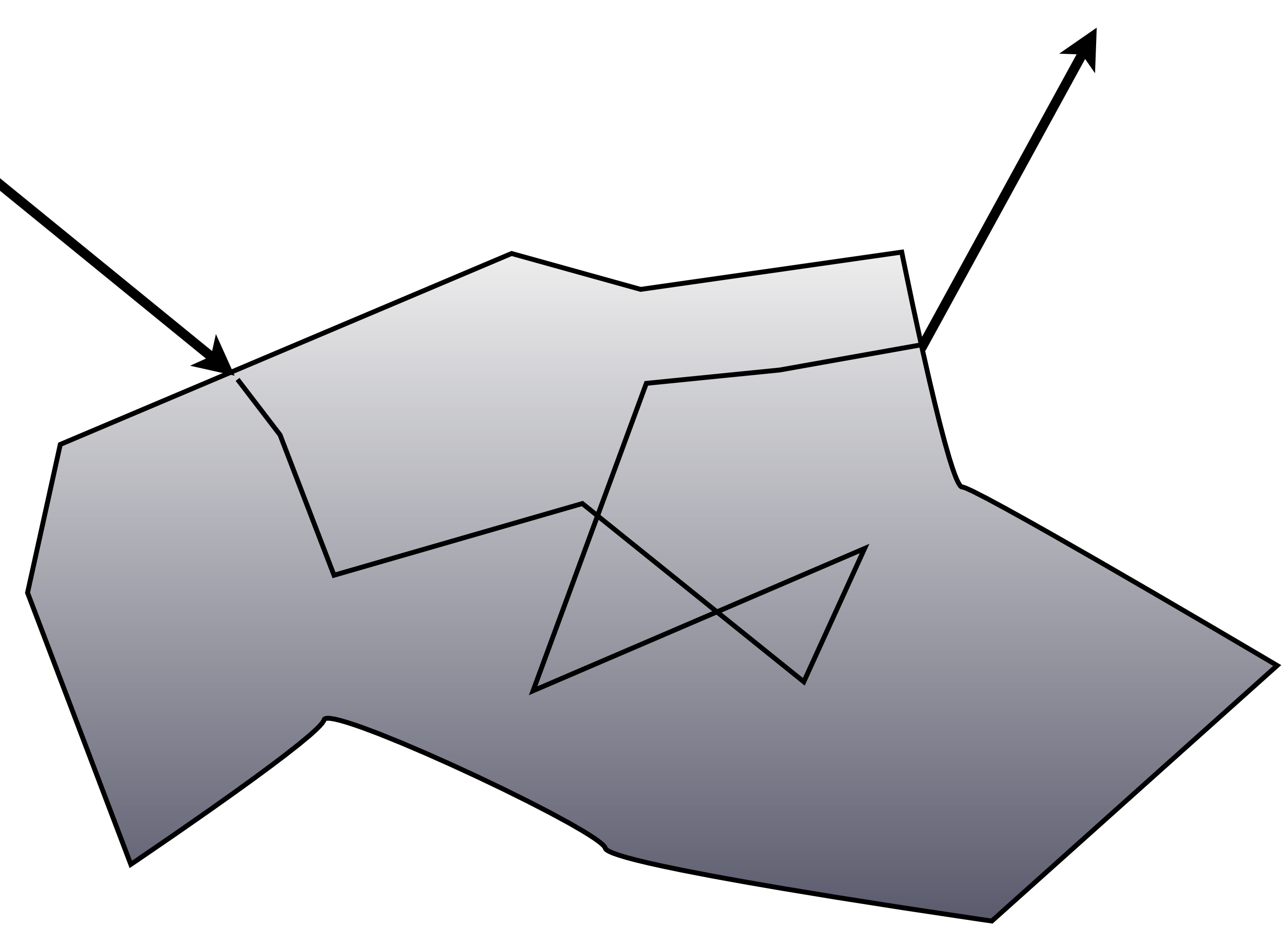
# Scattering functions

- **Generalization of BRDF; describes exitant radiance at one point due to incident differential irradiance at another point:**

$$S(x_i, \omega_i, x_o, \omega_o)$$

- **Generalization of reflection equation integrates over all points on the surface and all directions(!)**

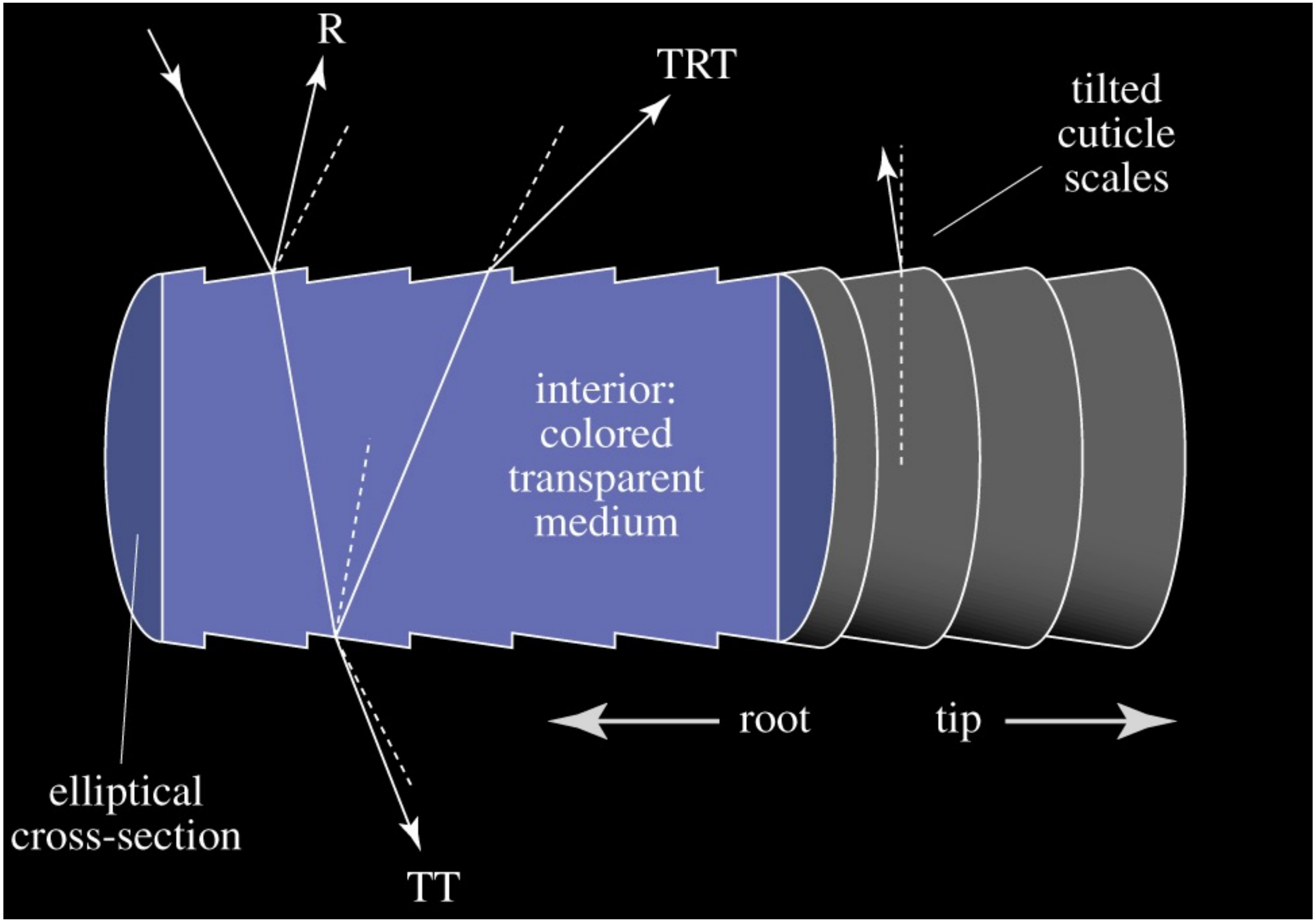
$$L(x_o, \omega_o) = \int_A \int_{H^2} S(x_i, \omega_i, x_o, \omega_o)\, L_i(x_i, \omega_i) \cos\theta_i \,\mathrm{d}\omega_i \,\mathrm{d}A$$



# BRDF

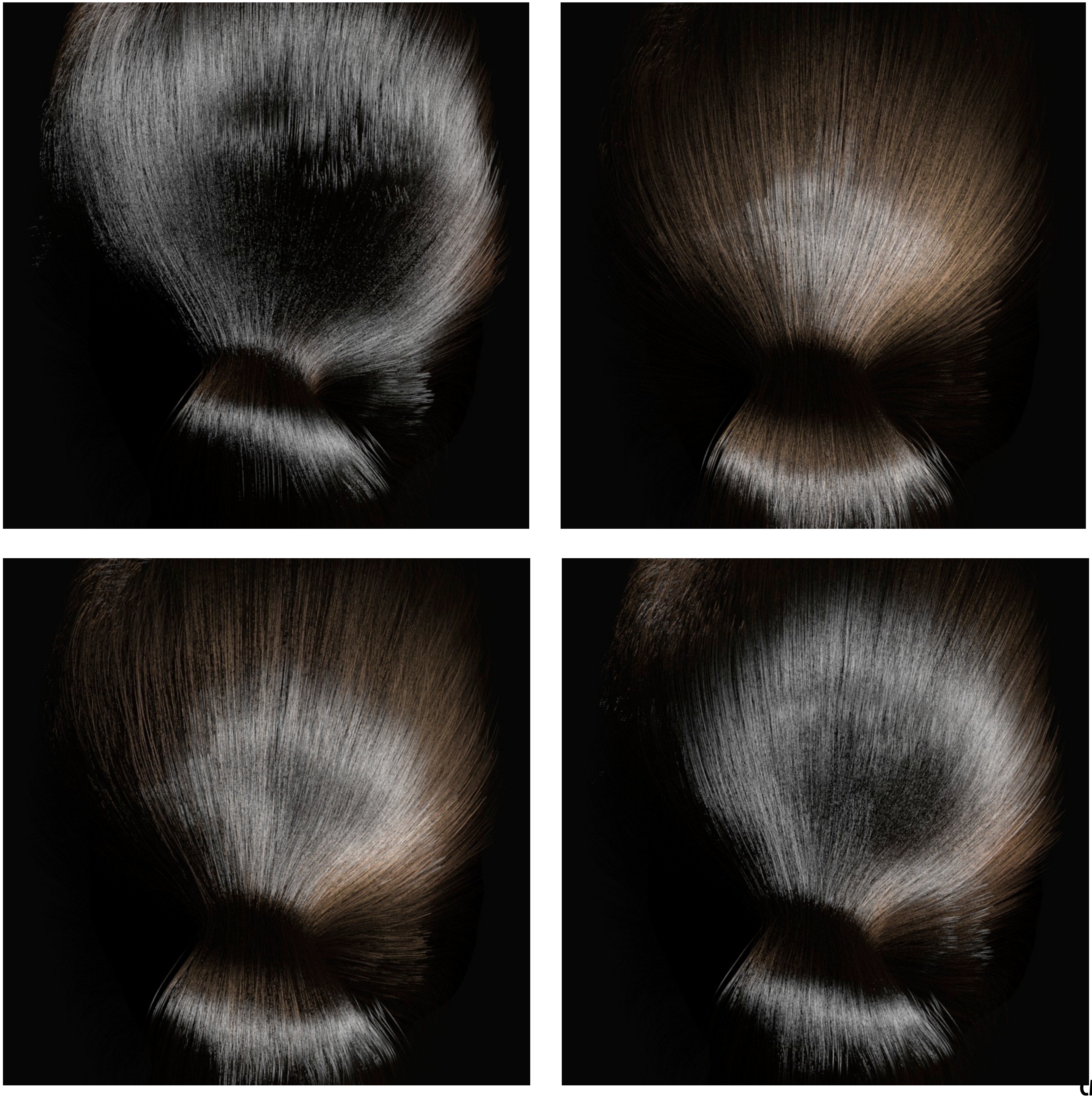
# BSSRDF

# Fiber model




[Marschner et al. 2003]

# Hair appearance



# Summary

- **BRDF describes how light reflects off a surface**
- **BRDF defines the fraction of energy incident on surface from direction $\omega_i$ that is reflected in the direction $\omega_r$**
- **Light is also transmitted through surfaces**
  - Snell's Law gives angle of transmitted ray
  - Amount of light reflected/transmitted is computed via Fresnel equations
  - Can think of BTDF (bidirectional transmission distribution function) describing directional distribution of transmission
- **Subsurface scattering**
  - Light exits surface at different point than it entered (e.g., skin)